Financial Contracting and Leverage Induced Over- and Under-Investment Incentives

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ABSTRACT

This paper investigates the effects of seniority rules and restrictive dividend convenants on the over- and under-investment incentives associated with risky debt. We show that increasing seniority of new debt decreases the incidence of under-investment but increases over-investment, and vice versa. Under symmetric information, the optimal seniority rule is to give new debtholders first claim on a new project without recourse to existing assets (i.e., project financing). Under asymmetric information, the optimal debt contract requires equating the expected return to new debtholders in the default state to the new project’s cash flow in the same rate. If this is not possible, the optimal seniority rule calls for strict subordination of new debt if the expected cash flow in default is small and full seniority if it is large. With regard to dividend convenants, we show that their effect depends on whether or not dividend payments are conditioned on future investments. When they are unconditioned, allowing more dividends increases the under-investment incentive. In contrast, conditional dividends decrease the under-investment incentive and increase the over-investment incentive.

IT IS WELL KNOWN that risky debt may induce conflicts of interest between stockholders and bondholders and lead to perverse corporate investment incentives. The resulting suboptimal investment decisions cause deadweight losses which are commonly referred to as agency costs of debt. The two best known examples of such costs are Jensen and Meckling’s (1976) asset substitution effect and Myers’ (1977) under-investment problem.

Various financial contracting methods have been examined for their potential in mitigating the agency problems. These methods can be broadly classified into three categories: 1) devise ways which allow a firm to eliminate existing debt or neutralize its impact prior to undertaking a new project;¹ 2) renegotiate prior contracts in order to resolve conflicts between different security holders;² and 3) design ex-ante debt contracts to mitigate the agency costs of debt. In this paper we focus on the third approach.

Ex-ante debt contracts which have been suggested to reduce the agency

¹ For example, Myers (1977) suggests reliance on short-term debt which matures before the project decision date; Bodie and Taggart (1978) and Chisea (1990) suggest the use of callable bonds which can be called before undertaking the project; and Jensen and Meckling (1976) and Green (1984) suggest convertible debt which is capable of neutralizing the conflict between stockholders and bondholders by giving the bondholders an equity claim.

problems include: collateralization of assets to make it difficult for shareholders to substitute low-risk projects with high-risk projects (Smith and Warner (1979)); restrictions on dividends and other cash distributions to force firms to retain funds and undertake investment projects (Kalay (1982) and Smith and Warner (1979)); and secured debt and leasing to alleviate the under-investment problem (Stulz and Johnson (1985) and Smith and Wakeman (1985)).

The common characteristic of these ex-ante contracts is that they specify the relative seniority of the claims of existing debtholders vis-à-vis present and future security holders. For example, retaining in bond indentures the option to finance new projects with secured debt or leasing allows the firm to grant new debtholders’ senior claims on new projects in the event of default. This seniority in turn reduces the promised interest rate on the new borrowing, inducing shareholders to undertake a value-increasing, risk-reducing project which they otherwise would have foregone.

It is important, however, to recognize that retaining the option of issuing non-subordinated (senior) debt has a double-edged effect: although it alleviates the under-investment problem, the low cost of borrowing also creates an incentive for excessive investment which may take the form of accepting negative net present value (NPV) projects. Thus, a financial contract designed to reduce the under-investment problem can end up increasing the total agency costs of debt.

To address this issue, we develop a model which allows for a simultaneous analysis of the under- and over-investment incentives. The model is used to examine how ex-ante seniority rules designed to mitigate under-investment problems are likely to affect the total agency costs of debt. The purpose is to derive optimal ex-ante contracting rules concerning the relative seniority between new and existing debt. The contracting rule is investigated under both symmetric and asymmetric information concerning the value of a new project.

We show that, with symmetric information, the optimal seniority rule is to issue debt through project financing, with the new debtholders being given first claim on a new project, but without recourse to existing assets. Specifically, we show that project financing does as well as any other seniority rule in alleviating the under-investment problem while it minimizes the over-investment problem. Intuitively, this is because project financing separates the new project as much as possible from the existing assets without making the new project into a separate and independent firm.

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3 Another solution to the under-investment problem is to design a forward debt contract. See Berkovitch and Greenbaum (1990).
4 In essence, the existing bondholders are allowing shareholders to capture part of the gains from the new project through the issuance of non-subordinated debt. This alleviates the Myers under-investment problem.
5 We define over-investment generally as any situation in which a firm undertakes a negative NPV project. This definition includes both Jensen and Meckling’s asset substitution effect in which a negative NPV project replaces a low risk project and situations in which a negative NPV project is added to a firm’s existing projects.
6 Obviously, if a new project has no economic or physical interdependence with existing assets, a complete separation via establishment of a new independent firm for the new project will yield the first-best solution, because it will enable the new project to be evaluated solely on its own merits. Thus the basic premise of our analysis is either that the synergy considerations arising from economic interdependency dominate the agency considerations or that the new project is physically inseparable from the existing project (e.g., upgrading existing facilities).
Our analysis also compares secured debt and leasing in terms of their effects on the total agency costs of debt. The results show that leasing is more similar to project financing than is secured debt. Everything else equal, leasing is more effective in mitigating the under-investment incentive while not exacerbating the over-investment incentive as much as secured debt.

The optimal seniority rules change when there is an informational asymmetry between stockholders and debtholders such that only the stockholders know the true value of the new project at the time of financing it. Under such an informational asymmetry, there are no seniority rules which are capable of reducing under-investment incentives without exacerbating over-investment. Project financing is no longer optimal. The optimal rule depends on the market’s perceived level of risk in the new project. If this perceived risk is sufficiently large (small), it is optimal to require (allow) that all new debt be strictly subordinated (senior) to the existing debt. Otherwise, the optimal rule is to give new debtholders sufficient seniority to equate their expected returns in default to the new project’s expected cash flows in the same state. This solution is analogous to project financing in that it obtains the maximum separation between claims on the new project and those on existing assets within the constraints imposed by information asymmetry.

An implication of the above results is that an important determinant of the ex-ante seniority rule is the collateral value of a new project. If the expected cash flow from a project upon default is low, then it is optimal to make new debt strictly subordinate; if the expected cash flow is sufficiently large, then it is optimal to retain the option to issue non-subordinated debt.

We also use our model to analyze the effects of dividend constraints on corporate investments. We first confirm the conjecture made by Smith and Warner (1979) and Kalay (1982) that restricting cash distributions to stockholders will reduce the under-investment incentive. However, when we allow cash distributions to be conditioned upon future investments, which Kalay shows is a standard feature in debt covenants, we reach quite different conclusions. Under such covenants, greater dividends alleviate the under-investment incentive while exacerbating the over-investment incentive. Thus, in general, the optimal level of cash distributions depends on the expected loss associated with over-investment vis-à-vis under-investment.

The paper is organized as follows. Section I introduces the basic model. We use this model to analyze the effects of ex-ante seniority rules in Section II. Section II.A analyzes the case of informational symmetry, while Section II.B examines the effects of informational asymmetry. Section III investigates the effects of dividend constraints. The final section contains conclusions.

I. The Basic Model

Consider a two-period model in which a firm has existing assets from investments in the previous period, time 0, and is evaluating a new investment opportunity at time 1. The firm will be liquidated at time 2. The gross returns at time 2 are dependent upon the state of the world. For the sake of simplicity, we assume
only two states, low \((L)\) and high \((H)\), with a probability of \(P\) and \((1 - P)\), respectively. The gross returns in each state are summarized below:

<table>
<thead>
<tr>
<th>State</th>
<th>Probability</th>
<th>Gross Returns from the Existing Project (project (x))</th>
<th>Gross Returns from a New Project (project (y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)</td>
<td>(P)</td>
<td>(X_L)</td>
<td>(Y)</td>
</tr>
<tr>
<td>(H)</td>
<td>(1 - P)</td>
<td>(X_H)</td>
<td>(Y + s)</td>
</tr>
</tbody>
</table>

The gross returns, \(X_L\), \(X_H\), \(Y\), and \(Y + s\), each represent the sum of cash flows from operation and proceeds from liquidation. Initially, we make no distinction between operating cash flows and liquidation values. We assume that all cash flows accrue at time 2 and that the firm makes no interim cash distribution to shareholders until time 2. We relax this assumption in Section III, where we examine the effects of restrictive covenants concerning dividends. Throughout the paper we assume that managers are perfect representatives of shareholders.

The information structure and the sequence of events are as follows. At time 0 the first project, project \(x\), is undertaken because its Net Present Value (NPV) is positive. All parameters concerning project \(x\) are known to everyone. The parameters of the second project, project \(y\), are unknown, but everyone shares a common probability distribution about them.

At time 1, shareholders discover the true values of project \(y\)'s unknown parameters and decide whether to accept the project. We initially assume that the market discovers the same information at the same time—i.e., symmetric information. In Section II.B we relax this assumption and examine the case of informational asymmetry. Figure 1 summarizes the above sequence of events.

