

# The Shape of Advertising Response Functions Revisited: A Model of Dynamic Probabilistic Thresholds

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Prior work in marketing has suggested that advertising *threshold effects*—levels beneath which there is essentially no sales response—are rarely encountered in practice. Because advertising policies settle into effective ranges through early trial and error, thresholds cannot be observed directly, and arguments for their existence must be based primarily on a “statistical footprint,” that is, on relative fits of a range of model types. To detect possible threshold effects, we formulate a switching regression model with two “regimes,” in only one of which advertising is effective. Mediating the switch between the two regimes is a logistic function of category-specific dynamic variables (e.g., order of entry, time in market, number of competitors) and advertising levels, nesting a variety of alternative formulations, among them both standard concave and S-shaped responses. A sequence of comparisons among parametrically related models strongly suggests: that threshold effects exist; that market share response to advertising is not necessarily globally concave; that superior fit cannot be attributed to model flexibility alone; and that dynamic, environmental, competitive, and brand-specific factors can influence advertising effectiveness. These effects are evident in two evolving durables categories (SUVs and minivans), although not in the one mature, nondurable category (liquid detergent) studied.

*Key words:* advertising; econometric models; product management; switching regression; dynamic models

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## Introduction

Advertising threshold effects, those observed when “some positive amount of advertising is necessary before any sales impact can be detected” (Hanssens et al. 2001, p. 113), have received a good deal of attention in the marketing literature (Lambin 1976, Bemmaor 1984). Although managers and advertising practitioners often profess some degree of belief in advertising threshold effects (Corkindale and Newall 1978, Ambler 1996), empirical evidence for their existence has been limited (Hanssens et al. 2001, Vakratsas and Ambler 1999). This raises the question of whether advertising thresholds exist and, if so, why they have proved difficult to observe or measure.

Several lines of argument have been developed to account for this paucity of evidence. The first, stemming from Bemmaor (1984), is that much of the

evidence “rejecting” the existence of a threshold effect is based on the adequate fit of a globally concave advertising response function (e.g., a double-log advertising response function, as in Lambin 1976), which implies decreasing returns to advertising, as opposed to the overt rejection of, say, an S-shaped response function. A second argument (Bronnenberg 1998, Simon and Arndt 1980, Steiner 1987, Vakratsas and Ambler 1999) proposes that most studies of advertising response have concerned themselves with frequently purchased, mature product categories, where the competitive environment is stable, advertising budgets are set, and the operating range of advertising expenditures for brand managers lies above the threshold point.

By contrast, given the situation typifying new-brand introductions, especially in emerging product

categories—a dynamic competitive environment, threats of competitive entry, active awareness building, high consumer uncertainty, and low expertise—thresholds may stand, relatively speaking, in bold relief. To this end, we develop a dynamic model of advertising response that encodes thresholds in a (latent) probabilistic manner in the style of Bemmaor (1984), but includes the type of dynamic decision and environmental variables considered in numerous prior studies (cf. Bowman and Gatignon 1996). In contrast to those studies, we apply the model to data on two evolving, durable categories as well as to one frequently purchased, nondurable one.

The model to be developed allows formal tests for the existence of threshold effects, for whether they are dynamic (time-varying), and for whether they vary across brands, in precisely the type of situation in which they are most likely to be detected. While several of these goals have been addressed by prior research, there has often been a presumption that threshold effects are either static, nonexistent, or unimportant, vantage points that the present study seeks to redress.

## Model Formulation and Discussion of Conjectures

As suggested by Bemmaor (1984), advertising threshold effects, as well as a method for their estimation, may be detected by recourse to a mixture of two concave advertising response functions; that is, a *two-regime* market share model. In one regime, where one operates below the critical threshold, advertising effects are negligible, whereas in the second regime advertising is (measurably) effective so that expenditures and share can be related through any of a number of monotonic functional forms. We call these, respectively, the “ineffective” and “effective” regimes. In real markets, demarcation points between the two regimes are difficult to pin down, let alone directly observe. Lack of direct threshold observability can be accommodated through a (latent) probabilistic mechanism, while the market share model in each of the two regimes takes a multiplicative form; prior research has confirmed the multiplicative specification to perform essentially as well as attraction models in accounting for market share effects (Brodie and de Kluyver 1984, Ghosh et al. 1984). We have deliberately called upon model formulations that have been widely validated empirically (such as the multiplicative and that of Bemmaor), and any claim to methodological novelty lies in their combination and joint estimation in a dynamic context.

If advertising thresholds indeed exist, the likelihood or “probability” of shift between the ineffective and effective regimes should clearly depend on advertising expenditure levels. We highlight the term

“probability” because it is important to note that the two-regime model operates in a manner similar to, for example, mixture models: While the model is itself *formally* probabilistic, the nature of response is not assumed to be.

Specifically, the model takes the following form:

$$\text{Regime 1, "ineffective:"} \quad M_{1it} = e^{K_1} D_{it}^{\alpha_D} P_{it}^{\alpha_P} E_i^{\alpha_E} T_{it}^{\alpha_T} e^{u_{1it}} \\ \text{with probability } p_{it} \quad (1)$$

$$\text{Regime 2, "effective:"} \quad M_{2it} = e^{K_2} A_{it}^{\beta_A} D_{it}^{\beta_D} P_{it}^{\beta_P} E_i^{\beta_E} T_{it}^{\beta_T} e^{u_{2it}} \\ \text{with probability } 1 - p_{it},$$

where, for brand  $i$  at time  $t$ :

- $M_{it}$  = Market share
- $K_r$  = Intercept (constant) term for regime  $r$
- $A_{it}$  = Advertising expenditures
- $D_{it}$  = Distribution (i.e., number of outlets)
- $P_{it}$  = Price
- $E_i$  = Order of entry
- $T_{it}$  = Time in market
- $u_{rit}$  = Error term for regime  $r$ ,  $N[0, \sigma_r^2]$ .

