

Monetary Policy Risk and the Cross-Section of Stock Returns*

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Abstract

The effects of monetary policy shocks on the equity premium and the cross-section of stock returns are analyzed in general equilibrium. Policy shocks affect real stock returns as a result of nominal product-price rigidities. Two opposite effects determine the impact of policy shocks on stock returns. A contractionary shock increases the marginal utility of consumption, reduces aggregate output and increases production markups. The output reduction requires a positive premium in expected returns. The markup increase acts as a consumption hedge and involves a negative premium. Low elasticities of intertemporal substitution of consumption and labor amplify the markup effect and can generate a negative net effect on the equity premium. In the cross-section, a contractionary shock reduces the relative output and expands the relative markup of a more rigid price industry with respect to a more flexible price industry. If the relative markup expansion dominates the relative output decline, the expected stock return of the more flexible price industry is higher than that of the more rigid price one. The policy-induced markup variation also generates time variation in expected returns. As the responsiveness of the policy to economic conditions increases, the effects of policy shocks on the equity premium and the cross-section decline.

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1 Introduction

Monetary policy in the United States is conducted to promote the goals of price stability and maximum employment. This mandate implies that monetary policy can affect real economic activity, and suggests that uncertainty in the policy can be a potential source of aggregate risk. Real stock returns can then be affected by this uncertainty, and stock investors may require a compensation for monetary policy risk. Since different economic sectors can be more or less sensitive to this risk, compensations may differ across industry stocks. Monetary policy is thus relevant to understand the economic foundations of stock prices. This paper provides a theoretical analysis of the effects of monetary policy risk on the aggregate stock market and the cross-section of stock returns.

Actions of policymakers are driven by changes in economic conditions. However, it is recognized that not all variation in monetary policy can be described as a response to the state of the economy. Christiano, Eichenbaum and Evans (1999) suggest some potential sources of policy uncertainty. They define this uncertainty as policy deviations from a systematic response to economic conditions. Deviations can be the result of exogenous changes in preferences of the monetary authority (e.g., shifts in the relative importance of inflation and unemployment), self-fulfilling shocks to private agents' expectations about the policy, or technical factors such as measurement error in the economic data available at the time the monetary authority makes decisions, among others. We abstract from the possible sources of risk and analyze an economic model where uncertainty in the policy is an exogenous shock affecting an interest-rate policy rule.

We model a two-industry production economy where monetary policy has real economic effects as the result of nominal price rigidities, as in Woodford (2003). We allow the two industries to differ in their degrees of price rigidity to analyze the effect of this difference on the cross-section of real stock returns.¹ Monetary policy is modeled as a rule to set a short-term nominal interest

¹There is ample evidence of infrequent adjustments in the prices of goods and services and significant differences in the degree of price rigidity across industries. Bils and Klenow (2004) analyze 350 product categories. They report

rate. The rule reacts to the level of inflation and a measure of output, and is affected by exogenous policy shocks. In the presence of price rigidities, these shocks are a source of variability in the marginal utility of consumption, and industry profits. Households then require a premium for policy shocks to hold stocks in this economy. The magnitude and sign of the premium are sensitive to the response of monetary policy to economic conditions, depend on preference and production parameters, and vary over time.

The impact of policy shocks on the equity premium can be understood as the combination of two opposite effects. Production profits depend on the level of output, and the markup the producer charges over production costs. While the effect of policy shocks on output increases the premium, the effect on the markup reduces it. When prices are not perfectly flexible, a contractionary policy shock reduces aggregate production and thus increases the marginal utility of consumption, generating a positive premium. However, the contractionary shock also reduces the demand for production factors and their nominal cost. Since prices are sticky, real costs decline as well, and markups expand. The markup expansion in times of high marginal utility (countercyclical markup) generates a negative premium. Therefore, the sign of the net effect of policy shocks on the equity premium depends on whether the increase in the fraction of production distributed as profits offsets the reduction in production after a contractionary shock. It in turn depends on the elasticities of intertemporal substitution of consumption and labor, as they determine the responsiveness of the markup to the shock. In addition, the size of the premium increases with the degree of price rigidity, and decreases with the response to economic conditions in the policy rule. While a higher price rigidity increases the distortions of policy shocks on the real economy, stronger reactions to output and inflation in the policy rule have larger stabilization effects.

We apply a similar reasoning to understand the cross-sectional effects of policy shocks on

a median duration of prices between 4 and 6 months and a standard deviation of around 3 months. Nakamura and Steinsson (2007) exclude price changes related to sales and adjust this duration upwards to a range between 8 and 11 months. Blinder et al. (1998) conduct surveys on firms' pricing policies and summarize different theories for the existence of price rigidities based on the nature of costs, demand, contracts, market interactions and imperfect information.

stock returns. Cross-sectional differences in the model arise from differences in the degree of price rigidities across the two industries. The real output of an industry with more rigid prices is more sensitive to policy shocks than the output of an industry with more flexible prices. A policy shock that increases the marginal utility of consumption reduces production in the more rigid price industry by more than in the less rigid price industry, since the high relative price of the first one reduces its relative product demand. This effect generates a positive premium for holding stocks on the more rigid price industry over the more flexible price industry. Simultaneously, markups in the two industries have different sensitivities to policy shocks. A contractionary shock produces a high relative price in the more rigid price industry. It implies that markups in this industry expand relative to those in the more flexible price industry in times of high marginal utility of consumption. The relative markup effect then reduces the riskiness (and the premium) of stocks in the more rigid price industry with respect to those in the more flexible price industry. If the relative markup effect outweighs the relative output effect, expected stock returns in the more flexible price industry are higher than those in the more rigid price industry. The tradeoff between the two effects is determined by the elasticities of substitution of consumption, labor, and across industry goods. The magnitude of the expected return differences is affected by difference in price rigidities and the policy rule. Larger differences in price rigidities in the two industries are reflected in larger differences in expected returns in industry stocks. Policies that are more responsive to economic conditions reduce the effect of policy shocks on the two industries and then the differences in their expected stock returns.

The markup variation related to policy shocks also generates variability in conditional expected stock returns. Contractionary shocks increase the fraction of output that is distributed as profits and then the correlation between output and profits. The increased correlation translates into an larger output effect and a reduced markup effect on stock returns. Therefore, the equity premium increases after a contractionary shock, as well as the expected stock return of the more rigid price industry with respect to the more flexible price one.

We complement the theoretical results by constructing two empirical portfolios of industries with different underlying price rigidities, and analyzing their returns. CAPM and Fama-French regressions indicate that the portfolio of industries with high rigidities has a higher systematic risk than the portfolio of industries with low rigidities. The differences in systematic risk between the two portfolios are lower for a sample period that is thought to be characterized by a more responsive monetary policy.

Related Literature

The response of the stock market to monetary policy shocks has been studied by Thorbecke (1997), Bernanke and Kuttner (2005) and Rigobon and Sack (2004), among others. They all find a positive (negative) reaction in the stock market value to expansionary (contractionary) policy shocks. For instance, Bernanke and Kuttner (2005) find that a surprise cut of 25 basis points in the federal funds rate translates into an increase of 1.25% in the value of the aggregate stock market. They also find that stocks of different industries have different responses to policy shocks. In this paper, the reaction of the stock market to policy shocks is explained by price rigidities in production, and differences in industry responses are explained by differences in prices rigidities across industries. Models with price rigidities have also been used to understand the effects of monetary policy on bond prices, as in Bekaert, Cho and Moreno (2006). The link between policy shocks and other financial assets has also been studied. For instance, Cochrane and Piazzesi (2002) and Piazzesi (2005) analyze the effects of policy shocks on bond yield dynamics.

Significant work has been produced using macroeconomic variables to explain stock price dynamics. For instance, Chen, Roll and Ross (1986) use industrial production and inflation (among other variables), Cochrane (1996) uses investment growth, or Yogo (2006) uses durable and non-durable consumption. Some authors, in particular, have recognized the importance of labor income dynamics to understand time series and cross-sectional properties of stock returns (e.g., Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Santos and Veronesi (2006)). In this paper, monetary policy shocks generates risk in production and labor income that is reflected in

time-varying expected stock returns at aggregate and industry levels.

This paper is also related to the literature on market competition and stock returns. Thomadakis (1976) analyzes the role of monopolistic power on the firm's returns and systematic risk. Subrahmanyam and Thomadakis (1980) show that the CAPM beta of a firm decreases as its market power increases. Bhattacharyya and Leach (1999) show that market power generates risk spillovers across firms affecting their expected stock returns. More recently, Hou and Robinson (2006) show that stocks in more competitive industries have higher average returns than in less competitive ones. However, none of these papers consider the effect of monetary policy shocks in determining market power variation over time and across industries.

The paper is organized as follows. Section 2 presents some general implications of time-varying markups on expected asset returns, to facilitate the analysis of the economic model described in section 3. Section 4 presents the stock pricing implications of the model. For comparison purposes we present results for three different economies: an economy with flexible prices, and economies with homogeneous and heterogeneous price rigidities across industries, respectively. Section 5 presents an empirical analysis of two portfolios of industries with different price rigidities, and section 6 concludes. The appendix contains all proofs.

2 Production Markups and Expected Returns

This section presents an analysis of the implications of time-varying markups on expected excess returns for different financial claims. This analysis will be helpful to understand the asset-pricing implications of the economic model in section 3. We are mainly interested in claims on aggregate and sectoral profits to understand the potential effects of markups on the equity premium and the cross-section of stock returns, respectively. The cyclical properties of markups determine their effects on expected returns and generate time variation in expected excess returns. The analysis is limited to claims that pay off one period in the future to facilitate the exposition.

Let Y_t be the aggregate level of production at time t . Production is distributed as a compensation for production factors, W_t , and aggregate profits, Ψ_t , such that $Y_t = W_t + \Psi_t$. We refer to W_t as factor income. There is a potentially time-varying production markup μ_t such that factor income and profits are

$$W_t = \frac{1}{\mu_t} Y_t, \quad \text{and} \quad \Psi_t = \left(1 - \frac{1}{\mu_t}\right) Y_t,$$

respectively.

The discount factor for cashflows at time $t + 1$ is $M_{t,t+1}$. The risk-free rate, $R_{f,t}$, satisfies

$$\frac{1}{1 + R_{f,t}} = \mathbb{E}_t [M_{t,t+1}].$$

We assume that the discount factor, the production level and the markup follow conditional lognormal distributions. It follows that

$$1 + R_{f,t} = \exp \left[-\mathbb{E}_t [m_{t,t+1}] - \frac{1}{2} \text{var}_t (m_{t,t+1}) \right],$$

where $m_{t+1} \equiv \log M_{t,t+1}$.

We are interested in the price at time t of claims on production, factor income and profits at time $t + 1$. The price of a claim on production is

$$S_{y,t} = \mathbb{E}_t [M_{t,t+1} Y_{t+1}],$$

with associated one-period return, $R_{y,t+1}$, given by

$$1 + R_{y,t+1} = \frac{Y_{t+1}}{S_{y,t}}.$$

The conditional expected return of this claim can be written in terms of the risk-free rate as

$$\frac{\mathbb{E}_t [1 + R_{y,t+1}]}{1 + R_{f,t}} = \exp [-\text{cov}_t(m_{t,t+1}, y_{t+1})], \quad (1)$$

where $y_t \equiv \log Y_t$.² That is, the covariance between the discount factor and future production captures the expected excess return of the claim on production over the risk-free rate.

Similarly, the price of a claim on factor income is

$$S_{w,t} = \mathbb{E}_t [M_{t,t+1} W_{t+1}],$$

its one-period return, $R_{w,t+1}$, satisfies

$$1 + R_{w,t+1} = \frac{W_{t+1}}{S_{w,t}},$$

and its conditional expected return can be written as

$$\frac{\mathbb{E}_t [1 + R_{w,t+1}]}{1 + R_{f,t}} = \exp [-\text{cov}_t(m_{t,t+1}, w_{t+1})],$$

where $w_t \equiv \log W_t$. Since $w_t = -\log \mu_t + y_t$, the expected return on this claim can be written in terms of expected return on the production claim as

$$\frac{\mathbb{E}_t [1 + R_{w,t+1}]}{\mathbb{E}_t [1 + R_{y,t+1}]} = \exp [\text{cov}_t(m_{t+1}, \log \mu_{t+1})].$$

That is, the expected excess return of claims on factor income over claims on production depends

²To see this, notice that

$$\begin{aligned} S_{y,t} &= \exp \left[\mathbb{E}_t [m_{t,t+1} + y_{t+1}] + \frac{1}{2} \text{var}_t(m_{t,t+1} + y_{t+1}) \right] \\ &= (1 + R_{f,t})^{-1} \exp [\text{cov}_t(m_{t+1}, y_{t+1})] \mathbb{E}_t [Y_{t+1}]. \end{aligned}$$

on the covariance between the discount factor and the production markup. If factor income is a low fraction of production (high markup) during times when the discount factor is high, claims on factor income are riskier than claims on production and involve a higher expected return. The opposite will apply to claims on profits.

Consider a claim on profits with price

$$S_{\psi,t} = \mathbb{E}_t [M_{t,t+1} \Psi_{t+1}] = S_{y,t} - S_{w,t},$$

and one-period return, $R_{\psi,t}$, given by

$$1 + R_{\psi,t+1} = \frac{\Psi_{t+1}}{S_{\psi,t}}.$$

A claim on future profits can be replicated by buying a claim on future production and selling a claim on future factor income. It follows that the returns of these three claims satisfy

$$1 + R_{y,t+1} = v_t (1 + R_{w,t+1}) + (1 - v_t) (1 + R_{\psi,t+1}),$$

where

$$v_t = \frac{S_{w,t}}{S_{y,t}}.$$

The ratios v_t and $1 - v_t$ can be seen as the weights of claims on factor income and profits, respectively, in a portfolio that replicates a production claim. It can be shown that the expected return on a profit claim in terms of the expected return on a production claim is

$$\frac{\mathbb{E}_t [1 + R_{\psi,t+1}]}{\mathbb{E}_t [1 + R_{y,t+1}]} = \frac{1}{1 - v_t} \{1 - v_t \exp [\text{cov}_t(m_{t,t+1}, \log \mu_{t+1})]\}.$$

Since $0 < v_t < 1$, the expected return of a claim on profits is lower than the expected return of a claim on production if the markup and the discount factor are positively correlated. That is,

claims on profits are less risky than claims on production if profits represent a higher fraction of production when the discount factor is high. In addition, notice that time variation in the weight v_t represents a source of time variation in expected returns on profit claims. Consider

$$\frac{d}{dv_t} \left(\frac{\mathbb{E}_t [1 + R_{\psi,t+1}]}{\mathbb{E}_t [1 + R_{y,t+1}]} \right) = \frac{1 - \exp[\text{cov}_t(m_{t,t+1}, \log \mu_{t+1})]}{(1 - v_t)^2}.$$

An increase in v_t reduces the expected excess returns of claims on profits with respect to claims on production if the discount factor and the markup are positively correlated.