We assume that the firm finances project \(x\) with positive amounts of debt and equity.\(^7\) The debt has a zero coupon, with a face value of \(F_0\), and is due at time 2. We assume that there is a positive probability that the firm will default at time 2 and that the market value of equity at time 0 is positive. These assumptions can be summarized as follows:

\[(A1): 0 < X_L < F_0 < X_H.\]

The inequality \(X_L < F_0\) indicates that there is a positive probability that the firm will default at time 2, while the following inequality, \(F_0 < X_H\), implies a positive value for equity at time 0.\(^8\) Note that (A1) also assumes that the gross return from project \(x\) in state \(L\) is strictly positive. This last assumption is made for convenience.

To keep the model as simple as possible, we assume that the market is risk

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\(^7\) In this paper we do not model how firms choose their debt/equity ratios. We are implicitly assuming that there are some benefits associated with debt (e.g., taxes) which outweigh the disincentive to borrow due to the agency cost of debt. For a latest review of this literature, see Kim (1989).

\(^8\) Since \(F_0\) is determined endogenously, the last two inequalities in A1 should be obtained as results of more basic assumptions. In Appendix C we solve for \(F_0\), and show that the second inequality of A1 holds if the amount of debt raised, \(D_0\), is greater than \(X_L\). The last inequality of A1 follows from the assumption that project \(x\) yields positive NPV. Since the expression for \(F_0\) is rather messy, we derive the results of the paper using a generic \(F_0\) that obeys A1. All the results in the paper hold when the endogenously determined \(F_0\) in Appendix C is used in the analysis.
neutral and that the time value of money (i.e., the risk-free interest rate) is zero. We also ignore taxes.

We define the values of equity and debt which are associated with project \( x \), \( E_x \) and \( D_x \), as follows:

\[
E_x = (1 - P)(X_H - F_0),
\]
\[
D_x = PX_L + (1 - P)F_0. \tag{2}
\]

Note that \( E_x \) and \( D_x \) will become the market values of equity and debt only if the firm decides not to accept the new project.

The new project, project \( y \), requires an investment outlay of \( I_1 \) at time 1 and will yield a gross return of \( Y \) in state \( L \) and \( Y + s \) in state \( H \). We assume that the project is financed entirely by debt. Thus, the market value of the new debt, \( D_1 \), must be equal to the requirement investment outlay (i.e., \( D_1 = I_1 \)). Initially, we assume that the new debt is strictly subordinated to the existing debt; this assumption will be relaxed in the next section.\(^9\)

To be consistent with (A1), we also assume that

(A2): \( X_L + Y < F_0 + F_1 < X_H + Y + s \),

where \( F_1 \) is the face value of the new debt. The first inequality in (A2) stems from the assumption that the probability of default remains positive after undertaking the project. The second inequality is necessary to ensure that the market value of equity remains positive.

Notice that, once \( Y \) is observed at time 1, it represents the certain component of returns from project \( y \) because it is realized in all states of the world. The

\(^9\)The results in this section will not change even if we assume that the new project is financed by new equity. This is because for our analysis of seniority rules, equity is equivalent to new subordinated debt. That is, the claims of both equity holders and subordinated debtholders are junior to those of existing debtholders; thus, which fraction of the new project is financed by subordinated debt or new equity has no effect on the existing debtholders. In other words, once the firm decides to undertake a new project (i.e., the investment decision is given), a “me-first” rule that all subsequent debt be strictly subordinated to the existing debt ensures the validity of the Modigliani and Miller (1958) irrelevance theorem (see Fama and Miller (1972)). However, if an ex-ante financial contract allows violation of this “me-first” rule such that new debt does not have to be strictly subordinated to old debt, then debt financing gives more flexibility than equity financing and will in general result in a better outcome. We examine this issue in the next section.
uncertain component is captured by $s$, which is realized, with probability $(1 - P)$, only if state $H$ occurs. Also note from (A2) that $Y$ and $s$ can be either positive or negative with an upper bound on $Y$, $\bar{Y} = F_0 - X_L + F_1$, and a lower bound on $s$, $\underline{s} = X_L - X_H$. If $s$ is positive, the returns of old and new projects are positively correlated; if negative, the correlation is negative.

Given the above assumptions and definitions, the face value of new debt is the solution to

$$ I_1 = D_1 = PZ + (1 - P)F_1, \quad (3) $$

where

$$ Z = \max\{X_L + Y - F_0, 0\}. \quad (4) $$

Equation (3) equates the amount of borrowing to the expected returns to new debtholders. Equation (4) shows that new debt is subordinated to the original debt and that the new debtholders have limited liability. By rearranging (3) we obtain

$$ F_1 = (I_1 - PZ)/(1 - P). \quad (5) $$

If the firm undertakes the new project, the market value of equity will become

$$ E_1 = (1 - P)(X_H + Y + s - F_0 - F_1). \quad (6) $$

Equation (6) expresses the fact that stockholders receive positive cash flows only in state $H$. Comparing (1) and (6) we get

$$ E_1 = E_x + (1 - P)(Y + s - F_1). \quad (7) $$

Substituting for $F_1$ and rearranging yields

$$ E_1 = E_x + Y + S + P(Z - Y), \quad (8) $$

where

$$ S = (1 - P)s - I_1. \quad (9) $$

Notice that $Y + (1 - P)s$ is the present value of project $y$, and, hence, the expression $Y + S = Y + (1 - P)s - I_1$ represents the NPV of the new project. Thus, (8) states that the new project affects stockholders’ wealth not only by its NPV but also by an additional term, $P(Z - Y)$. If this term is positive, it is possible to have $E_1 > E_x$ even when the NPV is negative, thereby inducing the shareholders to accept negative NPV projects—i.e., over-invest. Conversely, if $P(Z - Y)$ is negative, the shareholders may forego some positive NPV projects—i.e., under-invest.

To demonstrate these agency problems, we set $E_1 = E_x$ in (8) and use (4) to obtain the following indifference lines:

$$ S = -(1 - P)Y, \quad \text{if } Y \leq F_0 - X_L, \quad (10.1) $$

$$ S = -Y + P(F_0 - X_L), \quad \text{if } Y > F_0 - X_L. \quad (10.2) $$

These indifference lines are depicted in Figure 2. The line $E_1 = E_x$, which is given by (10.1) and (10.2), represents all combinations of $Y$ and $S$ that will leave
shareholders indifferent between accepting and rejecting the new project. Any point above this line represents a project that will increase the shareholder wealth; any point below will decrease shareholder wealth.

The line with a slope of \(-1\) going through the origin, \(S = -Y\), which we also label as \(\text{NPV} = 0\), represents all combinations of \(S\) and \(Y\) that yield zero NPV. Any point above this line represents a project with a positive NPV; any point below, a negative NPV.

Comparing the two lines reveals the conditions under which shareholders will deviate from the NPV rule. When \(Y\) is negative, the \(\text{NPV} = 0\) line exceeds the \(E_1 = E_x\) line. Thus, any point between the two lines represents a negative NPV project which increases shareholders’ wealth. The situation is reversed when \(Y\) is positive: the second line \((E_1 = E_x)\) now exceeds the first line \((\text{NPV} = 0)\). Thus, if a project falls between the two lines, the shareholders will reject the project.
even though it has a positive NPV. The above discussion is summarized in the following proposition:

**PROPOSITION 1**: (i) Suppose \((Y, S)\) satisfy \(S > 0\) and \(Y^*(S) = -S/(1 - P) < Y < -S\). Then, \(E_1 > E_x\) and \(NPV = Y + S < 0\) for every \(Y\) in the interval \((Y^*(S), -S)\); that is, the new project will be undertaken even though its NPV is negative (the over-investment problem). (ii) Suppose \((Y, S)\) satisfy \(S < 0\) and \(-S < Y < Y^*(S)\). Then, \(E_1 < E_x\) and \(NPV = Y + S > 0\) for every \(Y\) in the interval \((-S, Y^*(S))\); that is, the project will not be undertaken even though its NPV is positive (the under-investment problem).

**Proof**: See Appendix A.

The source of these perverse incentives to over- and under-invest can be seen by examining the effect of the new investment on the market value of old debt. In Appendix B we show that, when the lowest possible value of the firm is non-negative (i.e., \(X_L + Y \geq 0\)), the new project changes the value of the old debt by \(-P(Z - Y)\), which is precisely the negative of the extra term in (8). Therefore, the amount the shareholders gain or lose in excess of the NPV is the amount the old debtholders lose or gain due to the new project.