Note that the specification for the two regimes is identical up to estimated coefficients once that for Advertising ( $A_{it}$ ) is set to zero for the “ineffective” regime, as it must be by definition.<sup>1</sup>

Although the share model for each regime implies diminishing returns to advertising spending, the probabilistic nature of the switching mechanism suggests a region where returns to advertising are *increasing*, with location depending on  $p_{it}$ .<sup>2</sup> The market share formulation (1) does not address  $p$ , the probability of falling into the ineffective regime, which can take a variety of forms. For example,  $p$  might lack a direct functional relationship to any quantities of managerial interest, and so would be zero (so that there is no ineffective regime) or a nonzero constant (either estimated or prespecified). Prior models have invoked such restrictions on  $p$ , and we will test the proposed model against them.

Aside from advertising itself, several specific factors—number of competitors, order of entry, and

<sup>1</sup> “Time in Market” for any particular brand is perfectly collinear with time, and so captures temporal trends on a brand-by-brand basis. Brand-specific constants have not been estimated, as there is an overall regime-specific constant and both brand-specific order-of-entry and time-in-market values. Bowman and Gatignon (1996), who used an equivalent formulation, discuss the rationale for doing so (p. 236).

<sup>2</sup> We refer to  $p_{it}$  as  $p$  where no ambiguity arises. Although this may seem equivalent to an S-shaped response function, it is considerably more flexible. It is well established (Little 1979, Mahajan and Muller 1986, Feichtinger et al. 1994, Feinberg 2001) that advertising threshold effects influence the shape of the advertising response function, and thereby affect the qualitative nature of the optimal advertising policy.

time in market—can enhance or diminish advertising expenditure effects. In the present framework they can be taken to affect advertising thresholds via the regime-switching probability,  $p$ . For the purposes of estimation,  $p_{it}$  is expressed as a logistic function of the relevant input variables:

$$p_{it} = (1 + \exp[-(c_0 + c_\alpha \alpha_{it} + c_\tau \tau_{it} + c_N N_{it} + c_E E_i)])^{-1} \quad (2)$$

where, at time  $t$  and for brand  $i$ ,  $\alpha_{it} = \log(A_{it})$ ,  $N_{it}$  = number of competitors, and  $\tau_{it} = \log(T_{it})$ .<sup>3</sup> The system (1)–(2) incorporates dynamic elasticities in much the same manner as that suggested by Bowman and Gatignon (1996), only in our case advertising elasticities are affected by control variables (order of entry, etc.) in a potentially nonmonotonic manner.<sup>4</sup>

In sum, the proposed model allows one to model and test for the existence of dynamic, brand-specific advertising thresholds, as well as to obtain advertising elasticities which account for brand, category, and competitive dynamics.

### Comparison with Extant Model Formulations

The proposed *probabilistic* thresholds model of (1)–(2) reduces to Bemmaor’s (1984) model, when  $p_{it}$  does not admit of explanatory variables. A relevant restricted subcase has  $p = 0$ , a single regime with globally concave response to all input variables. This “classical” specification is useful as a benchmark, to help gauge the additional explanatory power of a second, ineffective regime, irrespective of whether  $p$  admits of decision variables (as in the proposed model) or not (as in Bemmaor’s). Another important special case is one for which the regime-switching probability  $p$  is not constant, but follows a distribution with stable, estimated parameters, as in the one-dimensional cutoff (grid search) model:

$$p_{it} = \begin{cases} 1 & \text{if } A_{it} > \bar{A}, \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

where  $\bar{A}$  is the advertising threshold. The cutoff model presumes that although there is indeed a switch between the two regimes, it is static, deterministic, homogeneous (across brands), single-attribute/unidimensional, and that one operates with *perfect certainty* in either the effective or ineffective regime.

<sup>3</sup> Here we use the log form for advertising expenditure and time in market due to the wide range of these variables, to avoid claiming an overly good fit due to outlier leverage. Alternate runs based on using the nonlogged forms yielded appreciably inferior results in this regard.

<sup>4</sup> Specifically, the model given by (1) and (2) implies share elasticity to advertising is (proof available from authors)  $\eta_{it} = (1 - p)[(M_{it} - M_{2it})c_\alpha + M_{2it}\beta_A]/M_{it}$ , where  $M_{rit}$  is the share for brand  $i$  at time  $t$  under regime  $r$  and  $M_{it} = pM_{1it} + (1 - p)M_{2it}$ .

### Empirical Issues

In the proposed model, advertising threshold effects correspond to parameters  $\{c_\alpha, c_\tau, c_N, c_E\}$  which mediate advertising’s main effect on switching probability and its dependence on, respectively, ad expenditure level, time in market (dynamic factor), number of competitors (dynamic factor), and order of entry (brand-specific factor). We briefly consider these “main effects” in turn. A considerably more detailed account is provided by Bowman and Gatignon (1996) and Parker and Gatignon (1996).

**Advertising Expenditure Level.** Existence of a threshold suggests that, *ceteris paribus*, a higher level of advertising serves to *reduce* the probability ( $p$ ) of being in the ineffective regime. One therefore expects  $c_\alpha$  to be negative.

**Time in Market.** If a brand tends to advertise at all, the longer it has been in the market, the greater the likelihood that consumers have been repeatedly exposed to its messages. For established brands, the role of advertising becomes largely one of *reminding* consumers: reinforcing knowledge of brand attributes, rather than “informing” (Batra et al. 1996, Smith and Swinyard 1982, Smith 1993). In such cases, lower advertising levels may be sufficient to surpass the threshold, that is, to be within the effective regime. Thus, the longer a brand has been in the market, the lower its advertising thresholds are likely to be, and so  $c_\tau$  is expected to be negative.

**Number of Competitors.** Where noise is greater, so too is the effort one must expend to be heard above it (Webb 1979, Webb and Ray 1979). Specifically, a larger number of direct competitors should translate into elevated advertising thresholds, resulting in higher below-threshold probabilities, so that  $c_N$  is expected to be positive.

