

Bond Risk Premiums and Optimal Monetary Policy

Francisco Palomino *

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Abstract

The bond yield dynamics implied by a welfare-maximizing monetary policy and its credibility are explored in general equilibrium. Credibility is captured by a regime change from discretion to commitment. The policy determines the optimal output and inflation responses to a source of inflation risk. Bond yields contain compensations for this risk that depend on the policy. Credibility improvements reduce the exposure to inflation risk and bond risk premiums decline. A model calibration implies lower yield spreads, less volatile yields, and reduced deviations from the expectations hypothesis under commitment. The model suggests an explanation for changes in yield dynamics in the U.S. across different policy regimes.

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1 Introduction

Recent contributions to the term structure literature such as Dai and Singleton (2002) and Duffee (2002) provide models that successfully reproduce salient properties of bond yields. Upward-sloping yield curves, volatile long-term yields, and time-varying compensations for risk in bonds (risk premiums), can be captured by no-arbitrage affine term structure models. This achievement, however, has been mainly accomplished by specifying bond yields as functions of latent variables with no evident economic interpretation. That is, the economic sources of bond yield variation still need to be explained. Monetary policy, as an important determinant of economic performance, is a natural candidate to explain properties of bond yields. This paper analyzes the implications of optimal monetary policy on long-term bond yields. It asks how a welfare-maximizing policy and its credibility affect bond risk premiums, and shows that changes in monetary policy are a natural explanation for some of the observed changes in the dynamics of United States government bonds.

The empirical evidence in section 2 shows that different regimes in U.S. monetary policy are characterized by different output, inflation and bond yield dynamics. In particular, the average difference between yield levels for long- and short-maturity bonds varies significantly across regimes. This suggests a potential link between monetary policy, economic performance, and risk premiums in long-term bonds. The link is explored with a theoretical model where monetary policy is conducted to maximize welfare and its regime depends on the policy's ability to affect agents' expectations.

The model builds on the standard framework for monetary policy analysis presented in Clarida, Galí and Gertler (1999) or Woodford (2003) and extends it to price long-term bonds. Simplified and extended versions of the model are presented in sections 3 and 4, respectively. The simplified model provides the intuition to understand the main results. The

extended model is built to deliver a tractable Duffie and Kan (1996) affine term structure model capturing salient bond properties. Equilibrium bond yields are linear functions of macroeconomic factors, with factor loadings depending on the policy regime and preference and production parameters. Upward sloping yield curves and time-varying bond risk premiums are obtained by incorporating external habit formation in preferences, similar to the specifications in Abel (1990), Campbell and Cochrane (1999), or Wachter (2006).

A welfare-maximizing goal for monetary policy aims at the joint stabilization of output and inflation. Monetary policy influences real activity due to nominal rigidities in an environment of monopolistic competition in the production sector. These rigidities, in combination with shocks to production markups, generate a tradeoff between output and inflation stabilization. As a result, markup shocks have effects on output and inflation, whose size depends on the credibility of the policy regime. Following Kydland and Prescott (1977), credibility is captured by allowing monetary policy to be conducted under discretion or commitment. Under discretion, the monetary authority takes private sector's expectations as given. Under commitment, the policy is perfectly credible and affects these expectations. Equilibrium differences across the two regimes capture the effects of policy credibility on economic conditions and bond yields.

The implications of optimal monetary policy for output, inflation and bond yields are analyzed in section 5. Welfare maximization prescribes an optimal tradeoff between output and inflation stabilization. The inflation weight in the welfare function is the elasticity of substitution across goods (ESG) and determines the tradeoff: The monetary authority must react to a percentage point of inflation by reducing output (or output growth if the policy is under commitment) by a number of percentage points equal to the ESG. This tradeoff and the existence of markup shocks do not allow the perfect stabilization of inflation and output simultaneously. The marginal utility of consumption is then affected by markup

shocks and investors require a compensation for holding assets with returns sensitive to these shocks, such as nominal bonds. The sign and size of the compensation for markup shocks are determined by the policy and its credibility.

The optimal nature of the policy determines the net effect of markup shocks on the marginal utility of consumption and, thus, the sign of the compensation for these shocks in financial assets. A positive markup shock reduces output and increases inflation. Lower output and higher inflation have opposite effects on the marginal utility of nominal consumption. The positive output effect outweighs the negative inflation effect if the inflation weight in the welfare function is high enough. Specifically, if the ESG is higher than the elasticity of intertemporal substitution of consumption (EIS), the increase in marginal utility from a reduced real consumption is not offset by the increase in nominal values from a higher inflation. In this case, asset payoffs positively correlated with markup shocks are high in periods of high marginal utility, providing a consumption hedge. These assets involve a negative risk premium for markup shocks. The effect is the opposite if the EIS is greater than the ESG.

Policy credibility determines the size of the compensation for markup shocks. The compensation is larger under discretion than under commitment. Under commitment, anchoring private sector's expectations works as an additional mechanism to stabilize output and inflation. Then, the impact of markup shocks on economic performance declines and the compensation for this risk decreases.

Bond returns are affected by economic conditions and are sensitive to markup shocks. It follows that bond premiums for these shocks depend on the optimal policy and its credibility. The effects of credibility on bond risk premiums are analyzed based on the model calibration and the policy experiment in section 6. The model under discretion is calibrated to match some macroeconomic and bond yield properties of the U.S. economy for the period 1971-2007. The experiment consists in a regime switch from discretion to commitment keep-

ing unchanged the initial calibrated parameters. Lower average spreads between long- and short-term bonds are observed under commitment, reflecting reduced bond risk premiums. Investors demand lower compensations for holding long-term bonds under commitment since bond returns are less sensitive to markup shocks. In addition, bond yields and risk premiums are less volatile under commitment, implying reduced deviations from the expectations hypothesis. It is explained by the different response of inflation to markup shocks under commitment.

The discussion in section 7 is intended to provide a credibility-based explanation for changes in bond yield dynamics across different monetary policy regimes in the United States. High policy credibility may explain the low volatile yields and low bond spreads during the Bretton Woods agreement. On the other hand, the increased bond risk premiums observed during the Greenspan era sheds some doubts about an improved policy commitment during this period. However, an explanation based on policy credibility may be given to the Greenspan conundrum, since higher commitment in monetary policy reduces the response of long-term rates to economic shocks.

Related Literature

This paper joins a growing body of work that relates the term structure of interest rates to monetary policy. Diebold, Piazzesi and Rudebusch (2005) summarize recent attempts in empirical and theoretical grounds to understand the joint dynamics of the term structure of interest rates, macroeconomic variables, and monetary policy. For instance, Ang and Piazzesi (2003) and Piazzesi (2005) show how economic information and monetary policy help us improve the empirical fitting of the yield curve relative to successful latent factor models. In the spirit of Taylor (1993), Piazzesi (2005) introduces monetary policy to the analysis by incorporating policy rules to yield curve models. Bikbov and Chernov (2006) and Ang, Dong and Piazzesi (2007) recognize the absence of arbitrage in bond prices as an

additional restriction to identify and estimate monetary policy rules. However, the policy rule approach is silent about the underlying monetary policy objectives. In this sense, it is inadequate to gain insights into how monetary policy objectives affect bond yield properties

Theoretical contributions such as Bekaert, Cho and Moreno (2006), Hördahl, Tristani and Vestin (2006), Wu (2006) and Rudebusch and Wu (2007) link monetary policy and the term structure through structural macroeconomic models. This approach provides macro factors and additional restrictions for no-arbitrage term structure models. These structural models are not able to capture time-variation in risk premiums from first principles and, therefore, limited to explain deviations from the expectations hypothesis.

McCallum (1994) proposes that observed deviations from the expectations hypothesis are compatible with time-varying risk premiums and a monetary policy rule responding to bond yields. Following this direction, Gallmeyer, Hollifield and Zin (2005) explore models implying time-varying risk premiums which deliver affine term structures. Ravenna and Seppälä (2007) propose a New-Keynesian model that generates time-varying premiums to show that the systematic component of monetary policy can explain the rejection of the expectations hypothesis. Buraschi and Jiltsov (2005) find that a time-varying inflation risk premium and monetary policy shocks are important to explain the rejection of the expectations hypothesis. Rudebusch and Wu (2008) examine the recent shift in the dynamics of the term structure of interest rates and suggest a link between this shift and changes in the perception of the dynamics of the inflation target. These papers do not incorporate an explicit welfare objective for monetary policy or policy credibility.

2 Empirical Evidence

This section presents summary statistics of United States time series for consumption, consumer prices, and bond yields from 1952:2 to 2007:4. The statistics are computed for the whole sample period and different sub-samples. The sub-samples are labeled as “Bretton Woods,” “Fiat Money,” and “Greenspan,” to capture the idea of potentially different monetary policy regimes across the three subperiods. The comparison across subperiods shows differences in the observed properties of consumption growth, inflation, and bond yields for different policy regimes. A monetary policy explanation for these differences will be discussed in section 7 based on the implications of the theoretical model in section 4.

The consumption growth series was constructed using quarterly data on real per-capita consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series was constructed following the methodology used in Piazzesi and Schneider (2007) to capture inflation related to non-durables and services consumption only. The term structure series was obtained from quarterly data on bond yields for yearly maturities from 1 to 5 years from the Fama-Bliss discount bonds database found in CRSP, and the short-term nominal interest rate is the 3-month T-bill from the Fama risk-free rates database. Tables 1 and 2 show summary statistics for macroeconomic variables and bond yields, respectively.

Table 1 shows that the volatility of consumption growth was higher during the Bretton Woods period than for the whole sample, and the volatility of inflation was lower. This reduction in inflation volatility was accompanied by a reduced persistence in inflation. Consumption growth and inflation are negatively autocorrelated and the autocorrelation was more negative during the Fiat Money period. The Greenspan period is characterized by low volatility and persistence in inflation, and a reduced volatility in consumption growth in comparison to the Bretton Woods period.

Table 1: U.S. Consumption Growth (Δc) and Inflation (π) Statistics. Average levels and standard deviations are presented in annualized percentage terms. Autocorrelations are the first-order autocorrelation coefficients for quarterly data.

	1952:2-2007:4 (Post-Accord)	1952:2-1971:3 (Bretton Woods)	1971:3-2007:4 (Fiat Money)	1987:4-2007:4 (Greenspan)
Average Δc	2.05	2.30	1.92	1.78
Average π	3.70	2.36	4.40	2.98
Standard deviation Δc	1.88	2.16	1.71	1.34
Standard deviation π	2.49	1.49	2.62	1.28
Autocorrelation Δc	0.35	0.27	0.41	0.24
Autocorrelation π	0.83	0.68	0.82	0.54
Correlation Δc and π	-0.28	-0.17	-0.32	-0.16

Table 2 presents average levels, average spreads, and standard deviations for bond yields. The average yield curve is upward sloping, and yield volatility decreases with maturity. These two characteristics are robust across sub-samples. In particular, we can notice that the Bretton Woods period is characterized by lower levels, spreads, and volatilities than in the Fiat Money period. The reduction in the spread between long-term bonds and the short-term interest rates indicates lower bond risk premiums on average. The table also presents the Campbell and Shiller (1991) regression coefficients, $\beta^{(n)}$, for bond yields. The coefficients are given by

$$\frac{\beta^{(n)}}{n-1} = \frac{\text{cov}\left(i_{t+1}^{(n-1)} - i_t^{(n)}, i_t^{(n)} - i_t\right)}{\text{var}\left(i_t^{(n)} - i_t\right)}, \quad (1)$$

where $i_t^{(n)}$ is the yield at time t of a bond with maturity at time $t+n$, and i_t is the one-period interest rate. Under the expectations hypothesis of interest rates, bond spreads, $i_t^{(n)} - i_t$, only reflect expected changes in bond yields, and the Campbell-Shiller coefficients are equal to one. Deviations from the expectations hypothesis are usually interpreted as evidence of time

Table 2: U.S. Government Bond Yields Statistics. Average levels, spreads and standard deviations are presented in annualized percentage terms. Spreads are computed with respect to the 3-month T-Bill rate.

