

# **COORDINATED POSITIONING AND DESIGN OF PRODUCT LINES FOR MARKETS WITH HETEROGENEOUS PREFERENCES**

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## **ABSTRACT**

Successful design and development of a line of products requires balanced consideration of technical and market tradeoffs. Quantitative methods for selecting desirable product attributes based on conjoint survey data are useful, taken on their own, for many product types. However, products with substantial engineering content involve critical tradeoffs in the ability to actually achieve those desired attributes. These technical tradeoffs must be made with an eye toward their market consequences, particularly when heterogeneous market preferences make differentiation and strategic positioning critical to capturing a range of market segments and avoiding cannibalization.

We present a methodology for product line optimization that efficiently coordinates positioning and design models to achieve realizable firm-level optima. The approach leverages prior methods where possible, overcoming several shortcomings of current positioning models by incorporating a general (Bayesian) account of consumer preference heterogeneity, alleviating known issues of combinatorial complexity, facilitating efficient parallel computing, and avoiding infeasible solutions. The method is demonstrated for a line of dial-readout scales, using physical models and web-collected consumer choice data from the literature. Results show that the optimal number of products is not necessarily equal to the number of market segments, and the representational form for consumer heterogeneity has a substantial impact on the design and profitability of the resulting optimal product line – even for the design of a single product. The method is of great comparative managerial value, as it yields product line solutions efficiently, accounting for the distribution of preferences in the marketplace as well as technical constraints on the range of achievable product solutions.

**KEYWORDS:** *Product Line Design; Analytical Target Cascading; Hierarchical Bayes; Conjoint Analysis; Discrete Choice Analysis; Constrained Optimization; Design Optimization; Preference Coordination.*

## 1. Introduction

Marketplace globalization, the proliferation of niche markets driven by heterogeneity of preferences, increased competitive pressures, and demand for differentiated and customized products have rendered the practice of isolated design and production of individual products essentially obsolete. Across industries, it is now common practice to design lines or families of product variants that reduce cost by taking advantage of economies of scale and scope, while reaching multiple market segments and achieving strategic market coverage to deter competitors. Planning for these lines of products requires care, since each product competes for market share not only with competitor products, but also with other products in the line. Current methods for quantifying the degree of expected cannibalization and the additional revenue and cost associated with differentiation offer support for planning product lines; however the scope and applicability of current methods in product line optimization is limited: Engineering approaches to product line optimization focus on the tradeoff between increased commonality among products and the resulting decreased ability to meet (usually hypothetical) exogenous performance targets for each product variant. These approaches generally lack data-driven models of market preferences for product performance attributes and, consequently, focus on reducing cost by increasing part commonality, designing platforms, or increasing modularity for mass customization.

In contrast, approaches to product line optimization in the management science and marketing literatures do not, by-and-large, address product design details that are not directly observable by consumers. These approaches typically presume that any combination of product attributes in a conjoint study can somehow be attained by engineering designers *post hoc*. While such an assumption may be well-suited for many simple or well-established products, applicability is limited for products that include even moderately complex engineering tradeoffs. Furthermore, existing approaches have not taken advantage of the many advances in econometric modeling of consumer preference heterogeneity, in particular hierarchical Bayes methods, and generally require exogenous individual-level or homogeneous segment-level preference data. A few cross-disciplinary approaches have recently emerged that design engineering products under business

and marketing objectives; however, most focus on the design of a single product. In this article, we develop a novel method that bridges existing gaps in the product line literature for designing products with engineering complexity and preference heterogeneity.

We proceed by reviewing relevant literature on product line optimization from engineering, management and marketing, as well as conjoint choice modeling, before proceeding to describe the proposed approach. Because the scopes, perspectives, modeling methods, and objectives differ substantially among product development disciplines (Krishnan and Ulrich provide a detailed overview [1]), it is inevitable that some conflicts of terminology will exist: In this article, we take product *positioning* to be the process of choosing values for physical (as opposed to perceptual) product attributes that are observed directly by the consumer, whereas product *design* involves decisions made by the engineer that are not observed directly but that have influence on the product attributes observed by the customer.

### **1.1. Product Line Optimization in Engineering**

The bulk of the engineering literature relevant to product line design focuses on studying product commonality and product platforms [2, 3]. These efforts generally focus on the tradeoff between increased commonality among products in a line and the resulting decreases in the ability to meet distinct performance targets for each product variant. Authors use the terms “product family” and “product platform” rather than “product line” to emphasize the focus on commonality among the set of products. These approaches tend to be engineering-centric and do not model market preferences using data. Instead, most focus on hypothetical targets set exogenously for each product in the family. Those that do reference marketing pose hypothetical market segments [4] or define an intuitive market segment grid by enumerating levels of product performance [5]. None of these product family methods invoke econometric models of consumer choice built on market data.

Recent efforts have attempted to link engineering optimization models to conjoint preference models; however, these approaches generally focus on a single-product [6-9]. A notable exception is Li and Azarm [10], who pose an approach that addresses design of a product line by first generating a set of designs that are Pareto-optimal in engineering performance and then

selecting a product line from that set based on a conjoint model of demand using genetic algorithms. This sequential approach can be effective for product line design cases where preferences for product attributes are strictly monotonic across the consumer population (e.g., fuel economy, reliability, price, etc.) and individual consumers vary only in their preferred tradeoffs among desirable attributes. However, it is not clear how this method might be extended for non-monotonic cases, such as the dial-readout scale examined in Section 4, where individuals have different ideal points and a single Pareto set cannot be defined common to all consumers.

Engineering approaches are generally designed around gradient-based constrained nonlinear programming techniques to handle continuous or mixed formulations with continuous and discrete variables. The focus on continuous variables increases applicability for practical engineering problems and avoids combinatorial complexity found in many positioning approaches. One difficulty with integrating models from various product development disciplines is that the combined model can be quite large and complex, causing optimization difficulties. Gu *et al.* [11] proposed a method for maintaining separate models for marketing and engineering decisions, coordinating them using the collaborative optimization (CO) technique for multidisciplinary design optimization, although they do not propose details for modeling and data collection for the marketing component. Michalek *et al.* [9] proposed a similar decomposition approach using analytical target cascading (ATC) to coordinate marketing and engineering models for a single product, assuming consumer preferences to be homogeneous. They point out a preference for the ATC approach over CO because ATC is defined for an arbitrary hierarchy of subsystems, and convergence proofs ensure coordination will lead to a solution that is optimal for the firm [12-17]. Most of the research on ATC has been applied to the design of a single product; however, Kokkolaras *et al.* [18] proposed a method using the ATC formulation to coordinate the design of a family of products. This approach is desirable, as the design of each product is handled in a separate subsystem, and the subsystems are coordinated. However, as is common in engineering approaches, Kokkolaras *et al.* use a dummy objective function for positioning and do not explicitly model market preferences for the variants.

Engineering methods for product line design are powerful, but their main drawback is that the vast majority fail to incorporate consumer demand. Those that do are exceptionally simple in comparison to prevalent approaches in marketing; for example, they ignore preference heterogeneity entirely. Hence, a product deemed ‘optimal’ may be so with respect to performance, cost or efficiency, but not in terms of profitability.

## **1.2. Product Line Optimization in Management Science and Marketing**

Table 1 summarizes major contributions to the product line optimization literature, focusing on those in marketing and management science, but including two aforementioned engineering approaches that address product lines. Among the earliest conceptualizations for product line optimization was that of Green and Krieger [19], who posed the product line *selection* problem as a binary programming problem involving selection of products from a candidate set to be included in the line in order to maximize the seller’s (or buyers’) welfare. Here the set of candidate products with their associated utility values is determined exogenously, and product demand is predicted using a *first choice* model, where each individual is assumed to choose deterministically the alternative with the highest associated utility. Variants of the original model were later proposed by Dobson and Kalish [20] and by McBride and Zufryden [21], who offer alternative integer programming techniques and heuristics for solving the problem. Dobson and Kalish [22] also introduce fixed and variable costs for each candidate product and show equivalence of their formulation to the uncapacitated plant location problem, an NP-complete formulation which they solve using greedy interchange heuristics.