**Order of Entry.** Earlier entrants often serve as category prototypes (Carpenter and Nakamoto 1989) and tend to benefit more from advertising outlays, as information learned about them is treated as novel by consumers (Kardes and Kalyanaram 1992). Advertising of early entrants can therefore serve as a barrier to entry for later competitors (Bain 1956, Comanor and Wilson 1967) in the form of higher advertising thresholds, so that  $c_E$  is expected to be positive.

The four posited directional results ( $c_\alpha < 0$ ,  $c_\tau < 0$ ,  $c_N > 0$ ,  $c_E > 0$ ) can combine to render some comparisons indeterminate. For example, while  $c_\tau < 0$  suggests that the longer a brand has been in the market the lower advertising thresholds should be,  $c_E > 0$  suggests that an increase in the number of competitors should lead to an *increase* in advertising thresholds. Over time, a brand’s in-market duration *and* its number of competitors both tend to

increase, yet their directional effects on thresholds are predicted to oppose one another. Put more directly, although the effects of each explanatory variable (on the below-threshold probability,  $p$ ) are assumed monotonic in the logistic formulation (2), their interactions may well lead to nonmonotonic relationships.

## Empirical Application

### Data

As discussed at the outset, confirming advertising threshold effects may not be possible in categories for which data are most readily available: frequently purchased, low-cost goods from mature categories with stable ad budgets. Thus, for empirical purposes, we have made use of data from two durable product categories: For “Category 1” (Sport Utility Vehicles, 26 brands over 10 years), data begin five years after its introduction, with nine competitors already in the market; in “Category 2” (passenger minivans; 16 brands over 10 years), data begin at category inception. For comparison, we also estimate the model for a stable, frequently purchased category, liquid laundry detergent (“Category 3”). Over the observation period (1998–2000), there were 15 major brands, with no entries or exits. In our discussions, we will focus primarily on the durables categories, as numerous prior studies have examined frequently purchased consumer goods; however, we examine model implications for all three categories.

Despite outward differences, both durables categories (during the period covered by the data) are highly dynamic: They undergo considerable changes in terms of the overall number of competitors (and consequently their relative market-timing measures), as well as levels of their various marketing-mix activities. The data for all three categories consist of information on market share, advertising expenditures, price, distribution (number of outlets), time in market, order of entry, and the number of competitors at any given time, measured here in monthly intervals.<sup>5</sup> Finally, advertising information was provided quarterly in terms of dollar expenditures (in units of thousands) and covers print media and television advertising; within each quarter, a uniform distribution is used to convert to monthly values.

The durables categories differ in terms of advertising deployment: Category 2 (minivans) has a higher average expenditure level and a far greater proportion of advertising spending concentrated among the

Top Five advertisers than Category 1 (SUVs). This suggests that there are fewer proverbial big spenders in Category 2, yet who each appear to spend more, and with greater consistency (lower coefficient of variation), than their Category 1 counterparts. Further, there are fewer competitors overall in Category 2, due perhaps to its being at an earlier stage of development. Regardless, we would anticipate that either of the durables categories (1 or 2) would provide a clearer window through which to observe threshold effects than nondurable Category 3.

### Estimation and Empirical Results

The joint system (1)–(2) was estimated through maximum likelihood implemented in Gauss.<sup>6</sup>

Due to substantial differences between the estimated parameters for the two categories, the empirical analysis was performed separately for each. Parameter estimates for the model given by (1)–(2) appear in Table 1. We discuss the four posited effects in turn.

**Advertising Threshold Effects.** Advertising effects on the regime-switching probability are, as conjectured, negative and significant for all three categories ( $c_{\alpha 1} = -0.45$ ;  $c_{\alpha 2} = -0.95$ ;  $c_{\alpha 3} = -0.19$ ).<sup>7</sup> A higher ad-expenditure level therefore increases the probability of surpassing the threshold, and thus operating in the effective regime. We stress that this outcome is in no way guaranteed by the form of the model or the estimation method. In fact, a closer look at Category 3 (liquid detergent) indicates that although advertising level affects switching probability  $p$ , *there are nevertheless no advertising-based threshold effects*, for the following reason. In the durables categories, advertising coefficients in the effective regime are strongly distinguishable from zero ( $\beta_{A1} = 0.32$ ,  $t = 24.4$ ;  $\beta_{A2} = 0.24$ ,  $t = 10.6$ ). By contrast, in the nondurable category, advertising has negligible effect in

<sup>6</sup> The market share density for brand  $i$  at time  $t$  (for  $\phi$  the standard normal density,  $p_{it}$  given by (3)) and associated log likelihood, are:

$$f_{it} = f[\log(M_{it}) | \alpha, \beta, c, \sigma] \\ = p_{it} \phi \left[ \frac{\log(M_{it}) - x'_{1it} \alpha}{\sigma_1} \right] + (1 - p_{it}) \phi \left[ \frac{\log(M_{it}) - x'_{2it} \beta}{\sigma_2} \right],$$

where:

$$x'_{1it} = [K_1 \ln D_{it} \ln P_{it} \ln E_i \ln T_{it}], \\ x'_{2it} = [K_2 \ln A_{it} \ln D_{it} \ln P_{it} \ln E_i \ln T_{it}], \\ \alpha' = [\alpha_D \alpha_P \alpha_E \alpha_T], \quad \beta' = [\beta_A \beta_D \beta_P \beta_E \beta_T], \\ c' = [c_0 \ c_\alpha \ c_\tau \ c_N \ c_E], \quad \log \Lambda = \sum_{t=1}^T \sum_{i=1}^I \log [f_{it}].$$

<sup>5</sup> “Price” denotes vehicle base price as published annually by *Ward’s Automotive*, irrespective of dealer incentives, rebates, or optional equipment; see Bowman and Gatignon (1996, p. 230) for more detail. Quarterly data on “quality” were also available but because they did not prove significant for either category in the forthcoming exploratory model analysis, we make no further reference to it.