We also show in Appendix B that, when the lowest possible value of the firm is negative (i.e., \(X_L + Y < 0\)), there is no longer a one-to-one correspondence between the wealth effects on shareholders and old debtholders. Specifically, there is an extra gain to the firm due to limited liability which enables the firm to free-ride on the public by abandoning its assets when their value is negative. This externality gives the shareholders a non-leverage induced incentive to over-invest; thus, even an all-equity firm will deviate from the NPV rule and over-invest. Figure 2 depicts this possibility with a broken line.\(^{10}\)

Although this model is relatively simple, it captures the essential points in the agency problems associated with risky debt. In our model, the certainty component of the new project is represented by \(Y\), and the risky component by \((1 - P)s\). As the risky component \((1 - P)s\) increases, so does \(S = (1 - P)s - I_1\). Thus, when \(S\) is positive, the range of \(Y\) for which the over-investment incentive occurs increases as the uncertainty variable, \(S\), increases. Conversely, when \(Y\) is positive, the range of \(S\) for which the under-investment incentive occurs increases as the certainty variable, \(Y\), increases.

Also notice that the default risk of the existing debt is represented by \(P(F_0 - X_L)\), which is the expected amount of unfulfilled promise to debtholders due to default. Equation (10.2) shows that, for \(Y > F_0 - X_L\), the line \(E_1 = E_x\) has an intercept equal to \(P(F_0 - X_L)\); hence, the difference between the \(E_1 = E_x\) line and the NPV = 0 line in Figure 2 will increase as \(F_0 - X_L\) increases. Thus, the model implies that the under-investment problem becomes more severe when the default risk of the existing debt is large.

\(^{10}\) John and Senbet (1989) also analyze how limited liability creates the free-rider problem and an over-investment incentive. In particular, they focus on how the under-investment incentive associated with debt acts as a counter-balancing force against the over-investment incentive created by the free-rider problem. However, they do not consider the over-investment incentive created by existing debt, nor do they address the issue of how seniority rules and dividend constraints affect the over- and under-investment problems.
II. The Effects of Ex-Ante Seniority Rules on the Over- and Under-Investment Incentives

In the preceding section, we have assumed that bond covenants require all new debt to be strictly subordinated to the existing debt. Relaxing this restriction may alleviate the under-investment problem. For example, if the new debt is senior to the existing debt, the promised interest rate will be lower. This lower cost of borrowing will give shareholders a greater incentive to undertake new projects. Therefore, they will accept risk-reducing positive NPV projects which they would otherwise have foregone. However, if the new project is a risk-increasing type with a negative NPV, allowing stockholders to issue senior debt may exacerbate the over-investment incentive. In this section we derive the optimal ex-ante seniority rule which minimizes the total agency costs arising from over- and under-investments.

Giving seniority to new debtholders can take many different forms. The most obvious one is to give new debtholders full seniority over old debtholders on all cash flows as well as on the terminal value of the firm at time 2. An alternative is to give partial seniority, in which the new debtholders have first claim on the new project, but have no claim on existing assets. Such partial seniority is given to new debtholders in a project financing. Intermediate seniority arrangements between these two cases involve giving new debtholders equal or subordinated claims on existing assets and senior claims on new assets. Secured debt and leasing are examples of such an arrangement.

To generalize these alternative sets of seniority rules, we define a seniority rule \((e; n)\) as follows: in the event of default, new debtholders receive a fraction \(e\) of both the cash flows and terminal value of existing assets and a fraction \(n\) of those new projects, where \(e\) and \(n\) are between zero and one. If \(e = n = 1\), the new debt has full seniority over old debt; if \(e = 0\) and \(n = 1\), the new debt is issued as a project financing; and if \(0 < e < 1\) and \(n = 1\), the new debt falls into the intermediate case of secured debt or leasing.

A. Symmetric Information

Under the assumption that the true values of all parameters of project \(y\) are revealed to everyone at time 1, giving seniority to new debtholders does not change equations (1) through (9), except for \(Z\) in (4). The payoff to new debtholders in the case of bankruptcy with seniority \((e; n)\) is now given as follows:

\[
Z = \max\{0, \min(eX_L + nY, F_1)\}.
\]  

(11)

That is, in the event of bankruptcy new debtholders receive the maximum between zero and the cash flow they are entitled to under seniority rules \((e; n)\).\(^{11}\)

\(^{11}\) Another possible outcome that can be included in the right-hand side of (11) but was ignored for the sake of brevity is \(X_L + Y - F_0\), which is the return to new debtholders under a strict subordination. This case needs to be considered only if \(X_L + Y - F_0\) can be greater than \(Z\) in (11); that is, if a strict subordination provides new debtholders with a return greater than under a non-subordination. Such a case is rather implausible because it violates the economic meaning of subordination in defining the relative seniority between new and old debtholders.
Substituting (11) into (8) yields:

\[ E_1 = E_x + (1 - P)Y + S + PF_1, \quad \text{if } eX_L + nY \geq F_1 \]  
\[ E_1 = E_x + [1 - P(1 - n)]Y + S + PeX_L, \quad \text{if } 0 \leq eX_L + nY < F_1 \]  
\[ E_1 = E_x + (1 - P)Y + S, \quad \text{if } eX_L + nY < 0 \]  

(12.1)  
(12.2)  
(12.3)

Note that, when \( eX_L + nY \geq F_1 \), the new debtholders face no risk of default. Because by assumption the time value of money is zero and the new project is financed exclusively by debt,

\[ F_1 = D_1 = I_1, \quad \text{if } eX_L + nY \geq F_1. \]  

(13)

By using (13) in (12) and setting \( E_1 = E_x \), we obtain the following indifference lines for shareholders:

\[ S = -(1 - P)Y - PI_1, \quad \text{if } eX_L + nY \geq I_1 \]  
\[ S = -(1 - P)Y - PeX_L, \quad \text{if } 0 \leq eX_L + nY < I_1 \]  
\[ S = -(1 - P)Y, \quad \text{if } eX_L + nY < 0. \]  

(14.1)  
(14.2)  
(14.3)

Figures 3 and 4 depict these indifference lines for several values of \( e \) and \( n \). The case of full seniority (\( e = 1; n = 1 \)) is described in Figure 3 with a bold line, denoted by \( E^F_1 = E_x \). The indifference line \( E_1 = E_x \) under a strict subordination is also shown. It can be seen that the two indifference lines coincide when \( Y \leq -X_L \), subjecting the firm to the same over-investment problem. This is because \( Y \leq -X_L \) implies that the lowest possible value of the firm \( (X_L + Y) \) is negative, in which case debtholders exercise the limited liability and seniority becomes irrelevant.

When \( Y > -X_L \), however, granting new debtholders full seniority over old debtholders changes the shareholders’ incentive. Full seniority makes the new debt less risky, allowing the shareholders to finance the new project at a lower

To be mathematically precise, however, ruling out the possibility of \( X_L + Y - F_0 \) being greater than \( Z \) in (11) requires that \( eX_L + nY > X_L + Y - F_0 > 0 \). (If \( X_L + Y - F_0 < 0 \), \( X_L + Y - F_0 \) must be less than \( Z \) in (11).) Since by assumption \( X > 0 \), this condition will be satisfied if

\[ nY \geq X_L + Y - F_0. \]  

(*)

By drawing the values of \( nY \) and \( Y - (F_0 - X_L) \) as a function of \( Y \) on a two-dimensional graph, where \( 0 < n < 1 \) and \( F_0 - X_L > 0 \), it can be seen that (*) holds up to a certain positive value of \( Y \), at which point the inequality becomes reversed. Notice that this point of reversal increases with \( n \). Thus to ensure that (*) holds for the relevant range of \( Y \), we need to restrict \( n \) such that the maximum possible \( Y \), \( \bar{Y} \), occurs before the point of reversal.

From assumption (A2), \( \bar{Y} = F_0 + F_1 - X_L \). But at \( \bar{Y} \), both old and new debt become riskless, and hence, \( F_1 = D_1 = I_1 \). Substituting \( \bar{Y} = F_0 + I_1 - X_L \) into (*) and rearranging yield

\[ n \geq I_1/(F_0 + I_1 - X_L), \]

which is less than 1. Note that we use \( F_0 \) with strict subordination. With higher seniority for new debt, \( F_0 \) increases, reducing the minimal \( n \) needed. Thus, the above condition is sufficient for (*) to hold under any seniority rule. This restriction on \( n \) does not affect the generality of our result, because as we shall see later in this section, the optimal value of \( n \) is 1.
cost. Consequently, the shareholders will have a greater incentive to undertake the new project. Thus, for $-X_L < Y < 0$, the distance between the $E_1^P = E_x$ line and the NPV = 0 line is larger than in the previous case; in other words, the over-investment incentive is exacerbated. For $0 < Y < I_1$, the new indifference line now lies below the NPV = 0 line, which means that the lower cost of borrowing not only eliminates the under-investment incentive but also creates an over-investment incentive which does not exist under a strict subordination. Thus when $Y < I_1$, one cannot reduce the under-investment incentive by giving new debtholders full seniority without at the same time exacerbating the over-investment incentive.