While these initial methods assumed utility for each product alternative has been determined exogenously, Kohli and Sukumar [23] departed from this approach, using conjoint part-worths and introducing a single stage binary programming formulation that selects product lines directly from their product attribute levels. Chen and Hausman [24] extended prior approaches by introducing *choice-based* conjoint analysis and the multinomial logit model for predicting choice. Since the choice task is arguably most similar to that which consumers perform in practice, researchers have found it to be the best approach for extracting individual-level consumer preferences [25]. Chen and Hausman propose a binary programming formulation with properties

that allow the problem to be solved efficiently using nonlinear programming techniques. Because these properties do not hold when consumer preferences cannot be presumed homogeneous, their approach cannot be directly used for designing product lines to meet the disparate needs of most real consumer populations. Among the most recent contributions is that of Steiner and Hruschka [26], who use genetic algorithms to efficiently locate near-optimal product line designs. Other approaches have also been proposed to model products qualitatively in terms of abstract “quality levels”, although these models are primarily used to analyze structural properties, rather than offer computational decision support tools (Krishnan and Zhu provide a recent review [27]).

Despite these advances, a number of key problems remain. Specifically, current approaches to the product line problem: lack coordination with engineering terms of product feasibility; do not easily accommodate a sophisticated account of preference heterogeneity; entail substantial computational problems; and require changes from the ground up to deal with new structures and phenomena (e.g. channel structure, models of competition). Our proposed methodology resolves each of these issues, as we discuss in the following sections.

### **1.3. Conjoint Choice Models**

As is typical in marketing and econometrics, we adopt a random utility framework [28] for estimating market demand for the product line, where the utility of each product to each consumer is assumed to depend on the product’s attributes, the consumer’s idiosyncratic preferences for those attributes and a random error component. Most of the previous product line optimization formulations adopt a demand model where utility is written as a *deterministic* function of the product and consumer attributes, which offers computational benefits for rating or ranking conjoint data (e.g., [10, 19-23]). A random utility framework, however, can be expected to provide a more realistic representation of the consumer decision process, allowing as it does for uncertainties and factors unobservable to the analyst. Furthermore, a random utility model avoids the discontinuities intrinsic to a deterministic framework, allowing efficient gradient-based nonlinear programming optimization algorithms for optimal product line determination. Importantly, a random utility framework allows for explicit modeling of consumer taste distributions, or heterogeneity. As our results illustrate, the various representations available to

model taste differences can have a substantial, and substantive, impact on the final optimal product line and its profitability; this is especially so if one chooses an overly parsimonious representation. This is hardly unexpected, as accounting for differences in product preferences is of pivotal importance in product line optimization. Surprisingly, the impact of preference heterogeneity on line configuration has not received much attention in the product line literature.

#### **1.4. Proposed Methodology**

Prior approaches to product line optimization can work well for certain types of products and markets but have identifiable gaps for a wide range of real-world product lines. Engineering approaches have applied decomposition methods to efficiently coordinate the design of multiple products; however, few measure or even account for consumer preferences; indeed, the only prior method that coordinates conjoint and engineering design for product line optimization [10] requires monotonic preferences for attributes (which will, in fact, be rejected empirically in our forthcoming application). Management and marketing approaches require the assumption that all attribute combinations in the conjoint model can be achieved by some engineering design, and none take advantage of modern techniques (Bayesian and mixture model formulations) to explicitly model preference heterogeneity. Moreover, for all these approaches, combinatorial explosion limits the number of the line's products that can be studied practically.

In this article, we propose a coordinated methodology to design and position product lines. The method uses ATC to coordinate attribute selection for each of the products desired by a heterogeneous market, while ensuring they can actually be *achieved* by a set of realizable designs. In demonstrating the methodology, we compare explicitly throughout to a prior, single-product design example under homogeneous demand [9], adopting its product topology model. We can therefore not only demonstrate the superiority of the derived product line results, but also show that even for the single product case, assumption of preference heterogeneity can be problematic. The proposed approach avoids the combinatorial complexity of binary/integer formulations in the marketing literature while extending applicability to continuous or mixed formulations and avoiding the need to assume monotonic preferences. The decomposition-based ATC approach offers the organizational and computational benefits of maintaining separate subsystems for



positioning and design of each project, reducing the dimensionality of each subspace and allowing each subsystem to be efficiently solved in parallel [16, 17]. ATC also facilitates scaling up to large problems with many products.

Like Chen and Hausman [24], we invoke a number of assumptions to focus on product line optimization issues: (1) total market size is exogenously determined; (2) each customer purchases zero or one product; (3) customers do not directly influence one another; and (4) production can be scaled up or down to suit demand. As such, our formulation is well-suited to stable durables and is less appropriate for rapidly-developing product classes. Unlike prior research in the area, we invoke only mild parametric assumptions about how to represent consumer preferences.

The proposed product line design methodology entails three stages: First, consumers choose among products in a conjoint setting; second, heterogeneous demand models are estimated, with preference coefficients interpolated using splines; and third, ATC coordinates optimization over the space of feasible product designs to yield optimal product attributes. The first two stages are viewed as preprocessing for the ATC model, as shown schematically in Figure 1, with symbols rigorously defined later in the text. We proceed by defining the ATC methodology in Section 2, conditional on a model to predict demand; next we describe alternative discrete choice model specifications for demand prediction in Section 3; and finally, we demonstrate the methodology with an application to dial-readout scales using models and data from the literature in Section 4 and discuss results in Section 5.

## **2. ATC Coordination of Product Positioning and Design**

The ATC-based methodology presented here calls on established modeling traditions in engineering design and marketing. It is innovative in terms of formally coordinating them, and does not seek to “reinvent the wheel” in either case. Indeed, this modularity is among its chief strengths. ATC was conceived as a broad platform for large-scale engineering systems optimization. By viewing a system as a decomposable hierarchy of interrelated subsystems, ATC allows each subsystem to be modeled and optimized separately, coordinated, and then iteratively re-optimized to reach a system solution [29].

ATC requires a mathematical or computational model of each subsystem, and in practice these can be numerous. The modeler's task is to organize the various subsystem models into a *hierarchy*, where each element in the hierarchy represents a (sub)system that is optimized to match targets passed from the parent (super)system while setting targets that are attainable by subsystem child elements. For example, a vehicle design could be decomposed into systems such as body, chassis, and powertrain; the powertrain system could be decomposed into subsystems such as the transmission and engine; and the engine could be further decomposed into components such as the piston, crankshaft, etc. In our application of ATC, the joint product line positioning and design problem is (formally) decomposed into interrelated subsystems, which can then intercommunicate and algorithmically iterate. It is known that the iterative solution of decomposed ATC subsystems under specific coordination strategies will converge to the solution of the joint non-decomposed problem [13-17]. In the present case, market positioning and the engineering design of each product in the line can be solved separately and in parallel, producing a solution that is optimal for the joint problem [16, 17, 30]. In practice, the joint problem can be far more difficult to solve, sometimes impossibly so, owing to high dimensionality, scaling difficulties and the need for modeler expertise in all areas.