<sup>7</sup> Numerical subscripts, other than for constants  $K_1$ ,  $K_2$ , and  $c_0$ , denote category number. Significance levels for all reported parameters appear in Table 1.

**Table 1** Switching Regime Model Parameter Estimates (*t*-Statistics)

Variable	Category 1: Sport Utility Vehicles			Category 2: Passenger Minivans			Category 3: Liquid Laundry Detergent		
	Regime 1 Ineffect.	Regime 2 Effective	$\Delta$	Regime 1 Ineffect.	Regime 2 Effective	$\Delta$	Regime 1 Ineffect.	Regime 2 Effective	$\Delta$
Intercept*		4.33 (5.8)	—		8.59 (5.2)	—	7.35 (17.6)	1.28 (0.52)	6.07 (3.4)
Advertising { $\alpha_A, \beta_A$ }	—	0.32 (24.4)	<b>0.32</b> <b>(24.4)</b>	—	0.24 (10.6)	<b>0.24</b> <b>(10.6)</b>	—	0.005 (0.5)	<b>0.005</b> <b>(0.5)</b>
Distribution { $\alpha_D, \beta_D$ }	0.96 (26.0)	0.15 (4.1)	0.81 (22.0)	1.12 (8.5)	0.61 (12.0)	0.51 (5.1)	-1.5 (-12.2)	0.43 (2.4)	-1.93 (-12.6)
Price { $\alpha_P, \beta_P$ }	-1.57 (-13.3)	-0.85 (-22.5)	-0.72 (-8.2)	-2.13 (-9.5)	-1.82 (-10.5)	-0.31 (-1.5)	0.21 (9.7)	0.46 (5.1)	-0.25 (-3.8)
Order of entry { $\alpha_E, \beta_E$ }	0.11 (1.6)	0.15 (0.8)	-0.04 (-0.5)	-0.32 (-2.0)	-0.39 (-6.2)	0.07 (0.6)	-0.19 (-4.7)	-0.35 (-0.7)	0.16 (0.5)
Time in market { $\alpha_T, \beta_T$ }	-0.13 (-2.6)	0.42 (12.7)	-0.30 (-13.0)	-0.05 (-0.4)	0.21 (4.1)	-0.26 (-2.7)	-0.07 (-1.1)	-0.55 (-1.5)	0.48 (1.8)
Variance, $\sigma^2$	1.10	0.48		1.89	0.52		0.14	0.44	
Probability of Regime Switch**									
Intercept		-2.52 (-2.64)			9.49 (5.1)			-0.71 (-2.4)	
Advertising, $c_\alpha$		<b>-0.45</b> <b>(-10.0)</b>			<b>-0.95</b> <b>(-46.2)</b>			<b>-0.19</b> <b>(-5.9)</b>	
Time in market, $c_\tau$		-0.29 (1.4)			-2.51 (-4.8)			—	
Number of competitors, $c_N$		0.14 (4.6)			0.52 (3.9)			—	
Order of entry, $c_E$		0.07 (1.6)			-0.64 (-4.3)			—	
Number of observations		1,973			960			420	
Log likelihood		-2,682.3			-1,245.5			-122.59	

Notes. \*Because the intercepts in the two regimes did not differ significantly for either Category 1 or 2, they were estimated as a single value for each. \*\*For Category 3, three effects ( $c_\tau$ ,  $c_N$ ,  $c_E$ ) did not approach significance, and so were set to zero for re-estimation.

the second regime ( $\beta_{A3} = 0.005$ ,  $t = 0.5$ ). Apparently, advertising level simply mediates, through  $p$ , regime differences in the other dynamic variables, distribution, price, and order of entry. This is especially intriguing, as only advertising has nonnegligible influence in the effective regime itself (i.e.,  $c_\tau$ ,  $c_N$ , and  $c_E$  are nonsignificant).<sup>8</sup>

In terms of the dynamic and brand-specific effects on the below-threshold probability, the durable and nondurable categories differ markedly. Time in market is in the conjectured direction for both durables categories, though significantly so only for Category 2, the “younger” of the two ( $c_{\tau1} = -0.29$ ;  $c_{\tau2} = -2.51$ ;  $c_{\tau3} \equiv 0$ ). The proper interpretation of this for Category 2 is that the longer a brand is in the market, the higher its probability of being above threshold (for a given level of advertising expenditures), so that “older” brands have lower thresholds, all else being equal. As conjectured, number of competitors is significantly positive for both durables categories

( $c_{N1} = 0.14$ ;  $c_{N2} = 0.52$ ;  $c_{N3} \equiv 0$ ), suggesting that a larger number of competitors decreases the probability of surpassing the threshold.

Contrary to conjecture, order of entry effects either fail to significantly affect regime-switching probability ( $c_{E1} = 0.07$ ,  $c_{E3} \equiv 0$ ) or do so in a negative manner ( $c_{E2} = -0.64$ ). In other words, later entrants appear to have higher probability of being above threshold, at least in Category 2. A plausible explanation concerns the possibility of later entrants’ capitalizing on the “awareness-building” expenditures of those before them.<sup>9</sup> Still, we consider this an anomalous, if equivocal, finding.

Considering these effects for time in market and number of competitors, the data suggest that, at least for Category 2 (in which both significantly affect below-threshold probability), there is a trade-off of sorts taking place. Whereas the “time in market effect” suggests that thresholds decrease the longer a brand is in the market, the “number of competitors effect” serves to increase thresholds. Therefore,

<sup>8</sup> As such, we present model results for these three ( $c_\tau$ ,  $c_N$ , and  $c_E$ ) constrained to zero in Category 3.