	1952:2-2007:4 (Post-Accord)	1952:2-1971:3 (Bretton Woods)	1971:3-2007:4 (Fiat Money)	1987:4-2007:4 (Greenspan)
Average Levels				
3-month rate	5.13	3.42	6.02	4.49
Spread 1-year yield	0.39	0.34	0.43	0.41
Spread 2-year yield	0.59	0.41	0.69	0.72
Spread 3-year yield	0.75	0.55	0.86	0.98
Spread 4-year yield	0.87	0.62	1.02	1.18
Spread 5-year yield	0.95	0.67	1.11	1.31
Standard Deviations				
3-month rate	2.87	1.62	2.98	2.05
1-year yield	2.88	1.72	2.93	2.10
2-year yield	2.84	1.61	2.86	1.99
3-year yield	2.78	1.56	2.75	1.88
4-year yield	2.75	1.53	2.69	1.79
5-year yield	2.71	1.48	2.62	1.72
Campbell-Shiller Coefficients				
1-year yield	-0.73	-0.08	-0.78	1.23
2-year yield	-0.88	0.31	-0.91	0.54
3-year yield	-1.18	1.42	-1.28	-0.28
4-year yield	-1.31	1.67	-1.45	-0.47
5-year yield	-1.60	1.74	-1.74	-0.48

variation in compensations for risk in bond yields (bond risk premiums). The coefficients in the table are negative for the whole sample, suggesting a strong positive correlation between bond spreads and bond risk premiums. However, this relation is not stable across sub-samples. The coefficients are negative for the Fiat Money period, and close to zero or positive during the Bretton Woods and Greenspan regimes. This instability suggests that properties of bond risk premiums might be affected by monetary policy regimes.

3 A Simplified Model

This section presents a model in the spirit of Clarida, Galí and Gertler (1999) to characterize the link between compensations for risk in long-term bonds and optimal monetary policy. The model can be seen as a simplified version of the model in section 4.

Consider the problem faced by a monetary policy maker who determines the optimal levels of log-output, y_t , inflation, π_t , and the one-period nominal interest rate, i_t , to maximize welfare. The problem can be written as

$$\max_{\{x_t, \pi_t, i_t\}} - \frac{1}{2} \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t (\kappa x_t^2 + \theta \pi_t^2) \right] \quad (2)$$

subject to

$$e^{-i_t} = \mathbb{E}_t [M_{t,t+1}], \quad (3)$$

and

$$\pi_t = \kappa x_t + \beta \mathbb{E}_t [\pi_{t+1}] + u_t, \quad (4)$$

where $x_t \equiv y_t - y_t^f$ is the output gap that captures deviations of output from the potential output y_t^f (its precise definition is presented in the next section.). Equation (2) is the welfare function. It can be obtained from an approximation of the households' utility. For simplicity and consistency, the relative weights of the output gap and inflation are denoted by $\kappa > 0$ and $\theta > 1$, respectively. Equation (3) is the relevant optimality condition from the households' problem. It relates the one-period nominal interest rate, i_t , to the intertemporal marginal rate of substitution of consumption in nominal terms, $M_{t,t+1}$. The marginal rate of substitution is the discount factor that prices all financial assets and reflects the compensations for risk in the economy. Equation (4) is the optimality condition for firms.

This equation is obtained from the maximization of profits in an environment of monopolistic competition with price rigidities. It links inflation to the output gap, expected future inflation and cost-push shocks, u_t . These shocks follow the autoregressive process

$$u_t = \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t},$$

where $\varepsilon_{u,t} \sim \text{IID}\mathcal{N}(0, 1)$.

The policy maker can solve the maximization problem under discretion or commitment. Under discretion, the policy maker is unable to affect expectations of households and firms about future economic conditions. The policy problem, therefore, reduces to

$$\max_{\{x_t, \pi_t\}} -\frac{1}{2} (x_t^2 + \pi_t^2) \quad \text{subject to} \quad \pi_t = \kappa x_t + F_t,$$

where $F_t = \beta \mathbb{E}_t[\pi_{t+1}] + u_t$ is taken as given. Optimality implies solutions for the output gap and inflation given by

$$x_t^d = -\frac{\theta}{1 + \kappa\theta - \beta\phi_u} u_t, \quad \text{and} \quad \pi_t^d = \frac{1}{1 + \kappa\theta - \beta\phi_u} u_t,$$

respectively. Under commitment, expectations about future economic conditions are affected by the policy and thus the policy maker maximizes (2) subject to (4) for all t . It implies solutions for the output gap and inflation given by

$$x_t^c = \chi_x x_{t-1}^c + \chi_u u_t \quad \text{and} \quad \pi_t^c = \chi_x \pi_{t-1}^c - \frac{\chi_u}{\theta} \Delta u_t,$$

respectively, where

$$\chi_x = \frac{1 + \kappa\theta + \beta - \sqrt{(1 + \kappa\theta + \beta)^2 - 4\beta}}{2\beta} \quad \text{and} \quad \chi_u = -\frac{\theta}{1 + \kappa\theta - \beta\phi_u + \beta(1 - \chi_x)}.$$

The output gap and inflation processes are different across the two policy regimes. While the two processes are proportional to the shock u_t under discretion, they depend on the current and lagged values of the shock under commitment. In addition, the response of the two processes to the current shock is always lower under commitment than under discretion. This reduced response is critical to understand the implications of the two regimes on bond risk premiums.

We define bond risk premiums as the expected excess returns on long-term bonds over the nominal risk-free rate. They are the compensation required by bondholders for the economic risks affecting bond returns. We analyze the effects of the policy on the compensation for two-period bonds. The analysis for bonds of any maturity is presented in section 4.

Consider a two-period nominal bond with associated yield $i_t^{(2)}$, and price $b_t^{(2)} = \exp(-2i_t^{(2)})$. The equilibrium price for this bond can be obtained as the discounted price of a one-period bond one period in the future, $\exp(-i_{t+1})$. That is, the bond price satisfies

$$b_t^{(2)} = \mathbb{E} [M_{t,t+1} e^{-i_{t+1}}],$$

and its one-period return is $r_{t+1}^{(2)} = 2i_t^{(2)} - i_{t+1}$. The price equation above and a log-normal marginal rate of substitution imply the expected excess return

$$\mathbb{E}_t[r_{t+1}^{(2)} - i_t] + \frac{1}{2}\text{var}_t(r_{t+1}^{(2)}) = -\text{cov}_t(\log M_{t,t+1}, r_{t+1}^{(2)}).$$

It follows that the bond risk premium is given by the covariance between the marginal rate

of substitution and the bond return (ignoring the correction term $\frac{1}{2}\text{var}_t(r_{t+1}^{(2)})$). The premium is then determined by the sensitivity of the marginal rate of substitution to sources of risk, or market prices of risk (λ), and the sensitivity of the bond return to these sources (i). We analyze the effects of the monetary policy regime on prices of risk and risk sensitivity of bond returns.

Consider the (log) marginal rate of substitution obtained from power utility,

$$-\log M_{t,t+1} = -\log \beta + \gamma(\Delta y_{t+1}^f + \Delta x_{t+1}) + \pi_{t+1},$$

where γ is the coefficient of relative risk aversion, and Δ is the difference operator. For simplicity, assume that $\Delta y_t^f \equiv 0$ for all t . The only source of risk in the economy is given by cost-push shocks u_t and then the market price of risk is the compensation per unit of these shocks. Replacing the processes for the output gap and inflation for the two regimes in the equation above, the stochastic component of the marginal rate of substitution implies market prices of risk

$$\lambda^d = -\frac{\gamma\theta - 1}{1 + \kappa\theta - \beta\phi_u}, \quad \text{and} \quad \lambda^c = -\frac{\gamma\theta - 1}{1 + \kappa\theta - \beta\phi_u + \beta(1 - \chi_x)},$$

under discretion and commitment, respectively. Since $0 < \chi_x < 1$, the market price of risk is always lower (in absolute value) under commitment than under discretion. Inflation and output are less affected by the shock in the commitment regime, and therefore compensations for risk are lower.

Since $r_{t+1} = 2i_t^{(2)} - i_{t+1}$, the return sensitivity to cost-push shocks is the negative of the one-period rate sensitivity to these shocks, which can be obtained by solving the expectation

in (3). The return sensitivity is

$$i^d = -\frac{(1 - \phi_u)\gamma\theta + \phi_u}{1 + \kappa\theta - \beta\phi_u}, \quad \text{and} \quad i^c = \left(\frac{\lambda^c}{\lambda^d}\right) i^d + \frac{1 + (\gamma\theta - 1)\chi_x}{1 + \kappa\theta - \beta\phi_u + \beta(1 - \chi_x)},$$

under discretion and commitment, respectively. Under discretion, a positive shock always increases the one-period nominal rate, reducing the return on the two-period bond. The gains from commitment are reflected in a reduced response of the short-term rate to the cost-push shock (in fact, for some parameter values the one-period rate decreases after a positive shock). It translates into a reduced bond return sensitivity with respect to the discretionary case.

The bond risk premium can be expressed in terms of the price of risk and the bond return sensitivity as $rp_t = \text{cov}_t(-\log M_{t,t+1}, r_{t+1}) = \lambda \text{var}_t(u_{t+1})$. It follows that the premium is

$$rp_t^d = \frac{(\gamma\theta - 1)[(1 - \phi_u)\gamma\theta + \phi_u]\sigma_u^2}{(1 + \kappa\theta - \beta\phi_u)^2}, \quad \text{and} \quad rp_t^c = \left(\frac{\lambda^c}{\lambda^d}\right)^2 rp_t^d - \frac{(\gamma\theta - 1)[1 + (\gamma\theta - 1)\chi_x]\sigma_u^2}{[1 + \kappa\theta - \beta\phi_u + \beta(1 - x_m)]^2}.$$

under discretion and commitment, respectively. Consider first the case of $\gamma\theta > 1$. The risk premium under discretion is positive, and the risk premium under commitment is lower and can be negative. Under discretion, positive cost-push shocks increase the marginal utility of consumption while reducing bond returns. The effects are less significant under commitment as a result of the additional stabilization benefits on the economy. Therefore, bondholders require a lower compensation for risk under commitment. If $\gamma\theta < 1$, a positive shock reduces the marginal utility of consumption, the risk premium is negative under discretion, and it is less negative or even positive under commitment. In summary, the stabilization benefits on output and inflation from commitment translate into lower market prices of risk and reduced risk premiums (in absolute value) in two-period bonds.

4 The Extended Model

The simplified model in section 3 provides the necessary intuition to understand the main theoretical results. However, it is unable to capture important properties of observed bond yield dynamics. This section presents an equilibrium model with nominal rigidities in the spirit of Woodford (2003), with two important features: the specification of habit persistence in household preferences, and time variation in shock volatility. These features allow a tractable affine representation for bond yields, and capture the average upward sloping curves and time-varying bond risk premiums that characterize the data in section 2.

Consider an economy populated by households that derive utility from the consumption of an aggregate of differentiated goods and disutility from labor. Households provide labor to firms that maximize profits in a monopolistic competitive setting characterized by price rigidities and a labor-only technology. Monetary policy is conducted to maximize welfare under one of two possible regimes: discretion or commitment. Bond risk premiums reflect compensations for three sources of uncertainty: preference, productivity, and markup shocks, which for simplicity are assumed to be uncorrelated. The equilibrium properties of the economies under discretion and commitment are presented in this section.

4.1 Households

Households exhibit preferences over an infinite horizon on consumption, C_t , and labor, $h_t(j)$, which is provided to a continuum of firms owned by the households and indexed by $j \in [0, 1]$. The consumption good is the Dixit-Stiglitz aggregate of a continuum of differentiated goods, $C_t(j)$, given by

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta_t-1}{\theta_t}} dj \right]^{\frac{\theta_t}{\theta_t-1}}, \quad (5)$$

where $\theta_t > 1$ is the potentially time-varying elasticity of substitution between differentiated goods, with expected value θ . The expected utility of households is represented by

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\gamma} \frac{C_t^{1-\gamma}}{Q_t} - \frac{1}{1+\omega} \int_0^1 h_t(j)^{1+\omega} dj \right) \right], \quad (6)$$

where γ^{-1} and ω^{-1} are the elasticities of intertemporal substitution of consumption and labor, respectively. The first and second terms in equation (6) are the utility from consumption and the disutility from labor, respectively. The utility derived from consumption is affected by the external habit Q_t . The process for the (log) habit $q_t \equiv \log Q_t$ is

$$q_{t+1} = q_t + \eta \Delta \tilde{c}_t + (1 + K_q \Delta \tilde{c}_t)^{1/2} \sigma_q \varepsilon_{q,t+1}, \quad (7)$$

where $\Delta \tilde{c}_t$ is aggregate consumption growth, and $\varepsilon_{q,t} \sim \text{IIDN}(0, 1)$ are preference shocks. If $\sigma_q \equiv 0$, the habit becomes $Q_t = C_t^\eta$ which is equivalent to the ‘‘catching up with the Joneses formulation’’ in Abel (1990) and Fuhrer (2000). The parameter η is the sensitivity of changes in the habit to lagged consumption. The preference shock generates time-varying volatility in the habit, where K_q captures the dependence of volatility on lagged consumption growth. This specification is similar to the ‘‘sensitivity function’’ for the surplus-consumption ratio in Campbell and Cochrane (1999). It generates time variation in risk premiums.