In general, ATC can accommodate an arbitrarily large hierarchy, where parent elements set targets for child elements. For example, the methodology has been demonstrated for large hierarchical systems such as vehicle design [31] and architectural design [32]. In the product line case, each design in the line could be decomposed into a set of subsystems and components, or additional marketing models could be included, say, for promotion and distribution. In this article, we address product line optimization by introducing a set of engineering design subsystems, one for each product in the line, along with a positioning formulation that sets targets for all products in the line using a heterogeneous model of demand. A schematic depiction of the process appears in Figure 2: The positioning subsystem involves determining price and (target) product attributes for the full product line to maximize a known objective function, which can be profit or some other measure of interest to the firm, while each design subsystem requires determining a feasible design – one conforming to known constraints – that exhibits product

attributes as close as possible to the targets set in the positioning subsystem. Decomposition into the ATC structure can be even more important in the product line case than in the single product case because including engineering models for the design of multiple products in a single optimization statement creates a high-dimensional, highly constrained space; by contrast, with ATC decomposition of the line, the space of each individual product design remains unchanged as new products are added to the line. The chief organizational benefit of ATC is that it segregates models by discipline: Marketers can build positioning models based on, say, conjoint analysis and new product demand forecasting; engineers can formulate models for product design and production; and other functional groups can focus on what they know how to do well. No functional area need become an expert in modeling the others, since ATC coordinates models with well-defined interfaces. The following sections lay out the design and positioning subsystems explicitly.

## 2.1. Engineering Design Subsystems

The objective of each engineering subsystem is to find a feasible design that exhibits product attributes matching the targets set during market positioning as closely as possible. Here the vector of product attributes  $\mathbf{z}_j$  for product  $j$  represents a set of objective, measurable aspects of the product, observable by the customer, resulting from engineering design decisions. In each engineering design subsystem  $j$ , search is conducted with respect to a vector of design variables  $\mathbf{x}_j$ , which represents decisions made by the designer that are not directly observable by consumers but that affect the attributes that consumers do observe  $\mathbf{z}_j$ . An engineering analysis simulation response function  $\mathbf{r}(\mathbf{x}_j)$  is used to calculate attributes  $\mathbf{z}_j^E$  as a function of  $\mathbf{x}_j$ . The design variable vector  $\mathbf{x}_j$  is restricted to feasible values by a set of constraint functions  $\mathbf{g}(\mathbf{x}_j) \leq \mathbf{0}$  and  $\mathbf{h}(\mathbf{x}_j) = \mathbf{0}$ , and so values for product attributes  $\mathbf{z}_j^E = \mathbf{r}(\mathbf{x}_j)$  are implicitly restricted to values that can be achieved by a feasible design. While construction of  $\mathbf{x}$ ,  $\mathbf{r}(\mathbf{x})$ ,  $\mathbf{g}(\mathbf{x})$  and  $\mathbf{h}(\mathbf{x})$  to represent a particular engineering design system is necessarily case-specific, general principals and guidelines are well established in the literature [33, 34]. The objective of each engineering design subsystem is to minimize deviation between the attributes achieved by engineering  $\mathbf{z}_j^E$  and the positioning targets  $\mathbf{z}_j^M$  set by the positioning subsystem, which are held constant in each engineering design

subsystem. This deviation objective originates from a relaxation of the consistency condition ( $\mathbf{z}_j^M - \mathbf{z}_j^E = \mathbf{0}$ ) by instead introducing a consistency constraint relaxation function  $\pi(\mathbf{z}_j^M - \mathbf{z}_j^E)$ . This relaxation can be handled in a variety of ways including penalty functions [12, 14, 29], Lagrangian relaxation [15] and augmented Lagrangian relaxation [13, 16, 17]. In particular, the diagonal quadratic approximation approach applied by Li *et al.* [16] produces separable subsystems and allows each design in the product line to be optimized in parallel, dramatically improving computational efficiency. The engineering optimization problem for product  $j$  can then be written as

$$\begin{aligned} & \underset{\mathbf{x}_j}{\text{minimize}} \quad \pi(\mathbf{z}_j^M - \mathbf{z}_j^E) \\ & \text{subject to} \quad \mathbf{g}(\mathbf{x}_j) \leq \mathbf{0}, \mathbf{h}(\mathbf{x}_j) = \mathbf{0}, \\ & \text{where } \mathbf{z}_j^E = \mathbf{r}(\mathbf{x}_j) \end{aligned} \quad (1)$$

## 2.2. Market Positioning Subsystem

The market positioning objective is to maximize profit  $\Pi$  with respect to the price  $p_j$  and the vector of product attribute targets  $\mathbf{z}_j^M$  for each product  $j$  in the product line  $j = \{1, 2, \dots, J\}$ . Although firms can specify arbitrarily sophisticated profit functions based on their experience, internal accounting and historical demand, we use a simple profit ( $\Pi$ ) formulation here – revenue minus cost – so that

$$\Pi = \sum_{j=1}^J \left( (p_j - c_j^V) q_j - c_j^I \right), \quad (2)$$

where  $p_j$  is the (retail) price of product  $j$ ,  $c_j^V$  is the unit variable cost of product  $j$ ,  $c_j^I$  is the investment cost for product  $j$ , which represents all costs of setting up a manufacturing line for product  $j$ , and  $q_j$  is quantity of product  $j$  sold (demand), which is a function of the product attributes  $\mathbf{z}_{j'}^M$  and price  $p_{j'}$  of all products  $j' = \{1, 2, \dots, J\}$ . We presume that product commonalities enabling investment cost sharing and improving economies of scale do not exist, so each new product design requires new manufacturing investment. In general,  $c_j^V$  and  $c_j^I$  can be considered functions of market conditions or engineering decisions, although in the example they are taken as constants. Given a demand model to calculate  $q_j$  for each product  $j$  as a function of  $\mathbf{z}_{j'}^M$  and  $p_{j'}$

for all products  $j'$ , which will be developed in the Section 3, the profit function is fully defined. The objective function also involves the consistency constraint relaxation function  $\pi(\mathbf{z}_j^M - \mathbf{z}_j^E)$  for minimizing deviation from attributes achievable by engineering  $\mathbf{z}_j^E$ , which are held constant in the positioning subsystem. Finally, the positioning subsystem for a single-producer scenario, conditional on a model for demand, is written as:

$$\begin{aligned} & \underset{\mathbf{z}_j^M, p_j; \forall j \in \{1, 2, \dots, J\}}{\text{maximize}} \quad \sum_{j=1}^J \left( (p_j - c_j^V) q_j - c_j^I - \pi(\mathbf{z}_j^M - \mathbf{z}_j^E) \right) \\ & \text{where } q_j = q_j(\mathbf{z}_{j'}^M, p_{j'}; \forall j') \end{aligned} \quad (3)$$

In Section 3, we address how conjoint analysis, discrete choice modeling and Bayesian (MCMC) methods can be used to represent the functional relationship between demand  $q$  and the variables  $\mathbf{z}^M$  and  $p$  for positioning a product line.

### 2.3. Complete ATC Formulation

Figure 2 conveys both a mathematical and verbal description of the complete formulation, showing the flow of the ATC-based product line optimization model for a single producer, where the number of products in the line  $J$  is determined through a parametric study: i.e.,  $J$  is held fixed during optimization, separate optimization solutions are found for each value of  $J = \{1, 2, \dots\}$ , and the value of  $J$  that produces the solution with the highest profit is selected. Coordination of the subsystems can be handled in a variety of ways. The most efficient method according to a recent study [16, 17] is the truncated diagonal quadratic approximation augmented Lagrangian approach. This method uses  $\pi(\mathbf{z}_j^M - \mathbf{z}_j^E) = \boldsymbol{\lambda}^T(\mathbf{z}_j^M - \mathbf{z}_j^E) + \|\mathbf{w} \bullet (\mathbf{z}_j^M - \mathbf{z}_j^E)\|_2^2$ , where  $\boldsymbol{\lambda}$  is the Lagrange multiplier vector,  $\mathbf{w}$  is a weighting coefficient vector, and  $\bullet$  is the Hadamard product (i.e.:  $(\mathbf{A} \bullet \mathbf{B})_i = \mathbf{A}_i \mathbf{B}_i$ ). The coordination procedure is:

1. Initialize all variables
2. Solve the positioning subsystem and each design subsystem in parallel
3. Update linearization of the cross component of the augmented term at the new point
4. Update  $\boldsymbol{\lambda}$  and  $\mathbf{w}$  using the method of multipliers
5. If converged, stop, else return to step 2

A review of alternative coordination methods for ATC is provided in [16, 17].

