Reconciling Value Estimates from the Discounted Cash Flow Model and the Residual Income Model

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Editorial Note by Jerry Feltham: This paper discusses research and implementation issues that are illustrated by the analyses in Penman and Sougiannis (1998), Courteau *et al.* (1999), and Francis *et al.* (2000). The first and third papers have been published, while the second is a working paper that was presented at the *Contemporary Accounting Research Conference* in October, 2000. Russell Lundholm was a discussant of the Courteau *et al*. paper. The key elements of Russell's remarks are included in this paper with Terry O'Keefe. We are publishing this paper in lieu of publishing Russell's discussants remarks, due to the fact that the issues raised occur in more than one paper and are likely to be of interest to anyone involved in financial statement analysis.

We thank Jim Ohlson for suggesting a more forceful way to make our point in this paper. We also thank Richard Brief, George Racette, Doug Skinner, Richard Sloan, Joseph Tham and the workshop participants at Syracuse University and at the Contemporary Accounting Research 2000 conference for helpful comments.

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In this paper we investigate why practitioners and researchers get different estimates of equity value when they use a discounted cash flow (CF) model versus a residual income (RI) model. Until recently this question did not arise because the CF model was the dominant model in practice and the only valuation approach that stood on solid theoretical ground. But recently the RI model has become a popular alternative to the CF model in practice, and a commonly-used model in research. And while both models are derived from the same underlying assumption -- that price is the present value of expected future net dividends discounted at the cost of equity capital – in practice they frequently yield different estimates. For this reason, a number of studies compare the models' accuracy in forecasting the actual price, generally concluding that the RI model is superior to the CF model (see Courteau, Kao and Richardson 1999; Francis, Olsson and Oswald 2000; and Penman and Sougiannis 1998).

This paper has two purposes. First, we argue that any claim of the RI model's superiority over the CF model is mistaken. While all authors acknowledge that the models are equivalent in theory, they proceed to compare them anyway based loosely on the idea that in a practical implementation or a large-sample study, the models can vary. But we argue that even in a practical implementation or large-sample study, the models should still be equivalent –for every firm in every year. The fact that the price estimates frequently differ between the two models illustrates the difficulty in consistently applying the same input assumptions to the different models. It does not provide any evidence about one model's superiority over another.

Nonetheless, given that it is not uncommon in research or in practice to get two different price estimates from the two different models, the second purpose of our paper is to identify some common inconsistencies in the implementation of the models. Each inconsistency is deceptively easy to make – we have made each of these mistakes ourselves – and each leads to surprisingly large differences in the RI model and CF model price estimates. We interpret the results of the empirical papers that compare the RI and CF models as documenting the size of the error these seemingly small inconsistencies cause.

The models we consider divide the forecasting and valuation problem up into two distinct periods: a finite forecasting period where financial statements are explicitly forecasted and each year's valuation attribute (residual income or free cash flow) is separately discounted, and a terminal period where the financial statement forecasts and valuation attributes are represented in some succinct way, typically by assuming they behave as a growing perpetuity. The first error we identify, labeled the inconsistent forecasts error, is caused by starting the perpetuity of valuation attributes off with the wrong amounts. In particular, we show that the correct starting value is rarely (1+g) times the last residual income or cash flow in the finite forecasting period, where g is the terminal growth rate. We also show that making such an assumption causes errors in the RI and CF models that are in the opposite directions and of different magnitudes, and that this error explains a large portion of the observed differences between the model estimates reported in the prior literature.

The second common implementation error we discuss occurs when the value of the equity is computed by first valuing the whole firm, discounted using a weighted average cost of capital, and then subtracting the value of the debt claim. This approach to valuation is the most common implementation of the CF model. What is not commonly known is that the appropriate discount rate is only a weighted average of the cost of equity and the cost of debt under certain conditions and, even then, the weights are not arbitrary. Failure to meet these conditions results in a discount rate that is inconsistent with the basic dividend discounting model, causing differences in the estimated value of the CF model and the RI model. We label this error the incorrect discount rate error.

The final mistake we discuss is labeled the missing cash flow error. The most simple version of this error occurs if the financial statement forecasts do not satisfy the clean surplus relation (i.e. net income less net dividends does not completely reconcile the change in shareholders' equity). This causes the dividend series implied by the residual income model to differ from the explicitly forecasted series. And, as we show later, a more subtle version of this mistake arises in the computation of free cash flow in the CF model.

Each of these implementation inconsistencies is present in at least one of the research papers cited above that compare the RI and CF models. We also cite evidence from some Harvard cases to support our contention that these inconsistencies are commonly observed in practice.

In the next section we make our first point – that there is no basis for the conclusion that the RI model is empirically superior to the CF model. In section III we develop the different valuation models in one common notation and in section IV we present the three implementation errors with examples of each from practice and in prior research. We conclude in section V.

II. EMPIRICAL COMPARISIONS OF THE RI AND CF MODELS

Three papers in the accounting literature have focused on comparing the RI model with the CF model. All three start with a single set of forecasted financial statements, either from ValueLine or from ex post realizations, but by inconsistent application of the forecasts to the valuation models, end up with different value estimates.

Francis et. al. (2000) use five years of ValueLine estimates for the finite forecasting period and assume the terminal period growth rate is either zero or four percent. They motivate their work by stating "we try to replicate the typical situation facing an investor using a valuation model to calculate an estimate of the intrinsic value of a firm (p. 1)." They conclude that "RI value estimates dominate value estimates based on free cash flow or dividends (p. 1)."[1] Later we estimate that the inconsistent forecasts error will cause the CF model to exceed the RI model by $6.75/share, on average, for their sample. In the empirical results, the CF estimate actually exceeds the RI estimate by $5.86/share, on average. The incorrect discount rate and missing flows errors are also present, accounting for the remaining difference in the model estimates.

The main purpose of Courteau, Kao and Richardson (1999) is to empirically assess whether the different valuation models are equivalent when a price-based terminal value calculation is used. In particular, they show that by using ValueLine's forecast of the future price in the terminal value calculation the models yield very similar estimates. Later in the paper they also present a horse-race between the CF and RI models using only forecasted financial statement data as input, much like Francis et. al. (2000), concluding that "RI is superior to CF in situations where terminal price forecasts are not available (p. 24)." But the only real question is why the estimates where not exactly the same for each model, regardless of whether or not the ValueLine terminal price forecast is used. We show later that the

[1] To be consistent, in all the quotes that follow we have substituted the acronym CF for cash flows and RI for residual income (also called abnormal earnings), even though the authors did not use these acronyms.

differences in their model estimates are due to the inconsistent forecasts error and the missing cash flows error.

Penman and Sougiannis (1998) use portfolios of ex post realizations of financial statement data as their ex ante forecasts and observe differences in model estimates that range from 70% to 170% of the actual share price. They state that their paper "examines these techniques with a focus on a practical issue: dividend , cash flow, and earnings approaches are equivalent when the respective payoffs are predicted to infinity, but in practice, forecasts are made over finite horizons (p. 346)." Their stated purpose is to examine which technique works best over a finite horizon, concluding that "techniques based on forecasting GAAP accrual earnings yield lower valuation errors that those based on forecasting dividends or cash flows (p. 347)." The RI and CF price predictions differ in this paper because of the inconsistent forecasts error and the incorrect discount rate error.