Households choose contingent consumption and labor streams to maximize their expected utility subject to the intertemporal budget constraint

$$\mathbb{E} \left[\sum_{t=0}^{\infty} M_{0,t} P_t C_t \right] \leq \mathbb{E} \left[\sum_{t=0}^{\infty} M_{0,t} \left(\int_0^1 w_t(j) h_t(j) dj + \int_0^1 \Psi_t(j) dj - \tau_t \right) \right],$$

where $M_{t,t+n} > 0$ is the factor that discounts nominal cashflows at time $t+n$ to time t , and P is the price of the consumption good. The present value of consumption is constrained by the

present values of labor income, $w_t(j)h_t(j)$, profits from the firms supplying the differentiated goods, $\Psi_t(j)$, and taxes, τ_t .

The solution to the household's problem implies a discount factor given by the marginal rate of substitution of consumption in nominal terms. It is

$$M_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma} \left(\frac{Q_{t+n}}{Q_t} \right)^{-1} \left(\frac{P_{t+n}}{P_t} \right)^{-1}. \quad (8)$$

The price at time t of a nominal bond with maturity at $t+n$ is

$$b_t^{(n)} = \mathbb{E}_t [M_{t,t+n}]. \quad (9)$$

In particular, the price of the one-period nominal bond with associated rate i_t satisfies

$$e^{-i_t} = \mathbb{E}_t [M_{t,t+1}] = \mathbb{E}_t [\exp(\log \beta - \gamma \Delta c_{t+1} - \Delta q_{t+1} - \pi_{t+1})], \quad (10)$$

where $\pi_{t+1} \equiv \log P_{t+1} - \log P_t$ is the inflation rate. Habit persistence makes the short-term interest rate depend not only on expectations about future consumption growth and future inflation but also on the current level of consumption growth and preference shocks.

To complete the analysis of the household's problem, the intratemporal marginal rate of substitution

$$\frac{w_t(j)}{P_t} = h_t(j)^\omega C_t^\gamma Q_t \quad (11)$$

provides the tradeoff between consumption and labor that must be satisfied in equilibrium.

4.2 Production Sector

The production of differentiated goods is characterized by monopolistic competition and price rigidities, and is affected by productivity and markup shocks. A continuum of suppliers of differentiated goods $j \in [0, 1]$ have market power to set their product prices. However, following Calvo (1983) staggered price setting, only a fraction $1 - \alpha$ of suppliers is able to change prices optimally in a given period, while a fraction α keeps last period prices adjusted by the long-term average inflation Π^* . The optimal price set at time t solves the profit-maximization problem

$$\max_{P_t(j)} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} [(1 + \tau) P_t(j) (\Pi^*)^{T-t} Y_{T|t}(j) - w_{T|t}(j) h_{T|t}(j)] \right] \quad (12)$$

where $Y_{T|t}(j)$, $w_{T|t}(j)$, and $h_{T|t}(j)$ denote product, wages and labor at time T , respectively, when the last price adjustment was at time t . The maximization problem is subject to the product demand function and the production function. The product demand function is

$$Y_{T|t}(j) = Y_T \left(\frac{P_t(j) (\Pi^*)^{T-t}}{P_T} \right)^{-\theta_T}, \quad (13)$$

where $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta_t-1}{\theta_t}} dj \right]^{\frac{\theta_t}{\theta_t-1}}$ is the aggregate output. The production function is

$$Y_{T|t}(j) = A_T h_{T|t}(j), \quad (14)$$

where growth in labor productivity, $\Delta a_t \equiv \log A_t - \log A_{t-1}$, is modeled as the autoregressive process

$$\Delta a_{t+1} = (1 - \phi_a) g_a + \phi_a \Delta a_t + (1 + K_a \Delta a_t)^{1/2} \sigma_a \varepsilon_{a,t+1}, \quad (15)$$

with $\varepsilon_{a,t} \sim \text{IID}\mathcal{N}(0, 1)$. The parameter K_a captures time variation in the volatility of the productivity shocks.

Appendix A shows that optimality in the production sector implies

$$\pi_t - \pi^* = \kappa(y_t - y_t^f) + \frac{\kappa}{\omega + \gamma}(q_t - q_t^f) + \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*], \quad (16)$$

where $\pi^* \equiv \log \Pi^*$, $y_t \equiv \log Y_t$, and y_t^f and q_t^f are, respectively, the output and habit in an economy where prices are perfectly flexible. The coefficients β_g and κ are defined in the appendix. The flexible-price output is

$$y_t^f = \frac{1}{\omega + \gamma} \left[-q_t^f + (1 + \omega)a_t - \epsilon_t \right]. \quad (17)$$

It depends on labor productivity, the external habit, and the markup shock, ϵ_t . Habit formation in preferences is reflected in additional persistence in production. The markup shock captures deviations of prices from marginal costs. It is defined as

$$\epsilon_{t+1} = \phi_\epsilon \epsilon_t + (1 + K_\epsilon \epsilon_t)^{1/2} \sigma_\epsilon \varepsilon_{\epsilon,t+1}, \quad (18)$$

with $\varepsilon_\epsilon \sim \text{IID}\mathcal{N}(0, 1)$ and the volatility of the shocks depending on the level of ϵ_t .

The forward-looking equation (16) captures the tradeoff between output and inflation induced by price rigidities. Inflation is driven by expectations on future inflation, and current and lagged deviations of output from the flexible-price output, as captured by $y_t - y_t^f$ and $q_t - q_t^f$, respectively.

To facilitate the exposition of the monetary policy problem, we define the output gap as $x_t \equiv y_t - y_t^f - (\omega + \gamma)^{-1} \epsilon_t$, and the habit gap, $l_t \equiv q_t - q_t^f$. The habit gap captures deviations

of lagged outputs from their flexible-price counterparts. As a result equation (16) becomes

$$\pi_t - \pi^* = \kappa x_t + \frac{\kappa}{\omega + \gamma} l_t + \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*] + \frac{\kappa}{\omega + \gamma} \epsilon_t. \quad (19)$$

This optimality condition is known as the New-Keynesian Phillips curve. It is similar to equation (4), where markup shocks play the role of cost-push shocks, and incorporates the effect of the habit on inflation.

4.3 Monetary Policy

Monetary policy is conducted to maximize households' welfare. The monetary authority is a social planner that sets the levels of output, inflation, and the nominal one-period interest rate, and conducts a policy under discretion or commitment. Under discretion, households' and firms' expectations cannot be affected by the monetary authority. Under commitment, these expectations are affected by the policy, leading to policy credibility.

The welfare maximization problem is

$$\max_{\{\pi_t, y_t, l_t\}} \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\gamma} \frac{Y_t^{1-\gamma}}{Q_t} - \frac{1}{1+\omega} \int_0^1 \left(\frac{Y_t(j)}{A_t} \right)^{1+\omega} dj \right) \right] \quad (20)$$

subject to (10), and (19), for all t . The welfare function is given by the household's expected utility, and the optimality conditions for households and firms have to be satisfied. Appendix B shows that maximizing welfare is equivalent to targeting inflation and output to minimize the loss function

$$\frac{1}{2} \mathbb{E} \left\{ \sum_{t=0}^{\infty} \beta_g^t \left[\left(x_t + \frac{1}{\omega + \gamma} l_t \right)^2 + \frac{\theta_g}{\kappa} (\pi_t - \pi^*)^2 \right] \right\},$$

where the relative weight of inflation, $\theta_g \equiv \theta \left(\frac{1-\alpha\beta_g}{1-\alpha\beta} \right)$, depends on the elasticity of substitution across goods. The lowest loss in welfare is obtained when inflation reaches the target π^* , and output and habit gaps reach their zero targets. Habit formation in preferences captures the idea that deviations from the target output in the past affect household's welfare today.

4.4 Equilibrium

The solution to the monetary policy problem provides us with equilibrium output, inflation, and interest rates. Equilibrium outcomes depend on whether the monetary authority is committed to the long-term objective of welfare maximization. The following two results present the equilibrium for output and inflation for policies under discretion and commitment, respectively.

Result 1. *Optimal monetary policy under discretion implies that output growth and inflation follow the processes*

$$\Delta y_t^d = \frac{1}{\omega + \gamma} [-\Delta q_t^d + (1 + \omega)\Delta a_t] - \theta_g \Delta \pi_t^d, \quad (21)$$

and

$$\pi_t^d = \pi^* + \nu_\epsilon^d \epsilon_t, \quad (22)$$

respectively, where $\nu_\epsilon^d = \frac{\kappa}{(\omega + \gamma)(1 + \kappa\theta_g - \beta_g\phi_\epsilon)}$.

Proof. See appendix C. □

If the monetary policy is conducted under discretion, deviations of inflation from the target are proportional to markup shocks. It implies that the persistence of inflation is equal

to the persistence of the shocks. Output growth depends on lagged output growth and is negatively affected by changes in inflation, with the size of the impact determined by the elasticity of substitution of differentiated goods.

Result 2. *Optimal monetary policy under commitment implies that output growth and inflation follow the processes*

$$\Delta y_t^c = \frac{1}{\omega + \gamma} [-\Delta q_t^c + (1 + \omega)\Delta a_t] - \theta_g(\pi_t^c - \pi^*), \quad (23)$$

and

$$\pi_t^c = (1 - \phi_\pi^c)\pi^* + \phi_\pi^c\pi_{t-1}^c + \nu_\epsilon^c\Delta\epsilon_t, \quad (24)$$

respectively, where

$$\phi_\pi^c = \frac{1}{2\beta_g} \left[1 + \kappa\theta_g + \beta_g - \sqrt{(1 + \kappa\theta_g + \beta_g)^2 - 4\beta_g} \right], \text{ and } \nu_\epsilon^c = \frac{\kappa}{(\omega + \gamma)(1 + \kappa\theta_g + \beta_g(1 - \phi_\pi^c - \phi_\epsilon))}.$$

Proof. See appendix C. □

Under commitment, output growth is affected directly by the level of inflation, and inflation is determined by lagged inflation and changes in markup shocks. Also, the persistence of inflation is no longer equal to the persistence of the markup shocks.

4.5 The Term Structure of Interest Rates

Equilibrium bond yields and bond risk premiums for all maturities can be obtained from the equilibrium processes for output growth and inflation described in results 1 and 2. Bond prices in (9) depend on the discount factors $M_{t,t+n}$ in equation (8). Since the (log) discount factors depend linearly on consumption growth and inflation, bond yields are linear functions

of these macroeconomic variables. This linearity is particularly convenient to derive bond yields in an affine term structure framework similar to the one in Duffie and Kan (1996). Bond yields are then linear functions of state variables with economic interpretation, where the loading coefficients are determined by preference and production parameters and the monetary policy regime. This section describes the affine framework, and the equilibrium bond yields and risk premiums under discretion and commitment.

Consider the vector of state variables, \mathbf{s}_t , following the autoregressive process

$$\mathbf{s}_{t+1} = \psi + \Phi \mathbf{s}_t + \Psi_c \Psi(\mathbf{s}_t) \Sigma^{1/2} \varepsilon_{t+1}, \quad (25)$$

where $\Sigma^{1/2} = \text{diag}\{\sigma_a, \sigma_\epsilon, \sigma_q\}$, and the vector of innovations $\varepsilon_t = (\varepsilon_{a,t}, \varepsilon_{\epsilon,t}, \varepsilon_{q,t})^\top$ contains the productivity, markup, and preference shocks. The vector ψ , the autoregressive coefficients in Φ , and the constant and time-varying components of the volatility, Ψ_c and $\Psi(\mathbf{s}_t)$, describe the dynamics of the state variables. These dynamics are constrained by the equilibrium characteristics of the two policy regimes.