Figure 3 also shows that when $Y > I_1$ the $E_1^P = E_x$ line lies above the NPV = 0 line, which implies that it is not possible to solve all under-investment problems via seniority rules alone. This limitation of seniority rules is due to the fact that when $Y > I_1$, the new project is so safe that not only does the new debt become
riskless, but the old debt also becomes safer. Therefore, some of the benefits of the new project are captured by old debtholders, creating a wealth transfer from stockholders to old debtholders.12 Because such a wealth transfer is the source of the under-investment problem, not all under-investment problems can be solved via ex-ante seniority rules alone.13

12 The new senior debt first becomes riskless when $Y > I_1 - X_L$, at which point $F_1 = I_1 = D_1$ and hence the default premium of new debt is zero. Thus beyond this point, allowing the shareholders to issue senior debt does not reduce the default premium. However, as long as $Y < I_1$, the old debtholders are indirectly subsidizing this low cost of borrowing, because the new project reduces the old debtholders' return in state $L$ (the state of default) from $\max(0, X_L)$ to $\max(0, X_L + Y - F_1) = \max(0, X_L + Y - I_1)$. Finally, if $Y > I_1$, $X_L + Y - I_1$ is greater than $X_L$, and the new project increases the old debtholders' return in default and hence engenders the type of wealth transfers from stockholders to old debtholders described in Appendix B.

13 Finally, note that $E'_F = E_L$ line can never lie above the $E_1 = E_x$ line, because the maximum value that $Y$ can take in our framework is $F_0 - X_L + I_1$. 

Figure 4. Over- and under-investment incentives when new debt is issued under project financing. The bold line represents the indifference line with project financing. The figure also shows other seniority rules.
Figure 4 illustrates the case of a project financing \((e = 0; n = 1)\) with a bold line. The figure also shows indifference lines for other values of \(e\) and \(n\): \(e = n = 1\) (full seniority), \(0 < e < 1\) and \(n = 1\) with a broken line (secured debt and leasing), and \(e = 0\) with \(n < 1\). Comparing project financing with full seniority, we see that the two indifference lines coincide for high and low values of \(Y\), i.e., \(Y > I_1\) and \(Y < -X_L\). However, for \(-X_L < Y < I_1\), the indifference line under project financing lies above that obtained with full seniority. More importantly, for this range of \(Y\) project financing does not exacerbate the over incentive problem while completely eliminating the under-investment problem.

When the seniority arrangement falls into the intermediate range of \(0 < e < 1\) and \(n = 1\) (secured debt and leasing), the indifference line falls between the lines representing the full seniority and project financing. Thus, secured debt financing and leasing still exacerbates the under-investment problem, although the effect is smaller than that under full seniority.

The above analysis suggests that the optimal seniority rule is project financing. That is, the optimal rule is to give the new debtholders the first claim on the new project and no claim whatsoever on the existing assets. This is summarized in Proposition 2.

**Proposition 2:** Under symmetric information, the seniority rule \((e = 0; n = 1)\), i.e., project financing, is the optimal ex-ante rule.

**Proof:** See Appendix A.

The intuition underlying Proposition 2 is relatively simple. When the cash flows and the terminal value of a new project can be separated from those of the existing assets, it is best to separate the two claims as much as possible. This will minimize the potential wealth effect of a new project on the old debtholders and hence will minimize the potential wealth transfers between stockholders and old debtholders, which are the source of the perverse investment incentives.\(^{14}\)

This raises the issue of why the shareholders do not simply separate the new project from existing assets by making the new project an independent firm. In that case the new project would be evaluated solely on its own merits without any regard to the existing debt; consequently, there would be no under- or over-investment incentive.

A complete separation will be optimal if there is no economic dependency (synergy) between the new project and existing assets. However, if the project is worth more as a part of the firm such that the value of synergy is greater than the expected deadweight costs caused by the perverse investment incentives, it will be optimal to keep the ownership of the project within the firm. For example, when a firm upgrades existing facilities or uses its managerial talents and/or technical know-how to start a new project, synergy considerations may dominate the agency cost consideration.

Thus, Proposition 2 implies that when a firm undertakes a new project internally, it is optimal to rely on project financing. However, not all projects can utilize project financing. For project financing to be feasible, both the cash

\(^{14}\) Note that the project financing also dominates equity financing, because as noted in footnote 9 equity financing is equivalent to issuing strictly subordinated debt.
flows and the terminal value of a new project must be separable from those of assets already in place. One category of investments which meets this criterion is commercial real estate projects. These projects are physically separable, have distinguishable cash flows, and offer lenders tangible security. Thus, it is not surprising that new commercial real estate projects are often financed by non-recourse mortgage loans, i.e., borrowing through project financing. Another example is British Petroleum's development of the Forties Field in the North Sea, in which the lenders were given first claims to the oil produced by the field without recourse to the parent firm (see Brealey and Myers (1988)).

If the cash flows of a new project are not clearly distinguishable from those of existing assets, the contracting and implementing costs that are necessary to separate cash flows may outweigh the benefits of project financing. Thus, when only part of the cash flow which is clearly distinguishable from the existing assets is the project's liquidation value, secured debt or leasing may be the preferred means of borrowing.\footnote{When a significant portion of the cash flows and the terminal value of a new project is not separable from those of existing assets, the cost of separating cash flows may make it optimal for the firm to assign seniority on the sum of cash flows rather than on individual cash flows. In such a case, \( e \) must be equal to \( n \). With the additional constraint that \( e = n \), it is easy to show that the optimal seniority rule requires that \( 0 < e = n < 1 \), and the indifference line intersects the line for optimal financing (\( e = 0; n = 1 \)). Thus, the incidence of over- and under-investments will be greater and the optimal seniority rule will be only a second-best condition.}

It is important to note that secured debt and leasing differ in several crucial respects (see Smith and Wakeman (1985)). First, when the claims of the secured debtholders are not fully satisfied in bankruptcy, they still have unsecured claims on the firm's other assets \((X_L)\), whereas the lessor's claim is limited to one year's lease payment. Second, it is generally less costly for a lessor to repossess a leased asset than it is for a secured debtholder to repossess the collateral. In the context of our model, these differences imply that leasing has both a smaller \( e \) (closer to 0) and a larger \( n \) (closer to 1) than secured debt financing. Consequently, leasing is closer to project financing than secured debt financing; thus, compared with secured debt financing, leasing is more effective in both mitigating the under-investment incentive and not exacerbating the over-investment incentive.\footnote{An exception to this generalization occurs if the terminal value of the new project is not clearly distinguishable from existing assets. In that case, contracting and monitoring costs may make secured debt the less costly alternative.}

\section*{B. Asymmetric Information}

In this section we relax the assumption of symmetric information and investigate the optimal seniority rule under informational asymmetry. Specifically, we assume that at time 1 only stockholders discover the true values of \( Y \) and \( s \), i.e., the NPV of the new project. Outside investors have only a common joint cumulative distribution function on \( Y \) and \( s \), \( F(Y, s) \), until after the investment decision is made by the stockholders. We assume \( I_1 \) is known to everyone and retain assumptions (A1) and (A2).

In this world of informational asymmetry, there exists only a pooling equilib-
rium in the debt market at time 1. This is due to our implicit assumptions that firms issue only debt and equity and that there are no costs (e.g., bankruptcy costs) which separate high quality firms from low quality firms. Consequently, outside investors cannot price new debt as accurately as they do with symmetric information. Instead, they price new debt based on the average quality of new projects. Thus, we introduce a new variable, \( \mu_z(e; n) \), which represents the expected cash flows to new debtholders in state \( L \) given seniority rule \( (e; n) \).

### B.1. Strict Subordination

As in the earlier sections, we begin with the case in which new debtholders’ claims are strictly subordinated to those of existing debtholders. In this case, equations (1) through (3), and (5) through (9) are still valid if \( Z \) is replaced by \( \mu_z(0; n) \), the expected cash flows to new debtholders when new debt is strictly subordinated to old debt:

\[
\mu_z(0; n) = E[\max(X_L + Y - F_0, 0)],
\]

where \( E \) is the expectations operator and \( n \) is the minimum \( n \) such that for any \( n \) above which the claims of new debtholders become higher than strict subordination (see footnote 11). In addition, we redefine \( E_1 \) as the private value of equity (as opposed to the market value) known only to the stockholders. This requires the additional assumption that no trade will take place before the market discovers the true values of \( Y \) and \( s \).

Replacing \( Z \) with \( \mu_z \) in (8) and setting \( E_1 = E_x \) yields the following indifference line:

\[
S = -(1 - P)Y - P\mu_z.
\]

Note that \( \mu_z \) is strictly positive as long as the market believes that there is a positive probability that new debtholders will receive positive returns in state \( L \) (i.e., \( X_L + Y > F_0 \) for some realizations of \( Y \)). Thus, the indifference line (16) has a negative intercept.