Each of the three papers conclude that, in an empirical implementation, the RI model is a more accurate predictor of current value than the CF model. And citations of this literature, of which there are many, unwaveringly view the work as empirical evidence that the RI model is superior to the CF model in practice.

But is there anything useful to learn from an empirical comparison of theoretically equivalent models? Can practical implementation issues create differences in the models that make an empirical comparison of them a useful exercise? We believe the answer to each of these questions is no. Starting with one set of forecasted pro forma financial statements, as is uniformly done in practice and in the aforementioned research papers, there is only one implied series of future net dividends; discounting this series at the exogenously-given cost of equity capital results in a unique value estimate. The trick to recovering this unique value estimate when discounting residual income flows or free cash flows is to make sure that the forecasts and discount rates are consistent with the pro forma financial statement forecasts and the original cost of equity capital assumption. While this may sound obvious, in practice it is quite difficult to do, as evidenced by the existence of the three previously-discussed research papers – if the authors always got the same value estimate from each model they would have never been tempted to compare the models empirically.

To make this point perfectly clear, the following example constructs the "horse-race" between the residual income model and the most simple cash flow model – dividend discounting -- using the ValueLine forecasts for the Home Depot Corporation. We will make use of ValueLine's terminal price forecast in the first comparison and use a perpetual growth assumption in the second comparison. The basic data is as follows:

Home Depot 1999 ValueLine Forecasts[2]

	t=1	*t=2*	*t=3*	*t=4*	*t=5*	$E_0(P_5)$
SE_{t-1}	5.36	6.45	7.82	9.80	11.79	80.00
$+NI_t$	1.25	1.55	2.16	2.17	2.85	
$-D_t$	0.16	0.18	0.18	0.18	0.24	
$=SE_t$	6.45	7.82	9.80	11.79	14.40	

where SE_t, NI_t and D_t are shareholders' equity, net income and net dividends at time t, respectively.

A. Comparing the models using ValueLine's terminal price forecast

Starting with the dividend discounting model, the value estimate at time 0 is

[2] Forecast for years 3 and 4 are constructed by interpolation between years 2 and 5, as described in footnote 14 of Courteau et. al..

$$P_0 = \sum_{t=1}^{5}(1+r_e)^{-t}E_0(D_t) + \sum_{t=6}^{\infty}(1+r_e)^{-t}E_0(D_t)$$

where D_t is net dividends, r_e is the cost of equity capital, and we divide the summation into two parts in order to make a distinction between the five year finite forecasting period and the remaining terminal period. The same model implies that $P_5 = \sum_{t=1}^{\infty}(1+r_e)^{-t}E_0(D_{t+5})$ so without any additional assumptions we have

$$P_0 = \sum_{t=1}^{5}(1+r_e)^{-t}E_0(D_t) + (1+r_e)^{-5}E_0(P_5).$$

Using a 10% cost of capital and inserting the Home Depot data gives

$$P_0 = \frac{.16}{(1+.10)} + \frac{.18}{(1+.10)^2} + \frac{.18}{(1+.10)^3} + \frac{.18}{(1+.10)^4} + \frac{.24}{(1+.10)^5} + \frac{80}{(1+.10)^5} = 50.375.$$

Now for the residual income model. Assuming the forecasts are such that the clean surplus relation $SE_t = SE_{t-1} + NI_t - D_t$ holds, as is the case in the Home Depot data above, the residual income model restates the dividend discounting model as

$$P_0 = SE_0 + \sum_{t=1}^{5}(1+r_e)^{-t}E_0(RI_t) + \sum_{t=6}^{\infty}(1+r_e)^{-t}E_0(RI_t),$$

where $RI_t = NI_t - r_eSE_{t-1}$. The same model implies that $P_5 = SE_5 + \sum_{t=1}^{\infty}(1+r_e)^{-t}E_0(RI_{t+5})$ so without any additional assumptions we have

$$P_0 = SE_0 + \sum_{t=1}^{5}(1+r_e)^{-t}E_0(RI_t) + (1+r_e)^{-5}\,E_0(P_5 - SE_5).$$

Inserting the Home Depot data gives the same valuation as before:

$$P_0 = 5.36 + \frac{.7140}{(1+.10)} + \frac{.9050}{(1+.10)^2} + \frac{1.378}{(1+.10)^3} + \frac{1.190}{(1+.10)^4} + \frac{1.671}{(1+.10)^5} + \frac{80-14.40}{(1+.10)^5} = 50.375$$

With this example in place, consider some common misperceptions. First, Courteau et al. assert that comparing the models is still an empirical issue because "the market's stock price expectations are not observable, and we use ValueLine's terminal price forecasts as a surrogate (p. 2)." They also state that "the equivalence of models may not hold empirically if the market does not efficiently price near-term earnings, cash flows or dividends (p. 3)." To refute these claims, suppose we use the number of planets in the solar system – 9 – as the terminal price forecast for the Home Depot instead of the $80 ValueLine forecast. This is unlikely to be a good representation of the market's expectations for Home Depot. Nonetheless, because each model inserts 9 instead of 80 for $E_0(P_5)$, each still yields exactly the same price estimate. The dividend discounting model now gives

$$P_0 = \frac{.16}{(1+.10)} + \frac{.18}{(1+.10)^2} + \frac{.18}{(1+.10)^3} + \frac{.18}{(1+.10)^4} + \frac{.24}{(1+.10)^5} + \frac{9}{(1+.10)^5} = 6.289$$

and the residual income model now gives

$$P_0 = 5.36 + \frac{.7140}{(1+.10)} + \frac{.9050}{(1+.10)^2} + \frac{1.378}{(1+.10)^3} + \frac{1.190}{(1+.10)^4} + \frac{1.671}{(1+.10)^5} + \frac{9-14.40}{(1+.10)^5} = 6.289 .$$

Clearly changing the terminal price forecast changes the resulting value estimate, but the change is exactly the same for both models. Similarly, we can insert <u>anything</u> in for the five years of D_t, NI_t and SE_t estimates; we could use the number of kids in the neighborhood, dogs on the porch or cats in the yard. As long as the estimates satisfy $SE_t = SE_{t-1} + NI_t - D_t$ the algebra holds and the value estimates will be the same for each model. The equivalence of the models has nothing to do with how efficiently the market prices earnings or dividends, or how accurately the ValueLine estimates represent the market's beliefs.

The reason Courteau et. al. get different value estimates from the different models when they use the ValueLine terminal price forecast is because 12% of their sample violates the clean surplus requirement (as they report in footnote 16). They also create some "dirty surplus" by imposing a cap on residual income equal to net income. In either case the resulting "dirt" becomes a missing cash flow – the dividend series indirectly implied by the NI and SE estimates is different from ValueLine's direct dividend estimates. But this is completely reconcilable – the difference is the present value of the "dirt" – and it says nothing about the relative accuracy of one valuation model over the other.