The one-period discount factor (8) is

$$-\log M_{t,t+1} = \Gamma_0 + \Gamma_1^\top \mathbf{s}_t + \lambda^\top \Psi(\mathbf{s}_t) \Sigma^{1/2} \varepsilon_{t+1}, \quad (26)$$

where the last term contains the discount-factor sensitivity to economic shocks, $\lambda^\top \Psi(\mathbf{s}_t)$, or market prices of risk. The analysis of this term is fundamental to understand the effects of policy credibility on bond risk premiums. Notice that the prices of risk depend on the time-varying component of the volatility of the state variables, $\Psi(\mathbf{s}_t)$. This component generates time variation in bond risk premiums.

Appendix D shows that equations (9), (25), and (26) imply the linear representation for

the n -period bond yield, $i_t^{(n)} = -\frac{1}{n} \log b_t^{(n)}$, given by

$$i_t^{(n)} = \frac{1}{n} (\mathcal{A}_n + \mathcal{B}_n^\top \mathbf{s}_t), \quad (27)$$

for all n . The coefficients \mathcal{A}_n and \mathcal{B}_n are obtained recursively as presented in the appendix. The one-period return on this bond is $r_{t+1}^{(n)} = -(n-1)i_{t+1}^{(n-1)} + ni_t^{(n)}$. It follows from equation (27) that the return sensitivity to the economic shocks is time varying, given by $-\mathcal{B}_{n-1}^\top \Psi_c \Psi(\mathbf{s}_t)$. When bond returns are affected by the state of the economy ($\mathcal{B}_n \neq \mathbf{0}$), long-term bonds incorporate compensations for risk. This notion is formalized with the definition of bond risk premiums. The risk premium for an n -period bond, $\xi_t^{(n)}$, captures the deviation of the n -period bond yield from its pure expectations hypothesis component. Explicitly,

$$\xi_t^{(n)} \equiv i_t^{(n)} - \frac{1}{n} \left[i_t + (n-1) \mathbb{E}_t i_{t+1}^{(n-1)} \right]. \quad (28)$$

It is easy to show, using equation (27), that bond risk premiums follow the linear representation

$$\xi_t^{(n)} = \xi_{\mathcal{A},n} + \xi_{\mathcal{B},n}^\top \mathbf{s}_t, \quad (29)$$

with coefficients $\xi_{\mathcal{A},n}$ and $\xi_{\mathcal{B},n}$ defined in the appendix. The risk premium satisfies

$$n\xi_t^{(n)} = -\frac{1}{2} \text{var}_t(r_{t+1}^{(n)}) - \text{cov}_t(\log M_{t,t+1}, r_{t+1}^{(n)}). \quad (30)$$

Therefore, it is determined by the co-movement of the marginal rate of substitution and bond returns. This co-movement depends on the policy regime. In addition, if the state variables are affected by time-varying volatility, the risk premium is time varying ($\xi_{\mathcal{B},n} \neq \mathbf{0}$).

The effects of time-variation in bond risk premiums are captured by the Campbell-Shiller coefficients in equation (1). The appendix shows that these coefficients are determined in the affine framework as

$$\beta^{(n)} = 1 - n \frac{\text{cov} \left(i_t^{(n)} - i_t, \xi_t^{(n)} \right)}{\text{var} \left(i_t^{(n)} - i_t \right)}. \quad (31)$$

It follows that constant risk premiums imply coefficients of 1. Coefficients different from one in the model imply that both bond spreads and risk premiums are affected by the state of the economy. The co-movement between spreads and premiums depends on the policy regime.

Results 3 and 4 characterize the affine framework under discretion and commitment, respectively. Under discretion, the dynamics of bond yields and risk premiums can be explained in terms of the dynamics of three macroeconomic factors: changes in labor productivity, inflation, and output growth. Under commitment an additional factor is required, e.g., markup shocks, to explain these dynamics.

Result 3. *Optimal monetary policy under discretion implies equilibrium bond yields and risk premiums described by equations (25), (26), (27), and (29), when*

$$\mathbf{s}_t \equiv \left(\Delta a_t, \quad \pi_t^d, \quad \Delta y_t^d \right)^\top,$$

$$\psi = \left((1 - \phi_a)g_a, \quad (1 - \phi_\epsilon)\pi^*, \quad \left(\frac{1 + \omega}{\omega + \gamma} \right) (1 - \phi_a)g_a - (1 - \phi_\epsilon)\theta_g\pi^* \right)^\top,$$

$$\Phi = \begin{bmatrix} \phi_a & 0 & 0 \\ 0 & \phi_\epsilon & 0 \\ \frac{1+\omega}{\omega+\gamma}\phi_a & \theta_g(1 - \phi_\epsilon) & -\frac{\eta}{\omega+\gamma} \end{bmatrix}, \quad \Psi_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \nu_\epsilon^d & 0 \\ \frac{1+\omega}{\omega+\gamma} & -\theta_g\nu_\epsilon^d & -\frac{1}{\omega+\gamma} \end{bmatrix},$$

$$\begin{aligned}
\Psi(\mathbf{s}_t) &= \text{diag} \left\{ (1 + K_a \Delta a_t)^{1/2}, \left(1 - K_\epsilon \frac{\pi^*}{\nu_\epsilon^d} + \frac{K_\epsilon}{\nu_\epsilon^d} \pi_t^d \right)^{1/2}, (1 + K_g \Delta y_t^d)^{1/2} \right\}, \\
\Gamma_0 &= -\log \beta + (1 - \phi_\epsilon)(1 - \gamma \theta_g) \pi^* + \frac{1 + \omega}{\omega + \gamma} (1 - \phi_a) \gamma g_a, \\
\Gamma_1 &= \left(\frac{1 + \omega}{\omega + \gamma} \gamma \phi_a, \quad \phi_\epsilon + \gamma \theta_g (1 - \phi_\epsilon), \quad \frac{\eta \omega}{\omega + \gamma} \right)^\top, \\
\text{and } \lambda &= \left(\gamma \frac{1 + \omega}{\omega + \gamma}, \quad \nu_\epsilon^d (1 - \gamma \theta_g), \quad \frac{\omega}{\omega + \gamma} \right)^\top. \tag{32}
\end{aligned}$$

Proof. The processes for Δa_t and Δy_t are obtained from equations (15) and (21), respectively. The process for π_t is obtained by replacing (18) into (22). The process for the discount factor is obtained by replacing equation (25) into (26). \square

Result 4. *Optimal monetary policy under commitment implies equilibrium bond yields and risk premiums described by equations (25), (26), (27), and (29), when*

$$\begin{aligned}
\mathbf{s}_t &\equiv (\Delta a_t, \quad \epsilon_t, \quad \pi_t^c, \quad \Delta y_t^c)^\top, \\
\psi &= \left((1 - \phi_a) g_a, 0, (1 - \phi_\pi^c) \pi^*, \frac{1 + \omega}{\omega + \gamma} (1 - \phi_a) g_a + \theta_g \phi_\pi^c \pi^* \right)^\top, \\
\Phi &= \begin{bmatrix} \phi_a & 0 & 0 & 0 \\ 0 & \phi_\epsilon & 0 & 0 \\ 0 & -\nu_\epsilon^c (1 - \phi_\epsilon) & \phi_\pi^c & 0 \\ \frac{1 + \omega}{\omega + \gamma} \phi_a & \nu_\epsilon^c \theta_g (1 - \phi_\epsilon) & -\theta_g \phi_\pi^c & -\frac{\eta}{\omega + \gamma} \end{bmatrix}, \quad \Psi_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \nu_\epsilon^c & 0 \\ \frac{1 + \omega}{\omega + \gamma} & -\nu_\epsilon^c \theta_g & -\frac{1}{\omega + \gamma} \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\Psi(\mathbf{s}_t) &= \text{diag}\{(1 + K_a \Delta a_t)^{1/2}, (1 + K_\epsilon \epsilon_t)^{1/2}, (1 + K_q \Delta y_t^c)^{1/2}\}, \\
\Gamma_0 &= -\log \beta + [1 - \phi_\pi^c(1 - \gamma \theta_g)]\pi^* + \frac{1 + \omega}{\omega + \gamma}(1 - \phi_a)\gamma g_a, \\
\Gamma_1 &= \left(\frac{1 + \omega}{\omega + \gamma}\gamma \phi_a, \quad -(1 - \phi_\epsilon)\nu_\epsilon^c, \quad \phi_\pi^c(1 - \gamma \theta_g) \cdot \frac{\omega \eta}{\omega + \gamma} \right)^\top, \\
\text{and } \lambda &= \left(\gamma \frac{1 + \omega}{\omega + \gamma}, \quad \pi_\epsilon^c(1 - \gamma \theta_g), \quad \frac{\omega}{\omega + \gamma} \right)^\top.
\end{aligned} \tag{33}$$

Proof. The processes for the state variables are obtained from equations (15), (18), (23), and (24). The process for the discount factor is obtained replacing equation (25) into (26). \square

5 Analysis

There is a specific mechanism linking bond yields and risk premiums to optimal monetary policy in the extended model. Optimal policy and its credibility determine the effects of markup shocks on output and inflation. Then, the price of risk of markup shocks, the bond return sensitivity to these shocks, and their associated bond risk premiums depend on whether the policy is conducted under discretion or commitment. This section describes the macroeconomic and bond yield implications of the policy to understand the mechanism.

5.1 Output and Inflation

Equations (21) to (24) summarize the main properties of output growth and inflation under discretion and commitment. The average output growth and inflation are $\frac{1 + \omega}{\omega + \gamma + \eta}g_a$ and π^* , respectively, under both regimes. It also can be noticed that the only source of inflation risk is markup shocks, the sensitivity of output to markup shocks is determined by the

policy, and habit formation adds persistence to the markup shock effects on output. The credibility improvements associated to commitment are reflected in a reduced sensitivity of output and inflation to markup shocks ($\nu_\epsilon^c < \nu_\epsilon^d$). Therefore, the tradeoff between inflation and output volatility improves under commitment, which is reflected in higher social welfare. The persistence of inflation also changes across regimes. Under discretion, inflation inherits the autocorrelation properties of the markup shocks. Under commitment, the autocorrelation is reduced by the fact that inflation reacts to changes in the level of markup shocks and not to the level of the shocks. This is formalized in the following result.

Result 5. *The first-order autocorrelation of inflation is ϕ_ϵ under discretion, and $\phi_\epsilon - \frac{1}{2}(1 - \phi_\pi^c)(1 + \phi_\epsilon)$ under commitment. Therefore, a policy conducted under commitment always reduces the persistence of inflation.*

Proof. The result follows from algebraic manipulations of the unconditional autocovariance and variance of the inflation processes in 3 and 4. □

The effects of credibility improvements on the unconditional variance and autocorrelation of output growth, and the correlation of output growth and inflation are difficult to explore analytically. Different model parameterizations suggest that the variability of output growth under commitment is always lower than under discretion, and the autocorrelation of output growth tends to increase with improvements in credibility. Parameterizations involving highly persistent markup shocks imply less negative correlation between output growth and inflation under commitment. However, the effect is reversed as the persistence of the shocks is reduced.

Summarizing, enhancements in policy credibility increase macroeconomic stability. The change is driven by a reduced reaction of inflation and output to markup shocks. Not surprisingly, this change also drives the change in bond yield dynamics.