Consider now differences in expected returns on claims across economic sectors. Let $Y_{1,t}$ and $Y_{2,t}$ be the production levels of economic sectors 1 and 2, respectively. Their associated factor income are $W_{1,t}$ and $W_{2,t}$, with markups $\mu_{1,t}$ and $\mu_{2,t}$, such that $Y_{k,t} = \frac{W_{k,t}}{\mu_{k,t}}$, for $k = \{1, 2\}$.³ Expected returns on individual claims on sectoral production are related by

$$\frac{\mathbb{E}_t [1 + R_{y,1,t+1}]}{\mathbb{E}_t [1 + R_{y,2,t+1}]} = \exp[-\text{cov}_t(m_{t+1}, y_{1,t+1} - y_{2,t+1})].$$

Claims on production are more risky in the sector where the relative production with respect to the other sector has a negative covariance with the discount factor. Applying the analysis above for claims on aggregate profits to claims on sectoral profits, the difference in expected returns on profit claims across the two sectors can be expressed as

$$\frac{\mathbb{E}_t [1 + R_{\psi,1,t+1}]}{\mathbb{E}_t [1 + R_{\psi,2,t+1}]} = \frac{\frac{1-v_{1,t} \exp(\text{cov}_t(m_{t,t+1}, \log \mu_{1,t+1}))}{1-v_{1,t}} \mathbb{E}_t [1 + R_{y,1,t+1}]}{\frac{1-v_{2,t} \exp(\text{cov}_t(m_{t,t+1}, \log \mu_{2,t+1}))}{1-v_{2,t}} \mathbb{E}_t [1 + R_{y,2,t+1}]},$$

where $v_{1,t}$ and $v_{2,t}$ are the sectoral counterparts of v_t . To gain some intuition, suppose $v_{1,t} = v_{2,t} = v_t$, and apply a log-linearization around $\log \mu_{1,t+1} = \log \mu_{t+1}$ and $\log \mu_{2,t+1} = \log \mu_{t+1}$. It results in

$$\frac{\mathbb{E}_t [1 + R_{\psi,1,t+1}]}{\mathbb{E}_t [1 + R_{\psi,2,t+1}]} \approx \frac{\mathbb{E}_t [1 + R_{y,1,t+1}]}{\mathbb{E}_t [1 + R_{y,2,t+1}]} \exp[-\text{cov}_t(m_{t,t+1}, \log \mu_{1,t+1} - \log \mu_{2,t+1})d_t], \quad (2)$$

³Sectoral production and factor income are measured in units of aggregate production.

where

$$d_t = \frac{\exp[\text{cov}_t(m_{t,t+1}, \log \mu_{t+1})] v_t}{1 - \exp[\text{cov}_t(m_{t,t+1}, \log \mu_{t+1})] v_t} > 0.$$

The difference in expected returns on profit claims is determined by the difference in expected returns on product claims and the covariance of the discount factor with the difference in sectoral markups. This covariance can amplify or reduce the effect of difference in sectoral outputs. As a result, the difference in riskiness of claims on sectoral profits depends on the joint dynamics of the discount factor, sectoral outputs and sectoral markups. The relative riskiness of output in one sector can potentially be offset by a positive correlation between the discount factor and the relative sectoral markup with respect to the other sector. Notice also that differences in the time-varying weights $v_{1,t}$ and $v_{2,t}$ are a source of time variation in the difference in expected returns across sectors.

Section 3 presents an economic model where the equilibrium joint dynamics for the discount factor, sectoral outputs and markups depend on monetary policy. The analysis above will allow us to explore implications of monetary policy on the equity premium and the cross-section of stock returns.

3 The Model

We model a production economy where households derive utility from the consumption of a basket of two goods and disutility from supplying labor for the production of these goods. The two goods are produced in two different industries characterized by monopolistic competition and nominal price rigidities. We allow for heterogenous degrees of price rigidity in the two industries to learn about the effects of different rigidities on the cross-section of stock returns.⁴

Nominal rigidities generate real effects of monetary policy. When some producers are not able

⁴Aoki (2001) studies a particular case for this economy in which one of the industries has perfectly flexible prices. His analysis focuses on the implications for optimal monetary policy in this economy, and does not explore any asset pricing implications.

to adjust prices optimally, inflation generates distortions in relative prices that affect production decisions. Since inflation is determined by monetary policy, different policies have different implications for real activity, and may affect the returns on financial claims linked to production (e.g. stocks). We model monetary policy as an interest-rate policy rule that reacts to inflation and deviations of output from a target.

3.1 Households

A representative household maximizes its expected total utility

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{N_t^{1+\omega}}{1+\omega} \right) \right], \quad (3)$$

where C_t is the consumption of a final good, and N_t is the supply of labor at time t . The final good is a basket of two intermediate goods produced in two industries. We refer to these industries as $I = \{H, L\}$, to indicate industries with high and low price rigidities, respectively. The consumption of each industry's good is $C_{I,t}$, and the final good is given by

$$C_t = \left[\varphi^{1/\eta} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\varphi)^{1/\eta} C_{L,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad (4)$$

where φ is the weight of industry H in the basket, and $\eta > 1$ is the elasticity of substitution between industry goods. Each industry good is a Dixit-Stiglitz aggregate of a continuum of differentiated goods, defined as

$$C_{I,t} = \left[\int_0^1 C_{I,t}(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \quad (5)$$

where, the elasticity of substitution across differentiated goods is $\theta > 1$.

Households supply labor $N_{I,t}(j)$ for the production of the differentiated good j in industry I .

The total labor supplied to industry I is aggregated as,

$$N_{I,t} = \left[\int_0^1 N_{I,t}(j)^{1+\omega} dj \right]^{\frac{1}{1+\omega}},$$

for $I \in \{H, L\}$, where ω is the inverse of the elasticity of intertemporal substitution of labor.

Aggregate labor can be written as

$$N_t = [\varphi^{-\omega} N_{H,t}^{1+\omega} + (1 - \varphi)^{-\omega} N_{L,t}^{1+\omega}]^{1/(1+\omega)}.$$

The intertemporal budget constraint faced by households is

$$\mathbb{E} \left[\sum_{t=0}^{\infty} M_{0,t}^{\$} P_t C_t \right] \leq \mathbb{E} \left[\sum_{t=0}^{\infty} M_{0,t}^{\$} \sum_{I \in \{H, L\}} \left(\int_0^1 w_{I,t}(j) N_{I,t}(j) dj + P_t \int_0^1 \Psi_{I,t}(j) dj \right) \right], \quad (6)$$

where $M_{0,t}^{\$} > 0$ is the nominal pricing kernel that discounts nominal cash flows at time t to time 0, P_t is the price of the final good, and $w_{I,t}(j)$ and $\Psi_{I,t}(j)$ are, respectively, the nominal wage and the firm's real profit related to the production of the differentiated good j in industry I .

The maximization of (3) subject to (6) provides us with the intertemporal marginal rate of substitution of consumption for the economy. The intertemporal marginal rates of substitution of consumption between period t and period $t + n$ in real and nominal terms are

$$M_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma}, \quad (7)$$

and

$$M_{t,t+n}^{\$} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma} \left(\frac{P_{t+n}}{P_t} \right)^{-1}, \quad (8)$$

respectively. From these two equations we can compute the real and nominal one-period risk-free

rates as

$$1 + R_{f,t} = \frac{1}{\mathbb{E}_t [M_{t,t+1}]} \quad (9)$$

and

$$1 + R_{f,t}^{\$} = \frac{1}{\mathbb{E}_t [M_{t,t+1}^{\$}]} \quad (10)$$

respectively. The real risk-free rate will be important to compute excess real returns on stocks. The one-period nominal risk-free rate is the instrument of monetary policy.

The intratemporal marginal rate of substitution between labor and consumption is

$$\frac{w_{I,t}(j)}{P_t} = \varphi_I^{-\omega} N_{I,t}(j)^{\omega} C_t^{\gamma}, \quad (11)$$

where $\varphi_H = \varphi$ and $\varphi_L = 1 - \varphi$. This equation provides us with real wages once we determine the levels of labor and output from the production problem.

3.2 Firms

The production of differentiated goods is characterized by monopolistic competition and price rigidities in two different industries. Producers have market power to set the price of their differentiated goods in a Calvo (1983) staggered price setting. A producer is unable to change the product price at any period of time, with some positive probability. We allow for different probabilities across industries to capture heterogeneous degrees of price rigidities.

The probability of not changing the price of a differentiated good at a particular time in industry I is α_I . When the producer is able to set a new price for the good, the price is set to maximize the expected present value of all future profits, taking into account the probability of

not changing that price in the future. The maximization problem is

$$\max_{\{P_{I,t}(j)\}} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \alpha_I^{T-t} M_{t,T}^{\$} (P_{I,t}(j) Y_{I,T|t}(j) - w_{I,T|t}(j) N_{I,T|t}(j)) \right], \quad (12)$$

subject to the demand function (see appendix A for its derivation)

$$P_{I,t}(j) = P_{I,T} \left(\frac{Y_{I,T|t}(j)}{Y_{I,T}} \right)^{-1/\theta},$$

and the production function

$$Y_{I,T|t}(j) = A N_{I,T|t}(j), \quad (13)$$

where $Y_{I,T|t}(j)$ is the level of output of firm j in industry I at time T when the last time the price was reset was at t . A similar definition applies to $N_{I,T|t}(j)$ and $w_{I,T|t}(j)$. We assume constant labor productivity, A .

The output of industry I is $Y_{I,t}$, and the aggregate output of the final good is Y_t . We denote deviations in aggregate output from the flexible-price output, or “output gap”, by

$$x_t \equiv \log Y_t - \log Y^f,$$

where Y^f is the constant aggregate output when prices are perfectly flexible. The flexible-price output is a reference point for monetary policy. Its equilibrium value is presented in section 4.

Inflation in industry I is

$$\pi_{I,t} \equiv \log P_{I,t+1} - \log P_{I,t},$$

and the relative price of the good from industry H with respect to the good in industry L is

$$p_{R,t} \equiv \log P_{H,t} - \log P_{L,t}.$$

Appendix A shows that the solution to the firm's maximization problem implies

$$\pi_{I,t} = \kappa_I x_t + \frac{\kappa_I}{\zeta} \left(\frac{1 + \eta\omega}{1 + \theta\omega} \right) \varphi_{-I} p_{R,t} + \beta \mathbb{E}_t[\pi_{I,t+1}], \quad (14)$$

where $\varphi_{-H} = -(1 - \varphi)$, and $\varphi_{-L} = \varphi$. The constants κ_I , and ζ are defined in the appendix. Inflation in the aggregate price index, $\pi_t \equiv \log P_{t+1} - \log P_t$, is a weighted average of inflation in the two industries, and the difference in industry inflations captures changes in the relative price across time. It follows that the industry inflation equations (14) can be written as

$$\pi_t = \bar{\kappa} x_t + b_\varphi p_{R,t} + \beta \mathbb{E}_t[\pi_{t+1}], \quad (15)$$

and

$$b_R p_{R,t} = \underline{\kappa} x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}], \quad (16)$$

where $\bar{\kappa}$, b_φ , and b_R are constants defined in the appendix. Equations (15) and (16) summarize the optimality conditions for the production sector of the economy. They depend on the aggregate output gap, inflation and the relative price. It allows us to observe the link between real activity and inflation generated by price rigidities. The inflation level in the economy depends on the output gap, the relative price between the two goods and expected future inflation. In particular, it implies that the relative price affects production decisions in both industries. This effect will be important to understand the cross-section of stock returns in this economy.

3.3 Monetary Authority

We model a monetary authority that sets the level of a short-term nominal interest rate. For simplicity, we define the continuously compounded one-period nominal rate, $i_t \equiv \log(1 + R_{f,t}^s)$.

Monetary policy is described by the policy rule

$$i_t = \bar{i} + \iota_\pi \pi_t + \iota_x x_t + u_t,$$

where the interest rate is set responding to aggregate inflation, the output gap, and a policy shock u_t . The shock follows the process

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1}, \tag{17}$$

with $\varepsilon_u \sim \text{IIDN}(0,1)$. Policy shocks are the only source of uncertainty in the economy and, therefore, expected returns on financial assets only reflect compensations for this risk.

3.4 Equilibrium

The equilibrium processes for inflation and real activity at aggregate and industry levels depend on monetary policy. Their dynamics can be described in terms of reactions to policy shocks u_t , and the relative price $p_{R,t}$. The dependence of output on monetary policy is the result of price rigidities. Output is affected by policy shocks and the response to economic conditions in the policy rule. The relative price plays a role in equilibrium when the degree of price rigidity is different across industries. Since the real discount factor $M_{t,t+1}$ is the marginal rate of substitution of consumption, it is affected by monetary policy. Therefore, policy shocks represent a source of systematic risk whose price of risk depends on the policy rule. Production profits are also affected by the policy and thus expected returns contain a compensation for policy shocks. We characterize important macroeconomic and asset pricing aspects of the equilibrium in this section. The implications of equilibrium on stock returns is the main focus of section 4.