B. Comparing the models using a perpetual growth terminal value assumption

Now suppose we are in the more common situation where there is no forecasted terminal price. In this case we need to represent our beliefs about the flow of valuation attributes from year six to infinity using a perpetuity expression. This is with little loss of flexibility. A terminal growth rate of g = –1 implies that the flows are zero after the next period and as g approaches the discount rate r_e the value of the perpetual flow approaches infinity. In this case the dividend discount model expresses value as

$$P_0 = \sum_{t=1}^{5}(1+r_e)^{-t} E_0(D_t) + (1+r_e)^{-5}\frac{E_0(D_6)}{r_e - g}$$

and the residual income model expresses value as

$$P_0 = SE_0 + \sum_{t=1}^{5}(1+r_e)^{-t} E_0(RI_t) + (1+r_e)^{-5}\frac{E(RI_6)}{r_e - g}.$$

A common misperception is that the models are somehow "ad hoc" when terminal price forecasts are unavailable and therefore they may not be equivalent. For instance, Courteau et. al. state "Of course, for a finite horizon, the equivalence [in models] is only possible with terminal stock price forecasts (p. 4)." But a perpetual growth approach is no less valid than using terminal price forecasts; it is simply a different way to express beliefs about what will happen from years six to infinity. Another version of this misperception is that if the market's belief about when the firm will reach a "steady state" differ from when the model assumes the terminal period starts, then the models can vary. Consequently, it is concluded, an empirical comparison of the models is meaningful. But regardless of when the market believes the firm will reach a constant growth rate, as long as the correct starting values are used in the perpetuity expressions, the models will yield identical estimates. An assumption that is inconsistent with the market's belief will cause the models to be inaccurate, but it will not cause them to differ. The trick is to construct values for $E_0(D_6)$ and $E_0(RI_6)$ in the numerators of the perpetuities that represent <u>the same</u> beliefs. As we show later, it is generally <u>not</u> the case that $E_0(D_6) = (1+g)E_0(D_5)$ implies $E_0(RI_6) = (1+g)E_0(RI_5)$ or visa versa.

To get consistent starting values for the perpetuities, compute each from common financial statement forecasts. Because the financial statement forecasts express a unique belief about the future, all valuation expressions that are derived from them should result in the same value estimate. Assuming that SE_t and NI_t grow at four percent after year five for the Home Depot gives $NI_6 = (1+.04)2.85 = 2.964$ and $SE_6 = (1+.04)14.40 = 14.976$, so that $E_0(D_6) = NI_6 - (SE_6 - SE_5) = 2.388$ and $E_0(RI_6) = NI_6 - r_eSE_5 = 1.524$. With these starting values, the dividend discounting model expresses value as

$$P_0 = \frac{.16}{(1+.10)} + \frac{.18}{(1+.10)^2} + \frac{.18}{(1+.10)^3} + \frac{.18}{(1+.10)^4} + \frac{.24}{(1+.10)^5} + \frac{2.388}{(1+.10)^5(.10-.04)} = 25.41$$

and the residual income model expresses value as

$$P_0 = 5.36 + \frac{.7140}{(1+.10)} + \frac{.9050}{(1+.10)^2} + \frac{1.378}{(1+.10)^3} + \frac{1.190}{(1+.10)^4} + \frac{1.671}{(1+.10)^5} + \frac{1.524}{(1+.10)^5(.10-.04)} = 25.41$$

Note that $E_0(D_6) \neq (1+.04)E_0(D_5)$ and $E_0(RI_6) \neq (1+.04)E_0(RI_5)$. If we mistakenly make this assumption we commit the inconsistent forecasts error and get a price estimate of \$3.28 from the dividend discount model and \$27.62 from the residual income model.

The idea that the models are ad hoc without terminal price forecasts appears to be the motivation behind Penman and Sougiannis (1998). They argue that, in practice, financial statement forecasts are only for a finite horizon, so it is useful to compare the accuracy of the different models based only on the finite horizon of forecasts. But this is effectively assuming that all valuation attributes -- residual income, dividends or free cash flows – become zero after some year T. These assumptions cannot all be consistent with the same underlying financial statement forecasts. And making this erroneous assumption creates an ex ante predictable difference between the model estimates.

For example, if RI_t is assumed to be zero after year five, the expected D_t is <u>not</u> generally zero after year five; rather, the present value of expected D_t from years six to infinity equals $\frac{SE_5}{(1+r_e)^5}$.[3] So in the Home Depot example, Penman and Sougiannis would

[3] To see this, note that $RI_{T+1} = 0$ implies that $NI_{T+1}=r_eSE_T$, so $D_{T+1} = NI_{T+1}-(SE_{T+1}-SE_T) = r_eSE_T - (1+g)SE_T+SE_T = (r_e - g)SE_T$. Now suppose SE_t grows at some arbitrary rate g starting in year T+1, but we don't know g. With this, the present value of the terminal growing perpetuity is computed as $(r_e - g)SE_T/(r_e - g) = SE_T$, regardless of the unknown amount g. This refutes another alleged benefit of the RI model – that it allows the user to stop forecasting as soon as the RI_t forecasts reach zero, whereas CF models require longer forecast horizons. If the

compute the dividend discount model based on only the five years of forecasted D_t, yielding P_0=\$0.7014; and the RI model based on only the five years of forecasted RI_t, yielding P_0=\$9.643. But the difference in these two estimates is exactly equal to the discounted value of $SE_5 = 14.40/(1+.10)^5$ = \$8.9413. The only reason the RI model wins the horse-race in Penman and Sougiannis is because the dividend discount model is artificially constrained to record zero for terminal value when the true value is known and computable using exactly the same data that the RI model uses.[4]

The final misperception we want to refute is the “intuition” that the RI model wins the empirical horse-race because accrual accounting brings the recognition of value forward in time and therefore relies less on imprecise terminal value estimates. All three papers make this claim. First, we would not be tempted to develop intuition to explain the RI model’s superiority over the CF model if each model always yielded the exact same value estimate. Second, while it is true that accrual accounting recognizes value creation as NI_t before it gets paid out as D_t, it is also true that NI_t and ΔSE_t completely recover D_t. Simply doing algebra on these variables cannot lower our uncertainty. How the future will unfold is uncertain, but the user’s beliefs about this uncertain process are completely embodied in the infinite series of forecasted financial statements and in the discount rate. Translating these forecasts into

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RI model user assumes RI_t are zero after year T the dividend discount model user can stop forecasting in year T as well, computing the present value of the terminal period dividends as $\frac{SE_T}{(1+r_e)^T}$.

[4] While the Penman and Sougiannis results provide no evidence of the RI model’s superiority over CF models, the results do demonstrate that accrual accounting captures relevant information not present in cash flows alone (in the spirit of Dechow 1994). In particular, to compute the true value of D_t in the terminal period the user must know SE_T – the past series of D_t alone is insufficient (we thank Steve Penman for pointing this out).

residual income flows or free cash flows or net dividend flows does not change the forecasts or the uncertainty surrounding them.[5]

As a simple example that illustrates the illogical nature of this claim, consider a $100 equity investment that is forecasted to pay $10 cash each year in perpetuity, which will be paid out in dividends, with a cost of equity capital of 10%. The accounting book value of this investment is $100, resulting in zero residual income each year. Using the RI model to value the investment, we have $100 in initial book value plus zero in residual income forever after, yielding a value estimate of $100. Examining the same investment, the CF model computes value as a perpetuity of $10 cash flows, finding once again that the value of the investment is $100 (=10/.10). Whether we view the value of the investment as being its book value of $100 or the present value of its series of $10 payments, the forecasted values underlying these two models are the same. We should be no more confident in our $100 book value than in our series of $10 payments – they are two sides of the same coin.