### 3. Models of Product Demand

Green and Krieger's comparative study of alternative conjoint methods for eliciting consumer preferences concluded that choice-based conjoint offers the best method for the extraction of individual-level consumer preferences [25]. We use it as follows: Respondents are presented with a series of questions or "choice sets"  $t = \{1, 2, \dots, T\}$ . In each choice set  $t$ , the respondent is presented a set of product alternatives  $j \in \mathcal{J}_t$ , with attributes set at discrete levels and systematically varied [35, 36]. The resulting data are each respondent's observed choices in each choice set:  $\Phi_{ijt}$ , where  $\Phi_{ijt} = 1$  if respondent  $i$  chooses alternative  $j$  in choice set  $t$ , and  $\Phi_{ijt} = 0$  otherwise. These data  $\{\Phi_{ijt}\}$  are then used to estimate the parameters of the choice model for the positioning subsystem, as illustrated in Figure 1.

In the random utility choice model, individuals  $i = \{1, 2, \dots, I\}$  derive from each product  $j = \{1, 2, \dots, J\}$  some utility value  $u_{ij}$  that is composed of an observable, deterministic component  $v_{ij}$  and an unobservable random error component  $\varepsilon_{ij}$ , so that  $u_{ij} = v_{ij} + \varepsilon_{ij}$ . Each individual will choose the alternative that gives rise to the highest utility (i.e., alternative  $j$  is chosen by individual  $i$  if  $u_{ij} > u_{ij'}$  for all  $j' \neq j$ ). The deterministic utility  $v_{ij}$  derived by individual  $i$  from product  $j$  is written as

$$v_{ij} = \sum_{\zeta=0}^Z \sum_{\omega=1}^{\Omega_{\zeta}} \beta_{i\zeta\omega} \delta_{j\zeta\omega}, \quad (4)$$

where the binary dummy  $\delta_{j\zeta\omega} = 1$  indicates alternative  $j$  possesses attribute  $\zeta$  at level  $\omega$ , and  $\beta_{i\zeta\omega}$  is the part-worth coefficient of attribute  $\zeta$  at level  $\omega$  for individual  $i$ , which is estimated from the conjoint choice data  $\Phi$ . In  $\delta_{j\zeta\omega}$  the elements of the product attribute vector  $\mathbf{z}_j^M$  are enumerated  $\zeta = \{1, 2, \dots, Z\}$ , and price  $p$  is included in  $\delta_{j\zeta\omega}$  and labeled as element  $\zeta = 0$ . Note that each product attribute  $\zeta$  is either intrinsically discrete or is discretized into  $\Omega_{\zeta}$  levels,  $\omega = \{1, 2, \dots, \Omega_{\zeta}\}$ ; this is crucial, as it does not presume linearity with respect to the underlying continuous variables. The probability  $P_{ij}$  that alternative  $j$  is chosen by individual  $i$  depends on the assumed error distribution. The most common distributions for  $\varepsilon_{ij}$  are the normal and double exponential, resulting in the standard probit and logit models, respectively [28]. It is well-known that very large samples are required to distinguish results produced by the logit and probit specifications

[37]. Finally, we index the “no choice option” (the outside good) as alternative 0, with error  $\varepsilon_{i0}$  and attraction value  $v_{i0}$  for individual  $i$ , where  $v_{i0} = 0; \forall i$  for identification purposes.

The representation of differences in consumer tastes, as given by  $\beta_i$ , where  $\beta_i$  contains the elements  $\beta_{i\zeta\omega}$ , can be expected to be important in product line optimization, as heterogeneity in preferences should give rise to differentiated product offerings. Failure to correctly model this heterogeneity can lead to biased parameter estimates, inaccurate predictions [38, 39] and, consequently, suboptimal product line designs. Furthermore, when heterogeneity is not adequately accounted for it is well-known that the independence from irrelevant alternatives (IIA) problem is exacerbated [40]. We therefore specify a very general continuous distributional form for  $\beta_i$  by using a mixture of normal distributions [41, 42]. The approach assumes that there are a finite number of groups or segments, in which individuals are similar – though, importantly, not identical – with respect to their preferences and tastes. To be more specific, we have

$$\beta_i \sim \sum_{b=1}^B s_b N(\theta_b, \Lambda_b), \quad (5)$$

where  $s_b$  is the fraction of the market in “segment” (or mixing component)  $b$ ;  $b = 1, \dots, B$ . Here  $\theta_b$  is the vector of means for  $\beta_i$  and  $\Lambda_b$  is a full variance-covariance matrix. This model provides a very general specification that combines both discrete and continuous heterogeneity and includes several well known heterogeneity models as special cases: (i) when  $B=1$  the well-known standard random-effects model arises, which, in combination with Bayesian estimation, enables individual-level estimates by pooling information among individuals via “shrinkage” [42]; (ii) when  $\Lambda_b = \mathbf{0}$  for all  $b = 1, \dots, B$  the standard latent class or finite mixture model arises [43], and individuals within a segment  $b$  are assumed to have identical preferences  $\theta_b$ ; and (iii) when  $\Lambda_b = \mathbf{0}$  and  $B = 1$  it is assumed that all individuals have the same preference  $\theta_1$ . The last, homogeneous case (iii) is overly simplistic, and demand models that assume homogenous tastes can be expected to perform poorly in terms individual specific part-worth recovery and market predictions. Andrews *et al.* [44] suggest that models with continuous (case i) and discrete (case ii) representations of heterogeneity recover parameter estimates and predict choices about equally well, except when the number of choices  $J$  is small, in which case the discrete heterogeneity

model (ii) outperforms the continuous model (i). One objective of the proposed research is to examine whether optimization results conditional on each of these models will also produce similar results.

For the general case, model parameters are estimated via standard Markov chain Monte Carlo (MCMC) techniques [45, 46]<sup>1</sup>. We generally specify conjugate priors, and the full conditional distributions for the MCMC sampler can be derived straightforwardly (e.g. [41, 42]). In order to choose the number of mixture components  $B$  in the mixture representation for  $\beta_i$ , we use the Deviance Information Criterion (DIC) statistic proposed by Spiegelhalter *et al.* [47]. DIC is particularly suited to complex hierarchical (Bayesian) models in which the number of parameters is “not clearly defined” [48], because the DIC statistic determines the “effective number of parameters” entailed by the model specification itself, unlike measures such as AIC.

