Long Run Labor Income Risk*

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November 2010

Abstract

We present a model in which the investors derive utility over both consumption and leisure. Using the preferences in Epstein and Zin (1989) and Weil (1989), we solve for asset prices when consumption and labor income exhibit long run risks as in Bansal and Yaron (2004). We show that incorporating leisure, and by corollary, labor income, generates significant improvements in the model. In particular, we show that the model can reproduce equity return moments with low risk aversion and persistence in long run risks that are empirically plausible. We also show that the model generates predictability in asset returns that are more consistent with the data than that in a model where dividends do not depend on the dynamics of wage growth.

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1 Introduction

The role of labor income in determining asset price dynamics and expected returns has garnered substantial attention in the recent asset pricing literature. Labor income appears to be important for understanding the cross-section of expected returns (Jagannathan and Wang (1996) and Campbell (1996)), predictability in asset returns (Lettau and Ludvigson (2001), Santos and Veronesi (2006)), and the joint dynamics of consumption growth, market returns, and human wealth (Lustig and VanNieuwerburgh (2008)). However, the explicit role of labor income as a claim to a firm's assets, the resulting joint dynamics of consumption, labor income, and dividends, and the implications of these dynamics in an exchange economy model of asset prices has been less explored. This omission is possibly due to the assumption in the canonical asset pricing model of Lucas (1978) that a representative agent consumes the output of firms in the economy, represented as dividends.

A focus on dividends in the asset pricing literature is natural, as dividends represent equity holders' cash flow claim to firm output. However, the consumption identity in general equilibrium suggests that consumption, as the residual of output after investment, comprises multiple claims to a firm's output. In addition to dividends, which Abel (1999) notes are a levered claim on firm output, consumption consists of labor income and interest paid on positive net supply corporate debt. National accounts data suggest that, on average, 76% of aggregate real per capita consumption in the U.S. derives from wage and labor income, compared to an average 5% from personal dividend income.[1] Moreover, since an increase in wages paid by the firm represents a decrease in cash flows available to pay dividends, the dynamics of labor income directly impact the dynamics of dividends. These facts warrant a closer look at the role that labor income plays in determining equilibrium asset prices.

In this paper, we examine a model of asset prices in which labor income plays a central role in determining an asset's equilibrium required rate of return. We follow Eichenbaum, Hansen, and Singleton (1988) in modeling agents' utility as a function of the consumption of both goods and leisure. Since wages incentivize agents to substitute leisure for consumption, labor income, and its joint dynamics with dividends, plays an important role in determining asset prices. We model agents' lifetime utility using the preferences specified in Epstein and Zin (1989) and Weil (1989), where utility is now defined over non-separable consumption and leisure. Further,

[1] Based on total per capital consumption expenditures, wage and salary disbursements, and personal dividend income from the NIPA tables at the Bureau of Economic Analysis. Data are deflated using the personal consumption expenditure deflator.

we follow Bansal and Yaron (2004) in specifying consumption dynamics with long run risk; expected consumption growth follows a persistent time series process. We augment their long run risks model with long run risks in labor income, which yield interesting implications for asset prices and their dynamics.

Our motivation for exploring the role of labor income in determining asset prices derives not only from theory, but also from empirical considerations. A critical assumption in the long run risks model is the presence of very persistent, yet low volatility component in consumption growth. Unfortunately, this component is quite difficult to detect in the data. In Table 1, Panel A, we report estimates of a first-order autoregressive model for quarterly growth in per capita real consumption.[2] As shown, persistence in consumption growth is quite small, as indicated by the autoregressive parameter estimate $\hat{\rho} = 0.326$. A more persistent conditional mean is implied by an ARMA(1,1) process for consumption growth, as in Bansal and Lundblad (2002). However, while the autoregressive parameter estimate displays considerably more persistence ($\hat{\rho} = 0.759$) and the specification is preferred by the log likelihood function, the implied monthly parameter remains well below the assumed $\rho = 0.979$ in Bansal and Yaron (2004).

Labor income growth displays similar dynamics to consumption growth. In Panel B, we present estimates of an AR(1) and ARMA(1,1) process for growth in real per capita labor income.[3] Again, a simple autoregressive model for labor income growth suggests somewhat low persistence of the conditional mean ($\hat{\rho} = 0.536$), while an ARMA process suggests greater persistence ($\hat{\rho} = 0.697$). When dividends are determined by the joint dynamics of two processes with persistent means, such as labor income and consumption, we show that the persistence in these dynamics necessary to generate the moments of the asset return data are more empirically reasonable than those needed by long run risk in consumption growth alone.