In sum, there is nothing to be learned from an empirical comparison of theoretically-equivalent models – algebra alone cannot generate testable hypotheses. And there is nothing impractical about avoiding implementation errors in a large-sample study. However, implementation errors are deceptively easy to make. In the remainder of the paper we present some common mistakes.

III. THE MODELS

A. The Forecasted Financial Statements

We begin with the pro forma financial statements shown in figure 1. The forecasts start with the balance sheet at time zero and then describe a series of income statements and

[5] Even if we considered a more general set of valuation models that priced uncertainty, as considered in Feltham

balance sheets extending into the future. The first T years are the finite forecasting period and consist of explicit and exogenous forecasts. Year T+1 begins the terminal period, consisting of an infinite series of future forecasts described by a perpetual growth at rate g. In the terminal period, for each line item on the balance sheet and income statement, the year T+1 value equals the year T value times (1+g), and so on, forever.[6]

To express the RI and CF models in one notation we need two components of the balance sheet and two components of income. We will use lowercase t to denote any generic period and uppercase T to denote specifically the last period in the finite forecasting period (i.e. when t=T). Divide the balance sheet at time t into net operating assets, denoted OA_t, and interest-bearing liabilities, denoted L_t, so that total shareholders' equity is given by $SE_t = OA_t - L_t$. Note that OA_t is total assets less all non-interest-bearing liabilities.[7]

Divide the income statement for the period ending at time t into net operating income, denoted OI_t, and interest on interest-bearing liabilities, denoted I_t, where each is net of tax. This gives net income of $NI_t = OI_t - I_t$.

It is crucial that all assets and liabilities get allocated between OA_t and L_t and all sources of net income get allocated between OI_t and I_t. Failure to do so will result in errors in the valuation estimates and, more to the point of this paper, different errors in the cash flow models and the residual income models. It is assumed that all valuation models take the forecasts shown in figure 1 as given.

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and Ohlson (1999), each model would still be based on the same distribution of financial statement outcomes and hence would arrive at the same valuation.

[6] The growth rate g can lie anywhere in (-1, r), where r is the discount rate used in the valuation model. One might imagine that individual line items can grow at rates different from g while still preserving shareholders' equity and net income growth at g. But it is easy to show that if a sum of line items grows perpetually at rate g, and each line item grows at some constant rate, then each line item must also grow at rate g.

[7] If financial assets are present they should be netted against the interest-bearing liabilities in the definition of L_t and interest revenue on the financial assets should be netted against interest expense in the definition of I_t.

To link the financial statements to valuation we also require the following clean surplus relations in the accounting system. First, we require that

$$SE_t = SE_{t-1} + NI_t - D_t, \quad (1)$$

where D_t is net dividends for the period. It is the net cash flows from all transactions with common equity providers; it is positive when common dividends are paid or the firm repurchases shares and it is negative when new capital is provided to the firm by equity investors. Second, denote all changes in OA_t that are not due to OI_t as free cash flows C_t, so that we have the clean surplus relation

$$OA_t = OA_{t-1} + OI_t - C_t. \quad (2)$$

Rearranging (2) gives the common expression for free cash flows $C_t = OI_t - \Delta OA_t$.[8] Further, subtracting (1) from (2) we get $L_t = L_{t-1} + I_t + D_t - C_t$ which can be rearranged to get another common expression for free cash flows: $C_t = D_t + I_t - \Delta L_t$. This expresses the free cash flows as the sum of the net transactions with the equity investors, D_t, and the net transactions with the debt investors, $I_t - \Delta L_t$.

Note the minimal details required for the financial statement forecasts: two line items each. This is far less detailed than a typical analyst report and is easily available from the ValueLine forecasts.

B. The Valuation Models

[8] In many textbooks, OI_t is earnings before interest and taxes times one minus the marginal tax rate, and ΔOA_t is computed as capital expenditures less depreciation.

We present a number of valuation expressions in figure 2 that are used to estimate the value of equity P_e; in the (not-to-be-published) appendix we provide a brief derivation of each. The expressions differ in their valuation attribute, either cash flows or residual income flows, shown as the two rows of the figure. The first column of the figure gives the models that compute P_e directly as the flow of valuation attributes to equityholders. The expression labeled CF(equity) is the starting point for all equity valuation models considered here; it computes the value of the equity P_e as the discounted present value of expected future dividends. The expression RI(equity) is the model recently popularized by Ohlson (1995).

The second and third columns in figure 2 give component expressions for valuing P_e indirectly in a two-step process. In these models, the value of the equity is computed as the value of the whole firm P_f less the value of the debt claim P_d. Using these building blocks, the most common version of the discounted cash flow model is to compute P_f using the CF(firm) model and compute P_d using the RI(debt) model, where it is also frequently assumed that the residual interest RIT_t is zero at all future dates so that $P_d = L_0$ (i.e. the debt is valued at its book value).

All expressions decompose the valuation exercise into the value over the finite forecasting period and the value over the terminal period (labeled the terminal value).[9] A special case considered in Penman and Sougiannis (1998) assumes the valuation attribute (dividends, residual income or free cash flow) is zero in the terminal period, so that the terminal value is zero.

A critical feature of the CF(firm) and RI(firm) models, and one that can lead to errors in valuation, is the use of a weighted average cost of capital r_w. Without making any statement about the exact weights at this point, we assert only that there exists some r_w between r_e and r_d such that computing $P_e = P_f - P_d$ yields a value estimate that is equivalent to the other models (with proof in an unpublished appendix).[10]

We stress again that each model is derived from the CF(equity) model with no additional assumptions, and each model takes as its input the infinite sequence of forecasted financial statements given in figure 1. As such, each should yield the exact same estimate of P_e, provided one of the three errors we discuss next does not arise.

IV. APPLICATION ERRORS

In this section we describe three commonly observed errors in the implementation of the RI and CF models. The first involves inconsistent forecasts at the start of the terminal period, the second involves an inconsistency between the cost of equity capital and the weighted average cost of capital, and the third involves missing cash flows in one or both models.