Allocations and prices are obtained by solving a system of rational expectations equations. These equations are the relevant optimality conditions for households and firms, and the monetary

policy rule presented above. The system can be summarized as

$$e^{-i_t} = \mathbb{E}_t [\exp(\log \beta - \gamma \Delta x_{t+1} - \pi_{t+1})], \quad (18)$$

$$\pi_t = \bar{\kappa} x_t + b_\varphi p_{R,t} + \beta \mathbb{E}_t [\pi_{t+1}], \quad (19)$$

$$b_R p_{R,t} = \underline{\kappa} x_t + p_{R,t-1} + \beta \mathbb{E}_t [p_{R,t+1}], \quad (20)$$

$$i_t = \bar{i} + \iota_x x_t + \iota_\pi \pi_t + u_t, \quad (21)$$

and $u_t = \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t},$

where equation (18) is the households' optimality condition (10), equations (19) and (20) are the profit maximization results (15) and (16), equation (21) is the monetary policy rule, and the last equation is the assumed process for the policy shocks. The market clearing conditions, $C_{I,t} = Y_{I,t}$, and $C_t = Y_t$, apply in equilibrium.

Appendix B shows that equilibrium dynamics for inflation, the relative price and the output gap depend on the lagged relative price and the policy shock. That is,

$$\pi_t = \bar{\pi} + \pi_p p_{R,t-1} + \pi_u u_t, \quad (22)$$

$$p_{R,t} = \bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t, \quad (23)$$

and $x_t = \bar{x} + x_p p_{R,t-1} + x_u u_t, \quad (24)$

where all coefficients ($\bar{\pi}, \pi_p, \pi_u, \bar{\rho}, \rho_p, \rho_u, \bar{x}, x_p, x_u$) depend on preference, production and policy parameters as described in the appendix.⁵

The real discount factor is obtained from equation (7) and the equilibrium process for the output gap in equation (24). The log of the real discount factor, $m_{t,t+1} \equiv \log M_{t,t+1}$, can be

⁵Similar processes are obtained for industry inflation and output gaps.

written in terms of the relative price and the policy shock as

$$m_{t,t+1} = \log \beta - \gamma x_p \Delta p_{R,t} + \gamma x_u (1 - \phi_u) u_t - \gamma x_u \sigma_u \varepsilon_{u,t+1},$$

where Δ is the difference operator. It follows that policy shocks are a source of risk in the real discount factor. The sensitivity of the marginal rate of substitution to the policy shocks provides us with their market price of risk, λ , given by

$$\lambda = \gamma x_u \sigma_u. \tag{25}$$

The market price of risk depends on the loading of the output gap on the policy shocks, x_u . This loading drives the conditional variance of output in the model. It depends on deep economic parameters and monetary policy. In particular, it is sensitive to the responses to inflation, ι_π , and the output gap, ι_x , in the policy rule. Figure 1 plots the market price of risk as a function of this responses.⁶ Weak responses to inflation and the output gap in monetary policy generate significant volatility in volatility and output which translate into a higher price of risk for policy shocks.

The one-period real rate from equation (9) is

$$1 + R_{f,t} = \frac{1}{\beta} \exp \left[-\frac{1}{2} \gamma^2 x_u^2 \sigma_u^2 + \gamma x_p \Delta p_{R,t} - \gamma x_u (1 - \phi_u) u_t \right].$$

The risk-free rate depends on expected consumption growth and thus is affected by the dynamics of the output gap. It follows that monetary policy becomes a determinant of the dynamics of the real risk-free rate.

⁶The figure shows the negative of the market price of policy shocks, which can be interpreted as the market price of risk for shocks to the output gap, since a positive policy shock has a negative effect on the output gap.

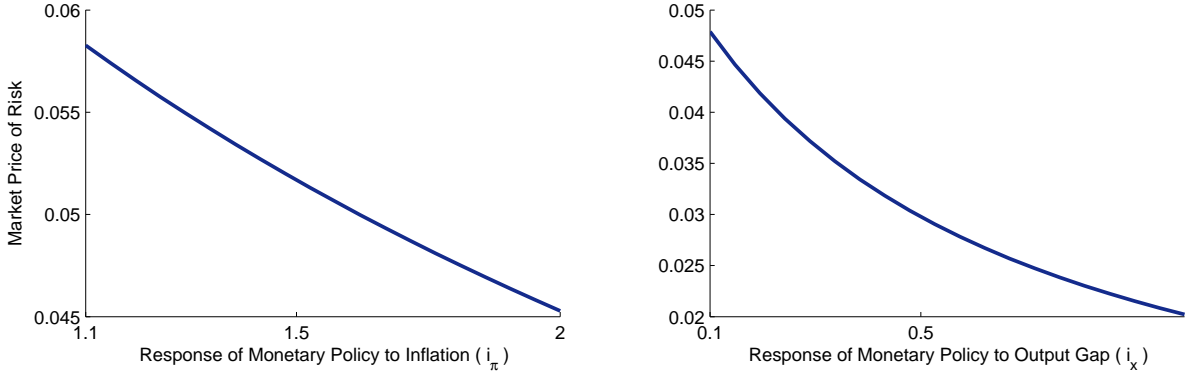


Figure 1: Market price of risk ($-\gamma x_u \sigma_u$) as a function of the response of monetary policy to inflation and the output gap. The baseline parameter values are presented in table 1 (except α_L and θ , which are $\alpha_L = \alpha_H = 0.66$, and $\theta = \eta = 2.5$).

4 Analysis of Stock Returns

This section presents the stock pricing implications of the model. The analysis shows that price rigidities generate a premium for policy shocks in expected stock returns. This premium depends on the reaction to economic conditions in the policy rule, and varies across the two industries when their price rigidities are different. We provide an explanation for the results based on the markup analysis in section 2.

We define industry stocks as financial claims on all future profits in the industry. The real stock price for industry I at time t , $S_{I,t}$ is given by

$$S_{\psi,I,t} = \mathbb{E}_t \left[\sum_{n=1}^{\infty} M_{t,t+n} \Psi_{I,t+n} \right], \quad (26)$$

with associated one-period real return, $R_{\psi,I,t+1}$, given by

$$1 + R_{\psi,I,t+1} = \frac{\Psi_{I,t+1} + S_{\psi,I,t+1}}{S_{\psi,I,t}}. \quad (27)$$

The aggregate stock market is a claim on aggregate profits, with price

$$S_{\psi,t} = S_{\psi,H,t} + S_{\psi,L,t}.$$

We analyze two simplified economies before turning to the analysis of an economy with different price rigidities across industries. The economies are one where all prices are perfectly flexible and one where the two industries have the same level of price rigidity. The economy with flexible prices provides us with a benchmark to understand the effects of price rigidities. The economy with the same level of price rigidity across industries allows us to understand the implications of price rigidities on the aggregate stock market. Finally, the economy with different degrees of price rigidities in the two industries allows us to characterize differences in expected returns for industry stocks.

4.1 Flexible-Price Economy

Monetary policy does not have any effects on real activity when prices are perfectly flexible ($\alpha_H = \alpha_L = 0$). Inflation is determined by the policy rule and reacts to policy shocks. However, production decisions are completely unlinked from inflation (zero output gap) and are insensitive to policy shocks. Aggregate output is constant, given by

$$Y^f = \left(\frac{A^{1+\omega}}{\mu} \right)^{\frac{1}{\omega+\gamma}},$$

where

$$\mu = \frac{\theta}{\theta - 1},$$

is the constant markup resulting from monopolistic competition, implying that labor income and profits are constant shares of production. In particular, real profits in industry I are given by

$$\Psi_{I,t}^f = \frac{\varphi_I}{\theta} Y^f,$$

and the real stock price is

$$S_{\psi,I,t}^f = \frac{\varphi_I}{\theta} \left(\frac{\beta}{1-\beta} \right) Y^f.$$

Since real stock prices are not affected by policy shocks, no compensations for risk are required to hold stocks. It follows that real stock returns in all industries are constant, equal to the real risk-free rate. That is,

$$R_{\psi,I,t} = R_{f,t} = \frac{1}{\beta},$$

for all I , and t .

4.2 Homogeneous Price Rigidity Across Industries and the Equity Premium

We analyze an economy where the two industries have the same degree of price rigidity ($\alpha_H = \alpha_L$) to gain insight into the effect of price rigidities on the equity premium. The two industries share the same dynamics, since the only difference between the two industries in the model is the degree of price rigidity. Inflation and output gaps at aggregate and industry levels are functions of policy shocks only (the relative price, $p_{R,t}$, does not play a role in equilibrium). Price rigidities and policy shocks generate distortions in aggregate output and markups. The distortions are reflected in the discount factor and production profits. As a result, claims on profits incorporate a compensation for policy shocks in expected returns. This compensation depends on the joint output and markup dynamics, and is sensitive to the parameters in the policy rule.

We assume the same elasticity of substitution of goods within an industry as across industries

($\theta = \eta$). Inflation in the aggregate price index (and in the two industries) is

$$\pi_t = \bar{\pi} + \pi_u u_t,$$

where

$$\begin{aligned} \bar{\pi} &= \frac{\kappa}{\kappa(1 - \iota_\pi) - \iota_x(1 - \beta)} \left[\log \beta + \bar{i} + \frac{1}{2} \left(\frac{\gamma}{\kappa} (1 - \beta \phi_u) + 1 \right)^2 \pi_u^2 \sigma_u^2 \right], \\ \text{and } \pi_u &= - \frac{\kappa}{\kappa(\iota_\pi - \phi_u) + \iota_x(1 - \beta) + \gamma(1 - \beta \phi_u)(1 - \phi_u)}, \end{aligned} \quad (28)$$

for $\kappa \equiv \bar{\kappa} = \kappa_H = \kappa_L$. The aggregate output gap is

$$x_t = \frac{1}{\kappa}(1 - \beta)\bar{\pi} + \frac{1}{\kappa}(1 - \beta \phi_u)\pi_u u_t.$$

We can obtain insightful information from equation (28). Under reasonable assumptions (i.e., $\iota_\pi > \phi_u$ and $\iota_x > 0$), inflation declines when a policy shock increases the nominal short-term rate ($\pi_u < 0$). More severe price rigidities (lower κ) imply higher variability in inflation, while a stronger response to economic conditions in the policy rule (higher ι_π , or ι_x) reduces it. The same applies to the output gap, since policy shocks generate a positive correlation between inflation and output. The price of risk from equation (25) becomes

$$\lambda = \frac{\gamma}{\kappa}(1 - \beta \phi_u)\pi_u \sigma_u,$$

making it clear that the magnitude of the market price of risk decreases as the response of monetary policy to inflation and the output gap increases. Policy shocks have a reduced effect on the marginal rate of substitution of consumption and investors require a lower compensation for holding assets exposed to this risk.

The real one-period rate is

$$1 + R_{f,t} = \frac{1}{\beta} \exp \left[-\frac{1}{2} \left(\frac{\gamma}{\kappa} (1 - \beta \phi_u) \pi_u \sigma_u \right)^2 - \frac{\gamma}{\kappa} (1 - \beta \phi_u) (1 - \phi_u) \pi_u u_t \right].$$

A policy shock that increases the nominal rate also increases the real rate. The effect on the real rate decreases for a stronger response to economic conditions in the policy rule.

Consider now the equity premium in this economy. It is convenient to start the analysis describing the aggregate markup, μ_t . This markup is

$$\mu_t = \mu \exp [-(\omega + \gamma)x_t], \tag{29}$$

as shown in appendix D. Time variation in the markup is the result of price rigidities and policy shocks. The markup is countercyclical with respect to the output gap. High markups are observed when output is low. That is, when the marginal rate of substitution of consumption is high. A policy shock that increases the real rate reduces output demand. When prices are perfectly flexible, producers reduce the price, and produce to obtain the optimal markup μ . In the presence of price rigidities, lower demand translates into lower real wages, which implies a higher markup than μ . We can also notice that the markup variability with respect to the output gap variability depends on the elasticities of intertemporal substitution of consumption and labor, γ^{-1} and ω^{-1} , respectively. The markup variability is high when intertemporal elasticities are low. Consumers with preferences for smooth consumption and labor are willing to work for lower wages during bad times, increasing the markup.

Countercyclical markups play then an important role in determining the properties of stock returns in this economy. In order to understand its effects we consider first simple claims that pay off profits only one period in the future to apply the analysis in section 2. This analysis is complemented with comparative statics for expected returns on stocks (claims on all future profits). The latter analysis is based on a numerical solution.