5.2 Market Prices of Risk

The discount factor in (26) contains prices of risk for technology, markup, and preference shocks, as captured by the vector $\lambda^\top \Psi(\mathbf{s}_t)$. It is convenient to analyze the components λ and $\Psi(\mathbf{s}_t)$ separately. The effects of optimal policy on the average price of risk are captured by λ under the two regimes, as presented in the following result.¹

Result 6. *The price-of-risk components λ for optimal monetary policies under discretion and commitment are given by equations (32) and (33), respectively. Therefore, the average prices of risk for technology and preference shocks are not affected by the credibility of the policy, and the magnitude of the price of risk for markup shocks is always lower under commitment than under discretion.*

Proof. The result follows directly from equations (32) and (33), and the fact that $\nu_\epsilon^d > \nu_\epsilon^c$. \square

Optimal monetary policy and its credibility affect the magnitude and sign of the price of risk of markup shocks. Since the sensitivity of output and inflation to markup shocks is lower under commitment than under discretion, the marginal utility of consumption is less affected by this shock and, therefore, the required compensation for facing this risk is lower (in absolute value). The sign of the price of risk depends on the relation between the intertemporal elasticity of substitution of consumption, γ^{-1} , and the (growth-adjusted) elasticity of substitution of differentiated goods, θ_g . The latter is the relative weight of inflation in the welfare function. Consider a positive markup shock that increases inflation and reduces output. While the higher inflation reduces the marginal utility of consumption in nominal terms, the lower output has the opposite effect. If the elasticity of intertemporal

¹To see this, consider $\mathbb{E}[\text{var}_t(\log M_{t,t+1})] = \lambda^\top \Sigma \mathbb{E}[\Psi(\mathbf{s}_t)^2] \lambda$. The matrix $\Psi(\mathbf{s}_t)^2$ is a diagonal matrix with exogenous components Δa_t and ϵ_t in the first two rows and an endogenous component Δy_t in the third row. Since the unconditional expectation of output growth is independent of monetary policy, changes in the policy only affect the average properties of the prices of risk through λ .

substitution of consumption is low in comparison to the elasticity of substitution across goods ($\gamma\theta_g > 1$), the negative output effect in marginal utility outweighs the positive inflation effect. As a result, asset returns that are positively affected by markup shocks are high in periods of high marginal utility, provide a consumption hedge, and investors are willing to hold them for negative compensations for risk. The opposite occurs if $\gamma\theta_g < 1$.

The term $\Psi(\mathbf{s}_t)$ is the time-varying component of the volatilities of productivity, markup, and preference shocks. It generates time variation in prices of risk. Time variation in the price of productivity shocks is driven entirely by the level of productivity growth, Δa_t , and thus is not affected by monetary policy. Time variation in the price of markup shocks is linked to the level of the shocks, ϵ_t . It is reduced by improvements in policy credibility since inflation and output are less sensitive to these shocks. Time variation in the price of preference shocks is given by $\sqrt{1 + K_q \Delta y_t}$. Since output growth is affected by credibility in monetary policy, the volatility of the premium is also affected. For a countercyclical risk premium ($K_q < 0$), when a positive markup shock affects the economy, output growth under discretion declines more than under commitment and, as a result, the preference premium under discretion increases more than under commitment.

In summary, the price of risk for markup shocks depends on the optimal policy since output and inflation sensitivities to markup shocks are affected by the policy. Improvements in policy credibility reduce the effects of markup shocks on inflation and real activity, and thus compensations for this risk in asset prices decline and are less volatile.

5.3 Bond Yields and Bond Risk Premiums

Results 3 and 4 show that bond yields and risk premiums can be expressed as functions of macroeconomic variables. The loading coefficients on these variables depend on deep

economic parameters and are affected by the policy regime. The decreased sensitivity to markup shocks in the economy under commitment shifts the level and volatility of short- and long-term bond yields. Changes in the shape of the yield curve reflect changes in the compensation for markup shocks in bond risk premiums. The reduced volatility in these premiums is reflected in diminished deviations from the expectations hypothesis.

The different dynamics of the short-term rate i_t under discretion and commitment capture the effects of policy credibility on the level of the yield curve. The response of short-term rate to economic conditions (output gap and inflation) differs across regimes as shown in appendix E. In the spirit of Taylor (1993), these differences in interest rates can be interpreted as different interest rules to implement the two policies. Result 7 shows that the difference in the average short-term rates between discretion and commitment is linked to the sensitivity of the economy to markup shocks in the two regimes. The average interest rate is always lower under discretion than under commitment. The reduced volatility under commitment implies a reduced precautionary savings motive that increases the average interest rate.

Result 7. *The difference in the average one-period interest rates implied by monetary policies under discretion and under commitment is*

$$\mathbb{E} [i_t^d - i_t^c] = -\frac{1}{2}(1 - \gamma\theta_g)^2[(\nu_\epsilon^d)^2 - (\nu_\epsilon^c)^2]\sigma_\epsilon^2.$$

Proof. See appendix E. □

The volatility of short- and long-term bonds is also affected by policy credibility. Different model parameterizations imply a volatility for the short-term rate that is higher under commitment, while the opposite is true for the volatility of long-term bond yields.

Policy credibility affects bond risk premium dynamics through changes in the co-movement of market prices of risk and bond returns, as can be seen from equation (30). The effects of

credibility on the prices of risk were analyzed above. To see the effects of credibility on bond returns, consider the sensitivity of returns to the sources of risk, given by $-\mathcal{B}_{n-1}^\top \Psi_c \Psi(\mathbf{s}_t)$. In general, the analytical solutions for these terms are complicated. However, the analysis for the sensitivity of the two-period bond ($\mathcal{B}_1^\top \Psi_c$) in result 8, and the numerical properties of this sensitivity for long-term bonds provide the necessary intuition to understand the effects of credibility on the average premiums.

Result 8. *The difference in the $\mathcal{B}_1^\top \Psi_c$ component between discretion and commitment is*

$$\begin{pmatrix} 0, \\ \gamma\theta_g\nu_\epsilon^d + (1 - \phi_\pi^c)\nu_\epsilon^c + \left[(1 - \gamma\theta_g)\phi_\epsilon - \theta_g \left(\frac{\omega}{\omega+\gamma} \right) \left(\eta - \frac{1}{2} \frac{\omega}{\omega+\gamma} K_q \sigma_q^2 \right) \right] (\nu_\epsilon^d - \nu_\epsilon^c) \\ -\frac{1}{2}(1 - \gamma\theta_g)^2 K_\epsilon \sigma_\epsilon^2 ((\nu_\epsilon^d)^2 - (\nu_\epsilon^c)^2), \\ 0 \end{pmatrix}^\top.$$

Therefore, the average 2-period term premium for productivity and preference shocks is not affected by the credibility of the policy.

Policy credibility only affects the average sensitivity of the 2-period bond returns to markup shocks. As a result, the compensation for markup shocks in bond risk premiums depends on the policy. The analysis can be complemented with numerical calculations of $\mathcal{B}_{n-1}^\top \Psi_c$ for both policies and different maturities, $n > 2$, to see that the only component that is affected by the policy is the average risk premium for markup shocks. The effects of credibility improvements on the time variation of bond risk premiums can be captured by the difference in the Campbell-Shiller coefficients in equation (31) across discretion and commitment. Coefficients that are closer to 1 under commitment than under discretion imply a reduced co-movement between bond spreads and risk premiums as a result of reduced sensitivity of the economy to markup shocks.

Table 3: Parameter Values

Parameter	Description	Value
β	Subjective discount factor	0.99
γ	Curvature parameter	3.51
η	Habit parameter	-0.19
ω	Inverse of elasticity of substitution of labor	10.59
σ_q	Habit volatility parameter	0.30
K_q	Habit stochastic volatility parameter	-35.40
θ	Elasticity of substitution of goods	3.00
α	Price stickiness probability	0.67
$\pi^* \times 10^2$	Average inflation	1.10
$g_a \times 10^3$	Average productivity growth	5.75
ϕ_a	Autocorrelation of productivity growth	0.18
$\sigma_a \times 10^3$	Productivity growth volatility parameter	6.71
K_a	Productivity growth stochastic volatility parameter	0.00
ϕ_ϵ	Autocorrelation of markup shock	0.82
$\sigma_\epsilon \times 10$	Markup shock volatility parameter	3.01
K_ϵ	Markup shock stochastic volatility parameter	3.32

6 Policy Experiment

Further insights into the differences in bond yields and risk premiums between discretion and commitment in monetary policy can be obtained from a policy experiment. The experiment is based on a model calibration using the U.S. quarterly data described in section 2 from 1971:3 to 2007:4 (the period labeled as “Fiat Money”). The main assumption in the calibration is that the Federal Reserve conducted a policy under discretion during the period and, therefore, the equilibrium properties of the model under discretion characterize the economy. The policy experiment evaluates changes in the macroeconomic and bond properties of the economy when the policy under commitment is implemented. Table 3 contains the parameter values implied by the calibration. The details of the calibration are presented in appendix F. The calibration focuses on capturing the slope of the yield curve, while matching important properties of inflation. In particular, the persistence of inflation is important to characterize

Table 4: Model-Implied Macroeconomic Properties. Average levels and standard deviations are presented in annualized percentage terms. Autocorrelations are the first-order autocorrelation coefficients for quarterly data.

	Data	Model	
	1971:3-2007:4	Discretion	Commitment
$\mathbb{E}[\Delta c_t] \times 4$	1.92	1.92	1.92
$\mathbb{E}[\pi_t] \times 4$	4.40	4.40	4.40
$\sigma(\Delta c_t) \times 4$	1.71	9.38	8.58
$\sigma(\pi_t) \times 4$	2.62	2.62	0.95
$\text{corr}(\Delta c_t, \Delta c_{t-1})$	0.41	0.00	0.08
$\text{corr}(\pi_t, \pi_{t-1})$	0.82	0.82	0.49
$\text{corr}(\Delta c_t, \pi_t)$	-0.32	-0.15	-0.34

the differences between discretion and commitment. The model captures the slope of the curve with a relatively low value for γ , negative values for η and K_q , and a positive K_ϵ . This is consistent with habit formation in preferences, countercyclical risk aversion, and inflation volatility that increases with the level of inflation.

Tables 4 and 5 show some statistical properties of the calibration under discretion and the implied properties under commitment. Quantitatively, the results of the calibration evidence the difficulty to simultaneously capture bond yield and consumption growth properties of the U.S. economy. This is consistent with Rudebusch and Swanson (2008) who find that habits in a production economy do not capture significant bond risk premiums unless a significant distortion in labor dynamics is allowed. The distortion is captured by a very inelastic labor supply (high ω) and is reflected in a high volatility of consumption growth and the short-term rate. Under commitment, consumption growth and inflation are less volatile and there is less persistence in inflation, the spread between long-term bond yields and the short-term rate is lower, the volatility of interest rates decreases for long-term maturities, and the Campbell-Shiller coefficients are closer to one. Lower spreads reflect a

Table 5: Model-Implied Bond Yield Properties. Average levels, spreads and standard deviations are presented in annualized percentage terms.