A. Inconsistent Forecasts Error

This error occurs when the starting value for terminal value perpetuity is incorrect (the starting value is the amount in the numerator in the final term of the value expressions shown in figure 2). Avoiding the error is easy – first construct the financial statement forecasts in the terminal period using the terminal growth rate (as shown in figure 1) and then construct the relevant valuation attribute in year T+1 using the forecasted financial statement

[9] The terminal value is always computed using the formula for a growing perpetuity. In particular, assuming a discount rate of r, the present value of a growing perpetuity starting in year T+1 with the value X and growing at rate g is $\frac{X}{(r-g)(1+r)^T}$.

data (as shown at the bottom of figure 2). Because the financial statement forecasts provide a unique expression of beliefs about what will happen in the future, all valuation models computed using these forecasts as inputs will be internally consistent and yield identical estimates. But, as we illustrated with the Home Depot example in section II, mistakenly assuming that RI_{T+1} equals $(1+g)RI_T$ or $D_{T+1} = (1+g)D_T$ effectively feeds forecast data to the models that is inconsistent with the underlying financial statement forecasts and causes errors in the valuation models.[11]

i. Calculating the inconsistent forecasts error

Consider the RI(equity) model. The difference between the correct and incorrect values for RI_{T+1} is

$$RI_{T+1} - (1+g)RI_T = -r_e[SE_T - (1+g)SE_{T-1}]. \tag{3}$$

Because they are in the finite forecasting period, the SE_t values in years T and T-1 are completely unconstrained, so there is no reason to expect the RHS to be zero. Further, since this error affects the entire perpetuity of RI_t flows after year T+1, the entire effect is much greater than this single period flow. Denoting the growth in SE_t between T-1 and T as g_T, the difference between the incorrect P_e estimate and the correct P_e estimate is

$$RI(equity)\ error = \frac{r_e\left[SE_T - (1+g)SE_{T-1}\right]}{(r_e - g)(1+r_e)^T} = \frac{r_e(g_T - g)SE_{T-1}}{(r_e - g)(1+r_e)^T}\ . \tag{4}$$

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[10] The errors we identify are not driven by any tax effects, so for simplicity we define the r_d as the after-tax cost of debt rather than carry the notation $(1\text{-}t)r_d$ throughout the paper.

[11] Another version of this mistake is to discount the first T-1 years individually and then start the perpetuity with the last forecasted value (e.g. RI_T). Either approach yields the same present value.

For example, if the growth in SE_t between T-1 and T exceeds the terminal growth rate (i.e. $g_T - g > 0$), the incorrect RI(equity) model will be overstated.

The inconsistency above is between the forecasted value of SE_T and the value implied by incorrectly assuming $RI_{T+1} = (1+g)RI_T$. But one could argue that, by assuming $RI_{T+1} = (1+g)RI_T$ directly, the value of SE_T never actually gets used in the calculation; $(1+g)SE_{T-1}$ is effectively used instead. Although the two forecasts may be inconsistent, the model has successfully ignored the SE_T amount. The problem with such an approach is that the CF model still uses the SE_T value; recall that $D_T = NI_T - (SE_T - SE_{T-1})$. Thus, there is an inconsistency between the forecasts that are effectively being used by the different models and they no longer reconcile.

To illustrate this point, consider the CF(equity) model in figure 2. The difference between the correct and incorrect values for D_{T+1} in this case is

$$D_{T+1} - (1+g)D_T = SE_T - (1+g)SE_{T-1}. \tag{5}$$

Unless $SE_T = (1+g)SE_{T-1}$, starting the perpetuity in year T+1 with $(1+g)D_T$ results in an inconsistency with the forecasted financial statements, just as in the RI(equity) model. The difference between the incorrect P_e estimate and the correct P_e estimate in the CF(equity) model is

$$CF(equity)\ error = -\frac{SE_T - (1+g)SE_{T-1}}{(r_e - g)(1+r_e)^T} = -\frac{(g_T - g)SE_{T-1}}{(r_e - g)(1+r_e)^T}, \tag{6}$$

recalling that g_T is the growth rate in SE_t between T-1 and T. Note that the sign and size of the error for the CF(equity) model is different than in the RI(equity) model. In the RI(equity) model, if the g_T is greater than g, the resulting value estimate for the RI(equity) model is overstated. In the CF(equity) model, if g_T exceeds g then the value estimate is understated, and by a larger amount. A higher value of SE_T decreases the properly-computed RI_{T+1} amount, all other things equal, as the "normal" level of income r_eSE_T is increased. But a higher value of SE_T increases the properly-computed D_{T+1} amount because, all else equal, it implies a smaller change in SE_{T+1} so more of NI_{T+1} will be paid out in dividends. In sum, by starting the perpetuity with (1+g) times the last explicitly forecasted value in the finite forecasting period both the RI(equity) and CF(equity) models are in error, and the errors are in the opposite direction and of different magnitude.[12]

These results also apply to a slightly different question. Suppose there are no explicit assumptions about the financial statements after year T. Rather, it is simply asserted that one of the valuation attributes, RI_t for example, behaves as a growing perpetuity starting in year T+1, taking as a result whatever financial statements this might imply for the future. If the problem is stated this way then the result is that the corresponding cash flow attribute (D_t in this case) only behaves as a growing perpetuity starting in year T+1 when the growth between years T-1 and T is also at rate g. If the forecasts in the finite forecasting period are inconsistent with this implicit growth assumption, there will be inconsistencies between the models.[13]

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[12] The same type of conditions apply to the RI and CF expressions for P_f and P_d. If the valuation attribute in the numerator of the terminal value in any of the expressions in figure 2 is computed as (1+g) times the year T amount, or equivalently, if the perpetuity starts in year T with the year T amount, then the differences between the incorrect estimate and the correct estimate are as follows:

$$CF(firm)\,error = -\frac{OA_T - (1+g)OA_{T-1}}{(r_w - g)(1+r_w)^T},\ RI(firm)\,error = \frac{r_w\left[OA_T - (1+g)OA_{T-1}\right]}{(r_w - g)(1+r_w)^T},$$

$$CF(debt)\,error = \frac{L_T - (1+g)L_{T-1}}{(r_d - g)(1+r_d)^T},\ \text{and}\ RI(debt)\,error = -\frac{r_d\left[L_T - (1+g)L_{T-1}\right]}{(r_d - g)(1+r_d)^T}.$$

In all cases note that the CF error is one over the relevant cost of capital higher than the corresponding RI error, in absolute value.

[13] Defining $ROE_T \equiv NI_T/SE_{T-1}$ and perpetual ROE as $(1+g)NI_T/SE_T$, it is straightforward to show that $g_T = g$ if and only if $ROE_T = ROE$.

ii. Does the inconsistent forecasts error occur?

All the papers discussed in section II appear to contain this error. For example, equation 10 in Penman and Sougiannis (1998) shows that they take a fixed number of years of realized data for their forecasts, and their notation indicates that the last value in the series is the starting value for the perpetuity. This will only yield correct and internally consistent results in the unlikely event that the actual realized growth rates in OA_t and L_t in the last year of the series equals the assumed growth rate in the perpetuity (either zero, two or four percent). This is unlikely to occur even on average, but certainly did not occur in each of their randomly selected portfolios.