Let $R_{\psi,t+1}^{(1)}$ be the return of a claim on aggregate profits Ψ_{t+1} . Equations (1) and (2) in section 2 allow us to express the conditional expected excess return of this claim over the risk-free rate as

$$\frac{\mathbb{E}_t \left[1 + R_{\psi,t+1}^{(1)} \right]}{1 + R_{f,t}} = \frac{1 - v_t \exp [\text{cov}_t(m_{t,t+1}, \log \mu_{t+1})]}{1 - v_t} \exp [-\text{cov}_t(m_{t,t+1}, y_{t+1})],$$

where $1 - v_t$ is the ratio between the price of this claim and the price of a similar claim on aggregate output. The expected excess return of this rate depends on two opposite effects. First, the expected excess return of a claim on aggregate output over the risk-free rate, captured by the term $-\text{cov}_t(m_{t,t+1}, y_{t+1}) = \gamma \text{var}_t(x_{t+1}) > 0$. Second, the positive covariance between the real discount factor and the countercyclical markup, $\text{cov}_t(m_{t,t+1}, \log \mu_{t+1}) = \gamma(\omega + \gamma) \text{var}_t(x_{t+1})$. The second effect decreases the riskiness of claims of profits with respect to claims on output, given that markups are high during periods of high marginal utility. Depending on deep economic parameters and monetary policy, the markup effect can offset the output effect and the profit claim may involve lower expected returns than the risk-free rate. In this case, profit claims are a hedge for consumption risk, since the markup increase is such that profits increase when output declines. The riskiness of these claims can be characterized in terms of the expected future markup. It can be shown that expected returns on the profit claim are higher than the risk-free rate if ⁷

$$\mathbb{E}_t[\mu_{t+1}] < \left(\frac{e^{\gamma(1+\omega+\gamma)\text{var}_t(x_{t+1})} - 1}{e^{\gamma\text{var}_t(x_{t+1})} - 1} \right) \exp [(1 + \omega)(\omega + \gamma)\text{var}_t(x_{t+1})]. \quad (30)$$

If the expected markup is low enough (expected profits are a small fraction of expected output), the reduction in expected returns resulting from the countercyclical markup is not enough to offset the negative effect of policy shocks on output. The expected markup depends on preference and

⁷To see this, notice that

$$v_t = \frac{\mathbb{E}_t \left[M_{t,t+1} \frac{Y_{t+1}}{\mu_{t+1}} \right]}{\mathbb{E}_t [M_{t,t+1} Y_{t+1}]} = \mathbb{E}_t \left[\frac{1}{\mu_{t+1}} \right] \exp [-\text{cov}_t(m_{t,t+1} + y_{t+1}, \log \mu_{t+1})].$$

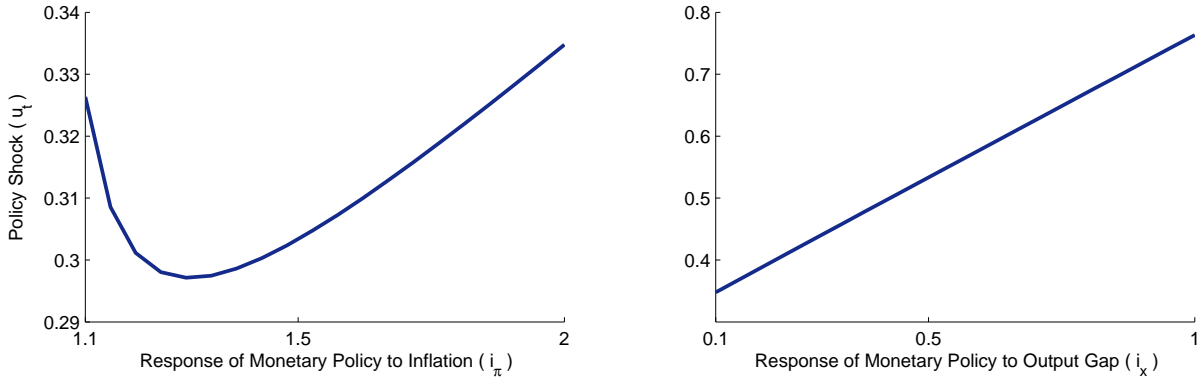


Figure 2: Policy shock u_t required to make the expected return of claims on profits Ψ_{t+1} equal to the risk-free rate, as a function of the response of monetary policy to inflation and the output gap. The baseline parameter values are presented in table 1 (except α_L and θ , which are $\alpha_L = \alpha_H = 0.66$, and $\theta = \eta = 2.5$).

production parameters such as the degree of price rigidity, elasticities of substitution of consumption, labor, and across goods. It also depends on the policy rule and its effects on the variability of the output gap. Figure 2 shows the value of the shock u_t at which the two effects are canceled out such that the expected return of the profit claim equals the risk-free rate.⁸ The figure presents this value for different responses to the output gap and inflation in the policy rule, based on a baseline parameterization. Policy shocks greater than this value generate a large expected markup that outweighs the negative output effect. Since policy shocks increase the marginal utility of consumption, claims on profits become hedging instruments for consumption and offer expected returns lower than the risk-free rate. The figure shows that the policy shock that makes the expected return equal to the risk-free rate increases monotonically as the response to the output gap in the policy rule increases. The value is not monotonically increasing with respect to the response to inflation in the policy rule. In summary, the analysis shows that the riskiness of claims on profits can be significantly affected by the systematic response to economic conditions in the policy rule, and the uncertainty in the policy.

⁸This value is obtained when the expected markup is equal to the right hand side of the inequality in (30), by expressing the expected markup in terms of the policy shock.

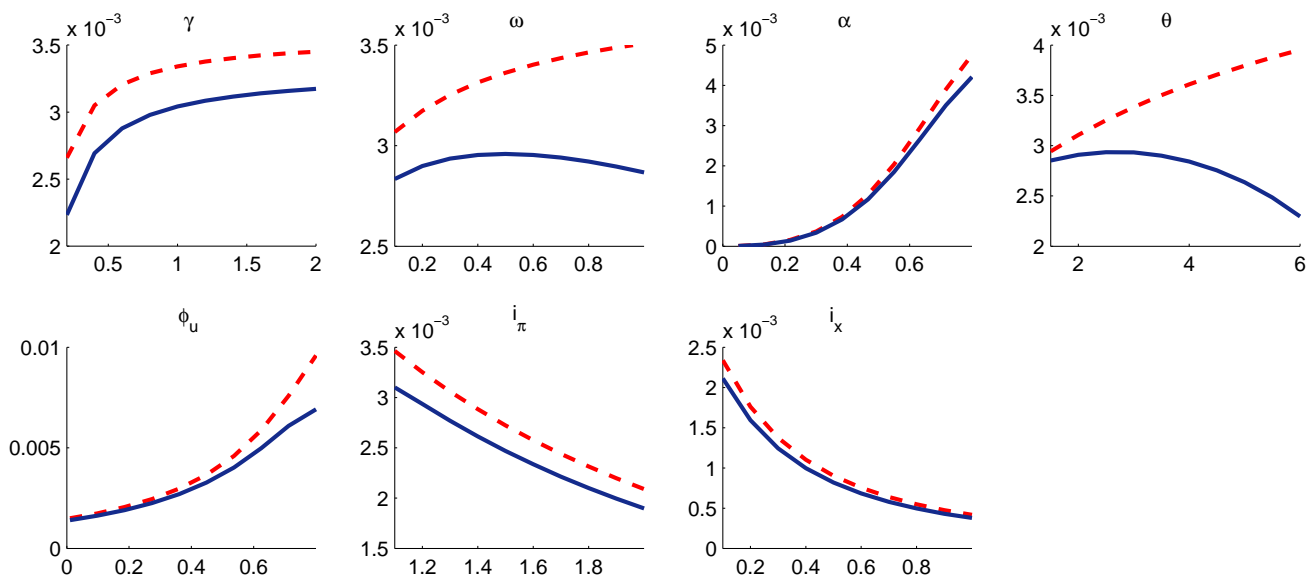


Figure 3: Expected excess stock return over the risk-free rate as a function of different parameters (solid line). The expected excess return is computed as $\frac{\mathbb{E}[1+R_{\psi,t+1}]}{\mathbb{E}[1+R_{f,t}]} - 1$. The dashed line is the expected excess return of claims on all future aggregate output. The baseline parameter values are presented in table 1 (except α_L and θ , which are $\alpha_L = \alpha_H = 0.66$, and $\theta = \eta = 2.5$).

We complete this section with an analysis of unconditional and conditional expected excess stock returns with respect to the risk-free rate. Figure 3 shows comparative statics for unconditional expected excess returns on stocks in figure. Stocks are claims on all future aggregate profits. The expected return on stocks were computed numerically as shown in appendix E. The figure shows that the expected excess returns on stocks are always lower than expected excess returns on claims on all future output. This is the result of the reduction of risk in profits from the counter-cyclical markup. However, the output effect outweighs the markup effect for this parameterization, and expected excess stock returns are positive. They increase with the degree of risk aversion, the degree of price rigidity and the persistence of the policy shock, since the distortions in output (and markups) caused by policy shocks increase. The opposite effect is observed for stronger responses to inflation and output in the policy rule, as the variability of the output gap declines. A lower elasticity of intertemporal substitution of labor (larger ω) can increase or decrease expected excess

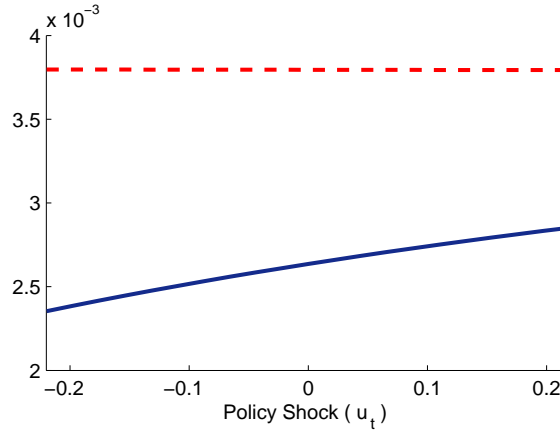


Figure 4: Conditional expected excess stock return over the risk-free rate as a function of policy shocks (solid line). The conditional expected excess return is computed as $\frac{\mathbb{E}_t[1+R_{\psi,t+1}]}{1+R_{f,t}} - 1$. The dashed line is the conditional expected excess return of claims on all future aggregate output. The parameter values are presented in table 1 (except α_L and θ , which are $\alpha_L = \alpha_H = 0.66$, and $\theta = \eta = 2.5$).

returns. For low ω 's, the higher riskiness of output is reflected in more risky profits. However, as the elasticity of substitution decreases, the markup effect tends to dominate. Households have stronger preferences for smooth labor supply over time and are willing to work for lower real wages during periods of high marginal utility, which expands markups.

Figure 4 plots conditional expected excess stock returns as function of the level of the policy shock. Expected excess returns for claims on all future output are almost constant. However, conditional expected excess returns on stocks are time-varying as a result of policy shocks affecting expected future markups. Positive policy shocks increase expected markups. Larger expected markups imply that a larger fraction of output will be distributed as profits (on average), which increases the correlation between profits and output. As a result, the riskiness of output has a stronger effect on the riskiness of profits, while the offsetting effect of the countercyclical markup decreases. Therefore, positive policy shocks increase the conditional expected excess return on stocks. Since marginal utility is high following a positive policy shock, expected excess stock returns are higher in states of the world with high marginal utility.

4.3 Heterogenous Price Rigidity Across Industries and the Cross-Section of Stock Returns

We turn now to the analysis of the economy where the two industries have different price rigidities ($\alpha_H \neq \alpha_L$). The main purpose is to understand differences in expected stock returns of industries with high and low price rigidities. We apply first the analysis in section 2 to simple claims on profits one period in the future, and complement it with an analysis for expected returns on stocks (claims on all future profits) relying on numerical solutions. Differences in expected returns across industries are explained by differences in industry outputs and markups resulting from policy shocks. Output in the industry where prices are more sticky (industry H) is more risky than in the industry with more flexible prices (industry L). On the other hand, markups in industry H expand relative to markups in industry L when policy shocks increase the marginal rate of substitution of consumption. The markup difference then acts as a hedge for profits in industry H with respect to industry L , and reduces the difference in riskiness in the two industries generated by differences in output. In particular we show that the markup effect can offset the output effect, such that expected returns on stocks in industry L can be higher than those of industry H .

Real profits in industry I , $\Psi_{I,t}$, can be written as

$$\Psi_{I,t} = \left(1 - \frac{1}{\mu_{I,t}}\right) Y_{I,t}^{real},$$

where $\mu_{I,t}$ is the industry markup, and $Y_{I,t}^{real}$ is the industry output in units of aggregate product. Appendix D shows that the markup is

$$\mu_{I,t} = \mu_t \exp [-(1 + \eta\omega)\varphi_{-IPR,t}],$$

and the real product is

$$Y_{I,t}^{real} = \frac{P_{I,t}}{P_t} Y_{I,t} = \varphi_I Y^f \exp [x_t + (\eta - 1)\varphi_{-I} p_{R,t}],$$

where μ_t is the aggregate markup as in (29). Differences in markups and real output in the two industries satisfy

$$\log \mu_{H,t} - \log \mu_{L,t} = (1 + \eta\omega)p_{R,t}, \quad (31)$$

and

$$y_{H,t}^{real} - y_{L,t}^{real} = \log\left(\frac{\varphi}{1 - \varphi}\right) - (\eta - 1)p_{R,t}, \quad (32)$$

respectively, where $y_{I,t}^{real} \equiv \log Y_{I,t}^{real}$. It follows that differences in real profits in the two industries can be explained in terms of the relative price.

The analysis in section 2 can be applied to understand the effects on expected returns. Consider a claim whose pay off is industry profits $\Psi_{I,t+1}$, with associated return $R_{\psi,I,t+1}^{(1)}$. Given the industry outputs and markups above, we can apply the approximation in equation (2) to obtain

$$\frac{\mathbb{E}_t \left[1 + R_{\psi,1,t+1}^{(1)} \right]}{\mathbb{E}_t \left[1 + R_{\psi,2,t+1}^{(1)} \right]} \approx \exp [-\gamma(\eta - 1)\text{cov}_t(x_{t+1}, p_{R,t+1})] \exp [\gamma(1 + \eta\omega)\text{cov}_t(x_{t+1}, p_{R,t+1})d_t],$$

where

$$d_t = \frac{\exp [\gamma(\omega + \gamma)\text{var}_t(x_{t+1})] v_t}{1 - \exp [\gamma(\omega + \gamma)\text{var}_t(x_{t+1})] v_t}.$$

The difference in expected stock returns in the two industries depends on two effects. First, the difference in expected returns on claims in industry output captured by $-\gamma(\eta - 1)\text{cov}_t(x_{t+1}, p_{R,t+1})$ and, second, an adjustment for the difference in markups captured by $\gamma(1 + \eta\omega)\text{cov}_t(x_{t+1}, p_{R,t+1})d_t$. Therefore, differences in expected stock returns can be explained as the result of differences in the covariance of the output gap and the product prices in the two industries, that is, the covariance of the output gap and the relative price, $\text{cov}_t(x_{t+1}, p_{R,t+1})$. This covariance is negative as the

Table 1: Baseline parameterization.

Parameter	Description	Value
A	Productivity	1
β	Subjective discount factor	0.974
γ	Inverse of EIS of consumption	0.7
ω	Inverse of EIS of labor	0.3
α_H	Price rigidity in industry H	0.66
α_L	Price rigidity in industry L	0.05
θ	Elasticity of substitution of differentiated goods	5.0
η	Elasticity of substitution of industry goods	2.5
ϕ_u	Autocorrelation of policy shock	0.4
σ_u	Conditional volatility of policy shock	0.045
$\bar{\tau}$	Constant in the policy rule	0.003
ν_π	Response to inflation in the policy rule	1.2
ν_x	Response to output gap in the policy rule	0
φ	Weight of industry H good in the basket	0.5

result of policy shocks. A positive policy shock reduces output demand and prices should decline. A higher price stickiness in industry H means a lower reduction in $p_{H,t}$ with respect to $p_{L,t}$, and thus an increase in the relative price $p_{R,t}$. Given this negative covariance, claims on output from industry H are more risky than those from industry L . However, markups in industry H expand with respect to those in industry L , reducing the riskiness of profits in industry H with respect to industry L . If the effect from the difference in markups is stronger than the effect from the difference in outputs, expected returns in the more flexible price industry are higher than those in the more rigid price industry. The magnitudes of the two effects depend on preference and production parameters, and the monetary policy rule.