Figure 5 depicts (16) by a bold line, denoted by \( E_1^* = E_x \), and compares it with the indifference line under symmetric information. The figure shows that as long as \( Y < F_0 - X_L + \mu_z \), firms are more willing to undertake projects under asymmetric information than under symmetric information. This is because the pooling equilibrium in the debt market allows low quality firms to borrow at a lower rate than that under symmetric information. This lower cost of borrowing gives the lower quality firms a greater incentive to invest, resulting in either exacerbation of the over-investment problem or reduction of the under-investment problem.

When \( Y > F_0 - X_L + \mu_z \) the marginal firm is now a relatively high quality firm that subsidizes low quality firms. Consequently, the firm’s cost of borrowing is higher than it should be under symmetric information, and hence, its under-investment incentive is greater.

---


18 In the absence of liquidity traders, no uninformed investors will trade and hence no trade will take place until the informational symmetry disappears.
Figure 5. Over- and under-investment incentives under asymmetric information. The bold line represents the indifference line under asymmetric information, and the $E_1 = E_x$ line is the indifference line under symmetric information shown in Figure 2.

**B.2. Non-Subordinated Debt**

If new debtholders are given non-subordinated claims, the indifference line changes because $\mu_z$ changes. Under the general seniority rule $(e; n)$, $\mu_z$ can be written as

$$\mu_z(e; n) = E[\max(0, \min(eX_L + nY, F_1))].$$

Note that under informational asymmetry, $e$ and $n$ affect shareholders' wealth only through their effect on $\mu_z$; thus, a seniority rule $(e; n)$ is equivalent to a seniority rule $(e'; n')$ if both result in the same $\mu_z$. This allows us to transform the two dimensional seniority rule $(e; n)$ into a one-dimensional seniority rule $(t)$ where $0 \leq t \leq 1$ specifies the fraction of total returns in state L that accrues to the new debtholders. For example, $e = n = 1$ would be equivalent to $t = 1$, \[780\]
which is the highest seniority that new debtholders can have. Using this simplified seniority rule, (17) can be rewritten as

$$
\mu_z(t) = E\{\max[0, \min[t(X_L + Y), F_1]]\}.
$$

(18)

A higher value of $t$ means a higher seniority to new debtholders and, as can be seen from (18), a higher $\mu_z$. Therefore, giving new debtholders a more senior claim causes a downward parallel shift of the indifference line in Figure 5.

Figure 5 shows that a downward shift of the indifference line (depicted by the bold line) will have two opposing effects: it reduces the under-investment incentive but exacerbates the over-investment incentive. Thus, giving a greater seniority to new debtholders (a higher $t$) mitigates the under-investment incentive but only at the expense of exacerbating the over-investment incentive. This will lower the average quality of new projects undertaken by firms, increasing the incidence of accepting negative NPV projects. Conversely, making new debt less senior will make borrowing more expensive and discourage investments, which will in turn decrease the over-investment incentive and increase the under-investment incentive.

Under asymmetric information, therefore, the optimal seniority rule requires trading off expected deadweight costs due to the incidence of over-investments against expected deadweight costs due to under-investments. To make the analysis tractable, we assume that investors believe $Y$ and $S$ are distributed uniformly over $(Y, \bar{Y})$ and $(\bar{S}, \bar{S})$; that is,

$$
f(Y, S) = \begin{cases} 
 c, & \text{if } Y \in [Y, \bar{Y}] \text{ and } S \in [\bar{S}, \bar{S}], \\
 0, & \text{otherwise},
\end{cases}
$$

(19)

where $f$ is the density function and $c = 1/(\bar{S} - S)(\bar{Y} - Y)$. The total expected deadweight costs can now be written as

$$
TC = \int_Y^{Y_2} \int_{h(\mu_z)}^{-Y} - c(Y + S) \ dSdY
+ \int_{Y_2}^{\bar{Y}} \int_{-Y}^{h(\mu_z)} c(Y + S) \ dSdY - K,
$$

(20)

where

$$
h(\mu_z) = -(1 - P)Y - P\mu_z.
$$

(21)

$$
K = \int_Y^{-X_L} \int_{S=-1-PY + PX_L}^{Y} c(Y + S) \ dSdY.
$$

(22)

The first double integral in (20) represents the deadweight costs due to over-investments, $-(Y + S) > 0$, which result from accepting a negative NPV project. Such incidents occur when $Y$ is in the range $(Y, \mu_z)$ and $S$ moves from $h(\mu_z)$ to $-Y$. Similarly, the second double integral represents the costs associated with

19 Note that at the time of writing the contract only the expected cash flow matters under informational asymmetry; hence, seniority rules $(e, n)$ and $t$ are equivalent for pricing debt.
under-investments, which equals the NPV times the probability of under-investments. Finally, $K$ represents the gains from free-riding on the public by exploiting the fact that both stockholders and debtholders have limited liabilities.

Differentiating $TC$ with respect to $\mu_z$, we obtain

$$
\frac{\partial TC}{\partial \mu_z} = P^2(\mu_z - EY)/(S - S),
$$

where $EY = (\bar{Y} + Y)/2$ is the expected value of $Y$.

**Proposition 3:** Under asymmetric information, the optimal seniority rule is to give new debt:

1. **strict subordination**, if $\mu_z$ with strict subordination is greater than $EY$;
2. **full seniority**, if $\mu_z$ with full seniority is less than $EY$; and
3. a seniority between strict subordination and full seniority which satisfies $\mu_z = EY$, if neither of the above conditions holds.

**Proof:** From (23), we see that $\partial TC/\partial \mu_z > 0$ for all values of $EY$ if the minimum $\mu_z > EY$. Thus, we have a corner solution in which the optimal seniority rule is the one that minimizes $\mu_z$. Since $\mu_z$ is minimized when new debt is strictly subordinated, the optimal rule is to make new debt strictly subordinated to old debt. Similarly, if the maximum $\mu_z < EY$, then $\partial TC/\partial \mu_z < 0$ for all values of $EY$. This gives another corner solution in which the optimal seniority rule is to give full seniority to new debt. When the value of $EY$ lies between these two cases, the optimal seniority rule is the one which makes $\mu_z = EY$ such that $\partial TC/\partial \mu_z = 0$. Q.E.D.

To explain the intuition underlying Proposition 3, recall that $Y$ represents the certainty component of the project to the stockholders. Thus, as $Y$ decreases relative to $S$, the project gets riskier and the over-investment incentive increases. Knowing that stockholders have this incentive, investors will become more concerned with over-investments if they expect $Y$ to be relatively low. One way to discourage over-investments is to increase the cost of new borrowing at time 1, which can be achieved by specifying in the ex-ante debt contract (at time 0) that all new debt be strictly subordinated to the existing debt.

Conversely, if the market expects $Y$ to be large relative to $S$, the under-investment problem becomes more important. Thus, both the firm and the investors may find it optimal to lower the cost of financing future projects by retaining in the bond indentures the option to issue non-subordinated debt. In the extreme, the optimal rule is to give full seniority to new debt.

When the expected value of $Y$ is such that an interior optimum is obtained, the optimal seniority rule equates new debtholders’ expected return in default ($\mu_z$) to the new project’s expected return in default ($EY$). Note that this is analogous to project financing under symmetric information, in which new debtholders’ return in default is equal to $Y$ (when $Y > 0$). Since $Y$ is also the new project’s return in default, the only difference between the two cases is that

---

20 See Appendix B and, in particular, (B8) and (B9) for the limits of integrals in (22).
under asymmetric information the returns are replaced by expected returns.\textsuperscript{21} Thus, the intuition for this interior solution is similar to that of project financing: a seniority rule which makes $\mu_s$ equal to $EY$ obtains the maximum separation between the claim on the new project and those on existing assets within the constraints imposed by informational asymmetry. This will minimize the potential wealth transfers between stockholders and existing debtholders and thereby minimizes the expected total agency costs associated with over- and under-investments.

To illustrate the implications of Proposition 3, consider the following two projects: one with a high collateral value, the other with zero liquidation value. The first project has a high expected value upon default (i.e., $EY$ is large), while the second project’s $EY$ is zero. In the case of the first project, the proposition states that it is optimal to increase $\mu_s$ by giving a non-subordinated claim, such as secured debt, leasing, or senior debt, to new debtholders. In the case of the second project, the proposition implies that the optimal seniority rule is to minimize $\mu_s$ by making new debt strictly subordinated. Examples of the second category of projects include advertising, research, and development expenditures, and a renovation of existing facilities into more firm-specific assets which reduces their resale value. If a firm’s future investment opportunities are primarily of this nature, the optimal ex-ante seniority rule is to make all future debt strictly subordinate to the existing debt.\textsuperscript{22}

Finally, it follows from Propositions 2 and 3 that the optimal debt contract depends on whether informational asymmetry exists regarding the new project. If there is little difference of opinion between the borrower and the lender regarding the new project, project financing may be optimal. If, on the other hand, the new project is not well understood by general investors, the optimal seniority rule depends on the new project’s expected cash flows in default states. If this cash flow is sufficiently small, the optimal rule is to make the new debt strictly subordinated. In general, under informational asymmetry the optimal seniority rule is project-specific, and therefore we observe a greater variety of seniority rules.