Francis et. al. compare the RI(equity) model with the traditional free cash flow model that computes P_e as P_f computed from the CF(firm) model less P_d computed from the RI(debt) model, with the added assumption that future residual interest RIT_t is zero, so that $P_d = L_0$. They assume that the forecasted amounts are constant in years three through five, so the forecasted growth in SE_5 and OA_5 is zero, as compared to a four percent terminal period growth rate in one of the cases they consider. Francis et. al. do not report mean SE_t or OA_t values, but since their sample is taken from ValueLine in a similar time period as in Courteau et. al., we use the year zero values from Courteau et. al. table 1 as a rough estimate of SE_4 = \$12.84 and OA_4 = \$21.64 per share, on average. Using their average estimates of a 13% cost of equity capital and a 11.8% weighted average cost of capital, this gives an error on the RI(equity) model of

$$\frac{r_e\left[SE_5-(1+g)SE_4\right]}{(r_e-g)(1+r_e)^5}=\frac{.13\left[12.84-(1+.04)12.84\right]}{(.13-.04)(1.13)^5}=\$.40/share\text{ understated}$$

and an error on the CF(firm) model of

$$-\frac{OA_5-(1+g)OA_4}{(r_e-g)(1+r_e)^5}=\frac{.118[21.64-(1+.04)21.64]}{(.118-.04)(1.118)^5}=\$6.35/share \text{ overstated.}$$

In sum, we estimate that the inconsistent forecasts error will cause the CF(firm) model to exceed the RI(equity) model by $6.75/share, on average. In the actual data, the CF(firm) model exceeds the RI(equity) model by $5.86/share, on average. The sign and approximate magnitude of the difference in the model estimates is consistent with the presence of the inconsistent forecasts error. And this error is significant; the average stock price in the sample is $31.27.[14] [15]

The inconsistent forecasts error is one reason why CF model estimates might differ from RI model estimates, either at the equity level or at the whole firm level. The error creates a gap between models using cash flows as the valuation attribute (shown in the first row of figure 2) and models using residual income flows as the valuation attribute (shown in the second row of figure 2). The next section describes an error that creates an inconsistency between valuing the firm's equity directly versus valuing the equity as the whole firm value less the debt value. That is, it creates a gap between the first column of figure 2 and the second and third columns.

B. The Inconsistent Discount Rate Error

The second application error we identify involves an inconsistency between the cost of equity capital used when valuing the equity directly and the weighted average cost of

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[14] Courteau et.al. begin the perpetuity of cash flows using (1+g) times the year 5 amount from ValueLine. Since the growth rate in the balance sheet items is unconstrained between years 4 and 5, the cash flow valuation model will suffer the inconsistent forecasts error.

[15] A practical example comes from the teaching note that accompanies the Schneider-Square D case in Palepu, Healy and Bernard (2000). In one scenario they compute the CF(equity) model with a terminal growth rate of 7%. But they start the perpetuity in the terminal period using (1+.07) times the last cash flow in the finite

capital used when valuing the equity indirectly as the whole firm value less the value of the debt. We make no claim about the theoretically correct cost of equity or debt, or whether these amounts should be treated as constants. If the constant cost of equity and cost of debt assumptions are incorrect it will cause both approaches to be inaccurate, but it will not create differences between them.[16] Our point is that, assuming the cost of equity is constant, as in the RI(equity) and CF(equity) models, and assuming the after-tax cost of debt is a constant, there is only one internally consistent discount rate r_w to use in the RI(firm) or CF(firm) models. Failure to use this rate will cause the equity value computed directly as P_e and the equity value computed indirectly as P_f - P_d to differ.

There are actually two parts to this problem. First, denoting the after-tax cost of debt capital as r_d and the weight on the equity cost of capital as α, the standard definition of the weighted average cost of capital is

$$r_w = \alpha r_e + (1-\alpha) r_d \text{, where } \alpha = \frac{P_e}{P_e + P_d}. \qquad (7)$$

In section II we asserted (and proved in an unpublished appendix) that there exists some value of r_w that will reconcile the value of P_e computed as P_f - P_d and the value of P_e computed directly. What is at issue now is whether the value of r_w that equates these models is the weighted average value shown in (7). Although not commonly appreciated, this definition of r_w is correct only when the firm's tax rate and market value leverage ratio α are constant through time (Miles and Ezzell 1980). A constant market value leverage ratio

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period while the shareholders' equity growth in the last period of the finite horizon is only 5%. Consequently, the growth condition in proposition 1 is not met and the estimate contains the inconsistent forecasts error.

[16] If market participants are risk neutral and the term structure of interest rates is flat and non-stochastic then the cost of equity capital is the constant risk-free rate (Ohlson 1990). However, when market participants are risk-averse, the theoretically correct valuation approach is to risk-adjust the flows; it is not generally correct to use a risk-adjusted discount rate (Feltham and Ohlson 1999). Nonetheless, using a risk-adjusted discount rate is ubiquitous in practice and in research.

implies that, if we computed P_e and P_d at each future date based on the accumulated past realizations and forecasted future values, we would get the same α. And this requirement is not just in expectation; it assumes that the firm actively rebalances its debt and equity after each year's realizations to maintain a constant α. Given that the forecasts through year T are arbitrary, neither the constant tax rate condition nor the constant α condition will be met in general.[17]

Second, even if the constant tax rate and leverage ratio conditions are met, the weights on r_e and r_d are not arbitrary; they must be determined by the estimated values P_e and P_d. They can be found by using the CF(equity) or RI(equity) model to estimate P_e, using CF(debt) or RI(debt) model to estimate P_d (see figure 2), and then using these estimates to compute α. Even with constant tax rates and leverage ratio, differences between the model estimates will result if anything other than this internally consistent weight α is used. In particular, estimating α using the book values of equity and debt, or using a target capital structure as is recommended in Copeland et al. (1994 p. 242), will be incorrect.

In general, when the constant tax rate and leverage ratios conditions are not met, the only way to find the internally consistent r_w is to first estimate P_e and P_d directly and then iteratively search for the r_w in the P_f calculation that solves $P_e = P_f - P_d$.

Both Francis et. al. and Penman and Sougiannis use a weighted average cost of capital to compute the CF(firm) model. Francis et. al. uses target weights based on ValueLine's long term predictions; Penman and Sougiannis state only that r_w was calculated using "standard techniques." But neither the ValueLine's forecasts used in Francis et. al. nor

[17] One way to meet the constant tax rate and market value leverage ratio conditions is to assume the valuation attributes are perpetuities starting in year one (i.e. there is no finite forecasting period). Perpetuities have the unique property that at all points in the future, regardless of the past, the value of the equity and the value of the liabilities are each constant.

the actual realizations used in Penman and Sougiannis are constrained to meet the constant tax rate and leverage conditions, and neither paper backs into the internally consistent r_w that reconciles the model estimates. Consequently, both papers suffer the inconsistent discount rate error. Courteau et al. avoid this problem by valuing the equity directly using the cost of equity capital.[18]

C. The Missing Cash Flow Error

The final source of inconsistency we consider is due to calculating the valuation attributes in an internally inconsistent way. An obvious case occurs when the financial statement forecasts do not satisfy the clean surplus relation (i.e. $SE_t \neq SE_{t-1} + NI_t - D_t$). For example, the ValueLine forecasts violate the clean surplus condition 12% of the time in the Courteau et. al. study. In these cases, the D_t series that is forecasted directly in ValueLine is inconsistent with the residual income series, which is computed based only on the forecasted NI_t and SE_t. Because they also use ValueLine data without correcting for dirty surplus, the Francis et. al. study also suffers the missing cash flow error.