	Data	Model	
	1971:3-2007:4	Discretion	Commitment
$\mathbb{E}[i_t] \times 4$	6.02	6.02	6.21
$\mathbb{E}[i_t^{(4)} - i_t] \times 4$	0.43	0.82	0.59
$\mathbb{E}[i_t^{(8)} - i_t] \times 4$	0.69	1.01	0.70
$\mathbb{E}[i_t^{(12)} - i_t] \times 4$	0.86	1.06	0.78
$\mathbb{E}[i_t^{(16)} - i_t] \times 4$	1.02	1.08	0.83
$\mathbb{E}[i_t^{(20)} - i_t] \times 4$	1.11	1.09	0.87
$\sigma(i_t) \times 4$	2.98	9.35	9.67
$\sigma(i_t^{(4)}) \times 4$	2.93	5.89	4.19
$\sigma(i_t^{(8)}) \times 4$	2.86	4.40	3.15
$\sigma(i_t^{(12)}) \times 4$	2.75	3.38	2.62
$\sigma(i_t^{(16)}) \times 4$	2.69	2.68	2.19
$\sigma(i_t^{(20)}) \times 4$	2.62	2.19	1.84
$\beta^{(4)}$	-0.78	0.83	0.90
$\beta^{(8)}$	-0.91	0.87	0.91
$\beta^{(12)}$	-1.28	0.90	0.91
$\beta^{(16)}$	-1.45	0.91	0.91
$\beta^{(20)}$	-1.74	0.92	0.91

reduction in average risk premiums resulting from reduced compensations for markup shocks. Changes in the Campbell-Shiller coefficients can be understood from the impulse responses to markup shocks in figure 1. The response of bond spreads to a positive markup shock under commitment is larger than under discretion, while the opposite is true for bond risk premiums. The difference can be explained by the reaction of inflation to markup shocks under the two regimes. Lack of policy credibility under discretion implies persistent inflation which increases bond risk premiums and spreads. Under commitment, an initial increase in inflation is followed by a period of disinflation, such that the average inflation is zero. Policy credibility implies a low inflation risk and thus investors do not require large changes in

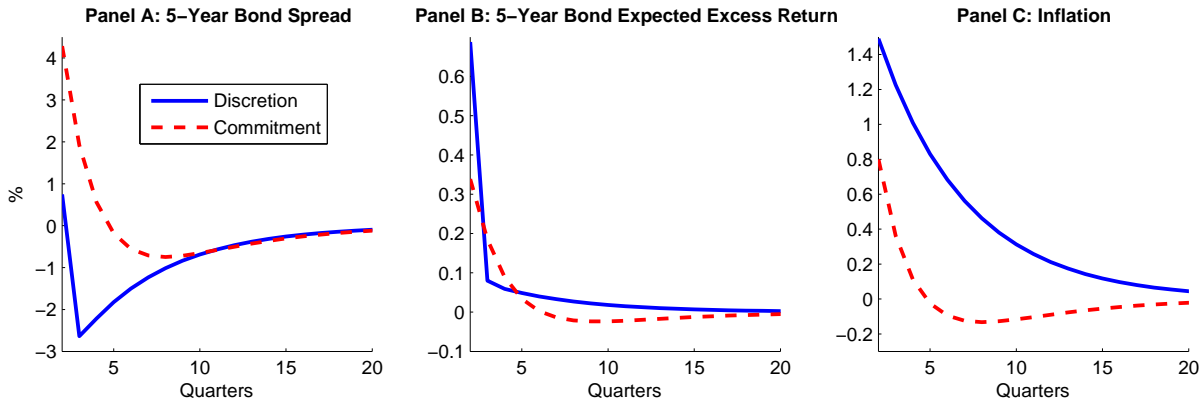


Figure 1: Impulse responses to a positive markup shock.

the compensation for holding long-term bonds. Simultaneously, bond spreads widen since there is significant decline in the short-term rate. In summary, smaller deviations from the expectations hypothesis are observed under commitment because the positive co-movement between bond spreads and risk premiums is reduced by policy credibility.

7 Discussion

The model's implications in sections 5 and 6 can be used to understand whether observed changes in bond yields across U.S. monetary policy regimes are consistent with differences in regime credibilities. The analysis is far from simple given the difficulties to determine differences in credibility across regimes, and the possibility of developments in bond yield dynamics that are not related to monetary policy. Given these limitations, this section presents arguments suggesting a high or low credibility for different regimes, and suggest credibility changes as a potential explanation for observed changes in yield dynamics. Alternatively, the analysis can be seen as providing additional information from the yield curve to identify the degree of policy credibility of a particular regime.

Bretton Woods, the Fiat Money Regime, and the Greenspan Era

The United States promised to fix the price of gold at \$35 per ounce under the Bretton Woods agreement. According to McKinnon (1996), the golden nominal anchor served to restrain U.S. policy makers. The agreement collapsed in 1971 when the government ended the convertibility from dollar to gold, and Bretton Woods was followed by a “Fiat Money Regime.” It can be argued that the Bretton Woods system had a higher credibility than the Fiat Money Regime, since the former had the implicit commitment of convertibility. According to the model’s implications, tables 1 and 2 provide support to this hypothesis. Inflation was less volatile and less persistent during the Bretton Woods system than during the Fiat Money Regime. The spread between the 5-year bond yield and the 3-month T-bill was 67 bps for the Bretton Woods period, significantly lower than the 111 bps during the Fiat Money Regime. Also, the Campbell-Shiller coefficients shifted from positive to negative across the two periods. The evidence then provides additional support to the idea of a higher policy credibility during the Bretton Woods system and suggests an explanation for the change in the dynamics of bond yields during the Fiat Money Regime.

The Fiat Money Regime covers the “Great Inflation” period and Paul Volcker’s and Alan Greenspan’s appointments as chairmen of the Federal Reserve. The relatively low and stable inflation observed during the Greenspan era points in the direction of another regime change. For instance, Clarida, Galí and Gertler (2000) provide empirical evidence suggesting a policy change from a passive stance in the pre-Volcker years toward a stronger anti-inflationary stance during the Volcker-Greenspan era. This increase in the response of the Federal Reserve to inflation can be seen as a greater commitment to low inflation. Table 1 provide support to this idea. Inflation and consumption growth are less volatile and there is a significant reduction in the autocorrelation of inflation during the Greenspan era. However, the bond yield statistics in table 2 show an increase in average bond spreads that cannot be

explained by improvements in policy credibility. This increase can be interpreted as a fact shedding doubts about credibility improvements during the Greenspan era or, alternatively, as an increase in the riskiness of long-term bonds due to technological changes or market developments not related to monetary policy.

The Greenspan Conundrum

The Greenspan Conundrum can be defined as a difficult-to-explain reduction in long-term rates accompanying a contractionary monetary policy. We can use the model's predictions and explain the conundrum as a consequence of credibility improvements in the policy. Consider the impulse responses in figure 1. Under discretion, an inflationary shock is followed by an increase in the short-term rate and wider bond spreads. Under commitment, spreads tend to decline after an inflationary shock. Since the policy is credible, investors perceive an increase in the short-term rate as a measure to decrease inflation in the future. It induces a reduction in the compensation for inflationary risk that reduces the slope of the curve. Similar explanations have been provided by Backus and Wright (2007), Cochrane and Piazzesi (2008), and Atkeson and Kehoe (2008).

8 Conclusion

This paper provides a structural affine term-structure model that links bond yield dynamics to optimal monetary policy and macroeconomic risk. The model is used to understand the effects of a welfare-maximizing policy and its credibility on bond risk premiums. The main finding is that monetary policy affects these premiums through a very specific channel. When a source of inflation risk does not allow the monetary authority to simultaneously stabilize output and inflation, bondholders demand a compensation for this risk. The sign of this premium depends on the optimal output-inflation tradeoff implied by the policy. The

premium is positive if the output distortion on the marginal utility of consumption outweighs the inflation distortion. The size of the compensation is unambiguously reduced by gains in policy credibility. Credibility improves the output-inflation stabilization tradeoff, the economy becomes less vulnerable to inflation risk, and investors then require lower compensations for risk to hold bonds. This link represents a natural explanation for some of the observed changes in bond risk premiums across policy regimes in the United States.

The framework also can be useful to understand fundamental issues in finance and macroeconomics. For instance, the portfolio choice implications of inflation, the hedging properties of real and nominal bonds, or the bond-market channel of transmission of monetary policy can be explored in this framework.

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Appendix

A Profit Maximization under Price Rigidities

This appendix contains the derivation of Equation (19). It is convenient to derive first the aggregate output of a hypothetical economy with full price flexibility. This output is known as the natural rate of output or potential output and is a reference point to conduct monetary policy. Denoting this output by Y_t^f , the profit-maximization problem is

$$\max_{P_t(j)} (1 + \tau)P_t(j)Y_t^f(j) - w_t(j)h_t(j)$$

subject to (13) and (14). The parameter τ can be seen as a subsidy to production provided to eliminate the inefficiency of the potential output due to monopolistic competition. The solution to this problem implies $\frac{P_t(j)}{P_t} = \frac{1+\tau}{\mu_t} s_t(j)$, with the time-varying markup $\mu_t \equiv \frac{\theta_t}{\theta_t - 1}$ arising from market power. In order to eliminate (on average) the inefficiency of the natural rate of output, taxes are set such that

$$1 + \tau = \frac{\theta}{\theta - 1} = \mu,$$

where θ is the average elasticity of substitution across goods and μ is the implied long-term markup. Denoting the deviation of the markup from the long-term markup by $\epsilon_t \equiv \log \mu_t - \log \mu$, the optimality condition can be written as

$$\frac{P_t(j)}{P_t} = e^{\epsilon_t} s_t(j).$$

The markup shock ϵ_t follows the process (18).

Using equations (11) and (14), the real marginal production cost of a differentiated good $s_t(j) \equiv \frac{1}{P_t} \frac{\partial(w_t(j)h_t(j))}{\partial Y_t(j)}$ is

$$s_t(j) = \frac{1}{Y_t(j)} \left(\frac{Y_t(j)}{A_t} \right)^{1+\omega} Y_t^\gamma Q_t. \quad (34)$$

Under price flexibility, $P_t(j) = P_t$, $Y_t(j) = Y_t$ and the real marginal cost for all suppliers becomes $s_t^f = e^{-\epsilon_t}$. Using this condition and equation (34), we obtain the flexible-price output (17), where $y_t^f \equiv \log Y_t^f$.

Consider now, the firm's problem (12). Writing the marginal rate of substitution (11) as $M_{t,T} = \beta^{T-t} \Lambda_{t,T}$, and noticing that

$$\frac{\partial \Psi_{T|t}(j)}{\partial P_t(j)} = \frac{Y_{T|t}(j)}{P_t(j)} e^{-\epsilon_T} \theta_T [P_t(j)(\Pi^*)^{T-t} - e^{\epsilon_T} S_{T|t}(j)],$$

the first order condition for the firm is

$$\begin{aligned} & \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \frac{\Lambda_T}{\bar{\Lambda}^{T-t} e^{\bar{\Delta}y}} \frac{Y_T}{Y_t} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} \theta_T \frac{P_t^*}{P_t} \right] \\ &= \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \frac{\Lambda_T}{\bar{\Lambda}^{T-t} e^{\bar{\Delta}y}} \frac{Y_T}{Y_t} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} e^{\epsilon_T} \frac{S_{T|t}}{P_t(\Pi^*)^{T-t}} \right], \end{aligned} \quad (35)$$

where $\beta_g = \beta \bar{\Lambda} e^{\bar{\Delta}y} \Pi^*$ and $\bar{\Delta}y$ is the long-run one-period output growth. It was used the fact that all producers who change prices optimally at t face the same problem, and therefore $Y_{T|t}(j) = Y_{T|t}$, $P_t^*(j) = P_t^*$ and $S_{T|t}(j) = S_{T|t}$. Here $\bar{\Lambda}^{T-t}$ can be seen as the steady state of Λ_T . Applying the Taylor expansion $a_t b_t = \bar{a} \bar{b} + \bar{b}(a_t - \bar{a}) + \bar{a}(b_t - \bar{b})$ to both sides of the equation around a steady-state with $P_t^* = P_t$, $\Lambda_T = \bar{\Lambda}^{T-t}$, $S_{T|t} = P_t(\Pi^*)^{T-t}$ and $\epsilon_T = 0$, we obtain for the left hand side of the equation

$$\begin{aligned} & \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \frac{\Lambda_T}{\bar{\Lambda}^{T-t} e^{\bar{\Delta}y}} \frac{Y_T}{Y_t} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} \theta_T \left(\frac{P_t^*}{P_t} \right) \right] \\ &= \theta \sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} + \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \left(\frac{\Lambda_T}{\bar{\Lambda} e^{\bar{\Delta}y}} \frac{Y_T}{Y_t} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} \theta_T - \theta \right) \right] + \theta \left(\frac{P_t^*}{P_t} - 1 \right) \sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \end{aligned}$$

and for the right hand side

$$\begin{aligned} & \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \frac{\Lambda_T}{\bar{\Lambda}^{T-t} e^{\bar{\Delta}y}} \frac{Y_T}{Y_t} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} e^{\epsilon_T} \frac{S_{T|t}}{P_t(\Pi^*)^{T-t}} \right] \\ &= \theta \sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} + \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \left(\frac{\Lambda_T}{\bar{\Lambda} e^{\bar{\Delta}y}} \frac{Y_T}{Y_t} \frac{Y_{T|t}}{Y_T} e^{-\epsilon_T} \theta_T - \theta \right) \right] \\ &+ \theta \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \left(\frac{e^{\epsilon_T} S_{T|t}}{P_t(\Pi^*)^{T-t}} - 1 \right) \right]. \end{aligned}$$

Noting that the first and second terms in both sides of the equation are the same, equation (35) becomes

$$\frac{1}{(1 - \alpha\beta_g)} \frac{P_t^*}{P_t} = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} \frac{e^{\epsilon_T} S_{T|t}}{P_t(\Pi^*)^{T-t}} \right].$$