We gain additional insights from numerical solutions for expected stock returns for the two industries. We conduct impulse-response and comparative statics analyses. The details of the numerical procedure are presented in appendix E. Table 1 shows the parameter values used in the exercise. Figure 5 shows impulse responses to a positive policy shock. This shock increases the real short-term rate, $r_t \equiv \log(1 + R_{f,t})$ and decreases output, implying an increase in the marginal

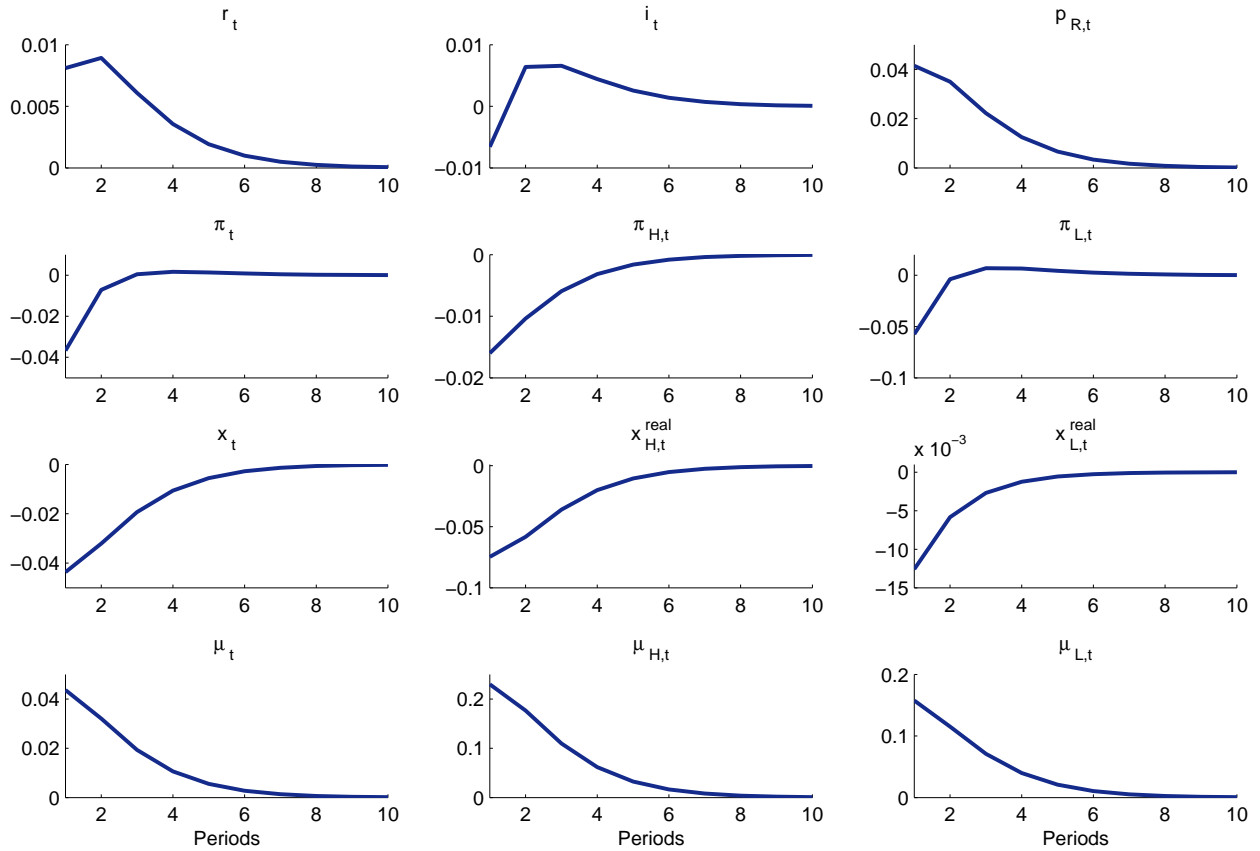


Figure 5: Impulse responses to a positive policy shock for different macroeconomic variables for the aggregate economy and the industries with high (H) and low (L) price rigidities. The parameter values are presented in table 1.

utility of consumption. Simultaneously, prices decline in both industries. The price decline in industry L is larger, increasing the relative price $p_{R,t}$. It translates into a real value of production in industry H more negatively affected than the real value of production in industry L . However, markups in the two industries expand. The expansion is larger in industry H than in industry L , which offsets the effect of differences in output. The net effect on stock returns can be observed in table 2. It shows the realized one-period returns for the two industries after the shock. The stock values for both industries decrease after the shock. However, profits (dividends) in both industries are high enough to provide positive one-period returns, and the return of the industry H stock is higher than the return of the industry L stock. It happens in a state of the world with high

Table 2: Realized industry stock returns after a positive policy shock.

Industry	Initial Stock Value	Final Stock Value	Dividend	Return
<i>H</i>	3.084	3.0345	0.1098	1.96%
<i>L</i>	3.503	3.4343	0.1069	1.09%

marginal utility of consumption (low output gap). Therefore, stocks of the industry with more sticky prices provide a hedge for consumption with respect to stocks of the industry with more flexible prices. Investors require then a higher expected return to hold stocks of industry *L*. The reduced riskiness of the industry *H* stock is the result of a markup effect that outweighs the output effect. The output effect can be observed from claims on real output in the two industries. A claim on real output in industry *H* has a lower return than a similar claim on real output in industry *L*, -2.61% and -2.15%, respectively. However, the markup in industry *H* is more countercyclical than the markup in industry *L*, which expands profits as fraction of output to reverse the output effect.

Figure 6 shows comparative statics for the difference in expected stock returns in the two industries. It allows us to make comparisons between the output and markup effects above. As the risk aversion coefficient increases (a lower elasticity of intertemporal substitution of consumption), the output effect becomes stronger and expected returns of industry *H* stock increase with respect to those of industry *L*. On the other hand, as the elasticity of intertemporal substitution of labor decreases (higher ω), the markup effect increases and the expected stock return of *L* increase with respect to *H*. An increase in the elasticity of substitution between industries (higher η) decreases the markup effect, increasing the expected return of industry *H* with respect to *L*. In addition, some comparisons can be made with respect to the magnitude of the difference in expected returns. A larger difference in price rigidity across industries, a higher persistence of the policy shock, or a higher elasticity of substitution within industry (higher θ) increase the difference in expected stock returns for the two industries. Finally, with respect to the response to economic conditions in the

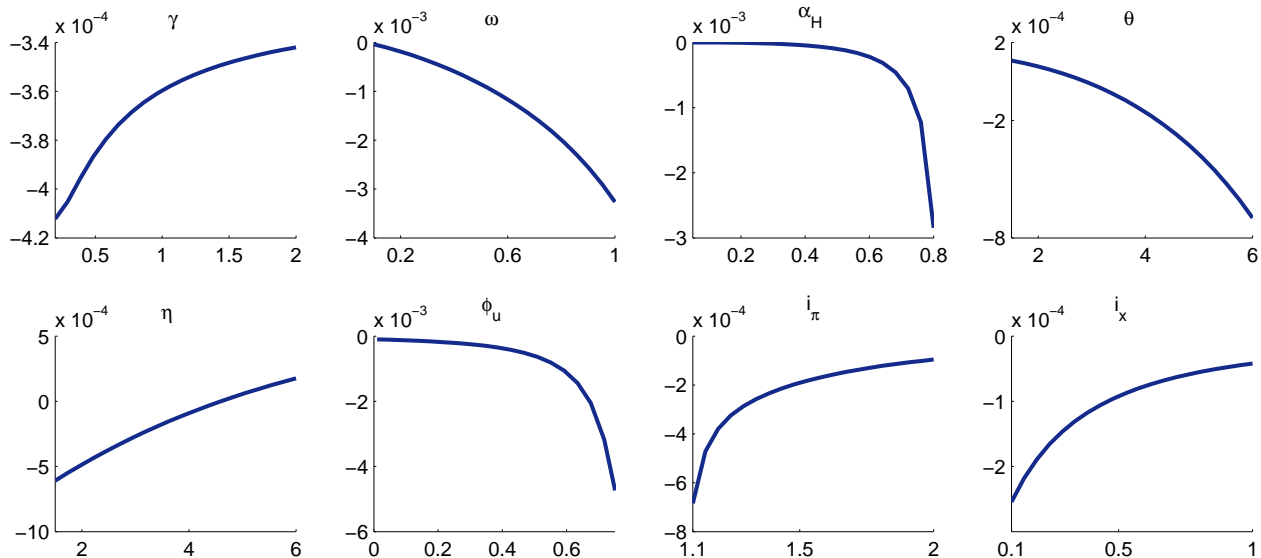


Figure 6: Expected excess return of industry H over industry L as a function of different parameters. The expected excess return is computed as $\frac{\mathbb{E}[1+R_{\psi,H,t+1}]}{\mathbb{E}[1+R_{\psi,L,t+1}]} - 1$. The baseline parameter values are presented in table 1.

policy rule, more aggressive responses to inflation and the output gap reduce the difference in expected returns. Policies that imply reduced distortions of policy shocks on production decrease the difference in output and markups across industries.

The effects of policy shocks on the conditional expected excess return of stocks in industry H over industry L can be observed in figure 7. Expected excess returns vary over time as a result of time-varying future expected markups in the two industries. This variation is driven by policy shocks and depends on the relative price. Positive policy shocks reduce the difference in expected returns in the two industries. A positive shock increases expected markups in the two industries, and thus the correlation between industry profits and output. It increases the positive effect of differences in output on the excess return, and decreases the negative effect of differences in markups. Similarly, the expected excess return increases with the relative price. A higher relative price reflects a larger difference in industry markups which should decrease the expected return in industry H with respect to industry L . However, a higher relative price also reflects a larger

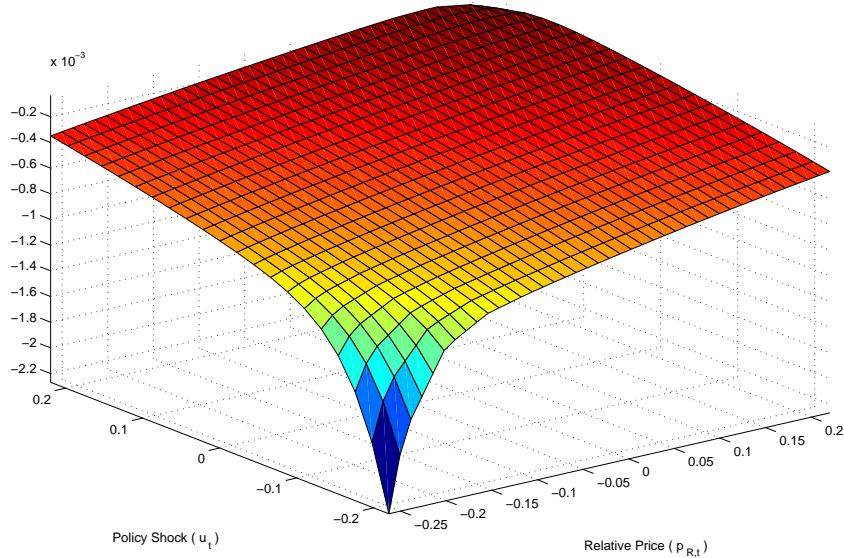


Figure 7: Conditional expected excess return of industry H over industry L as a function of the level of the policy shock and the relative price. The conditional expected excess return is computed as $\frac{\mathbb{E}_t[1+R_{\psi,H,t+1}]}{\mathbb{E}[1+R_{\psi,L,t+1}]} - 1$. The parameter values are presented in table 1.

difference in industry outputs which has the opposite effect. The second effect prevails in the numerical exercise, and the expected excess return increases with the relative price.

5 A First Approach to the Empirical Evidence

We compare in this section the realized returns of two stock portfolios with different implied product price rigidities. We compute the significance, the magnitude, and the sign of the difference in the return of these portfolios for historical periods with different monetary policy.⁹ The analysis suggests that portfolios of stocks with high price rigidities have a higher systematic risk than portfolios of stock with low price rigidities. The difference in systematic risk in the two portfolios declines for a period characterized by a more aggressive response to economic conditions in

⁹This analysis should not be seen as a formal test of the empirical predictions of the model. Such an analysis is beyond the scope of this paper and would require a joint examination of portfolio returns and monetary policy shocks.

monetary policy.

In order to construct the two portfolios of stocks with different price rigidities, we first assigned a degree of price rigidity to 35 industry portfolios based on the price rigidities reported by Bils and Klenow (2004).¹⁰ We sorted the 35 industry portfolios by their assigned price rigidity as shown in table 3, and construct two portfolios of industries with low and high price rigidities, respectively. The portfolio of industries with low price rigidities (portfolio L) is composed by the 17 industries with the lowest price rigidities (from Petroleum and Natural Gas to Electronic Equipment). The portfolio of industries with high price rigidities (portfolio H) has the 17 industries with the highest price rigidities (from Computer Software to Other). We excluded the Business Supplies industry to obtain two portfolios with the same number of industries. The average duration of prices in portfolios L and H are 3.1 and 8.6 months, respectively. We computed realized returns for the two portfolios using data for value-weighted and equal-weighted industry returns from the Center for Research in Security Prices (CRSP).