A more subtle variation on this error occurs when using the CF(firm) and CF(debt) models. If the user fails to allocate all the forecasted income to either OI_t or I_t and all the equity to either OA_t or L_t, then some implied cash flow will be lost. For example, Francis et. al. define operating income OI_t as the ValueLine forecasted amount

OI_t = (Sales - Operating Expenses - Depreciation Expense)(1-effective tax rate),

[18] As a practical example, the teaching note for The Gap case in Palepu, Healy and Bernard (2000) provides forecasts that do not suffer the inconsistent forecasts error. However, r_w is estimated based on a target capital structure, so the implemented CF(firm) model is inconsistent with the RI(equity) model. In particular, they estimate r_e=15% and r_d=5.5%, and using the target capital structure, estimate r_w = 13.2%. Using these amounts they estimate P_f with the CF(firm) model and estimate P_d as L_0, resulting in a value estimate of $17.26. In

with all other income flows therefore implicitly considered to be in I_t. But the I_t flows are never actually forecasted and valued because P_d is estimated as the book value of the debt. Thus, if ValueLine forecasts non-operating income it will be included in the Francis et. al. computation of income in RI_t but will not be included in either the P_f valuation or the P_d valuation.[19]

V. CONCLUSION

Our purpose here is to refute the commonly-held belief that practical implementation issues create differences in the theoretically equivalent RI and CF models, and that these differences make empirical comparisons of the models worthwhile. If the user starts with forecasted financial statements and an exogenous cost of equity capital then getting the same value estimate out of each model is only a matter of care. Reported differences in the model estimates in prior research merely point out the difficulty in conducting this exercise; it says nothing about the superiority of one model over another. Research efforts in valuation would be better spent on the study of how to make more accurate forecasts of financial statement data, not in how to represent and discount the resulting flows of value.

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contrast, their RI(equity) model estimate is $15.55. There exists a value of r_w that equates the two estimates -- approximately 14%.

[19] To be precise, when using the Value Line data I_t must be defined as the difference between OI_t, constructed as (Sales - Operating Expenses - Depreciation Expense)(1-effective tax rate), and the forecasted Net Profit.

Figure 1
Forecasted Financial Statements

for the period ending							
0	1	2 …	T-1	T	T+1	T+2	…
OA_0	OA_1	OA_2 …	OA_{T-1}	OA_T	$OA_T(1+g)$	$OA_T(1+g)^2$	…
$-L_0$	$-L_1$	$-L_2$ …	$-L_{T-1}$	$-L_T$	$-L_T(1+g)$	$-L_T(1+g)^2$	…
SE_0	SE_1	SE_2 …	SE_{T-1}	SE_T	$SE_T(1+g)$	$SE_T(1+g)^2$	…
	OI_1	OI_2 …	OI_{T-1}	OI_T	$OI_T(1+g)$	$OI_T(1+g)^2$	…
	$-I_1$	$-I_2$ …	$-I_{T-1}$	$-I_T$	$-I_T(1+g)$	$-I_T(1+g)^2$	…
	NI_1	NI_2 …	NI_{T-1}	NI_T	$NI_T(1+g)$	$NI_T(1+g)^2$	…

where

OA_t is the net operating asset balance, defined as total assets less all non-interest-bearing (i.e. operating) liabilities,

L_t is the interesting-bearing liability balance,

SE_t is the shareholders' equity balance, defined as the OA_t - L_t,

OI_t is the operating income for the period, defined as all income except interest expense on the interest-bearing liabilities, net of tax,

I_t is the interest expense on the interest-bearing liabilities for the period, net of tax,

NI_t is net income for the period, defined as OI_t - I_t, and

g is the perpetual growth rate beginning in year T+1.

Figure 2
Cash Flow and Residual Income Valuation Models

Valuation Attribute	*Equity Valued Directly as P_e*	*Equity Valued Indirectly as $P_e = P_f - P_d$*	
	Value of P_e	Value of Whole Firm P_f	Value of Debt P_d
Cash Flows	**CF(equity)** $P_e = \sum_{t=1}^{T} \frac{D_t}{(1+r_e)^t} + \frac{D_{T+1}}{(r_e - g)(1+r_e)^T}$	**CF(firm)** $P_f = \sum_{t=1}^{T} \frac{C_t}{(1+r_w)^t} + \frac{C_{T+1}}{(r_w - g)(1+r_w)^T}$	**CF(debt)** $P_d = \sum_{t=1}^{T} \frac{I_t - \Delta L_t}{(1+r_d)^t} + \frac{I_{T+1} - \Delta L_{T+1}}{(r_d - g)(1+r_d)^T}$
Residual Income	**RI(equity)** $P_e = SE_0 + \sum_{t=1}^{T} \frac{RI_t}{(1+r_e)^t} + \frac{RI_{T+1}}{(r_e - g)(1+r_e)^T}$ where $RI_t = NI_t - r_e SE_{t-1}$	**RI(firm)** $P_f = OA_0 + \sum_{t=1}^{T} \frac{ROI_t}{(1+r_w)^t} + \frac{ROI_{T+1}}{(r_w - g)(1+r_w)^T}$ where $ROI_t = OI_t - r_w OA_{t-1}$	**RI(debt)** $P_d = L_0 + \sum_{t=1}^{T} \frac{RIT_t}{(1+r_d)^t} + \frac{RIT_{T+1}}{(r_d - g)(1+r_d)^T}$ where $RIT_t = I_t - r_d L_{t-1}$.

<u>Notation</u>

OA_t is the operating asset balance at time t
L_t is the liability balance at time t
SE_t is the shareholders' equity at time t; $OA_t - L_t = SE_t$
OI_t is the operating income for the period ending at time t, net of tax
I_t is the interest expense for the period ending at time t, net of tax
NI_t is the net income for the period ending at time t; $NI_t = OI_t - I_t$
D_t is net dividends paid to common equity; $SE_t = SE_{t-1} + NI_t - D_t$

C_t is the free cash flow; $C_t = OI_t - \Delta OA_t$
r_e is the cost of equity capital
r_d is the after-tax cost of debt capital
r_w is the weighted average cost of capital
P_e is the value of the equityholders' claim at time 0
P_f is the value of the firm to all investors at time 0
P_d is the value of the debtholders' claim at time 0

<u>The correct values for the start of the perpetuities are as follows:</u>

$D_{T+1} = (1+g)NI_T - (1+g)SE_T + SE_T$,
$C_{T+1} = (1+g)OI_T - (1+g)OA_T + OA_T$,
$(I_{T+1} - \Delta L_{T+1}) = (1+g)I_T - (1+g)L_T + L_T$, and

$RI_{T+1} = (1+g)NI_T - r_e SE_T$,
$ROI_{T+1} = (1+g)OI_T - r_w OA_T$,
$RIT_{T+1} = (1+g)I_T - r_d L_T$.