<sup>9</sup> We are grateful to a reviewer for pointing this out.

as categories evolve—and both time in market and number of competitors tend to increase—the overall effect on threshold is indeterminate, pulled downward by the former and upward by the latter.

**Model Validation**

One can take the “standard” concave market share response model—that of the second (effective) regime—and augment it in a number of ways. To this standard (henceforth “base”) model, three constructs have been grafted: (1) an “ineffective” regime; (2) S-shaped response in advertising expenditure; and (3) dynamic, multivariate regime switching ( $p$ ). We explore whether such additions are warranted by benchmarking the proposed model against an inter-related set of similar ones<sup>10</sup>:

**M1: Single-Regime Concave (“Base”) Model.** Equivalent to (1) alone (or by (1) and (2) with  $p$  set to 0), i.e., assuming that there are no threshold effects, and thus only an effective regime.

**M2: Single-Regime Parsimonious S-Shaped Model.** Identical to the single-regime model, but with advertising entering (1) as  $\exp[-\beta_A/(1 + A_{it})]$  rather than  $A_{it}^{\beta_1}$ . This form is S-shaped only for  $\beta_A > 2$ , thus allowing for S-shaped response—without overtly requiring it—with the same number of parameters as the single-regime model, so that no parsimony is sacrificed.<sup>11</sup>

**M3: Single-Regime Logistic Advertising Model.** Identical to the single-regime model, but with advertising entering (1) as a linear-logistic transform,  $[1 + \exp(\gamma_0 + \gamma_1 A)]^{-1}$ , rather than  $A_{it}^{\beta_1}$ . This form allows for a more general type of S-shaped advertising response than either of the other single-regime models, but is less parsimonious.<sup>12</sup> The advertising specification is similar to that used by Mahajan and Muller (1987) and Feinberg (1992).

**M4: Two-Regime Constant  $p$  Model** (Bemmaor 1984). Equivalent to (1), but with  $p$  an estimated constant (that is, independent of exogenous variables).

**M5: Two-Regime Cutoff Model.** Equivalent to (1), but with the specification for  $p$  given by (4), i.e., a two-regime model in which thresholds are *deterministic*, and a single cutoff point is estimated (through a grid search) beneath which advertising is “ineffective.”

**M6: Two-Regime  $p = f(A)$  Model.** Equivalent to (1) and (2)—the full probabilistic thresholds model—but with the nonadvertising coefficients in (2),  $\{c_r, c_N, c_E\}$ , set to zero. The model therefore lacks category-specific and dynamic factors in the regime-switching probability (and is the model estimated for Category 3 in Table 1).

**M7: Two-Regime Probabilistic Thresholds Model.** (1) and (2).

Although each of the six other models is related to the probabilistic thresholds model, only M4 (Bemmaor’s two-regime constant  $p$  model) and M6 (the two-regime  $p = f(A)$  model) are strictly nested within it in the sense of parametric restriction.<sup>13</sup> The remaining two-regime models, M4 and M5, are special cases of the probabilistic thresholds model for boundary values of  $p$  (respectively, always  $p = 0$ , or variously 0/1 depending on  $A_{it}$ , as per Equation (4)), and thus we use BIC to compare relative model performance. It is helpful to anticipate what insights might be gained through various comparisons among seven models:

*Single- vs. two-regime models* (M1–M3 vs. M4–M7): Is a second regime warranted?

*S-shaped advertising response vs. concavity* (M1 vs. M2 vs. M3): Irrespective of threshold effects, is there evidence for S-shaped advertising response?

*Dependence of the regime-switching probability ( $p$ ) on advertising level* (M4 vs. M5–M7): Does the overall expenditure level help determine which of the two regimes is operational?

*Specification of advertising response* (M5 vs. M6–M7): If advertising level appears in the regime-switching probability ( $p$ ) specification, what form should the functional relationship take?

*Inclusion of dynamic marketing variables in specification for  $p$*  (M7 vs. M4–M6): Can a model that excludes time-varying and environmental factors capture the data adequately?

By structuring comparisons in this manner, a rationale for a model such as the probabilistic thresholds will either be “built up” or fail to be, with each component of the model spotlighted as to its marginal importance. The results of all such comparisons can be found in Table 2.

<sup>10</sup> As Category 3 lacks advertising thresholds, we limit model comparisons to the two durables categories.

<sup>11</sup> Because  $\partial^2 y / \partial A^2 = [y\beta_A(1 + A_{it})^{-4}][\beta_A - 2(1 + A_{it})]$  and  $A_{it}$  is non-negative, there is an inflection point iff  $\beta_A > 2$ .

<sup>12</sup> Model M3 is equivalent to M7, under the restrictions  $\{c_r, c_N, c_E\} = 0$  (M6),  $\beta_A = 0$ , and  $\alpha_i = \beta_i$ . To see that M3, M6, M7 and can capture S-shaped response, hold aside  $D_{it}^{\alpha_D} P_{it}^{\alpha_P} E_i^{\alpha_E} T_{it}^{\alpha_T}$  and suppress subscripts  $i$  and  $t$ :

Brand  $i$ ’s market share at time  $t$  is

$$M \propto \frac{1 + \exp[-(c_1 + c_\alpha \alpha)]}{1 + \exp[-(c_0 + c_\alpha \alpha)]}$$

where  $c_1 = c_0 - \log(K_1/K_2)$  and  $\alpha = \log[A]$ . Simple algebra shows  $\partial^2 M / \partial A^2 = 0$ , when  $A = \exp[-(\log(c_\alpha + 1) - \log(c_\alpha - 1) + c_0) / c_\alpha]$ . Thus, an inflection point occurs when  $c_\alpha < -1$ , so S-shape response is possible, but not inevitable, for M3, M6, and M7.

<sup>13</sup> It is possible to construe M1 (the single-regime concave model) in a similar manner by setting the four effects coefficients to zero and the constant term in Equation (2) to negative infinity. However, we apply the more conservative nonnested tests in this case.