Table 4 reports regression results for the excess returns of portfolio H with respect to portfolio L . We refer to a claim on these excess returns as the “rigidity” portfolio. The analysis covers the sample period 1970 – 2006, and the two subsamples 1970 – 1980 and 1980 – 2006. The selection of the two subsamples is based on evidence from Clarida, Galí and Gertler (2000) suggesting that the interest-rate policy rule followed by the monetary authority has been more sensitive to inflation since 1980. Therefore, differences in the properties of excess returns for these two periods may provide an idea of the implications of different policy rules on the cross section of returns. The table shows a negative average return of the rigidity portfolio which is statistically insignificant. We capture the exposure of the returns of this portfolio to sources of systematic risk using CAPM, and

¹⁰Specifically, Bils and Klenow (2004) provide the monthly frequency of price changes for 350 categories of consumer goods and services comprising around 70% of consumer expenditures from 1995 to 1997. We assign price rigidities to industry portfolios by mapping SIC codes for product categories to the SIC codes corresponding to the 49-industry classification from Kenneth French’s web site. We were able to assign price rigidities from 330 product categories to 35 industries. The price rigidity within an industry is the weighted average of the price rigidities of all product categories in that industry. The weight for the product categories is also provided by Bils and Klenow (2004).

Fama and French (1993) regressions. The CAPM regressions show a significant positive covariance between the rigidity portfolio and the market portfolio. It suggests that stocks of industries with high price rigidities are riskier than stocks of industries with low price rigidities since the return of the market portfolio is low during periods of low economic growth. In the Fama-French regressions, the significance of the covariance of the rigidity portfolio and the market portfolio decreases, and the *SMB* and *HML* provide additional explanatory power. In particular, the rigidity portfolio has positive and negative loadings on the *SMB* and *HML* returns, respectively. The magnitudes of these loadings and the loadings on *RMRF* are larger for the 1970 – 1979 period than for the 1980 – 2006 period, as well as for the adjusted R^2 's. This can be interpreted as lower differences in the returns of portfolios H and L as the response to inflation in the policy rule increases.

6 Conclusions

This paper provides a theoretical framework for the analysis of the effects of monetary policy on asset returns. We use this framework to analyze the implications of monetary policy shocks on the equity premium and the cross-section of stock returns. In the presence of price rigidities, monetary policy shocks are a source of risk in both aggregate consumption and real profits in the production sector. Price rigidities then generate a compensation for policy shocks in the equity premium, and differences in price rigidities across industries entail cross-sectional differences in expected stock returns. Two opposite effects determine the stock return implications of policy shocks. At the aggregate level, these shocks simultaneously increase the riskiness of aggregate output and generate a countercyclical variation in the aggregate markup that acts as a consumption hedge. If the markup effect outweighs the output effect, policy shocks reduce the equity premium. In the cross-section, policy shocks make the output in more rigid price industries more risky than the output in more flexible price industries. However, this riskiness is reduced and can be reversed by a countercyclical markup in the more rigid price industry that is more sensitive to policy shocks

than the markup in the more flexible price industry. The impact of the markup effect is determined by the elasticities of intertemporal substitution of consumption and labor, and the elasticity of substitution across industry goods. The analysis also shows that policies that are more responsive to economic conditions reduce the effect of policy shocks on stock returns.

The analysis can be extended in at least two dimensions. First, monetary policy may also affect the response of the economy to other sources of risk such as productivity or markup shocks, and therefore the compensation for these risks in expected asset returns. In particular, markup shocks generate an interesting tradeoff between inflation and output stabilization for the monetary authority. Then, a monetary authority that places significant weight on price stability may increase the compensation for real risks in financial assets. Second, endogenizing price rigidities would help us understand the link between monetary policy, pricing policy and the cost of capital for firms.

Finally, the markup mechanism driving the results evidences the importance of the dynamics of production costs to understand asset pricing behavior. More research is needed on this front.

Table 3: Price rigidity for industry portfolios

This table reports the percentage of monthly price changes (δ) and the corresponding average duration of prices (in months) for 34 industry portfolios. The average duration is calculated as $-1/\log(1 - \delta/100)$. The industry numbers correspond to the classification for the 49 industry portfolios in Kenneth French's website.

Industry Number	Description	Number of Products	% Price Change	Price Duration
30	Petroleum and Natural Gas	7	73.29	0.76
41	Transportation	8	43.06	1.78
31	Utilities	4	42.72	1.79
1	Agriculture	11	41.40	1.87
35	Computer Hardware	2	37.49	2.13
23	Automobiles and Trucks	5	36.42	2.21
2	Food Product	71	33.69	2.43
10	Apparel	41	32.73	2.52
3	Candy and Soda	9	27.59	3.10
45	Banking	2	25.24	3.44
21	Machinery	3	21.46	4.14
5	Tobacco Products	2	21.24	4.19
9	Consumer Goods	44	20.89	4.27
6	Recreation	14	20.09	4.46
22	Electrical Equipment	1	19.4	4.64
4	Beer and Liquor	4	19.31	4.66
37	Electronic Equipment	1	17.80	5.1
39	Business Supplies	4	17.51	5.20
36	Computer Software	1	16.50	5.55
20	Fabricated Products	1	16.10	5.70
33	Personal Services	33	16.04	5.72
13	Pharmaceutical Products	2	15.92	5.77
16	Textiles	6	15.9	5.77
46	Insurance	2	15.44	5.96
44	Restaurants, Hotels, Motels	9	15.28	6.03
32	Communication	4	13.37	6.97
14	Chemicals	1	13.30	7.01
15	Rubber and Plastic Products	3	13.13	7.11
17	Construction Materials	8	11.82	7.95
7	Entertainment	5	9.70	9.80
12	Medical Equipment	5	9.56	9.96
8	Printing and Publishing	7	8.28	11.57
11	Healthcare	6	7.65	12.56
34	Business Services	3	7.28	13.23
49	Other	1	4.90	19.90

Table 4: Performance-attribution regressions for the price-rigidity portfolio

This table reports the average return, the CAPM and Fama-French regression analysis for the price-rigidity portfolio. The Fama-French regression is $R_{H,t} - R_{L,t} = constant + \beta_{RMRF} * RMRF_t + \beta_{SMB} * SMB_t + \beta_{HML} * HML_t + \epsilon_t$, where $R_{H,t}$ and $R_{L,t}$ are the monthly realized returns for the portfolios of industries with high and low price rigidities, respectively. The CAPM regression is the Fama-French regression assuming $\beta_{SMB} = \beta_{HML} = 0$. The excess return of the value-weighted market portfolio is $RMRF_t$, and the returns on the zero-investment factor-mimicking portfolios that capture size and book-to-market are SMB_t and HML_t , respectively. The t-statistics are presented in parenthesis.

	Value-weighted					Equal-weighted				
	Constant	$RMRF$	SMB	HML	Adj. R^2	Constant	$RMRF$	SMB	HML	Adj. R^2
1970 - 2006										
Average	-0.10 (0.40)					-0.05 (0.18)				
CAPM	0.00 (-1.95)	0.11 (6.37)			0.08	0.00 (-1.50)	0.11 (7.00)			0.10
Fama-French	0.00 (-1.77)	0.05 (3.01)	0.19 (8.38)	-0.05 (-2.02)	0.23	0.00 (-0.52)	0.03 (2.15)	0.18 (9.04)	-0.13 (-5.69)	0.31
1970 - 1979										
Average	-0.12 (0.16)					-0.13 (0.14)				
CAPM	0.00 (-0.77)	0.20 (5.16)			0.18	0.00 (-0.96)	0.08 (2.92)			0.06
Fama-French	0.00 (-1.02)	0.02 (0.72)	0.47 (11.2)	-0.19 (-3.94)	0.61	0.00 (-0.55)	-0.03 (-1.22)	0.28 (7.61)	-0.21 (-5.01)	0.42
1980 - 2006										
Average	-0.09 (0.26)					-0.02 (0.06)				
CAPM	0.00 (-1.67)	0.07 (3.84)			0.04	0.00 (-1.23)	0.12 (6.46)			0.11
Fama-French	0.00 (-1.00)	0.03 (1.26)	0.08 (3.25)	-0.08 (-2.75)	0.11	0.00 (-0.4)	0.05 (2.81)	0.16 (6.49)	-0.11 (-3.84)	0.29

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Appendix

A Profit Maximization under Price Rigidities

Consider the Dixit-Stiglitz aggregate (5) as a production function, and a competitive “producer” of the industry good facing the problem

$$\max_{\{C_{I,t}(j)\}} P_{I,t}C_{I,t} - \int_0^1 P_{I,t}(j)C_{I,t}(j)dj$$

subject to (5). Solving the problem, we find the demand function

$$P_{I,t}(j) = P_{I,t} \left(\frac{C_{I,t}(j)}{C_{I,t}} \right)^{-1/\theta} \quad (33)$$

The zero-profit condition implies

$$P_{I,t}C_{I,t} = \int_0^1 P_{I,t}(j)C_{I,t}(j)dj = \int_0^1 P_{I,t}C_{I,t} \left(\frac{P_{I,t}(j)}{P_{I,t}} \right)^{-\theta} dj.$$

Solving for $P_{I,t}$, it follows that

$$P_{I,t} = \left[\int_0^1 P_{I,t}(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \quad (34)$$

which can be written as the demand function for each differentiated good in sector I

$$C_{I,t}(j) = \left(\frac{P_{I,t}(j)}{P_{I,t}} \right)^{-\theta} C_{I,t}. \quad (35)$$

Similarly, we can solve the profit maximization problem of the final good industry, which use goods from industry H and L as inputs. The demand function for industry I good is

$$C_{I,t} = \varphi_I \left(\frac{P_{I,t}}{P_t} \right)^{-\eta} C_t \quad (36)$$

where P_t is the final good price, defined as the aggregate price index. The zero profit condition of the final goods production implies

$$P_t = \left[\varphi P_{H,t}^{1-\eta} + (1-\varphi)P_{L,t}^{1-\eta} \right]^{1/(1-\eta)}.$$

Notice that these relations imply that consumption in both sectors is related by

$$C_{H,t} = \frac{\varphi}{1-\varphi} \left(\frac{P_{H,t}}{P_{L,t}} \right)^{-\eta} C_{L,t}.$$

Therefore, when prices are flexible, prices of the sector goods are the same and consumptions in the two sectors are proportional.

The profit maximization problem (12) is solved relying on a linear approximation around a “steady state”. The

steady state is defined as the solution of the profit maximization problem in an economy with perfectly flexible prices. It is convenient to analyze this problem for the hypothetical flexible economy first and then show the solution for the actual economy.

$$\max_{\{P_{I,t}(j)\}} P_{I,t}(j)Y_{I,t}^f(j) - w_{I,t}(j)N_{I,t}(j)$$

subject to (33) and (13). The solution to this problem implies

$$\frac{P_{I,t}(j)}{P_t} = \mu s_{I,t}(j)$$

where the markup $\mu = \frac{\theta}{\theta-1}$ over the real marginal cost $s_{I,t}(j) \equiv \frac{1}{P_t} \frac{\partial(w_{I,t}(j)N_{I,t}(j))}{\partial Y_{I,t}(j)}$ is the result of monopolistic power. By using the production function (13) and the marginal rate of substitution (11) we can write the real marginal production cost as

$$s_{I,t}(j) = \frac{\varphi_I^{-\omega}}{Y_{I,t}(j)} \left(\frac{Y_{I,t}(j)}{A} \right)^{1+\omega} Y_t^\gamma. \quad (37)$$

Since prices are flexible and firms are identical, $P_t(j) = P_t$, and $Y_t(j) = Y_t$. As a result, production in the flexible-price economy can be written as

$$y_t^f = \log Y_t^f = \frac{1}{\omega + \gamma} [(1 + \omega) \log A - \log \mu]. \quad (38)$$

Since the level of production is a constant, we drop the subscript t in Y_t^f . The real wage in industry I is the same for every firm

$$\frac{w_{I,t}}{P_t} = \varphi_I^{-\omega} A^{-\omega} Y_{I,t}^\omega C_t^\gamma = A^{-\omega} (Y^f)^{\omega+\gamma}.$$

The real profit of industry I is then given by

$$\begin{aligned} \Psi_I^f &= \varphi_I Y^f \left(1 - \frac{1}{\mu} \right) \\ &= \frac{\varphi_I}{\theta} Y^f \end{aligned}$$

The flexible-price output provides us with a ‘‘point’’ to approximate the solution to the profit maximization problem in the sticky price economy.

Let $M_{t,T}^S = \beta^{T-t} \Lambda_T$, and $S_{I,t} = P_t s_{I,t}$. Consider the derivative

$$\frac{\partial \Psi_{I,T|t}(j)}{\partial P_{I,t}(j)} = Y_{I,T|t}(j) \frac{1-\theta}{P_{I,t}(j)} [P_{I,t}(j) - \mu S_{I,T|t}(j)].$$

Therefore, the first order condition to the profit maximization problem (12) is

$$\mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t}(j) P_{I,t}^*(j) \right] = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t}(j) \mu S_{I,T|t}(j) \right]. \quad (39)$$

Since all producers in industry I who can change prices at t face the same optimization problem, $Y_{I,T|t}(j) = Y_{I,T|t}$, $P_{I,t}^*(j) = P_{I,t}^*$ and $S_{I,T|t}(j) = S_{I,T|t}$. Applying the Taylor expansion $a_t b_t = \bar{a}\bar{b} + \bar{b}(a_t - \bar{a}) + \bar{a}(b_t - \bar{b})$ to both sides of the equation around a steady-state with $\bar{P} = \mu\bar{S}$, we have for the left-hand side of the equation

$$\begin{aligned} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t} P_{I,t}^* \right] &= \bar{\Lambda Y} \bar{P} \sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} + \bar{P} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} (\Lambda_T Y_{I,T|t} - \bar{\Lambda Y}) \right] \\ &+ \bar{\Lambda Y} (P_{I,t}^* - \bar{P}) \sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t}, \end{aligned}$$

and for the right-hand side

$$\begin{aligned} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t} \mu S_{I,T|t} \right] &= \mu \bar{\Lambda Y} \bar{S} \sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} + \mu \bar{S} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} (\Lambda_T Y_{I,T|t} - \bar{\Lambda Y}) \right] \\ &+ \mu \bar{\Lambda Y} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} (S_{I,T|t} - \bar{S}) \right]. \end{aligned}$$

Noting that the first and second terms in both sides of the equation are the same, equation (39) becomes

$$\frac{1}{(1 - \alpha_I \beta)} P_{I,t}^* = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \mu S_{I,T|t} \right].$$