APPENDIX **(only for working paper – not to be published)**

Derivation of Valuation Expressions Shown in Figure 2

The CF(equity) and RI(equity) models

The CF(equity) model is the initial assumption for all valuations models. It states that P_e equals the present value of expected future dividends. To show that the RI(equity) model is equivalent to the CF(equity) model, rearrange (1) to get

$$D_t = SE_{t-1} - SE_t + NI_t \tag{A1}$$

and then substitute the definition of residual income into (A1) to get

$$D_t = SE_{t-1} - SE_t + RI_t + r_e SE_{t-1}. \tag{A2}$$

Since (A2) holds for all t, substitute it for all future D_t in the CF(equity) model to get

$$P_e = \sum_{t=1}^{\infty} (1+r_e)^{-t} \left(SE_{t-1} - SE_t + RI_t + r_e SE_{t-1} \right). \tag{A3}$$

Separating out the residual income terms RI_t and writing out the second part of the summation for a few terms gives the following sequence:

$$P_e = \sum_{t=1}^{\infty} (1+r_e)^{-t} RI_t + SE_0 - \frac{SE_1}{1+r_e} + \frac{SE_1}{1+r_e} - \frac{SE_2}{(1+r_e)^2} + \frac{SE_2}{(1+r_e)^2} - ..... \tag{A4}$$

Note that the terms involving SE_t alternate signs and cancel with each other, so (A4) simplifies to

$$P_e = SE_0 + \sum_{t=1}^{\infty} (1+r_e)^{-t} RI_t \ . \tag{A5}$$

Dividing the infinite series of terms into T years of the finite horizon plus a perpetuity of flows beginning in year T+1 gives the RI(equity) model as shown in figure 2.

Modeling Equity Value as Firm Value less Debt Value

We first show that P_f can be computed using either CF(firm) or RI(firm), and P_d can be computed using either CF(debt) or RI(debt). We then address the issue of how to combine the two estimates so that $P_e = P_f - P_d$ yields the same equity value as estimated from the CF(equity) or RI(equity) model. That is, we initially take as given that there exists a weighted average cost of capital r_w that sets $P_f - P_d$ equal to P_e.

The CF(firm) and RI(firm) models

Recall that the firm's free cash flows $C_t = D_t + I_t - \Delta L_t$. The D_t are net dividends to equityholders and $I_t - \Delta L_t$ are net distributions to debtholders. Consequently the sum C_t is the cash generated by the whole firm. This implies the CF(firm) model is

$$P_f = \sum_{t=1}^{\infty}(1+r_w)^{-t} C_t \; .$$

Dividing the infinite series of terms into T years of the finite horizon plus a perpetuity of flows beginning in year T+1 gives the CF(firm) model as shown in figure 2.

To show that the RI(firm) model is equivalent to the CF(firm) model, we use a similar set of algebra steps that converted the CF(equity) model to the RI(equity) model. Recall that

$$C_t = OI_t - \Delta OA_t. \tag{A6}$$

Now define residual operating income as $ROI_t = OI_t - r_w OA_{t-1}$ and then substitute this definition into (A6) to get

$$C_t = ROI_t + r_w OA_{t-1} - OA_t + OA_{t-1}. \tag{A7}$$

Since (A6) holds for all t, substitute it for all future C_t in the CF(firm) model to get

$$P_f = \sum_{t=1}^{\infty}(1+r_w)^{-t}\left(OA_{t-1} - OA_t + ROI_t + r_w OA_{t-1}\right). \tag{A8}$$

Separating out the residual operating income terms ROI_t and writing out the second part of the summation for a few terms gives the following sequence:

$$P_f = \sum_{t=1}^{\infty}(1+r_w)^{-t} ROI_t + OA_0 - \frac{OA_1}{1+r_w} + \frac{OA_1}{1+r_w} - \frac{OA_2}{(1+r_w)^2} + \frac{OA_2}{(1+r_w)^2} - \ldots.. \tag{A9}$$

Note that the terms involving OA_t alternate signs and cancel with each other, so (A9) simplifies to

$$P_f = OA_0 + \sum_{t=1}^{\infty}(1+r_w)^{-t} ROI_t \; . \tag{A10}$$

Dividing the infinite series of terms into T years of the finite horizon plus a perpetuity of flows beginning in year T+1 gives the RI(firm) model as shown in figure 2.

The CF(debt) and RI(debt) Models

By now the derivation should be pretty obvious. The cash flows to debt holders is $I_t - \Delta L_t$, so the present value of this series gives

$$P_d = \sum_{t=1}^{\infty}(1+r_d)^{-t}(I_t - \Delta L_t) \; . \tag{A11}$$

Dividing the infinite series of terms into T years of the finite horizon plus a perpetuity of flows beginning in year T+1 gives the CF(debt) model as shown in figure 2.

To show that the RI(debt) model is equivalent to the CF(debt) model, define residual interest as $RIT_t = I_t - r_d L_{t-1}$ and then substitute this definition into $I_t - \Delta L_t$ to get

$$I_t - \Delta L_t = RIT_t + r_d L_{t-1} - L_t + L_{t-1}. \tag{A12}$$

Since (A12) holds for all t, substitute it for all future $I_t - \Delta L_t$ in the CF(debt) model to get

$$P_d = \sum_{t=1}^{\infty} (1+r_d)^{-t} \left(L_{t-1} - L_t + RIT_t + r_d L_{t-1} \right). \tag{A13}$$

Using the exact same sequence steps as in the derivation of RI(equity) and RI(firm), (A13) can be rewritten as

$$P_d = L_0 + \sum_{t=1}^{\infty} (1+r_d)^{-t} RIT_t \ . \tag{A14}$$

Dividing the infinite series of terms into T years of the finite horizon plus a perpetuity of flows beginning in year T+1 gives the RI(debt) model as shown in figure 2.

The Weighted Average Cost of Capital

First we show that there exists some discount rate r_w such that $P_e = P_f - P_d$, where P_f is computed as CF(firm) or RI(firm) and P_d is computed as CF(debt) or RI(debt). Then we provide an example demonstrating that the internally-consistent r_w value is not always the weighted average cost of capital.

Recall that $D_t = C_t - I_t + \Delta L_t$. Substituting this into the CF(equity) model gives

$$P_e = \sum_{t=1}^{\infty} (1+r_e)^{-t} C_t - \sum_{t=1}^{\infty} (1+r_e)^{-t} (I_t - \Delta L_t) \ . \tag{A15}$$

What we need to show is that, taking r_e and r_d as given, there exists a value r_w such that

$$P_e = \sum_{t=1}^{\infty} (1+r_w)^{-t} C_t - \sum_{t=1}^{\infty} (1+r_d)^{-t} (I_t - \Delta L_t) \ . \tag{A16}$$

Fix P_e as a positive value. Assume the C_t series is composed of positive values after some finite period; if not then P_e cannot have a positive value. A sufficiently high r_w will make the first term in (A16) arbitrarily close to zero and a sufficiently low r_w term will make the first term in (A16) arbitrarily large. Since the first term in (A16) is also continuous in r_w, this term can take any positive value given the correct choice of r_w. The second term in (A16) is the present value of cash flows to debtholders and is generally positive. But even if this summation is negative, as long as its absolute value is less than P_e, by changing r_w the first term in (A16) can be set to cause the difference in the two terms to equal any fixed P_e.

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