Once the model parameters are estimated, we compute market demand for the positioning subsystem (Figure 2) in three steps: First, we generate a large set of  $\beta_i$  (say  $i = 1, \dots, I_D$ ) from the hierarchical model  $\{s_b, \theta_b, \Lambda_b\}$ , which describes the mixture distribution<sup>2</sup>. Secondly, we use *natural cubic splines* [9, 49] to interpolate  $\beta_i$  for intermediate values of product attributes and price. Specifically, natural cubic spline functions  $\Psi_{i\zeta}$  are fit through the discrete part-worth coefficients  $\beta_{i\zeta\omega}$  for each  $i$  and  $\zeta$ , where  $\omega = \{1, 2, \dots, \Omega_\zeta\}$  to interpolate the deterministic component of utility. Indexing attributes as  $\zeta = 1, \dots, Z$  and price as  $\zeta = 0$ , the interpolated value of the observable component of utility is

$$\hat{v}_{ij} = \Psi_{i0}(p_j) + \sum_{\zeta=1}^Z \Psi_{i\zeta}(\beta_{i\zeta\omega}, \mathbf{z}_{j\zeta}^M), \quad (6)$$

where  $\mathbf{z}_{j\zeta}^M$  indicates the  $\zeta^{\text{th}}$  element of the vector  $\mathbf{z}_j^M$ . These interpolated  $\hat{v}_{ij}$  give rise, through the random utility specification, to expected individual choice probabilities  $P_{ij}$ , which are computed using either a logit or a probit distribution for the errors. Finally, the individual choice probabilities are used to compute total market demand (Figure 2). The logit formulation increases

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<sup>1</sup> We are indebted to Peter Lenk for sharing both his GAUSS code and expertise.

<sup>2</sup> After estimating the model, we have a set of draws from the posterior distribution of  $\beta_{i\zeta\omega}$  for each survey respondent. One could then use this information to estimate market demand using this specific set of individuals. We take a Bayesian perspective and use the hyperparameters that describe the mixture distribution after the MCMC chain has converged, as these parameters can be viewed as giving rise to the individual-level  $\beta_i$  values. Hence, an arbitrarily large sample of new  $\beta_i$  values from this distribution can be drawn to describe the market.



optimization speed considerably, since choice probabilities can be calculated exactly and efficiently in each step of the ATC optimization, as opposed to the probit case, where choice probabilities must be approximated using numerical methods [37]. Calculating market demand for product  $j$  involves multiplying the probability  $P_{ij}$ , by the market potential  $S$  for each individual  $i=1, \dots, I_D$ , and averaging the resulting quantities across the individuals. Market potential is assumed to be exogenously determined through pre-market forecasting techniques [50].

## 4. Empirical Application

We apply the proposed methodology to design a line of dial-readout bathroom scales using engineering models and conjoint choice data from [9] for comparison. This example was originally posed for optimization of a single product using a homogeneous logit-based demand model. As we will show, even when a firm seeks to enter the market with a single product (a rarity in durable product categories) the presumption of homogeneity is troublesome.

The inherent modularity of the proposed methodology for product line design circumvents the need to build a joint model of the full product line for each case  $J = \{1, 2, \dots\}$ . Instead, a model of only a single product need be developed, and a duplicate can be created for each product  $j$  constituting the line<sup>3</sup>, as illustrated in Figure 2. It is important to note that the design space  $\mathbf{x}$  for this product does *not* map one-to-one with the attributes  $\mathbf{z}$  communicated to consumers. This comes about because the engineering design model specifies some product attributes as functions of interactions among design variables; that is, different designs may exhibit identical product attributes, as observed by the customer. A manager could enact any number of criteria *post hoc* to choose from among such a continuum, or detailed cost data and preferences for commonality could drive selection of a single engineering design among the set of possibilities, although we do not pursue such strategies here.

The product attributes  $\mathbf{z}$  seen by consumers are weight capacity  $z_1$ , aspect ratio  $z_2$ , platform area  $z_3$ , tick mark gap  $z_4$ , and number size  $z_5$ , in addition to price  $p$ . For the conjoint study, the range of values for each attribute was captured by five (discrete) levels. Each respondent ( $n =$

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<sup>3</sup> All models and results are available from the authors upon request.

184) made choices from 50 consecutive sets in a choice-based conjoint task, identical across respondents, each with three options (plus “no choice”), implemented through a web browser. With the ATC approach, it is not necessary, nor practical to pre-restrict choice sets to include only realizable products. The goal of the conjoint task is the effective and unbiased measurement of consumer preference drivers. Infeasible combinations of product attributes are implicitly avoided during optimization through coordination with the engineering design subsystem.

The demand/profit function requires (exogenous) estimates of several quantities, which are based here on manufacturer and publicly-available figures:  $c_j^V = \$3$  cost per unit,  $c_j^I = \$3$  million for initial investment, and market size  $S = 5$  million, the approximate yearly US dial-readout scale market. Being completely exogenous, these values are easily altered. The entire marketing subproblem is formulated as in Eq.(3), with the demand model specified in Eq.(4)-(6). The special cases of discrete mixture ( $\Lambda=\mathbf{0}$ ) and homogeneous ( $B=1, \Lambda=\mathbf{0}$ ) models are straightforward to estimate using maximum likelihood techniques [28]. For the mixture of normal distributions, estimates from a classical mixture of probits were used as starting values, and the Gibbs sampler was iterated until a stationary posterior was obtained. To mitigate autocorrelation, the data were thinned by retaining every 10<sup>th</sup> draw, after a burn-in of 50,000 iterations. Convergence was examined through iteration plots. Posterior marginals revealed no systematic convergence problems. In order to optimize over this posterior surface, Monte Carlo integration was applied, as follows: When the chain has stabilized, *new* values of  $\beta_i$  are generated as the chain continues to run, allowing hyperparameters to vary across the generated values. These are thinned to reduce serial correlations; specifically, 10,000 values are generated, and every 10<sup>th</sup> is retained. The resulting set of 1000  $\beta_i$  draws, with splines fit through the part-worth attribute levels of each draw, is used to represent the population (the posterior surface) throughout the optimization. Accuracy can be enhanced, if need be, by generating additional  $\beta_i$  values, although in the case study, tests of solution sensitivity to additional draws (up to 24,000) show that 1000 draws is sufficient to represent the surface.

## 5. Results

There are two main components to the approach advocated here: 1) econometric, for the extraction of individual-level preferences and generation of the preference splines, and 2) optimization-based, for the determination of the best number of products, their positioning and design conditional on the preference splines. We look at these in turn.

### 5.1. Demand Model Results

Table 2 lists DIC results for the normal mixture model and BIC results for the discrete mixture and homogeneous cases as well as classical log-likelihood values for reference. The latent class model identified by BIC consists of seven segments, while the mixture model with a diagonally-restricted covariance matrix identified by DIC has three mixing components, and the full-covariance mixture model has two. It is apparent that: (1) continuous heterogeneity (normal mixture) alone is superior to discrete heterogeneity (latent class) alone, up through a fairly large number of segments [42]; (2) a correlated (random) coefficients specification for the normal mixture is superior to an uncorrelated one; and (3) more than one segment in the normal mixture model is supported. In short, the most general specification fares best, and each of its attributes – correlated coefficients, and both discrete and continuous heterogeneity – is useful in accurately representing consumer preferences. In the following sections, we will refer primarily to this full model, calling on others peripherally for comparison purposes.

For illustration and a “reality check” we briefly examine the posterior means of part-worth coefficient vectors,  $\beta_i$ . The resulting splines are shown graphically in Figure 3, along with analogous splines for the discrete mixture and homogeneous cases. Recall that for identification purposes these values are scaled so that the sum in each set of attributes is zero, making for easier visual comparison. In each of the six attribute spline graphs the heterogeneous model is most “arched” or highly sloped, suggesting the presence of some consumers with relatively strong preference differentials across attribute levels. Of course, part-worth values have a nonlinear mapping onto choice probabilities, so an “averaged part-worth” is only a rough guide to comparing across heterogeneity specifications.

Although it is not our main focus here, a number of trends are apparent across these mean estimated coefficient values. Unsurprisingly, price appears to exert the strongest influence, and is decisively downward-sloping (this is true of the posterior means for each of the  $n = 184$  original participants). One might have expected similarly monotonic preferences for number size and weight capacity, but this is only true for the former; apparently, too high a capacity was viewed as “suboptimal” by the respondents, on average. Note that these  $\beta_i$  values reflect pure consumer preference, and *not* any sort of constraint resulting from infeasible designs, which can only arise from the engineering design subsystem. Preferences for the other three variables (platform area, aspect ratio (i.e., shape) and interval mark gap) all have interior maxima.