Since $S_{T|t} = s_{T|t} P_T$, replacing equation (37) in the equation above and re-arranging terms, we obtain

$$\frac{1}{(1 - \alpha_I \beta)} (P_{I,t}^*)^{1+\theta\omega} = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \mu P_T^{1+\theta\omega} Y_T^{\omega+\gamma} A^{-(1+\omega)} \right].$$

Dividing by $\bar{P}^{1+\theta\omega}$, the equation can be written in terms of the output gap $x_t = y_t - y_t^f$ as

$$\frac{1}{(1 - \alpha_I \beta)} \left(\frac{P_{I,t}^*}{\bar{P}} \right)^{1+\theta\omega} = e^{(\omega+\gamma)x_t} \left(\frac{P_t}{\bar{P}} \right)^{1+\theta\omega} + \frac{\alpha_I \beta}{1 - \alpha_I \beta} \mathbb{E}_t \left[\left(\frac{P_{I,t+1}^*}{\bar{P}} \right)^{1+\theta\omega} \right].$$

Letting $p_{I,t}^* = \log \frac{P_{I,t}^*}{\bar{P}}$ and using the approximation $e^x \approx 1 + x$, we obtain

$$\frac{1}{(1 - \alpha_I \beta)} (1 + (1 + \theta\omega)p_{I,t}^*) = 1 + (\omega + \gamma)x_t + (1 + \theta\omega)p_t + \frac{\alpha_I \beta}{1 - \alpha_I \beta} \mathbb{E}_t [1 + (1 + \theta\omega)p_{I,t+1}^*] \quad (40)$$

that simplifies to

$$p_{I,t}^* = \frac{\omega + \gamma}{1 + \theta\omega} x_t + p_t + \frac{\alpha_I \beta}{1 - \alpha_I \beta} \mathbb{E}_t [p_{I,t+1}^* - p_{I,t}^*]. \quad (41)$$

Since there are infinitely many firms in each industry, at each period, a measure α_I of firms will keep the last period's price and a measure $1 - \alpha_I$ of firms will set a new price by solving the above maximization problem. The

aggregate price index for industry I is

$$P_{I,t} = \left[(1 - \alpha_I) (P_{I,t}^*)^{1-\theta} + \alpha_I P_{I,t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

A first order Taylor approximation of the price index results in

$$p_{I,t} = (1 - \alpha_I) p_{I,t}^* + \alpha_I p_{I,t-1}.$$

It implies

$$p_{I,t}^* = \frac{\alpha_I}{1 - \alpha_I} \pi_{I,t} + p_{I,t}, \quad \text{and} \quad p_{I,t+1}^* - p_{I,t}^* = \frac{1}{1 - \alpha_I} \pi_{I,t+1} - \frac{\alpha_I}{1 - \alpha_I} \pi_{I,t}.$$

Replacing these equations in equation (41), we obtain

$$\pi_{I,t} = \kappa_I x_t + \frac{\kappa_I}{\zeta} (p_t - p_{I,t}) + \beta \mathbb{E}_t[\pi_{I,t+1}],$$

where $\kappa_I \equiv \frac{(1-\alpha_I\beta)(1-\alpha_I)}{\alpha_I} \zeta$ and $\zeta \equiv \frac{\omega+\gamma}{1+\theta\omega}$. We can write

$$p_t - p_{I,t} = \varphi_{-I} p_{R,t}$$

where $\varphi_{-H} \equiv -(1 - \varphi)$ and $\varphi_{-L} = \varphi$.

Equation (14) can be written in terms of aggregate inflation, the output gap and the relative price. Inflation in the aggregate price index, $\pi_t \equiv \log P_{t+1} - \log P_t$, can be written in terms of industry inflations as

$$\pi_t = \varphi \pi_{H,t} + (1 - \varphi) \pi_{L,t}.$$

As a result, by adding up the two equations (weighted by the industry weights) we obtain

$$\pi_t = \bar{\kappa} x_t + b_\varphi p_{R,t} + \beta \mathbb{E}_t[\pi_{t+1}].$$

where

$$\bar{\kappa} = \varphi \kappa_H + (1 - \varphi) \kappa_L, \quad \underline{\kappa} = \kappa_H - \kappa_L \quad \text{and} \quad b_\varphi = -\frac{\varphi(1-\varphi)}{\zeta} \left(\frac{1+\eta\omega}{1+\theta\omega} \right) \underline{\kappa}.$$

Therefore, if the degree of price rigidities in the two industries is the same ($\underline{\kappa} = 0$), aggregate inflation does not depend on the relative price between the two industries. In order to obtain an expression for the evolution of the relative price, we can subtract one of the equations (14) from the other one and obtain

$$b_R p_{R,t} = \underline{\kappa} x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}],$$

where

$$b_R = 1 + \beta + \frac{1}{\zeta} \left(\frac{1+\eta\omega}{1+\theta\omega} \right) [(1-\varphi)\kappa_H + \varphi\kappa_L].$$

This equation describes the evolution of the relative price in terms of the output gap, the one-period lag and the expected future relative prices.

B Equilibrium

Finding the equilibrium of the economy involves solving the system of equations

$$\begin{aligned}
e^{-i_t} &= \mathbb{E}_t [\exp(\log \beta - \gamma(\Delta y^f + \Delta x_{t+1}) - \pi_{t+1})], \\
\pi_t &= \bar{\kappa} x_t + b_\varphi p_{R,t} + \beta \mathbb{E}_t[\pi_{t+1}], \\
b_R p_{R,t} &= \underline{\kappa} x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}], \\
i_t &= \bar{i} + \iota_\pi \pi_t + \iota_x x_t + u_t, \\
\text{and } u_t &= \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t},
\end{aligned}$$

where $b_\varphi = -\frac{\varphi(1-\varphi)}{\zeta} \left(\frac{1+\eta\omega}{1+\theta\omega} \right) \underline{\kappa}$, $\bar{\kappa} = \varphi\kappa_H + (1-\varphi)\kappa_L$, $\underline{\kappa} = \kappa_H - \kappa_L$ and

$$b_R = 1 + \beta + \frac{1}{\zeta} \left(\frac{1+\eta\omega}{1+\theta\omega} \right) [(1-\varphi)\kappa_H + \varphi\kappa_L].$$

Equation (19) can be written as

$$x_t = \frac{1}{\bar{\kappa}} [\pi_t - b_\varphi p_{R,t} - \beta \mathbb{E}_t[\pi_{t+1}]] \quad (42)$$

and its first difference as

$$\Delta x_{t+1} = \frac{1}{\bar{\kappa}} [\Delta \pi_{t+1} - b_\varphi \Delta p_{R,t+1} - \beta (\mathbb{E}_{t+1}[\pi_{t+2}] - \mathbb{E}_t[\pi_{t+1}])]. \quad (43)$$

Replacing (42) in (20) we obtain

$$b_R p_{R,t} - p_{R,t-1} = \mathbb{K} (\pi_t - b_\varphi p_{R,t} - \beta \mathbb{E}_t[\pi_{t+1}]) + \beta \mathbb{E}_t[p_{R,t+1}], \quad (44)$$

where $\mathbb{K} = \frac{\underline{\kappa}}{\bar{\kappa}}$.

We can guess solutions for inflation and the relative price of the form

$$\pi_t = \bar{\pi} + \pi_p p_{R,t-1} + \pi_u u_t, \quad \text{and} \quad p_{R,t} = \bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t,$$

respectively. Replacing this solution in equation (44) and matching coefficients we obtain the sub-system of equations

$$b_\pi \bar{\rho} = \mathbb{K}(1-\beta)\bar{\pi} + \beta \bar{\rho}, \quad (45)$$

$$b_\pi \rho_p = 1 + \mathbb{K} \pi_p, \quad (46)$$

$$b_\pi \rho_u = \mathbb{K}(1-\beta\phi_u)\pi_u + \beta\rho_u\phi_u, \quad (47)$$

where

$$b_\pi = b_R + b_\varphi \mathbb{K} + \beta \mathbb{K} \pi_p - \beta \rho_p.$$

To complete the system of equations, replace (43) and (21) in (18). The guessed solutions imply log-normal

distributions for all variables and, therefore, we obtain

$$\begin{aligned}
-\bar{i} - \iota_\pi \pi_t - \iota_x x_t - u_t &= \log \beta - \frac{\gamma}{\bar{\kappa}} [(\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p)(\bar{\rho} + (\rho_p - 1)p_{R,t-1} + \rho_u u_t) \\
&\quad - (1 - \phi_u)(\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) u_t] \\
&\quad - \bar{\pi} - \pi_p(\bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t) - \pi_u \phi_u u_t \\
&\quad + \frac{1}{2} \text{var}_t \left(\frac{\gamma}{\bar{\kappa}} (\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) u_{t+1} + \pi_u u_{t+1} \right). \tag{48}
\end{aligned}$$

Matching coefficients we obtain the sub-system

$$\begin{aligned}
-\bar{i} - \iota_\pi \bar{\pi} - \frac{\iota_x}{\bar{\kappa}} [(1 - \beta)\bar{\pi} - (b_\varphi + \beta \pi_p)\bar{\rho}] &= \log \beta - \bar{\pi} - \frac{\gamma}{\bar{\kappa}} (\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p) \bar{\rho} - \pi_p \bar{\rho} \\
&\quad + \frac{1}{2} \left(\frac{\gamma}{\bar{\kappa}} (\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) + \pi_u \right)^2 \sigma_u^2, \tag{49}
\end{aligned}$$

$$-\iota_\pi \pi_p - \frac{\iota_x}{\bar{\kappa}} [\pi_p - (b_\varphi + \beta \pi_p) \rho_p] = \frac{\gamma}{\bar{\kappa}} (\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p) (1 - \rho_p) - \pi_p \rho_p, \tag{50}$$

$$\begin{aligned}
-\iota_\pi \pi_u - \frac{\iota_x}{\bar{\kappa}} [(1 - \beta \phi_u) \pi_u - (b_\varphi + \beta \pi_p) \rho_u] - 1 &= -\frac{\gamma}{\bar{\kappa}} (\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p) \rho_u \\
&\quad + \frac{\gamma}{\bar{\kappa}} (1 - \phi_u) (\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) \\
&\quad - \pi_p \rho_u - \pi_u \phi_u. \tag{51}
\end{aligned}$$

The complete system is given by equations (45)-(47) and (49)-(51). This system allows us to obtain the equilibrium parameters $\{\bar{\pi}, \pi_p, \pi_u, \bar{\rho}, \rho_p, \rho_u\}$. Notice that equations (46) and (50) only depend on π_p and ρ_p . Therefore, we can use these two equations to solve for these two parameters. After some algebra manipulations we obtain

$$\begin{aligned}
&\gamma \beta^2 \rho_p^4 - \beta [\bar{\kappa} + \gamma(1 + \beta + b_R) + \beta \iota_x] \rho_p^3 \\
&\quad + [\gamma(\beta + b_R + \beta(1 + b_R)) + \bar{\kappa} b_R + b_\varphi \bar{\kappa} + \iota_\pi \bar{\kappa} \beta + \iota_x \beta(1 + b_R)] \rho_p^2 \\
&\quad - [\bar{\kappa} + \iota_\pi (\bar{\kappa} b_R + b_\varphi \bar{\kappa}) + \iota_x (b_R + \beta) + \gamma(1 + b_R + \beta)] \rho_p + \iota_\pi \bar{\kappa} + \gamma + \iota_x = 0.
\end{aligned}$$

We choose the solution for this equation where $0 \leq \rho_p < 1$. The coefficient π_p can be obtained from

$$\pi_p = \frac{b_\varphi \rho_p [\gamma(1 - \rho_p) + \iota_x]}{[\gamma(1 - \rho_p) + \iota_x](1 - \beta \rho_p) + \bar{\kappa}(\iota_\pi - \rho_p)}.$$

Using equations (47) and (51) we find ρ_u and π_u . The sensitivity of inflation to the policy shock solves

$$\begin{aligned}
\pi_u &= \left[\bar{\kappa}^2 (\phi_u - \iota_\pi) - (\gamma(1 - \phi_u) + \iota_x) \bar{\kappa} (1 - \beta \phi_u) + \frac{1 - \beta \phi_u}{b_\pi - \beta \phi_u} \bar{\kappa} \right. \\
&\quad \left. \times (\pi_p (\gamma + \bar{\kappa}) + (b_\varphi + \beta \pi_p) (\gamma(1 - \rho_p - \phi_u) + \iota_x)) \right]^{-1} \bar{\kappa}^2
\end{aligned}$$

and the sensitivity of the relative price to policy shocks is

$$\rho_u = \frac{\mathbb{K}}{b_\pi - \beta \phi_u} (1 - \beta \phi_u) \pi_u.$$

From equations (45) and (49) we find $\bar{\rho}$ and $\bar{\pi}$. The constants are

$$\begin{aligned}\bar{\pi} &= \left[1 - \iota_\pi - \frac{\iota_x}{\bar{\kappa}}(1 - \beta) + \frac{\mathbb{K}}{b_\pi - \beta}(1 - \beta) \left(\frac{\gamma}{\bar{\kappa}}(\pi_p - b_\varphi \rho_p - \beta \pi_p \rho_p) + \pi_p + (b_\varphi + \beta \pi_p) \frac{\iota_x}{\bar{\kappa}} \right) \right]^{-1} \\ &\times \left[\bar{i} + \log \beta + \frac{1}{2} \left(\frac{\gamma}{\bar{\kappa}}(\pi_u - b_\varphi \rho_u - \beta \pi_p \rho_u - \beta \pi_u \phi_u) + \pi_u \right)^2 \sigma_u^2 \right]\end{aligned}$$

and

$$\bar{\rho} = \frac{\mathbb{K}}{b_\pi - \beta}(1 - \beta)\bar{\pi}.$$

The solution for the output gap is given by

$$x_t = \bar{x} + x_p p_{t-1} + x_u u_t,$$

where

$$\begin{aligned}\bar{x} &= \frac{1}{\bar{\kappa}} [(1 - \beta)\bar{\pi} - (b_\varphi + \beta \pi_p)\bar{\rho}], \\ x_p &= \frac{1}{\bar{\kappa}} [\pi_p - (b_\varphi + \beta \pi_p)\rho_p], \\ x_u &= \frac{1}{\bar{\kappa}} [\pi_u - (b_\varphi + \beta \pi_p)\rho_u - \beta \phi_u \pi_u].\end{aligned}$$