**Table 2 Model Validation and Nested Model Comparison**

Model	Number of Parameters		Log Likelihood		BIC and (Rank)	
	Category 1	Category 2	Category 1	Category 2	Category 1	Category 2
M1: Single-regime concave	7	7	-2,996.1	-1,594.7	-3,026.4 (6)	-1,618.7 (7)
M2: Single-regime parsimonious S-shape	7	7	-3,024.1	-1,591.4	-3,054.4 (7)	-1,615.4 (6)
M3: Single-regime logistic advertising	8	8	-2,891.9	-1,586.3	-2,926.0 (4)	-1,613.7 (5)
M4: Two-regime constant $p$	13	13	-2,918.8	-1,409.1	-2,971.9 (5)	-1,453.7 (4)
M5: Two-regime cutoff (grid search)	14	14	-2,792.5	-1,316.3	-2,853.2 (3)	-1,364.4 (3)
M6: Two-regime $p = f(A)$	14	14	-2,715.6	-1,276.0	-2,772.5 (2)	-1,324.1 (2)
M7: Probabilistic thresholds	17	17	-2,682.3	-1,245.1	-2,750.6 (1)	-1,303.5 (1)
Nested Model Comparisons	$\Delta df$		$-2(\Delta LL)$		$p$ -Value for $\chi^2_{\Delta df}$	
M6 vs. M4	1	1	406.4	266.2	3.3 E-16	3.8 E-15
M7 vs. M4	4	4	473.0	328.0	1.2 E-13	1.2 E-12
M7 vs. M6	3	3	66.6	61.8	2.3 E-14	2.4 E-13

**Model Validation Results**

As can be seen from the “BIC (and Rank)” column of Table 2, the pattern of ranking results for the BIC measure is remarkably similar across the two durables categories,<sup>14</sup> with the probabilistic thresholds model faring best (M7: BIC (within-category rank) = -2,750.6 (1); -1,303.5 (1)), followed by the two-regime  $p = f(A)$  model (M6: -2,772.5 (2); -1,324.1 (2)), and with the single-regime concave model near last in both categories (M1: -3,026.4 (6); -1,618.7 (7)). In order to provide structure to our discussion of model comparison, we follow the five questions put forth earlier. All comparisons are based on BIC (Table 2) unless otherwise noted (i.e., likelihood ratio test for nested models).

**Single- vs. Two-Regime Models (M1–M3 vs. M4–M7).** The two-regime models perform far better than their more parsimonious counterparts, particularly so in Category 2. This is not merely a matter of insufficiently penalizing nonparsimonious models; for example, in Category 1 the single-regime logistic advertising model (M3: -2,926.0 (4)) performs better than the two-regime constant  $p$  (M4: -2,971.9 (5)) model. However, this is but a single case, and all other comparisons provide ample evidence supporting the existence of an “ineffective” regime.

**S-Shaped Advertising Response vs. Concavity (M1 vs. M2 vs. M3).** To determine whether S-shaped response is warranted in the absence of a second regime, we compare the globally concave M1 to two variants for which advertising can be S-shaped. Within this set of three models, for both data sets a clear pattern emerges, with the globally concave model (M1: -3,026.4 (6); -1,618.7 (7)) and the

parsimonious S-shaped model (M2: -3,054.4 (7); -1,615.4 (6)) performing least well, and the logistic advertising model (M3: -2,926.0 (4); -1,613.7 (5)) besting each.

**Dependence of the Regime-Switching Probability ( $p$ ) on Advertising Level (M4 vs. M5–M7).**

Over and above any effects accounted for by the share models (1), regime-switching probabilities (2) are strongly affected by advertising-spending levels. For both categories, the BIC measure indicates a far poorer fit for the constant  $p$  model (M4: -2,971.9 (5); -1,453.7 (4)) than for the other two-regime models, the cutoff (M5: -2,853.2 (3); -1,364.4 (3)),  $p = f(A)$  (M6: -2,772.5 (2); -1,324.1 (2)) and the probabilistic thresholds (M7: -2,750.6 (1); -1,303.5 (1)) models (a simple Chi-square test on the nested models M4 vs. M6 indicates the enormity of the difference, with  $\chi^2(1 df) = 406.4$  and 266.2, respectively, for Categories 1 and 2). This is especially compelling evidence that merely including advertising levels in the response functions (1) alone fails to adequately account for market share dynamics in these categories. Given the role played by  $c_\alpha$  in the elasticity expression (Footnote 4), excluding advertising from the regime-switching probability expression can lead to biased elasticity measures.

**Specification of Advertising Response (M5 vs. M6–M7):** Having established that there should be two regimes and that switching probability depends on advertising levels, we wish to determine the nature of this functional dependence. Specifically, as ad spending diminishes, should the regimes switch abruptly or taper off? The cutoff model (M5) posits the former, seeking the point beneath which advertising ceases to be effective (given that this level depends on other variables as well). By contrast, the  $p = f(A)$  (M6) and probabilistic thresholds (M7)

<sup>14</sup> Both Kendall’s  $\tau$  ( $\tau = 0.810, p < 0.01$ ) and Spearman’s  $\rho$  ( $\rho = 0.929, p < 0.01$ ) reject a lack of rank-order association.

model entail continuous logistic relationship (2); in M6, advertising as the *sole* input variable. Again, there is a clear pattern, with both “continuous specification” models (M6:  $-2,772.5$  (2);  $-1,324.1$  (2); M7:  $-2,750.6$  (1);  $-1,303.5$  (1)) edging out the “discrete-jump” cutoff model (M5:  $-2,853.2$  (3);  $-1,364.4$  (3)).