## 5.2. Product Line Optimization Results

Conditional on the generated splines arising from the HB conjoint estimates (using the full normal mixture model), the design and positioning subsystems are solved iteratively until convergence. Optimization was carried out with each subsystem solved using sequential quadratic programming. The ATC hierarchy is solved for a fixed product line size  $J$ , and a parametric study is performed to determine the value of  $J$  that produces the most profitable overall product line. As is typical, local optima are generated, and global optima are sought using multi-start. Figure 4 shows the best resulting profit across several local minima found using ten runs with random starting points for each case  $J = \{1, 2, \dots, 7\}$ . It is clear that a product line with four products is most profitable. Table 3 shows the resulting product attributes for each heterogeneity specification. Several of the resulting scale designs are bounded by active engineering design constraints; this is necessary to ensure that the scale is physically tenable, e.g., that the dial, spring plate and levers fit in the case. Note as well that all the scales in the line lie well within the range available through online retailers, although resulting prices migrate to the upper bound due to the single-producer scenario. Looking across the table, and considering primarily marketing attributes, we might term the resulting products “large high-capacity, small-numbered square scale” (27.4% of the market), “large-number portrait scale” (21.0%), “small, low-capacity landscape scale” (18.6%) and “high-priced, middle-of-the-road” scale (11.3%). Note that these do

not add to 100%, given the presence of the “no choice” option, which allows some portion of the potential market to prefer no scale at all to any of the four in the final line configuration.

#### **5.2.1. Effectiveness of ATC Coordination**

A major contribution of the methodology presented here is to provide rigorous coordination between positioning and design models for a product line to find a joint solution that is optimal under consideration of both heterogeneous customer preferences and engineering feasibility. To demonstrate the importance of this coordination, the ATC solution was compared to the solution obtained through a disjoint sequential approach, which has been referred to as analytical target setting [9, 51]. In the disjoint scenario, price and product attribute positioning targets are set based on consumer preference data without engineering feasibility information (the positioning subsystem), and these are passed to engineering design teams. Each engineering team then designs a feasible product that meets its targets as closely as possible (the engineering subsystem) without further iteration.

In this disjoint scenario, marketing produces a plan for a line of four scales with a predicted market share of 83.4% and resulting profit of \$81.2 million. There is no reason to believe these products will be feasible, as they are based on consumer preferences alone. In the disjoint case, these (unachievable) targets are passed to engineering teams who each design a feasible product to achieve product attributes as close as possible to the targets requested by marketing without further iteration. The resulting products differ significantly from the initial plan and so have attributes less preferred by consumers, resulting in 70.5% market share and \$67.9 million profit: 16% less than marketing’s original (unachievable) prediction. If ATC is instead used to iteratively coordinate positioning and design, the resulting joint solution is a line of four different products, resulting in 78.2% market share and \$72.4 million profit. In this case, coordination resulted in a feasible product line with a predicted 6% improvement in profitability relative to disjoint decision-making. In the disjoint scenario marketing “leads” by developing the original plan and engineering design “follows” by attempting to meet product attribute targets. The reverse situation, where engineering “leads”, is possible when all consumers have monotonic preferences for product attributes by first designing a set of products that are Pareto-optimal in performance

and then allowing marketing to pick a line from that set of products [10]. However, in this example, preferences for attributes are non-monotonic, so no such common Pareto set exists, and without preference information, engineering design has no single well-defined optimization objective.

### 5.2.2. Heterogeneity Representation

It is critical to assess what impact the heterogeneity specification has on the resulting optimal solution, and whether simpler forms of heterogeneity might have sufficed for optimal line design. The simple homogeneous demand model is ill-suited for generating product lines, as the IIA property leads to lines with duplicate products. So, we compare the discrete mixture (latent class) model with the normal mixture model. Because the discrete mixture model is natively supported in many statistical packages, it might prove convenient for line optimization. Though fit statistics (Table 2) alone argue that the discrete mixture model is dramatically inferior to the normal mixture specification, this does not necessarily mean that, *conditional* on the resulting estimates, the resulting optimal line will be similarly inferior. Table 3 lists a comparison between the resulting profitability (evaluated *post hoc* with the full normal mixture model) of the best locally-optimal solutions found using the discrete and continuous mixture demand models over ten multi-start runs with random starting points for each value of  $J$ . Not only do the different heterogeneity specifications result in different product line solutions (a line of six products under the discrete mixture model vs. four under the normal mixture model), but the former suffers a profit decrement of 18.4%<sup>4</sup>. Furthermore, because the discrete mixture specification models all individuals within a segment as having identical preferences, the remaining within-segment IIA property can result in solutions with duplicate products, such as the one reported in Table 3. It is important to note that such solutions are artifacts of the econometric model and may be difficult to interpret for managerial use. For example, simply taking the solution resulting from the model and eliminating product duplicates to create a line of four products will not, in general, produce a locally optimal solution in the reduced space. Furthermore, the within-segment homogeneity of

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<sup>4</sup> The profit of the line generated under the discrete mixture model was evaluated for profitability post hoc using the normal mixture model, which offers better fit and generalizes all other models considered here.

preferences results in a profit surface containing pronounced local minima, which impedes the optimization process and makes global search difficult. Thus, even a relatively sophisticated heterogeneity representation can offer very different, and potentially sub-optimal, product line results.

While it may be unsurprising that simpler heterogeneity representations can lead to suboptimal product lines, it is less obvious whether a homogeneous model is sufficient for the design of a *single* product (as assumed, for example, in [9]). Our analysis strongly suggests that it is not. Table 3 lists single-product solutions under the three demand model scenarios. Although in this case the more restrictive models do a fairly good job predicting some of the optimal product attributes, this is not so for price, which is notably exaggerated (relative to the normal mixture model), resulting in a solution with a loss of 7% market share using the discrete mixture model and 14% using the homogeneous one. These results make sense because continuous heterogeneity allows for some consumers that are highly price sensitive, so that a single price need be lower to avoid losing them entirely. Simply put, even when making a ‘one size fits all’ product, a manufacturer should not presume that everyone is really the same. It had not initially been anticipated, based on any prior literature of which we are aware, that preference heterogeneity would be so important when only a single product is being produced. We believe that the issue of how heterogeneity specification affects contingent optimization results is worthy of further study on its own.

## 6. Conclusions

Firms work to position and design lines of products that best suit their market and profitability goals. Different functional entities within the firm can interpret this imperative idiosyncratically: measuring customer preferences and strategically positioning new products for marketers; maximizing performance under technological constraints for engineers. Considered independently, these goals often lead to conflict, both in practice and with respect to optimization models in each discipline, and disjoint sequential “throw it over the wall” approaches to resolving the conflict can lead to suboptimal decision-making. The proposed approach to product line design draws on previous research in product line optimization, analytical target cascading,

econometric discrete choice analysis, preference heterogeneity and hierarchical Bayesian techniques to offer a method for coordinating the profit-driven design and positioning of the line by modeling each separately in different subsystems and coordinating them via ATC. The resulting product line is a solution to the joint problem, which produces demonstrably better results than a simple sequential approach. The separation of the subsystems is advantageous both for organizational purposes, since each modeling group can focus on what it knows best and need not be an expert in all areas, and for computational purposes, since the individual subsystems can be solved in parallel within low dimensional spaces and with fewer constraints as compared to solving the full system without decomposition. This is a particularly important feature for product line design, since the coupling between products is relatively weak (primarily cannibalization calculations), while the coupling among variables within a product is strong. Iterative coordination of these decision-models then acts to reduce the need for more costly human iteration. Thus the product line problem is well suited for ATC decomposition, facilitating scalability of the problem to complex products, which can be represented as (sub)hierarchies themselves, or to large numbers of products, by simply adding more subsystems. The intrinsic modularity of the approach also readily accommodates additions, variations, and extensions. The combinatorial complexity associated with search over discrete levels in previous marketing approaches to product line design is reduced here because search occurs over a continuous domain, and efficient gradient-based algorithms can be called upon. The approach also does not require the set of attributes of interest to have monotonic preferences in the population, and so it can be useful for addressing products such as the dial-readout scale example.