C Inflation in Individual Industries

We can write the inflation within industry I as a function of the state variables:

$$\pi_{I,t} = \bar{\pi}_I + \pi_{I,p} p_{R,t-1} + \pi_{I,u} u_t. \quad (52)$$

We know that the first order Taylor expansion of the relative price relation is

$$p_t - p_{I,t} = \varphi_{-I} p_{R,t}$$

and the inflation in sector I is

$$\pi_{I,t} = \kappa_I x_t + \frac{\kappa_I}{\zeta} \left(\frac{1 + \eta\omega}{1 + \theta\omega} \right) \varphi_{-I} p_{R,t} + \beta \mathbb{E}_t [\pi_{I,t+1}].$$

Combined with the equilibrium conditions in Section 3.4, we find the coefficients for industry inflations as

$$\begin{aligned}\bar{\pi}_I &= \frac{\kappa_I}{1 - \beta} \left[\bar{x} + \frac{\varphi_{-I} \bar{\rho}}{\zeta(1 - \beta \rho_p)} \left(\frac{1 + \eta\omega}{1 + \theta\omega} \right) + \frac{\beta \bar{\rho} x_p}{1 - \beta \rho_p} \right], \\ \pi_{I,p} &= \frac{\kappa_I}{1 - \beta \rho_p} \left[x_p + \frac{\varphi_{-I} \rho_p}{\zeta} \left(\frac{1 + \eta\omega}{1 + \theta\omega} \right) \right], \\ \pi_{I,u} &= \frac{\kappa_I}{1 - \beta \phi_u} \left[x_u + \frac{\varphi_{-I} \rho_u}{\zeta(1 - \beta \rho_p)} \left(\frac{1 + \eta\omega}{1 + \theta\omega} \right) + \frac{\beta \rho_u x_p}{1 - \beta \rho_p} \right].\end{aligned}$$

D Real Labor Income and Real Profits

We denote by $W_{I,t}$ the real labor income at time t in industry I , given by

$$W_{I,t} = \int_0^1 \frac{w_{I,t}(j)}{P_t} N_{I,t}(j) dj.$$

Using equation (11), (35), and (36), real labor income can be written as

$$W_{I,t} = \frac{\varphi_I Y_t^{1+\gamma+\omega}}{A^{1+\omega}} \left(\frac{P_{I,t}}{P_t} \right)^{-\eta(1+\omega)} \int_0^1 \left(\frac{P_{I,t}(j)}{P_{I,t}} \right)^{-\theta(1+\omega)} dj.$$

We can substitute the output under flexible prices

$$\left(Y_t^f \right)^{\omega+\gamma} = \mu^{-1} A^{1+w},$$

to obtain

$$\begin{aligned} W_{I,t} &= \frac{\varphi_I Y_t^{1+\omega+\gamma}}{\mu (Y^f)^{\omega+\gamma}} \left(\frac{P_{I,t}}{P_t} \right)^{-\eta(1+\omega)} \int_0^1 \left(\frac{P_{I,t}(j)}{P_{I,t}} \right)^{-\theta(1+\omega)} dj \\ &= W^f X_t^{1+\omega+\gamma} \left(\frac{P_{I,t}}{P_t} \right)^{-\eta(1+\omega)} \int_0^1 \left(\frac{P_{I,t}(j)}{P_{I,t}} \right)^{-\theta(1+\omega)} dj. \end{aligned}$$

where $W^f = \frac{\varphi_I}{\mu} Y^f$ is the real labor income under flexible prices.

Decomposing the last term in the labor income equation we obtain

$$\begin{aligned} \int_0^1 \left(\frac{P_{I,t}(j)}{P_{I,t}} \right)^{-\theta(1+\omega)} dj &= \int_{j \in (1-\alpha_I)} \left(\frac{P_{I,t}^*}{P_{I,t}} \right)^{-\theta(1+\omega)} dj + \int_{j \in \alpha_I} \left(\frac{P_{I,t-1}(j)}{P_{I,t}} \right)^{-\theta(1+\omega)} dj \\ &= (1-\alpha_I) e^{-\theta(1+\omega)(p_{I,t}^* - p_{I,t})} + \alpha_I e^{-\theta(1+\omega)(p_{I,t-1} - p_{I,t})}. \end{aligned}$$

A first order Taylor approximation results in

$$\begin{aligned} \int_0^1 \left(\frac{P_{I,t}(j)}{P_{I,t}} \right)^{-\theta(1+\omega)} dj &\approx 1 - (1-\alpha_I)\theta(1+\omega)(p_{I,t}^* - p_{I,t}) - \alpha_I\theta(1+\omega)(p_{I,t-1} - p_{I,t}) \\ &= 1 - \theta(1+\omega) [(1-\alpha_I)p_{I,t}^* + \alpha_I p_{I,t-1} - p_{I,t}] = 1, \end{aligned}$$

where the second equality comes from the approximation

$$p_{I,t} = (1-\alpha)p_{I,t}^* + \alpha p_{I,t-1}.$$

Therefore, a first order approximation to labor income is

$$W_{I,t} = W^f \left(\frac{Y_t}{Y_t^f} \right)^{1+\omega+\gamma} \left(\frac{P_{I,t}}{P_t} \right)^{-\eta(1+\omega)} = W^f e^{(1+\omega+\gamma)x_t + \eta(1+\omega)\varphi_{-IPR,t}}.$$

We define the real output in industry I as

$$Y_{I,t}^{real} = \frac{Y_{I,t}P_{I,t}}{P_t} = \varphi_I Y_t \left(\frac{P_{I,t}}{P_t} \right)^{1-\eta}$$

where the second equality is implied by the demand function for industry I . Using the first order Taylor expansion, we get

$$Y_{I,t}^{real} = \varphi_I Y^f e^{x_t + (\eta-1)\varphi - I p_{R,t}}.$$

Therefore, the real profit in industry I at time t is given by

$$\Psi_{I,t} = Y_{I,t}^{real} - W_{I,t} = \left(1 - \frac{1}{\mu_{I,t}} \right) Y_{I,t}^{real},$$

where

$$\mu_{I,t} = \mu_t e^{-(1+\eta\omega)\varphi - I p_{R,t}}$$

is the time-varying markup for sector I , and $\mu_t = \mu e^{(\omega+\gamma)x_t}$ is the aggregate markup.

E Numerical Procedure to Compute Expected Returns

1. We compute the equilibrium for macroeconomic variables (aggregate and industry inflations and output gaps) using the solution in appendix B.
2. We discretize the policy shock process u_t using the Rouwenhorst (1995) procedure. It provides n_u grid values for the policy shocks, transition probabilities between values and unconditional probabilities for each value.
3. We discretize the relative price process $p_{R,t}$, which depends on the policy shocks. The n_p values are obtained based on the unconditional expected value and standard deviation for the relative price. The range is $[\mathbb{E}[p_{R,t}] - const * \sigma(p_{R,t}), \mathbb{E}[p_{R,t}] + const * \sigma(p_{R,t})]$ for a constant $const$. All values are equally spaced on the grid.

The objective is to compute expected returns for different assets. Let $R(z_t)$ be the return of a security at time t . This return depends on the state variables $z_t = (q_t, u_t)$, where $q_t \equiv p_{R,t}$ is the relative price, and u_t is the policy shock. The process for the relative price is

$$q_t = \bar{\rho} + \rho_p q_{t-1} + \rho_u u_t,$$

and the unconditional expected return can be written in terms of conditional expected returns as

$$\mathbb{E}[R(z_{t+1})] = \mathbb{E}[\mathbb{E}[R(z_{t+1})|z_t]].$$

The return can be written as

$$1 + R(z_{t+1}) = \frac{V(z_{t+1})}{V(z_t) - D(z_t)},$$

where $D(q_t, u_t)$ are the dividends paid off by the corresponding asset and

$$\begin{aligned} V(z_t) &= D(z_t) + \mathbb{E}[M(z_{t+1})V(z_{t+1})|z_t] \\ &= D(z_t) + \sum_{z_{t+1}} M(z_{t+1}|z_t)V(z_{t+1}|z_t)\mathbb{P}(z_{t+1}|z_t). \end{aligned} \quad (53)$$

We want to obtain a solution for $V(z_t)$ based on the recursive equation above. The solution provides $n_p \times n_u$ values for all possible combinations (q_t, u_t) .

The values for q_{t+1} are computed from all possible combinations of q_t and u_t on the grid, using the relative price process. It implies that, in general, the values for q_{t+1} are not part of the values on the grid for relative prices. Therefore, we approximate $V(z_{t+1}|z_t)$ such that it can be represented by a z_{t+1} on the grid. We use the linear approximation

$$V(z_{t+1}|z_t) = \sum_{k=1}^{n_p} w(q_k, u_{t+1}|q_t)V(q_k, u_{t+1}),$$

such that all q_k 's are values on the grid of the relative price and

$$q_{t+1} = \bar{\rho} + \rho_p q_t + \rho_u u_{t+1} = \sum_{k=1}^{n_p} w(q_k, u_{t+1}|q_t)q_k.$$

The weights $w(q_k, u_{t+1}|q_t)$ are computed as follows. If $q_{t+1} \leq q_1$, then $w(q_1, u_{t+1}|q_t) = 1$, and $w(q_k, u_{t+1}|q_t) = 0$ for all $k \neq 1$. If $q_{t+1} \geq p_{n_p}$, then $w(q_{n_p}, u_{t+1}|q_t) = 1$, and $w(q_k, u_{t+1}|q_t) = 0$ for all $k \neq n_p$. If $q_{t+1} \in (p_j, p_{j+1}]$, then

$$w(q_j, u_{t+1}|q_t) = \frac{p_{j+1} - q_{t+1}}{p_{j+1} - p_j}, \quad \text{and} \quad w(q_{j+1}, u_{t+1}|q_t) = 1 - w(q_j, u_{t+1}|q_t),$$

and $w(q_k, u_{t+1}|q_t) = 0$ for all $k \neq j$ and $k \neq j + 1$.

From this approximation, it follows that equation (53) is

$$V(q_t, u_t) = D(q_t, u_t) + \sum_{k=1}^{n_p} \sum_{i=1}^{n_u} M(\bar{\rho} + \rho_p q_t + \rho_u u_i, u_i|z_t)\mathbb{P}(u_i|u_t)w(q_k, u_i|q_t)V(q_k, u_i),$$

since the conditional probability $\mathbb{P}(z_{t+1}|z_t) = \mathbb{P}(u_{t+1}|u_t)$. The value for the pricing kernel $M(\bar{\rho} + \rho_p q_t + \rho_u u_i, u_i|z_t)$ can be obtained analytically. The equation above should hold for all combinations (q_t, u_t) on the grid for relative prices and policy shocks. Therefore, it provides a system of $n_p \times n_u$ equations. The solution of this system gives us the values $V(q_t, u_t)$.

The solutions $V(q_t, u_t)$ allow us to find conditional expected returns as

$$\mathbb{E}[1 + R(z_{t+1})|z_t] = \sum_{i=1}^{n_u} \mathbb{P}(u_i|u_t) \frac{V(\bar{\rho} + \rho_p q_t + \rho_u u_i, u_i)}{V(q_t, u_t) - D(q_t, u_t)},$$

where the values $V(\bar{\rho} + \rho_p q_t + \rho_u u_i, u_i)$ are obtained from linear interpolation over $V(q_t, u_i)$. We compute $n_p \times n_u$ conditional expected returns for all possible combinations (q_t, u_t) . It follows that the unconditional expected return is

$$\mathbb{E}[1 + R(z_{t+1})] = \sum_{k=1}^{n_p} \sum_{i=1}^{n_u} \mathbb{P}(q_k, u_i) \mathbb{E}[1 + R(z_{t+1})|q_k, u_i].$$

E.1 Numerical Approximation of $\mathbb{P}(q_t, u_t)$

We need to compute a numerical approximation of the joint distribution of the relative price and the policy shock, to obtain the expected returns above. The joint distribution can be approximated as

$$\begin{aligned}
\mathbb{P}(q_t = q, u_t = u) &= \sum_{k=1}^{n_p} \mathbb{P}(q_t = q, u_t = u, q_{t-1} = q_k) \\
&= \sum_{k=1}^{n_p} \mathbb{P}(q_t = q | u_t = u, q_{t-1} = q_k) \mathbb{P}(u_t = u, q_{t-1} = q_k) \\
&= \sum_{k=1}^{n_p} \mathbb{1}(\bar{\rho} + \rho_p q_k + \rho_u u = q) \sum_{i=1}^{n_u} \mathbb{P}(u_t = u, u_{t-1} = u_i, q_{t-1} = q_k) \\
&= \sum_{k=1}^{n_p} \sum_{i=1}^{n_u} \mathbb{1}(\bar{\rho} + \rho_p q_k + \rho_u u = q) \mathbb{P}(u_t = u | u_{t-1} = u_i, q_{t-1} = q_k) \mathbb{P}(u_{t-1} = u_i, q_{t-1} = q_k) \\
&= \sum_{k=1}^{n_p} \sum_{i=1}^{n_u} \mathbb{1}(\bar{\rho} + \rho_p q_k + \rho_u u = q) \mathbb{P}(u_t = u | u_{t-1} = u_i) \mathbb{P}(u_{t-1} = u_i, q_{t-1} = q_k).
\end{aligned}$$

where $\mathbb{1}$ is the indicator function, which is 1 if q is the closest relative price on the grid for the value $\bar{\rho} + \rho_p q_k + \rho_u u$, and 0 otherwise.

The equation above applies to the $n_p \times n_u$ combinations (q_t, u_t) on the grid. Since the joint distribution is stationary, it provides a system of equations to find the joint probabilities at all points of the grid. One of the equations in the system is linear dependent, and can be replaced with the condition

$$\sum_{k=1}^{n_p} \sum_{i=1}^{n_u} \mathbb{P}(q_t = q_k, u_t = u_i) = 1,$$

to find the solution.