Since $S_{T|t} = s_{T|t} P_T$, replacing equation (34) in the equation above and re-arranging terms, we obtain

$$\frac{1}{(1 - \alpha\beta_g)} (P_t^*)^{1+\theta\omega} = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha\beta_g)^{T-t} e^{\epsilon_T} \left(\frac{P_T}{P_t(\Pi^*)^{T-t}} \right)^{1+\theta\omega} Y_T^{\omega+\gamma} Q_T A_T^{-(1+\omega)} \right],$$

where the approximation $\theta_T \approx \theta$ was used. The equation can be written in terms of the output gap and the habit gap as

$$\frac{1}{(1 - \alpha\beta_g)} \left(\frac{P_t^*}{P_t} \right)^{1+\theta\omega} = e^{\epsilon_t + (\omega+\gamma)x_t + l_t} \left(\frac{P_t}{P_t} \right)^{1+\theta\omega} + \frac{\alpha\beta_g}{1 - \alpha\beta_g} \mathbb{E}_t \left[\left(\frac{P_{t+1}^*}{P_{t+1}} \right)^{1+\theta\omega} \left(\frac{P_{t+1}}{P_t \Pi^*} \right)^{1+\theta\omega} \right].$$

Letting $p_t^* = \log P_t^*$, $p_t = \log P_t$ and using the approximation $e^x \approx 1 + x$, we obtain

$$\begin{aligned} \frac{1}{(1 - \alpha\beta_g)} (1 + (1 + \theta\omega)(p_t^* - p_t)) &= 1 + \epsilon_t + (\omega + \gamma)x_t + l_t \\ &+ \frac{\alpha\beta_g}{1 - \alpha\beta_g} \mathbb{E}_t [1 + (1 + \theta\omega)(p_{t+1} - p_t - \pi^* + p_{t+1}^* - p_{t+1})]. \end{aligned} \quad (36)$$

A first order Taylor approximation of $P_t = \left[(1 - \alpha)(P_t^*)^{1-\theta_t} + \alpha(P_{t-1}\Pi^*)^{1-\theta_t} \right]^{\frac{1}{1-\theta_t}}$ results in

$$p_t = (1 - \alpha)p_t^* + \alpha(p_{t-1} + \pi^*).$$

Replacing this equation in equation (36) and noticing that $\pi_t = p_t - p_{t-1}$, the aggregate supply condition in equation (19) follows, where $\beta_g = \beta^* \exp\left(\frac{(1+\omega)(1-\gamma-\eta)}{\omega+\gamma+\eta} g_a\right)$, and $\kappa \equiv \frac{(1-\alpha\beta_g)(1-\alpha)(\omega+\gamma)}{\alpha(1+\theta\omega)}$.

B Loss Function

Let $\bar{Y}_t = Y_0 e^{t\bar{\Delta}y}$ be the steady-state output at time t , where Y_0 is defined as the output at time 0 and $\bar{\Delta}y$ is the long-run output growth. The steady-state output satisfies

$$\frac{\bar{Y}_t^{\omega+\gamma}}{\bar{Q}_t^{-1} \bar{A}_t^{1+\omega}} = 1, \quad (37)$$

where \bar{Q}_t and \bar{A}_t are the steady-state habit and productivity at time t . By defining $\hat{y}_t \equiv \log Y_t - \log \bar{Y}_t$ and $\hat{q}_t \equiv \log Q_t - \log \bar{Q}_t$, the utility from consumption in equation (20) can be written as

$$\frac{1}{1 - \gamma} \frac{Y_t^{1-\gamma}}{Q_t} = \frac{1}{1 - \gamma} \frac{\bar{Y}_t^{1-\gamma}}{\bar{Q}_t} e^{(1-\gamma)\hat{y}_t - \hat{q}_t}.$$

A second-order log-approximation of this equation results in

$$\begin{aligned} \frac{1}{1 - \gamma} \frac{Y_t^{1-\gamma}}{Q_t} &= \frac{1}{1 - \gamma} \frac{\bar{Y}_t^{1-\gamma}}{\bar{Q}_t} \left(1 + (1 - \gamma)\hat{y}_t - \hat{q}_t + \frac{1}{2}(1 - \gamma)^2 \hat{y}_t^2 + \frac{1}{2} \hat{q}_t^2 - (1 - \gamma)\hat{y}_t \hat{q}_t \right) \\ &+ \mathcal{O}(\|\hat{y}, \hat{q}\|^3). \end{aligned} \quad (38)$$

Replacing equation (13), the disutility from labor in equation (20) is

$$\int_0^1 \frac{h_t(j)}{1+\omega} dj = \frac{1}{1+\omega} \left(\frac{Y_t}{A_t} \right)^{(1+\omega)} \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\theta_t(1+\omega)} dj.$$

Since $\left(\frac{Y_t}{A_t} \right)^{(1+\omega)} = \left(\frac{\bar{Y}_t^*}{A_t} \right)^{(1+\omega)} e^{(1+\omega)(\hat{y}_t - \hat{a})}$ and $\left(\frac{\bar{Y}_t}{A_t} \right)^{(1+\omega)} = \frac{\bar{Y}_t^{1-\gamma}}{\bar{Q}_t}$, it follows that

$$\int_0^1 \frac{h_t(j)}{1+\omega} dj = \frac{1}{1+\omega} \frac{\bar{Y}_t^{1-\gamma}}{\bar{Q}_t} \int_0^1 e^{(1+\omega)[\hat{y}_t - \hat{a}_t - \theta_t \tilde{p}_t(j)]} dj,$$

where $\tilde{p}_t(j) = \log \frac{P_t(j)}{P_t}$. The term under the integral can be log-approximated by

$$\begin{aligned} \int_0^1 e^{(1+\omega)[\hat{y}_t - \hat{a}_t - \theta_t \tilde{p}_t(j)]} dj &= 1 + (1+\omega)(\hat{y}_t - \hat{a}_t) + \frac{1}{2}(1+\omega)^2(\hat{y}_t - \hat{a}_t)^2 - \theta_t(1+\omega)\mathbb{E}_j \tilde{p}_t(j) \\ &+ \frac{1}{2}\theta_t^2(1+\omega)^2[\text{var}_j(\tilde{p}_t(j)) + (\mathbb{E}_j \tilde{p}_t(j))^2] + \mathcal{O}(\tilde{p}^3) \\ &= 1 + (1+\omega)(\hat{y}_t - \hat{a}_t) + \frac{1}{2}(1+\omega)^2(\hat{y}_t - \hat{a}_t)^2 \\ &+ \frac{1}{2}\theta(1+\omega)[1 + \theta\omega]\text{var}_j(p_t(j)) + \mathcal{O}(\tilde{p}^3) \end{aligned} \quad (39)$$

where the second equality comes from the log-approximation $p_t = \int_0^1 p_t(j) dj + \frac{1}{2}(1-\theta)\text{var}_j(p_t(j)) + \mathcal{O}(p^3)$ and $\tilde{p}_t(j) = p_t(j) - p_t$.

Under Calvo staggered price setting, Woodford (2003), pages 399 and 400, shows that

$$\begin{aligned} \text{var}_j(p_t(j)) &= \alpha \text{var}_i(p_{t-1}(j)) + \frac{\alpha}{1-\alpha}(\pi_t - \pi^*)^2 + \mathcal{O}(p^3) \\ &= \frac{\alpha}{1-\alpha} \sum_{s=0}^t \alpha^{t-s}(\pi_s - \pi^*)^2 + \alpha^{t+1}\text{var}_j(p_{-1}(j)). \end{aligned} \quad (40)$$

In order to obtain a representation in terms of the output gap, define $\tilde{Y}_t^f \equiv Y_t^f \exp((\omega + \gamma)^{-1}\epsilon_t)$. It is the natural rate of output under flexible prices adjusted by markup shocks. Notice that $x_t = y_t - \tilde{y}_t^f$ where $\tilde{y}_t^f \equiv \log \tilde{Y}_t^f$ and optimality implies

$$\frac{\left(\tilde{Y}_t^f \right)^{\omega+\gamma}}{\left(Q_t^f \right)^{-1} A_t^{1+\omega}} = 1. \quad (41)$$

Combining equations (37) and (41) we can write \hat{a}_t in terms of $\hat{y}_t^f \equiv \log \tilde{Y}_t^f - \log \bar{Y}_t$ and $\hat{q}_t^f \equiv \log Q_t^f - \log \bar{Q}_t$, such that $x_t = \hat{y}_t - \hat{y}_t^f$ and $l_t = \hat{q}_t - \hat{q}_t^f$.

Notice that $\frac{\bar{Y}_t^{1-\gamma}}{\bar{Q}_t}$ does not depend on monetary policy and $\beta_g^t = \beta^t \bar{Y}_t^{1-\gamma} \bar{Q}_t^{-1}$. Given the log-approximations for the first and second term in equations (38), (39) and (40), and using the

assumption that the habit gap l_t is external for the monetary authority, the loss function is obtained.

C Equilibrium under Optimal Monetary Policy

C.1 Optimal Policy under Discretion

Let $z_t = x_t - (\omega + \gamma)^{-1}l_t$. Under discretion the monetary policy problem at time t reduces to

$$\max -\frac{1}{2} \left[z_t^2 + \frac{\theta_g}{\kappa} (\pi_t - \pi^*)^2 \right]$$

subject to

$$\pi_t - \pi^* = \kappa z_t + F_t$$

where

$$F_t = \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*] + \frac{\kappa}{\omega + \gamma} \epsilon_t$$

is taken as given. From the first order conditions, deviations of optimal inflation from the target can be written in terms of deviations from the target output and habit gaps as

$$\pi_t - \pi^* = -\frac{1}{\theta_g} z_t.$$

Replacing equation (42) in equation (19), we obtain the linear rational expectations equation

$$\pi_t - \pi^* = -\kappa \theta_g (\pi_t - \pi^*) + \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*] + \frac{\kappa}{\omega + \gamma} \epsilon_t.$$

Guess a solution for optimal inflation of the form $\pi_t = \pi^* + \nu_\epsilon^d$. Replacing this form in the equation above and matching coefficients, it can be seen that the coefficient ν_ϵ^d must satisfy the equation

$$(1 + \kappa \theta_g) \nu_\epsilon^d = \beta_g \phi_\epsilon \nu_\epsilon^d + \frac{\kappa}{\omega + \gamma},$$

with solution

$$\nu_\epsilon^d = \frac{\kappa}{(\omega + \gamma)(1 + \kappa \theta_g - \beta_g \phi_\epsilon)}.$$

In order to obtain the output growth process, notice that

$$\Delta z_t = -\theta_g \Delta \pi_t.$$

From the definitions of x_t , l_t and Δy_t^f , the process (21) follows.

Solving the problem for x_t instead of z_t and noticing that l_t can be written in terms of x_{t-1} provides two solutions. The first solution is consistent with the one presented above. The second solution depends on the lagged output gap x_{t-1} . This dependence survives even when the lagged term does not appear in the rational expectations equation (when there is no habit, $\eta = 0$), and

thus violates the minimum state variable (MSV) criterion. Following McCallum (1999), the second solution is ruled out.