Comparing solutions achieved under different heterogeneity specifications (based on the same choice data) demonstrates that the form of heterogeneity chosen by the modeler can have non-trivial impact on the optimal solution obtained. This suggests that a suitably general heterogeneity specification should be used, where possible, for product line optimization. In our case, the full-covariance finite normal mixture model was superior to the homogeneous and discrete mixture representations at representing underlying preferences, and use of a more restrictive model can lead to different solutions with substantial reductions in profitability. Importantly, the form of



heterogeneity also affects the optimal solution for the *single product solution*. Contrary to presumptions that homogeneous models might suffice for designing a single product, our application plainly suggests that accounting for preference heterogeneity can be critical.

In closing, several maxims are relevant for management, marketing and engineering design communities. First, although engineers are keenly aware, at every step of their work, of real, inviolable constraints, marketers tend to work to find desirable product attribute targets for exploring new markets. A tacit belief is that most, if not all, design constraints can be vanquished by ingenuity or sufficient capital. While this is sometimes true, often it is not. ATC encodes non-negotiable technological infeasibilities directly into its conceptual foundations. As such, marketers using their own models within an overarching ATC formulation can gain a gut feel for what will work, and what will not, in terms of actual, deliverable products, to supplement their intuitive understanding of the consumer marketplace. The flip side is that engineers can come to terms with the “consumer space”, every bit as real as the geometry and physics underlying their own models, and resolve tradeoffs among competing performance goals through coordination with marketing. Second, while it may appear simple to specify directly which product attribute combinations cannot co-exist, in practice it is often impractical: This infeasible hull can snake through the product attribute space in ways difficult to visualize or translate into meaningful consumer terms. ATC frees marketers from considering such issues when collecting consumer preferences; iterative coordination readily avoids infeasible product line configurations implicitly. Third, heterogeneity matters: it must be accounted for in sufficient generality, even for the design of a single product.

And finally, the question arises whether our proposed approach can be trusted alongside mainstays like conjoint analysis and discrete choice modeling to aid in product line design. The underlying ATC framework is proven, for a broad class of problems, to converge to joint optimality across its various subsystems. As such, it can literally guarantee better profitability, as in our application, than possible by sequentially optimizing the design and positioning subsystems. Given its scalability, efficiency, and ability to key into a wide variety of extant

modeling techniques, we hope to see this framework widely adopted as a cross-disciplinary platform for the design of complex product lines.

## FIGURES AND TABLES

Figure 1 Diagram of the modeling process

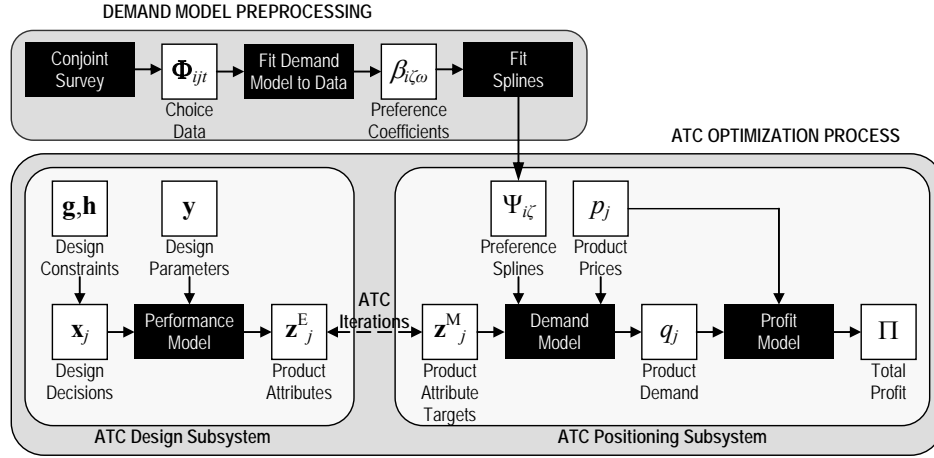
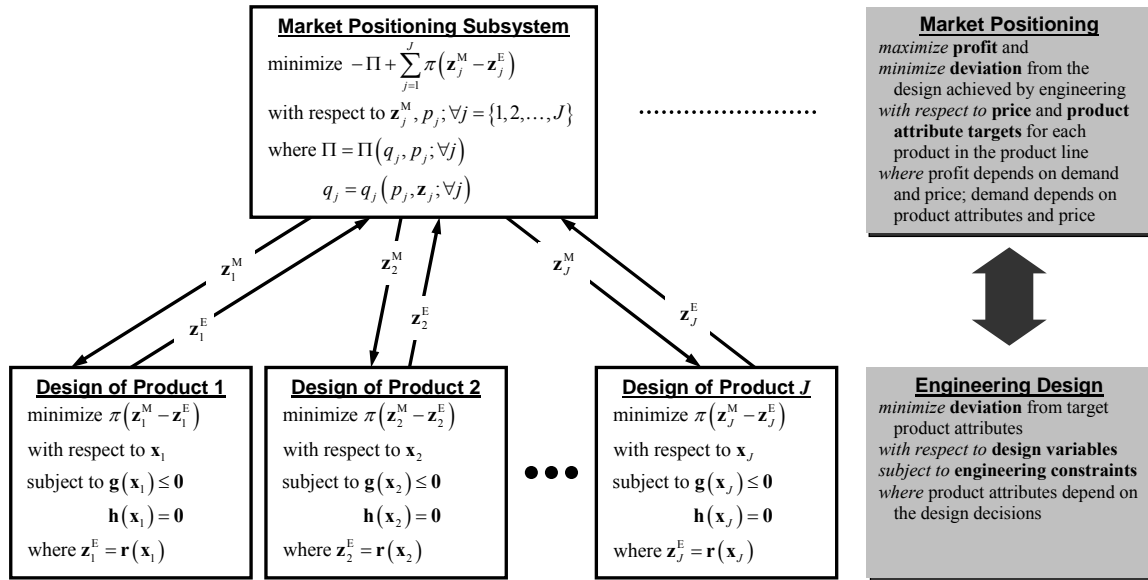
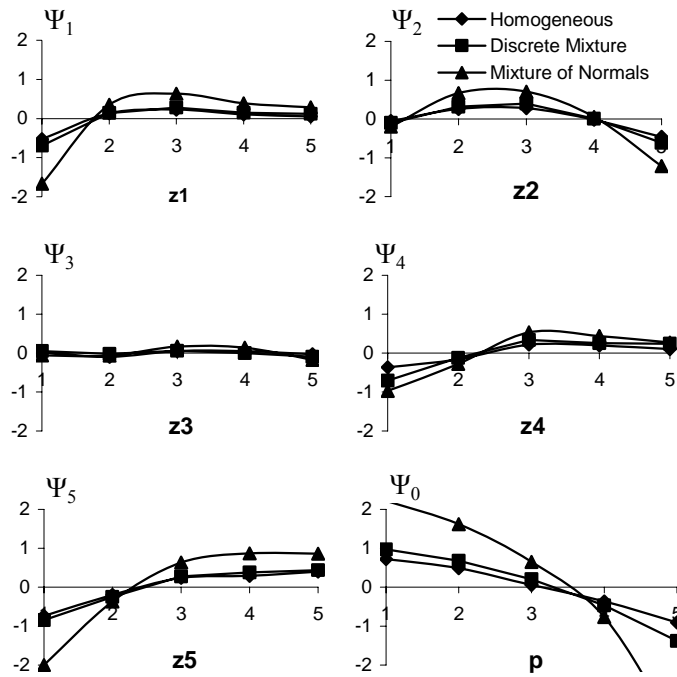