### III. Restrictive Covenants on Cash Distributions

As stated at the outset of this paper, other financial contracting methods which help control the under-investment incentive include restrictive covenants on cash distributions to shareholders. Such covenants apply to dividends, share repurchases, and liquidating dividends; for brevity, we shall refer to them as

\textsuperscript{21} When $EY < 0$, Proposition 3 implies that the optimal seniority rule is to make new debt strictly subordinated. Recall from Figures 3 and 4 that, when $Y < 0$, project financing and making new debt strictly subordinate are equivalent.

\textsuperscript{22} Note that the security with the lowest seniority is common stock. Thus, Proposition 3 implies that firms which invest heavily in advertising and in research and development activities will rely more heavily on equity. This prediction is consistent with Bradley, Jarrell, and Kim’s finding (1984) that there is a significant inverse relation between debt ratio and the level of advertising and R&D expenditures.
dividend covenants. Smith and Warner (1979) and Kalay (1982) show that dividend covenants usually establish a limit on cash distributions to stockholders by defining an inventory of funds available for cash distributions over the life of the bonds. They argue that such restrictions effectively impose a minimum level on investment expenditures and thus reduce the under-investment problem. Our model can be used to both verify the validity of this conjecture and examine the impact of more complicated dividend constraints on the over- and under-investment incentives.

In developing our model in Section I we assumed that no dividends would be paid until time 2. Thus, our analysis to this point has implicitly focused on the effect of seniority rules under the most severe dividend restriction. To examine the specific impact of dividend restrictions, we now assume that the firm is allowed to pay cash dividends up to an amount \( d \) before the debt matures at time 2.\(^{23}\) We consider two types of dividend covenants: 1) the amount of dividend payment, \( d \), is not conditioned on any particular action taken by the firm, and 2) the dividend payment is conditioned on accepting the new project, i.e., \( d \) cannot be paid unless the firm takes on a new project.

A. Unconditional Dividend Covenants

As in earlier sections, we begin with the case of strict subordination for new debt. For simplicity we assume that \( 0 < d < X_L \). Since the equations in Section I are based on the assumption that there are no interim cash distributions, allowing positive \( d \) requires the following modifications:

\[
E_x = (1 - P)[X_H - d - F_0(d)] + d, \tag{23}
\]
\[
D_x = P(X_L - d) + (1 - P)F_0(d), \tag{24}
\]

where \( F_0(d) \) is the face value of the existing debt given the level of \( d \). We show in Appendix C that \( F_0(d) \) is strictly increasing in \( d \), because higher \( d \) implies that the coverage for debtholders in default states is smaller. Note that (23) and (24) are written under the assumption that shareholders take the maximum amount \( (d) \) of cash dividend allowed. Indeed, doing so is a dominant strategy for shareholders, because debtholders will have the first claim on the firm’s assets in the event of default. We also assume, as in (A2), that state \( L \) implies bankruptcy, while state \( H \) does not, i.e.,

\[
(A3): X_L - d + Y < F_0(d) + F_1(d) < X_H - d + Y + s,
\]

where \( F_1(d) \) is the face value of the new debt when \( d \) is positive.

By repeating the analysis of Section I, it can be seen that equations (3)–(9) hold with two modifications. First, \( Z \) needs to be replaced by

\[
Z(d) = \max[X_L - d + Y - F_0(d), 0], \tag{25}
\]

which reflects our temporary assumption that new debt is strictly subordinated.

\(^{23}\) The cash dividends may be made from either operating cash flows or the sale of assets. For simplicity we assume that if liquidation of assets is needed, it reduces the terminal value of the firm by exactly the same amount.
Second, $E_1$ must be redefined to incorporate the positive dividend.

$$E_1 = (1 - P)[X_H - d + Y + s - F_0(d) - F_1(d)] + d. \quad (26)$$

By using (5), (23), and (25) in (26) and setting $E_1 - d = E_x - d$, we obtain the following indifference lines:

$$S = -(1 - P)Y, \quad \text{if } Y \leq F_0(d) - X_L + d, \quad (27.1)$$

$$S = -Y + P[F_0(d) - X_L + d], \quad \text{if } Y > F_0(d) - X_L + d. \quad (27.2)$$

Comparing (27) with indifference line (10), which assumes $d = 0$, we see that while (27.1) and (10.1) are identical, (27.2) lies to the right of (10.1) because $F_0(d) > F_0$ and $d > 0$. This difference is illustrated in Figure 6 where indifference line (27), denoted by $E_1(d) = E_x(d)$, is depicted by a bold line to the right of indifference line (10).

From Figure 6, we observe that allowing positive cash distributions increases the under-investment problem, and that the greater (smaller) the allowed interim cash distribution, the greater (smaller) is the under-investment problem. This result is consistent with the assertions by Kalay and Smith and Warner that restrictions on cash distributions reduce the under-investment problem.

**B. Conditional Dividend Payments**

In the above analysis we assume that the amount of dividend allowed is not conditioned on any particular actions of the firm. Clearly, this is overly simplistic. Kalay (1982), for example, shows that a typical dividend constraint can be viewed as a minimum investment constraint and that the amount of dividend a firm can pay depends, among other things, on the net cumulative investment. To analyze the incentive effect of making dividends conditional on investments within our framework, we assume that the firm can pay $d$ only if the new project is undertaken. This simple condition changes the above conclusion rather drastically. Now the dividend payment gives an incentive for shareholders to undertake the new project. Thus, as opposed to the unconditional dividend case, higher dividends reduce the under-investment incentive and aggravate the over-investment incentive.

With dividend payments conditioned on investments, $d$ cannot be paid unless the new project is undertaken. Thus, $E_x$ does not include $d$ and hence is described by (1) with $F_0(d)$ replacing $F_0$. The other terms, $E_1$ and $Z(d)$, are unchanged from (26) and (25). By substituting the revised (1) and (5) into (26), we obtain

$$E_1(d) = E_x(d) + (1 - P)Y + S + P[Z(d) + d]. \quad (28)$$

Note that because stockholders can take dividends irrespective of the investment decision, the investment decision is based on the ex-dividend value of equity.

It is also evident from Figure 6 that the over-investment problem is not affected by the dividend. This is due to the fact that, for $Y < F_0 - X_L$, the debtholders receive nothing irrespective of the magnitude of dividends (see(25)). Thus, for this range of $Y$ the pricing of new debt is not affected by dividends, which in turn means that dividends do not affect the cost of new borrowing for shareholders. Consequently, unconditional dividends do not change investment incentives when $Y < F_0 - X_L$. 
Substituting (25) into (28) and setting $E_1(d) = E_x(d)$ yield the following indifference lines:

$$S = -(1 - P)Y - Pd, \quad \text{if } Y \leq F_0(d) - X_L + d, \quad (29.1)$$

$$S = -Y + P(F_0(d) - X_L), \quad \text{if } Y > F_0(d) - X_L + d. \quad (29.2)$$

Comparing (29) with (10), we note that (29.1) lies below (10.1) because $d$ is positive and (29.2) lies to the right of (10.2) because $F_0(d) > F_0$ (see Appendix C). This difference is illustrated in Figure 7 where indifference line (29), depicted by a bold line, is included along with indifference line (10).

Figure 7 shows that an increase in $d$ generally increases the over-investment incentive and decreases the under-investment incentive. The intuition for this result is as follows: If undertaking the new project allows shareholders to take cash from the firm before the debt matures, it gives them an added incentive to undertake the project. If the project is an under-investment type with a positive NPV, the interim cash distributions reduce the gains to existing debtholders, thereby mitigating the under-investment incentive. However, the interim cash distributions also makes it easier for shareholders to expropriate wealth from the
old debtholders by accepting a risky, negative NPV project, exacerbating the over-investment incentive.\textsuperscript{26}

The implication of this result for ex-ante contracting is that dividend restrictions should reflect the market’s belief about the nature of the firm’s future investment opportunities. If it is more likely that the firm will encounter underinvestment incentives, the debt covenant should allow more cash distributions from future projects. If the firm is more likely to encounter over-investment incentives, the debt covenants should be more restrictive. Thus, in general, the optimal level of cash distributions allowed in dividend covenants depends on the expected loss associated with over-investments vis-à-vis under-investments.\textsuperscript{27}

\textsuperscript{26} This result does not hold when the new project is so safe that the old debtholders are guaranteed to receive $F_0$ even in the default state, i.e., $Y > F_0 - X_L + d$. In such a situation shareholders gain nothing from dividends, because what they receive in dividends reduces the cash flow they receive at time 2 by exactly the same amount. Moreover, allowing positive dividends in debt covenants ex-ante increases $F_0$ to $F_0(d)$; thus, the cash flow to shareholders at time 2 is reduced by more than $d$, with the difference going to the old debtholders. This reduces the incentive for shareholders to undertake new projects and exacerbates the under-investment incentive. This is shown in Figure 7 for $Y > F_0 - X_L + d$.