C.2 Optimal Policy under Commitment

Let $z_t = x_t - (\omega + \gamma)^{-1}l_t$. From equation (19), we obtain

$$z_t = \frac{1}{\kappa} \left(\pi_t - \pi^* - \beta_g \mathbb{E}_t[\pi_{t+1} - \pi^*] - \frac{\kappa}{\omega + \gamma} \epsilon_t \right).$$

Using this equation, we can replace z_t in the welfare function to obtain a function that depends only on current and expected future inflation. As a result, the first-order condition with respect to π_t is

$$\beta_g^t \left[\frac{1}{\kappa} z_t + \frac{\theta_g}{\kappa} (\pi_t - \pi^*) \right] - \beta_g^{t-1} \frac{\beta_g}{\kappa} z_{t-1} = 0.$$

It implies

$$\Delta z_{t-1} = -\theta_g (\pi_t - \pi^*).$$

Replacing this condition in equation (19), we obtain

$$-\frac{1}{\theta_g} (z_t - z_{t-1}) = \kappa z_t - \frac{\beta_g}{\theta_g} \mathbb{E}_t[z_{t+1} - z_t] + \frac{\kappa}{\omega + \gamma} \epsilon_t.$$

Guessing a solution of the form $z_t = \varphi_1 z_{t-1} + \varphi_2 \epsilon_t$ and matching coefficients, the rational expectations equation has to satisfy the system:

$$\begin{aligned} \varphi_1^2 - \frac{1}{\beta_g} (\kappa \theta_g + 1 + \beta_g) \varphi_1 + \frac{1}{\beta_g} &= 0, \\ \varphi_2 &= -\frac{\kappa \theta_g}{(\omega + \gamma)(1 + \kappa \theta_g + \beta_g - \beta_g \varphi_1 - \beta_g \phi_\epsilon)}. \end{aligned}$$

The equation for φ_1 has two solutions but only one solution is stable ($0 < \varphi_1 < 1$). We use the stable solution and define $\phi_\pi^c = \varphi_1$ and $\nu_\epsilon^c = -\frac{\varphi_2}{\theta_g}$. Using the definition for z_t we obtain the expressions for output and inflation in result 2.

D Affine Term Structure

The bond price equation (9) can be written recursively as

$$b_t^{(n)} = \mathbb{E}_t \left[M_{t,t+1} b_{t+1}^{(n-1)} \right]. \quad (42)$$

We conjecture the solution for bond yields of the form $i_t^{(n)} = \frac{1}{n} (\mathcal{A}_n + \mathcal{B}_n^\top \mathbf{s}_t)$. Replacing this form in equation (42), and noticing that the state variables follow a normal distribution, we obtain

recursive formulas for coefficients \mathcal{A}_n and \mathcal{B}_n given by

$$\begin{aligned}\mathcal{A}_n &= \Gamma_0 + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}^\top \psi - \frac{1}{2} \lambda_n^\top \mathbb{C} \Sigma \lambda_n, \\ \mathcal{B}_n^\top &= \Gamma_1^\top + \mathcal{B}_{n-1}^\top \Phi - \frac{1}{2} \lambda_n^\top \mathbb{K} \Sigma \text{diag} \{ \lambda_n \} \mathbb{A},\end{aligned}$$

with initial conditions $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}^\top$, where \mathbb{C} and \mathbb{K} satisfy $\Psi(\mathbf{s}_t)^2 = \mathbb{C} + \mathbb{K} \text{diag} \{ \mathbf{s}_t \mathbb{A} \}$, and $\lambda_n^\top \equiv \lambda^\top + \mathcal{B}_{n-1}^\top \Psi_c$.

Since bond yields are conditionally normally distributed, equation (42) becomes

$$e^{-ni_t^{(n)}} = \exp \left\{ \mathbb{E}_t \left[\log M_{t,t+1} - (n-1)i_{t+1}^{(n-1)} \right] + \frac{1}{2} \text{var}_t \left(\log M_{t,t+1} - (n-1)i_{t+1}^{(n-1)} \right) \right\},$$

then,

$$-ni_t^{(n)} = -i_t - (n-1) \mathbb{E}_t \left[i_{t+1}^{(n-1)} \right] + \frac{1}{2} \left[\text{var}_t \left(\log M_{t,t+1} - (n-1)i_{t+1}^{(n-1)} \right) - \text{var}_t \left(\log M_{t,t+1} \right) \right].$$

Comparing the result above with the definition of bond risk premium in equation (28) and the variance terms in equation (26), we obtain

$$\xi_t^{(n)} = \frac{1}{2n} \left[\lambda^\top \Psi(\mathbf{s}_t) \Sigma \Psi(\mathbf{s}_t)^\top \lambda - \lambda_n^\top \Psi(\mathbf{s}_t) \Sigma \Psi(\mathbf{s}_t)^\top \lambda_n \right]. \quad (43)$$

Solving, we obtain the affine form (29) with coefficients recursively determined as

$$\xi_{\mathcal{A},n} = -\frac{1}{2n} (\lambda + \lambda_n)^\top \mathbb{C} \Sigma (\lambda_n - \lambda) \quad \text{and} \quad \xi_{\mathcal{B},n}^\top = -\frac{1}{2n} (\lambda + \lambda_n)^\top \mathbb{K} \Sigma \text{diag} \{ \lambda_n - \lambda \} \mathbb{A}.$$

Consider now the Campbell and Shiller (1991) coefficients, $\beta^{(n)}$, implied by the regression

$$i_{t+1}^{(n-1)} - i_t^{(n)} = \alpha^{(n)} + \frac{\beta^{(n)}}{n-1} \left(i_t^{(n)} - i_t \right) + \varepsilon_{CS,t}^{(n)}. \quad (44)$$

These coefficients are the scaled projections of $i_{t+1}^{(n-1)} - i_t^{(n)}$ on $i_t^{(n)} - i_t$, and, therefore, are given by equation (1). Using the definition of risk premiums in (28), equation (31) follows.

E Short-Term Nominal Rates under Discretion and Commitment

Consider first the interest rate i_t^f observed in a hypothetical economy with flexible prices, and inflation at the target level π^* . This rate satisfies $e^{-i_t^f} = \mathbb{E}_t \left[\exp(\log \beta - \gamma \Delta y_t^f - \Delta q_t^f - \pi^*) \right]$,

which implies

$$\begin{aligned}
i_t^f &= -\log \beta + \pi^* + \gamma \frac{1+\omega}{\omega+\gamma} (1-\phi_a) g_a - \frac{1}{2} \gamma^2 \left(\frac{1+\omega}{\omega+\gamma} \right)^2 \sigma_a^2 - \frac{1}{2} \left(\frac{\omega}{\omega+\gamma} \right)^2 \sigma_q^2 - \frac{1}{2} \left(\frac{\gamma}{\omega+\gamma} \right)^2 \sigma_\epsilon^2 \\
&+ \gamma \frac{1+\omega}{\omega+\gamma} \left[\phi_a - \frac{1}{2} \gamma \frac{1+\omega}{\omega+\gamma} K_a \sigma_a^2 \right] \Delta a_t + \frac{\omega}{\omega+\gamma} \left[\eta - \frac{1}{2} \frac{\omega}{\omega+\gamma} K_q \sigma_q^2 \right] \Delta y_t^f \\
&+ \frac{\gamma}{\omega+\gamma} \left[1 - \phi_\epsilon - \frac{1}{2} \left(\frac{\gamma}{\omega+\gamma} K_\epsilon \sigma_\epsilon^2 \right) \right] \epsilon_t.
\end{aligned} \tag{45}$$

The interest rate under flexible prices depends labor productivity, output growth and markup shocks. The habit parameter $\eta < 0$ reduces the natural rate of output to dissuade households from excessive savings. A positive markup shock tends to increase the one-period rate given the reduction of output that results from higher market power. Time variation in the volatility of the shocks generate precautionary savings terms that reduce the natural rate of interest for high levels of productivity, output, and markup shocks.

When prices are not perfectly flexible, the interest rate differs from the natural rate of interest and depends on monetary policy. The equilibrium level of the interest rate depends on current changes in the output gap and inflation. If the policy is conducted under discretion, the one-period interest rate is

$$\begin{aligned}
i_t^d &= i_t^f + [(1-\gamma\theta_g)(1-\phi_\epsilon) - 1] \pi^* + \frac{\omega}{\omega+\gamma} \left[\eta - \frac{1}{2} \left(\frac{\omega}{\omega+\gamma} \right) K_q \sigma_q^2 \right] (y_t^d - y_t^f) \\
&+ [(1-\gamma\theta_g)\phi_\epsilon + \gamma\theta_g] \pi_t^d - \left(\frac{\gamma}{\omega+\gamma} \right) (1-\phi_\epsilon) \epsilon_t \\
&- \frac{1}{2} \left[(1-\gamma\theta_g)^2 (\nu_\epsilon^d)^2 - \left(\frac{\gamma}{\omega+\gamma} \right)^2 \right] (1 + K_\epsilon \epsilon_t) \sigma_\epsilon
\end{aligned} \tag{46}$$

If the policy is conducted under commitment, the interest rate is

$$\begin{aligned}
i_t^c &= i_t^f - (1-\gamma\theta_g) \phi_\pi^c \pi^* + \frac{\omega}{\omega+\gamma} \left[\eta - \frac{1}{2} \left(\frac{\omega}{\omega+\gamma} \right) K_q \sigma_q^2 \right] (y_t^c - y_t^f) + (1-\gamma\theta_g) \phi_\pi^c \pi_t^c \\
&- (1-\phi_\epsilon) \left[(1-\gamma\theta_g) \nu_\epsilon^c + \frac{\gamma}{\omega+\gamma} \right] \epsilon_t \\
&- \frac{1}{2} \left[(1-\gamma\theta_g)^2 (\nu_\epsilon^c)^2 - \left(\frac{\gamma}{\omega+\gamma} \right)^2 \right] (1 + K_\epsilon \epsilon_t) \sigma_\epsilon^2.
\end{aligned} \tag{47}$$

The two policies imply different reactions to inflation, the output gap and markup shocks in the two regimes.

F Calibration and Policy Experiment

The calibration consists of matching selected properties of the U.S. economy for the 1971:3 to 2007:4 period to the associated unconditional first and second moments of the equilibrium properties of the model under discretion.

The unconditional expectation of the state variables, \mathbf{s}_t , is $\mathbb{E}[\mathbf{s}_t] = (\mathbb{I} - \Phi)^{-1} \psi$, and the unconditional covariance matrix is given by the solution $\text{var}(\mathbf{s}_t)$ to the Lyapunov equation

$$\text{var}(\mathbf{s}_t) - \Phi \text{var}(\mathbf{s}_t) \Phi^\top = \Psi_c \Sigma (\mathbb{C} + \mathbb{K} \text{diag}\{\mathbb{E}[\mathbf{s}_t]\}) \Psi_c^\top.$$

This solution is an approximation since \mathbf{s}_t is constrained to take values that ensures that $\Psi(\mathbf{s}_t)^2$ is positive definite. The solution to this equation is given by

$$\text{var}(\mathbf{s}_t) = \sum_{t=0}^{\infty} \Phi^t \Psi_c \Sigma (\mathbb{C} + \mathbb{K} \text{diag}\{\mathbb{E}[\mathbf{s}_t]\}) \Psi_c^\top (\Phi^\top)^t.$$

Using the unconditional moments above and the appropriate formulas from the affine framework in Section 4.5, it is possible to compute unconditional moments for bond yields and risk premiums, and Campbell-Shiller coefficients. The calibration amounts then to find parameter values that minimize deviations of the theoretical unconditional moments from their empirical counterparts.

There are 16 parameters involved in the calibration: seven preference parameters, β , γ , η , ω , θ , K_q and σ_q , and nine production parameters, α , π^* , g_a , ϕ_a , ϕ_ϵ , K_a , K_ϵ , σ_a and σ_ϵ . In order to obtain values for ϕ_a , σ_a and K_a , the labor productivity series from Gomme and Rupert (2007) was used to fit an AR(1) model for growth in labor productivity as in equation (15). Since there is no statistical evidence of heteroscedasticity in the errors, K_a is set to 0. The value of g_a is chosen such that the unconditional expected consumption growth implied by the model perfectly matches the average consumption growth during the period. The values of β and θ were set at 0.99 and 3, respectively. The value for π^* was set equal to the average inflation for the sample. The persistence of markup shocks, ϕ_ϵ , is set equal to the first-order autocorrelation of inflation. This particular calibration seems appropriate to understand the effects of credibility improvements on the level of interest rates and risk premiums, given that, as shown in Section 5.1, inflation persistence is an important indicator of the level of credibility of the policy. The value of σ_ϵ was chosen such that the model matches the standard deviation of inflation. The values for γ , η , ω , K_q and K_ϵ were chosen to minimize a measure of deviations from the sample standard deviation of output growth, the average three-month T-Bill rate, the average slope of the curve and the volatility of the one-year bond yield. The stochastic volatility parameters, K_q and K_ϵ were constrained such that the volatility of the habit and the markup shock are well defined, respectively, after one standard deviation shocks. That is, $1 + K_q(\mathbb{E}[\Delta y_t] + \sigma(\Delta y_t)) \geq 0$ and $1 - K_\epsilon \sigma_\epsilon \geq 0$.