**Inclusion of Dynamic Marketing Variables in Specification for  $p$  (M7 vs. M4–M6).** Among our main goals is to determine the existence of thresholds and whether they are influenced by time-variant, brand-specific factors. The only model of the seven capable of capturing this (over and above the other constructs previously addressed) is M7, the probabilistic thresholds model. Based again on BIC, it performed far better than *all* other models, including the other two-regime comparison models, which are only slightly more parsimonious. It (M7:  $-2,750.6$  (1);  $-1,303.5$  (1)) offered performance well in excess of its nearest “competitor,” the  $p = f(A)$  model (M6:  $-2,772.5$  (2);  $-1,324.1$  (2)), a particularly telling comparison in that M6 is a direct parametric restriction of M7, obtained by setting the coefficients for time in market, order of entry, and number of competitors to zero. Chi-squared tests— $\chi^2(3 df) = 66.6$  for Category 1 and  $61.8$  for Category 2—argue strongly for the inclusion of these omitted, dynamic factors.

This sequence of five comparisons, taken together, systematically supports our main line of argument: that threshold effects exist; that market share response to advertising is not necessarily globally concave; that which regime one operates in is a function of advertising level; and that dynamic, competitive, and brand-specific factors affect the regime-switching probability.

A number of additional insights emerge. First, it is intriguing to note that the cutoff model (M5) performs better than the constant  $p$  model (M4) for both categories. This may be attributed to the fact that, in both the probabilistic thresholds (M7) and the cutoff model (M5), a core construct—either the logistic probability (2) or the cutoff point (4), respectively—*other than the market share model itself* is guided by the level of advertising expenditures. For example, in the constant  $p$  model of Bemmaor (M4), although  $p$  is estimated for each data set, it does not vary with the level of advertising expenditure. While this mechanism does encode two different effectiveness regimes, it assumes that the likelihood of being in one or the other *does not vary with expenditure*; by contrast, the cutoff model (M5) makes choice between the regimes *completely* a function of advertising level, in an all-or-nothing manner. In this way, the constant  $p$  and the cutoff model are rather like opposites, the former stating that regime-switching probability is not affected by advertising (or anything else), the

latter saying that being above some critical advertising threshold is literally all that matters. That neither performs as well as the proposed model may suggest that one should well allow for a *gradual* shift to the effective regime (e.g., as a function of market-level covariates, as in M7).

Consequently, the poor performance of the two-regime constant  $p$  model (in Category 1, where it barely bests the one-regime model) suggests not only the importance of a two-regime model, but of having the “switching function” (2) depend on strategic input variables such as advertising expenditure. A similar comparison can be made between the proposed probabilistic thresholds model (M7) and the only other model which approached its performance, the two-regime cutoff model (M5). Whereas the cutoff model takes account of advertising, it does so in a rigid, abrupt fashion. The proposed model, by contrast, allows for thresholds which are both dynamic and brand specific, assigning a probability of operating in the effective regime for any set of mix variable inputs. Indeed, the degree of dominance (based on BIC) of the probabilistic thresholds model over the other two-regime models is decisive, and speaks to the additional explanatory power, specifically, of including dynamic competitive and marketing-mix variables in the logistic probability specification (4). Taken as a group, the degree of dominance of the three two-regime models suggests that the inclusion of an ineffective regime more than compensates for the lack of parsimony involved.

Of all the model comparisons, the one we believe to be most salient is that to the single-regime S-shaped model, for several reasons. First, as shown above (Footnote 12), the probabilistic thresholds model is capable of producing S-shaped response to advertising spending. Because each model allows for S-shaped response, their comparison boils down to *whether including an ineffective regime is called for over and above S-shaped response*, a major point of justification for the present study. Second, because the S-shaped model requires the same number of parameters as the single-regime (concave) model, and far fewer than the probabilistic thresholds model, its fit statistics give some sense of the relative additional explanatory power of S-shaped response vs. a second, ineffective regime. Based on BIC, the data suggest unequivocally, for both categories, that S-shaped response alone provides comparatively little in terms of fit, compared with a second, ineffective regime: Taking the single-regime concave model—no S-shape, no ineffective regime—as a baseline (M1:  $-3,026.4$  (6);  $-1,618.7$  (7)), the differential in “adding” S-shaped response (M2:  $-3,054.4$  (7);  $-1,615.4$  (6)) is far less than further adding a second regime, as in the probabilistic thresholds model (M7:  $-2,750.6$  (1);  $-1,303.5$  (1)).



**Parsimoniousness vs. Flexibility.** The issue remains as to whether better fit of the two-regime model is attributable to threshold effects per se or just to greater “flexibility”—in the sense that the switching function allows the market share model to vary by firm, whereas the other models have a single function that is held fixed across firms—or its being less parsimonious. Thorough discussions of the relevant issues on model complexity can be found in Van Heerde et al. (2001) treatment of deal effects, as well as the “potentially nonmonotonic” spline approach to pricing of Kalyanam and Shively (1998). To rule out these possibilities, we estimate a less parsimonious S-shaped model, one which should provide better fit based solely on more parameters and at least equal, if not greater, flexibility. Such a model can be described compactly as  $M8 = pM1 + (1 - p)M3$ , that is, a logistically mediated combination of two single-regime models—M1 (single-regime convex model), M3 (single-regime logistic model)—creating a hybrid more flexible, and less parsimonious, than M7 (two-regime probabilistic thresholds model). A simple way to think about this is as a model which allows S-shaped response *directly* in the “effective” regime, thus accommodating a wide range of response forms. Interestingly, the less parsimonious M8 fit *worse* in both durables categories.<sup>15</sup> Specifically, we find that, in terms of log likelihoods, for SUVs  $M7 = -2,682.3$ ,  $M8 = -2,693.9$ ; and for minivans  $M7 = -1,245.1$ ,  $M8 = -1,251.0$ . Given this pattern of results for both categories, neither flexibility nor nonparsimoniousness *alone* accounts for the superior fit of the proposed probabilistic thresholds model.