\textsuperscript{27} A formal analysis of the tradeoffs between the deadweight costs associated with the over- and under-investment incentives is similar to that in Section II.B. Since the analysis itself does not add any new insight, it is not presented here.
C. Conditional Dividend Payments with Non-Subordinated Debt

In Section II.A we have shown that it is not possible to solve all under-investment problems via ex-ante seniority rules alone. The conditional dividend payment adds a new dimension in solving the under-investment problem because it benefits stockholders directly by giving them the first claim on a part \((d)\) of uncertain cash flows. Giving a higher seniority to new debtholders benefits stockholders indirectly by lowering the cost of new borrowing. Because the cost of borrowing cannot be further reduced once debt becomes riskless, the seniority rule becomes ineffective when a new project is so safe that it makes all debt riskless. To examine whether this limitation of seniority rules can be overcome by conditional dividends, we now analyze the joint effect of allowing both conditional dividends and non-subordinated new debt. To this end, we return to the general seniority rule \((e; n)\).

Incorporating conditional dividends into the general seniority rule requires a redefinition of \(Z(d)\) as follows:

\[
Z(d) = \max[0, \min[e(X_L - d) + nY, F_1(d)]].
\]

This definition of \(Z(d)\) is similar to that of \(Z\) in (11) in that new debtholders receive the maximum between zero and the cash flow they are entitled to under seniority rule \((e; n)\) in default. The only difference between (11) and (30) is that with the conditional dividend, shareholders pay the maximum dividend allowed upon undertaking the new investment. This reduces the new debtholders claim on the existing assets by \(d\).

It can be seen that (28) remains unchanged with seniority rule \((e; n)\). Thus, we substitute (30) into (28), set \(E_1(d) = E_2(d)\), and rearrange to obtain the following indifference lines:

\[
S = -(1 - P)Y - P(I_1 + d), \quad \text{if } e(X_L - d) + nY \geq I_1, \quad (31.1)
\]

\[
S = -(1 - P)(1 - n)Y - P[eX_L + (1 - e)d], \quad \text{if } 0 \leq e(X_L - d) + nY < I_1, \quad (31.2)
\]

\[
S = -(1 - P)Y - Pd, \quad \text{if } e(X_L - d) + nY < 0. \quad (31.3)
\]

Comparing the indifference lines in (31) with those in (14) reveals that while a positive conditional dividend \((d > 0)\) does not change any of the slopes of the lines, it does decrease the intercept for each of them. This means that with a positive conditional dividend, each of the lines drawn in Figure 4 will make a downward parallel shift, which in turn means that either the over-investment incentive increases or the under-investment incentive decreases. For example, in the case of project financing where \(e = 0\) and \(n = 1\), a positive dividend causes a downward parallel shift in the bold line in Figure 4, which means the following: (1) when \(Y \leq 0\), the over-investment incentive increases; (2) when \(0 \leq Y \leq I_1\), a new over-investment incentive is created which does not exist with \(d = 0\); (3) when \(Y \geq I_1\), the under-investment problem is reduced.

Thus, the level of under-investment is decreasing in \(d\) while the over-investment is increasing in \(d\). By increasing \(d\) sufficiently, it is possible to eliminate all under-investment incentives. Since this reduction in under-investment is achieved only at the expense of exacerbating the over-investment incentive, it is
not possible to eliminate all agency costs of debt by conditioning financial contracts on dividends and seniority rules.

IV. Conclusions

In this paper we propose a model which allows for a simultaneous analysis of both over- and under-investment incentives. We demonstrate how financial contracts which may alleviate the under-investment incentive affect the over-investment incentive. This analysis of interaction between the two opposing incentives yields several interesting results. First, project financing is the most effective method in minimizing the total agency costs associated with risky debt when there is no difference in information between lenders and borrowers.

Second, when there is asymmetric information about the return attributes of a new project, giving more seniority to new debt will alleviate the under-investment problem and at the same time will exacerbate the over-investment problem. Thus, under asymmetric information, the optimal seniority rule depends on the relative importance of the two opposing effects. In particular, when over-investment considerations dominate (are dominated by) under-investment considerations, the optimal ex-ante seniority rule is to make new debt strictly subordinated (senior).

Third, our analysis of dividend constraints confirms the generally accepted notion that restricting cash distributions to stockholders will help control the under-investment incentive. However, when cash distributions are conditioned upon future investments, which Kalay (1982) shows is a common feature in debt covenants, we reach a rather surprising conclusion: allowing greater conditional cash distributions in debt covenants alleviates the under-investment incentive while exacerbating the over-investment incentive.

In a broader context, our analysis of seniority rules can be viewed as an extension of the “me-first” rules (see Fama and Miller (1972), Kim, McConnell, and Greenwood (1977), Fama (1978), and Chen and Kim (1979)). When the me-first rule is applied to debtholders, it is equivalent to an ex-ante seniority rule that all new debt be strictly subordinated to existing debt. As we have demonstrated in this paper, such an ex-ante rule is optimal only in limited cases and can exacerbate the agency problem. More important, our analysis shows that there are certain cases of informational asymmetry in which it would be optimal to have a “me-last” rule (i.e., giving strict seniority to new debtholders). In general, we believe that most cases fall in intermediate categories in which neither the me-first nor the me-last rule is optimal.

Appendix A

A. Proof of Proposition 1

Proof: (i) \( Y^*(S) \), the value of \( Y \) at which the firm is indifferent between accepting and rejecting the project, is given by rearranging (10.1), yielding

\[
Y^*(S) = -S/(1 - P).
\]
Because $0 < P < 1$, $Y^*(S) < -S$. Hence, for every $Y$ in the interval $(Y^*(S), -S)$, $E_1 - E_x = (1 - P)Y + S > (1 - P)Y^*(S) + S = 0$ but the NPV = $Y + S < 0$.

(ii) To analyze the case when $S < 0$, first consider the case of $Y \leq F_0 - X_L$. In this case, the point of indifference for stockholders is $Y^*(S) = -S/(1 - P) > -S$. Thus, for every $Y$ in the interval $(-S, Y^*(S))$, $E_1 - E_x = (1 - P)Y + S < 0$, but NPV = $Y + S > 0$.

When $Y > F_0 - X_L$, the point of indifference is $Y^*(S) = -S + P(F_0 - X_L) > -S$ because $F_0 > X_L$ by (A1). Thus, for every $Y$ in the interval $(-S, Y^*(S))$, $E_1 - E_x = Y + S - P(F_0 - X_L) < Y^*(S) + S - P(F_0 - X_L) = 0$, but NPV = $Y + S > 0$. Q.E.D.

B. Proof of Proposition 2

Proof: To prove this proposition, we have to consider the effect of seniority rules on $F_0$. This can be done, indirectly, by using the following domination criterion: Seniority rule $(e; n)$ dominates seniority rules $(e'; n')$ if, for any $F_0$ and any realization of the parameters of project $y$ at time 1, seniority rule $(e; n)$ induces the firm to:

(i) reject all negative NPV projects which would be rejected under $(e'; n')$,
(ii) accept all positive NPV projects which would be accepted under $(e'; n')$, and
(iii) reject some negative NPV projects which would be accepted under $(e'; n')$ and/or accept some positive NPV projects which would be rejected under $(e'; n')$.

Compare the $(e = 0, n = 1)$ line with the other indifference lines for which $e > 0$ and $n = 1$ in Figure 4. It is obvious that when $Y \geq I_1$ and $Y \leq -X_L$, conditions (i) and (ii) are satisfied because the lines coincide. When $-X_L < Y < I_1$, there are situations in which negative NPV projects will be accepted when $e > 0$ but rejected under $e = 0$. Hence, condition (iii) is satisfied. It is also easy to see that conditions (i) and (ii) are satisfied when $-X_L < Y < I_1$. Therefore when $n = 1$, a seniority rule with $e = 0$ dominates all seniority rules with $e > 0$.

Similarly, comparing the bold line $(e = 0; n = 1)$ with the $(e = 0; n < 1)$ line, we can easily see that conditions (i) and (ii) are satisfied and that there are situations in which positive NPV projects will be rejected when $n < 1$ but accepted when $n = 1$, satisfying condition (iii). Thus, a seniority rule with $e = 0$ and $n = 1$ dominates all seniority rules with $e = 0$ and $n < 1$.