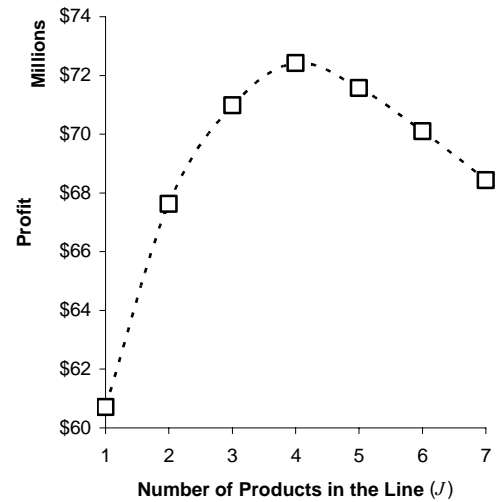
Figure 2 ATC Formulation of the Product Planning and Engineering Design Product Development Problem



**Figure 3 Plots of the average splines for each product attribute and price under the three demand models**



**Figure 4 Resulting Profit as a Function of the Number of Products in the Line**



**Table 1: A Summary of the Product Line Optimization Literature**

|                                 | Green and Krieger (1985) | McBride and Zufryden (1988) | Dobson and Kalish (1988) | Kohli and Sukumar (1990) | Dobson and Kalish (1993) | Chen and Hausman (2000) | Steiner and Hruschka (2002) | Li and Azarm (2002) | Kokkolaras et al. (2002) | Proposed Method |
|---------------------------------|--------------------------|-----------------------------|--------------------------|--------------------------|--------------------------|-------------------------|-----------------------------|---------------------|--------------------------|-----------------|
| <b>Product Selection</b>        | B                        | B                           | B                        | -                        | B                        | B                       | -                           | B                   | -                        | -               |
| <b>Product Attributes</b>       | -                        | -                           | -                        | B                        | -                        | -                       | B                           | M                   | -                        | M               |
| <b>Product Design</b>           | -                        | -                           | -                        | -                        | -                        | -                       | -                           | M                   | C                        | M               |
| <b>Preference Elicitation</b>   | -                        | -                           | -                        | CA                       | -                        | CBC                     | CBC                         | CA                  | -                        | CBC             |
| <b>Preference Model</b>         | FC                       | FC                          | FC                       | FC                       | FC                       | L                       | L                           | FC                  | -                        | L/P             |
| <b>Preference Heterogeneity</b> | -                        | -                           | -                        | -                        | -                        | -                       | -                           | -                   | -                        | BNM/LC          |
| <b>Solution Algorithm</b>       | H                        | H                           | H                        | H                        | H                        | NLP                     | GA                          | GA                  | ATC                      | ATC             |

Key: B=binary; C=continuous; M=mixed; CA=conjoint; CBC=choice-based conjoint; FC=first choice; L=logit; P=probit; LC=latent class  
BNM=Bayesian normal mixture; H=heuristics; NLP=nonlinear programming; GA=genetic algorithms; ATC=target cascading

**Table 2: Comparison of Heterogeneity Specifications: Discrete Latent Class vs. HB Random Parameters**

|                                  |      | B | Λ | LL           | BIC          |  |   | B    | Λ            | LL*          | DIC |
|----------------------------------|------|---|---|--------------|--------------|--|---|------|--------------|--------------|-----|
| Latent Class<br>Discrete Mixture | Homo | 1 | 0 | -10983       | 22194        | Hierarchical Bayes<br>Continuous Mixture | 1 | Diag | -3813        | 12432        |     |
|                                  |      | 2 | 0 | -10239       | 20944        |  | 2 | Diag | -3713        | 12073        |     |
|                                  |      | 3 | 0 | -9784        | 20271        |  | 3 | Diag | <b>-3656</b> | <b>11961</b> |     |
|                                  |      | 4 | 0 | -9537        | 20014        |  | 4 | Diag | -3638        | 12029        |     |
|                                  |      | 5 | 0 | -9336        | 19850        |  |   |      |              |              |     |
|                                  |      | 6 | 0 | -9187        | 19788        |  | 1 | Full | -4051        | 11742        |     |
|                                  |      | 7 | 0 | <b>-9059</b> | <b>19770</b> |  | 2 | Full | <b>-4016</b> | <b>11674</b> |     |
|                                  |      | 8 | 0 | -8948        | 19785        |  | 3 | Full | -4017        | 11745        |     |

\* Classical LL for the HB models was evaluated using posterior means as plug-in values and is included only for informal comparison to the Latent Class models.

**Table 3: Optimal Product Line Solutions under Each Demand Specification**

|       |                    |                 | Single Product Solutions |                     |                   | Product Line Solutions          |         |         |         |         |         |                |         |         |         |
|-------|--------------------|-----------------|--------------------------|---------------------|-------------------|---------------------------------|---------|---------|---------|---------|---------|----------------|---------|---------|---------|
|       |                    |                 | Homo-<br>geneous         | Discrete<br>Mixture | Normal<br>Mixture | Discrete Mixture (Latent Class) |         |         |         |         |         | Normal Mixture |         |         |         |
|       |                    |                 |                          |                     |                   | 1                               | 2       | 3       | 4       | 5       | 6       | 1              | 2       | 3       | 4       |
| Π     | Profit (Millions)* | \$              | \$54.10                  | \$58.30             | \$60.70           | \$59.10                         |         |         |         |         |         | \$72.40        |         |         |         |
|       | Market share*      | %               | 48.80%                   | 57.80%              | 65.00%            | 25.10%                          | 8.70%   | 8.70%   | 8.70%   | 6.90%   | 4.90%   | 27.40%         | 21.00%  | 18.60%  | 11.30%  |
| $z_1$ | Weight capacity    | lbs.            | 255                      | 254                 | 256               | 238                             | 257     | 257     | 257     | 253     | 248     | 292            | 262     | 200     | 255     |
| $z_2$ | Aspect ratio       | -               | 0.996                    | 1.047               | 1.002             | 1.045                           | 1.041   | 1.039   | 1.039   | 1.062   | 1.051   | 0.98           | 1.156   | 0.921   | 0.986   |
| $z_3$ | Platform area      | in <sup>2</sup> | 134                      | 127                 | 130               | 100                             | 131     | 131     | 131     | 123     | 114     | 140            | 122     | 105     | 135     |
| $z_4$ | Tick mark gap      | in.             | 0.116                    | 0.117               | 0.115             | 0.106                           | 0.116   | 0.116   | 0.116   | 0.114   | 0.111   | 0.103          | 0.116   | 0.121   | 0.116   |
| $z_5$ | Number size        | in.             | 1.334                    | 1.339               | 1.315             | 1.193                           | 1.341   | 1.337   | 1.337   | 1.316   | 1.268   | 1.221          | 1.351   | 1.293   | 1.331   |
| $p$   | Price              | \$              | \$26.41                  | \$24.21             | \$22.61           | \$23.96                         | \$30.00 | \$30.00 | \$30.00 | \$30.00 | \$29.37 | \$22.89        | \$24.53 | \$23.84 | \$30.00 |

\* as calculated post-hoc using the normal mixture demand model

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