**Sensitivity Analysis for the Switching Function.** One might question, if not the particular form of the switching probability, then the inclusion of three effects, order of entry (OE), time in market (TM), and number of competitors (NC), which may appear to intrinsically covary. To get a handle on this, we compared the “full” model (M7) to analogous models leaving out each of the variables, singly and in pairs.<sup>16</sup> In the minivan category, leaving out any of the effects very significantly decreases fit ( $p < 0.0001$ ), and for SUVs, each of the effects is at least marginally significant; moreover, most coefficients are stable across models. We stress that conclusions regarding multicollinearity can be misleading, as it is important to

distinguish dynamic effects *within* brand and cross-sectional effects *across* them. It is possible that, say, NC can decrease overall while TM *for a particular brand* increases. So, although the (marginal) effect of each of the OE, NC, and TM on  $p$  is monotonic, combinations of the three can lead to a nonmonotonic dynamic threshold effect over time.

For the SUV category, results are clear cut: The dynamic effects are monotonically increasing, and TM and NC are both important. The situation for minivans, by contrast, is quite “rich;” the full (OE, NC, and TM) model does markedly better than all the other combinations because the net effect is nonmonotonic, and there is a trade-off between maturity (TM lowers threshold) and competition (NC increases threshold). We strongly believe that all three effects belong in a compelling model and, moreover, that potentially positively correlated variables like TM and NC being able to have opposing effects is an important and useful property.

Taken together, then, the pattern of results for these categories suggests that none of the constructs—S-shaped advertising response, additional parameters, a second regime,  $p$  as a function of advertising level—*alone* accounts for the far better fit of the probabilistic thresholds model, although each certainly exerts some effect. Rather, the dynamic, brand-specific nature of the thresholds themselves appears to strongly affect our ability to accurately model market share fluctuations in these categories. It is unfortunate that the data available do not allow formal cross-category comparisons, particularly so in (nondurable) Category 3, where we find no evidence of advertising thresholds. We note, however, that cross-category data used for such purposes would need to be relatively free of category-specific idiosyncrasies, a daunting task both in terms of data requirements and modeling formulation.

## Summary and Conclusions

That there may be spending “regions” that produce little or no advertising response is an issue of clear practical import. When a brand enters a market, particularly a turbulent or developing one, it is typically the case that some form of variation (cycling, blitzing, or pulsing) is used. However, due to the dynamic complexity of the competitive environment, it is difficult to tell whether the “low” parts of the pulse are better than not advertising at all. The verdict of the extant literature in marketing, economics, and operations research has been that no such expenditure regions exist. Simply put: In terms of resulting share, some advertising is *always* better than no advertising at all. Our results question that unequivocal verdict.

<sup>15</sup> M8 requires 19 parameters; M7, 17. Because the models are nonnested—as they must be for the comparison to be a sensible test—this is entirely possible. Similar “convex combinations” of other models in the paper failed to improve results. Note as well that the comparison for the nondurable category (liquid detergent) is not relevant, as no advertising thresholds were detected in that case.

<sup>16</sup> We thank the area editor for suggesting this analysis. Full results are available from the authors.

We believe there to be at least two related reasons for prior studies' not having found compelling evidence of threshold effects. The first is that posited throughout: that the product classes for which data were available were simply not suitable for the task. We would concur that threshold effects play a modest role for frequently purchased goods in mature markets, and have therefore focused on categories in a state of flux, either new or characterized by entry and exit of major players. The second possible reason for nonconfirmation of thresholds has to do, we believe, with the specification of the so-called regime-switching probability. If the probability of falling into the "ineffective" region is fixed, or based *solely* on advertising level, one is essentially left with a single-regime model: All that really matters is advertising level, even if some *formal* probabilistic mechanism is involved. By making the regime-switching probability a function of dynamic and category-specific factors, threshold effects stand in comparatively bold relief, as evidenced by the consistently superior fit of the probabilistic thresholds model over restricted, static variants.

Unsurprisingly, spending more increases the likelihood of advertising "effectively" in the sense of surpassing the (dynamic) threshold. Similarly, the longer a brand is in the market, the higher its probability of being above threshold (for a given level of advertising expenditures), so that "older" brands have lower thresholds, all else equal. A larger number of competitors decreases the probability of surpassing the threshold, consistent with the argument regarding "clutter" and the difficulty of being heard above it. We find only modest evidence, in one of the three categories, that order of entry affects the likelihood of operating in the "effective" regime, and then in a direction contrary to expectations. We stress that each of these effects was confirmed—and only *could have been confirmed*—through a model supporting the existence of an ineffective regime, one which is partially determined by dynamic, competitive, and environmental factors.

Because thresholds lack direct observability, we estimated a variety of functional forms and model types. Contingent on the best of these, we then looked at constrained submodels (to determine whether *all* the variables in the switching regression formulation are really necessary) and less parsimonious ones (to rule out flexibility as the main driver of the threshold result). Across these sets of comparisons, there was unequivocal support for the influence of dynamic, brand-specific marketing and environmental factors in determining critical advertising threshold levels.

While there was some degree of support for the additional explanatory power of S-shaped response (of two distinct varieties) in a single-regime modeling

context, there was a far stronger indication in favor of adding a second, ineffective regime (which was shown able to capture S-shaped response on its own). There was also strong support for the influence of advertising both in the response *and* switching functions, suggesting that models which include advertising spending as a regressor in a single-regime market share model may exhibit an a priori misspecification.

There are several issues which our study raises but has not attempted to address. For example, we have not considered advertising as an *endogenous* firm decision variable. A possible extension of the present modeling framework would offer a formal method to account for this type of endogeneity, extending the seminal investigations in this area of Bass (1969) and Bass and Parsons (1969), as well as the recent work of Villas-Boas and Winer (1999). Further, the plausibility of any explanations involving life-cycle stage should be more formally investigated through a cross-category analysis of advertising thresholds. Coupled with corresponding profit data, a similar model could provide implementable, dynamic guidelines for allocating advertising funds.

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