Finally, if $e > 0$ and $n < 1$, the line with $e = 0$ and $n < 1$ will make a parallel shift to the left, intersecting with the bold line $(e = 0; n = 1)$. Thus, it is apparent that a seniority rule with $e > 0$ and $n < 1$ will lead the firm to accept some negative NPV projects which will be rejected if $e = 0$ and $n = 1$, and reject some positive NPV projects which will be accepted if $e = 0$ and $n = 1$. Thus, condition (iii) will again be satisfied. Q.E.D.
Appendix B

The Sources of Under- and Over-Investment Incentives

The sources of perverse incentives to under- and over-invest can be seen by examining the effect of new investment on the market value of old debt. With strict seniority, the post-investment market value of the old debt will be:

\[
D^1_X = \begin{cases} 
F_0, & \text{if } X_L + Y \geq F_0, \\
(1 - P)F_0, & \text{if } 0 \leq X_L + Y < F_0, \\
(1 - P)F_0, & \text{if } X_L + Y < 0. 
\end{cases}
\]  

(B1)

Using (2) in (B1) gives

\[
D^1_X = \begin{cases} 
D_X - P(X_L - F_0), & \text{if } X_L + Y \geq F_0, \\
D_X + PY, & \text{if } 0 \leq X_L + Y < F_0, \\
D_X - PX_L, & \text{if } X_L + Y < 0. 
\end{cases}
\]  

(B2)

Consider first the case where the lowest possible value of the firm is positive, i.e., $X_L + Y \geq 0$. Equation (B2) shows that the gain or loss to old debtholders, $(D^1_X - D_X)$, is equal to $-P(X_L - F_0)$ if $Y \geq F_0 - X_L$, and $PY$ if $Y \leq F_0 - X_L$. Hence, from (4), we can see that $D^1_X - D_X = -P(Z - Y)$. Recall that $P(Z - Y)$ is the extra term in (8) which gives shareholders the incentive to under- and over-invest. Thus, when $X_L + Y \geq 0$, there is a one-to-one correspondence between the wealth effects of the new investment on shareholders and old debtholders. That is, what the stockholders get in addition to the NPV comes at the expense of the old debtholders, and any portion of the NPV which does not accrue to the shareholders goes to the old debtholders. In other words, the perverse investment incentives arise because the new project engenders wealth transfers between stockholders and the old debtholders.

When the lowest possible value of the firm is negative ($X_L + Y < 0$), however, there is no one-to-one correspondence between $P(Z - Y)$ and the wealth effect on the old debtholders. Instead, we see from (B2) and (4) that there is an externality which is equal to

\[
P(Z - Y) + (D^1_X - D_X) = -P(X_L + Y) > 0.
\]  

(B3)

Thus, when $X_L + Y < 0$, the project generates an additional value that is distinct from its NPV. The new value does not represent a wealth transfer from old debtholders. Rather, it arises from the limited liability which enables stockholders and debtholders to abandon the firm when its value is negative and force the public to absorb the cost. In other words, the source of the value in (B3) is a loss to the public. Consequently, even an all-equity firm has an incentive to deviate from the NPV rule and over-invest.

To illustrate, suppose now that the firm has no existing debt (i.e., $F_0 = 0$). In that case, equations (1) and (4) change as follows:

\[
E^U_x = PX_L + (1 - P)X_H, 
\]  

(B4)

\[
Z = \max(X_L + Y, 0).
\]  

(B5)
Substituting (B4) and \( F_0 = 0 \) into (6) gives
\[
E^U_1 = E^U_x - PX_L + (1 - P)(Y + s - F_1).
\] (B6)

By substituting (5), (9), and (B5) into (B6) and rearranging, we obtain
\[
E^U_1 = \begin{cases} 
E^U_x + Y + S, & \text{if } X_L + Y \geq 0, \\
E^U_x + Y + S - P(X_L + Y), & \text{if } X_L + Y < 0. 
\end{cases}
\] (B7)

If there is no existing debt and the lowest possible value of the firm is positive \((X_L + Y > 0)\), \( E^U_1 \) consists of only \( E^U_x \) and the NPV of the new project. Thus, the NPV rule holds; a project will be accepted if its NPV is positive and will be rejected if negative.

This NPV rule does not hold for the all-equity firm if the lowest possible value of the firm is negative \((X_L + Y < 0)\). In this case, it is possible to have \( E^U_1 > E^U_x \) even though NPV = \( Y + S < 0 \). The additional term in (B7), \(-P(X_L + Y) > 0\), is equal to the sum of gains and losses to stockholders and old debtholders in (B3). Thus, this externality does not depend on the existence of risky debt. In other words, there is a non-leverage induced incentive to over-invest. This incentive arises from the limited liability which enables stockholders and new debtholders to free-ride on the public by abandoning a firm when the value of its assets is negative.

This over-investment incentive facing the all-equity firm can also be shown in Figure 2. By setting \( E^U_1 = E^U_x \) in (B7), we obtain
\[
S = -Y, \quad \text{if } Y \geq -X_L \tag{B8}
\]
\[
S = -(1 - P)Y + PX_L, \quad \text{if } Y < -X_L \tag{B9}
\]

(B8) is equal to the NPV = 0 line, and (B9) is depicted on the upper left corner of Figure 1 with a broken line. Thus, when a project falls between the NPV = 0 line and the broken line representing (B9), even an all-equity firm will undertake a negative NPV project because it increases the shareholder wealth. Such a situation will arise, of course, only when the project is very risky (e.g., \( S > X_L \) and \( Y < -X_L \)).

Note that the broken line representing (B9) gets closer to the NPV = 0 line as \( X_L \) increases. Thus, the incentive to free-ride on the public decreases as the value of the firm’s assets in place gets larger.

**Appendix C**

The Properties of the Face Value of Existing Debt \((F_0)\)

Given the amount of debt the firm needs to raise, \( D_0 \), the face value of debt is the solution to
\[ D_0 = (1 - P)F + P\mu_L, \]
where \( \mu_L \) is the expected cash flow in state \( L \), which can be written as
\[ \mu_L = (1 - r)X_L + rE_{Y/T}[\max\{0, \min\{F_0, X_L + Y\}\}], \]
where \( E_{Y/T} \) represents the expectation over \( Y \) given that the new project is taken,
and $r$ represents the probability that the new project will be taken. Since $X_L + Y < F_0$ with positive probability, $\mu_L < F_0$ and $F_0 > D_0$ (if $D_0 > X_L$ as we assume). Thus, the assumption $F_0 > X_L$ in the paper is consistent. Now, to simplify the analysis that follows, let us assume $F_0 \geq X_L + \bar{Y}$. (All the results hold without this assumption, but notations became more cumbersome). Then, $F_0$ can be written as follows:

$$F_0 = \frac{D_0}{1 - P} - \frac{P}{1 - p} [(1 - r)X_L + E_{Y/T}[r \max\{0, X_L + Y\}]] .$$

When we introduce unconditional dividends, $d$, we obtain

$$F_0(d) = \frac{D_0}{1 - P} - \frac{P}{1 - p} \mu_L(d),$$

where

$$\mu_L(d) = (1 - r(d))(X_L - d) + E_{Y/T}[r(d)\max\{0, X_L + Y - d\}] . \quad \text{(C1)}$$

Note that the probability of taking the new project, $r$, depends on $d$.

**PROPOSITION C1:** With unconditional dividends, $F_0(d)$ is increasing in $d$.

**Proof:** $F_0$ is increasing in $d$ if $\mu_L$ is decreasing in $d$. Using (C1), we can write $\mu_L(d)$ as

$$\mu_L(d) = [X_L - d + r(d)[E\{\max(0, X_L + Y - d)\} - (X_L - d)]] .$$

Consider $d, d'$ where $d > d'$, and check $\mu_L(d) - \mu_L(d')$.

Clearly, $X_L - d - (X_L - d') = d' - d < 0$. Moreover, it can be seen that the second term in the first brackets is larger than $|r(d) - r(d')|(d' - d) > d' - d$. Therefore, $\mu_L$ is decreasing in $d$ and $F_0$ is increasing in $d$. Q.E.D.

When the dividend is conditional on the new project, $\mu_L$ can be written as follows:

$$\mu_L(d) = (1 - r(d))X_L - E_{Y/T}[r(d)\max\{0, X_L + Y - d\}] . \quad \text{(C2)}$$

**PROPOSITION C2:** With conditional dividends, $F_0(d)$ is increasing in $d$.

**Proof:** Again, it is sufficient to show that $\mu_L(d)$ is decreasing in $d$. Since $r(d)$ is increasing in $d$, (C2) immediately implies this result. Q.E.D.

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