Our paper is related to a body of literature that examines the impact of labor income and leisure on asset prices. Perhaps most closely related are Uhlig (2007) and Yang (2010), who focus on the impact of leisure on a model of long run risk and asset prices. Uhlig finds that the effects of leisure in asset pricing dynamics can be as large as those from consumption. Yang calibrates a model of consumption and leisure data to empirical moments. Leisure growth is essentially mean zero, but exhibits persistent time-varying volatility. Consequently, and due to substitution between leisure and consumption, long run growth dynamics play less of a role in

[2] Consumption data are taken from the NIPA Tables at the Bureau of Economic Analysis. Aggregate consumption is the per capita consumption of nondurables and services, converted to real using the PCE deflator. Data cover the period Q1:1947 through Q4: 2009.

[3] Labor income data are also obtained from the NIPA Tables at the Bureau of Economic Analysis. We utilize per capita wage and salary income, deflated by the PCE deflator over the period Q1: 1947 through Q4: 2009.

determining equity prices and risk premia. In contrast, volatility in leisure drives the majority of the risk premia in the model. Our approach is complementary, focusing on labor income rather than leisure. The link between labor income and leisure is provided by the marginal rate of substitution between consumption and leisure. Unlike leisure, labor income exhibits persistent growth dynamics. As shown in Table 1, Panel B, the dynamics of labor are similar to those of consumption. When estimated as an autoregressive process, labor income displays relatively low and short-lived persistence. However, when mean dynamics are modeled as an ARMA(1,1) process, the mean of labor income growth exhibits considerably greater persistence.

We find that introducing labor income dynamics through preference over leisure has several interesting and important effects on asset pricing moments. Including labor income dynamics allows us to match equity return moments as well as the consumption-only long run risks model of Bansal and Yaron (2004) with lower risk aversion and lower persistence in the dynamics of consumption growth. Beeler and Campbell (2009) note that the long run risks model implies higher order autocorrelations that are inconsistent with the data. Our results suggest that incorporating leisure preference significantly improves the ability of the model to jointly capture equity return moments while matching the persistence of observed consumption growth. Moreover, if we do allow for increased persistence in consumption growth, the model is better able to capture the dynamics of the price-dividend ratio.

In addition to providing results on the model's ability to generate the moments of the equity and aggregate data, we present results on the model's ability to capture predictability in asset returns and aggregate growth rates. We find that allowing for dividend sensitivity to growth in wages has important effects on the ability of the model to capture the predictability of asset returns, consumption growth, and dividend growth. In particular, the inclusion of this sensitivity improves upon the model's ability to generate predictability of equity returns and reduces the predictability of the model for consumption and dividend growth. These results suggest that a model that incorporates sensitivity of dividends to wage growth substantially improves the ability of the model to capture the degree of predictability observed in the data.

The remainder of the paper is organized as follows. In Section 2, we discuss the theoretical framework of the model. We derive equilibrium asset prices under Epstein and Zin (1989) and Weil (1989) preferences when agents have preference over labor and leisure, as in Eichenbaum, Hansen, and Singleton (1988). Section 3 presents results of the calibration of our model for asset return moments. Section 4 discusses the implications of the model for predictability in asset returns. Section 5 concludes.

2 Utility in Consumption and Leisure

Consider an economy where a representative agent maximizes its utility determined by the preferences in Epstein and Zin (1989) and Weil (1989):

$$V_t = \left((1-\tilde{\beta}) U_t^{1-\frac{1}{\psi}} + \beta \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{1}$$

where β is the subjective time discount factor, $\tilde{\beta}$ is the adjusted subjective discount factor given growth in the economy, ψ is a parameter related to the elasticity of intertemporal substitution, and γ is a parameter related to the degree of relative risk aversion.[4] Following Eichenbaum, Hansen, and Singleton (1988), the intratemporal utility U_t is given by

$$U_t = C_t^{\alpha} (H - N_t)^{1-\alpha}, \tag{2}$$

where C_t denotes aggregate consumption, N_t is labor supplied by households to the production sector, and H is the endowment of "hours" that the representative agent distributes between labor and leisure. Notice that the intratemporal utility is non separable in consumption and labor.

The representative agent faces the intertemporal budget constraint

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s} C_{t+s} \right] \leq \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s} (W_{t+s} N_{t+s} + D_{t+s} + B_{t+s}) \right],$$

where $M_{t,t+s}$ is the pricing kernel that discounts cashflows from $t+s$ to time t, W_t is the wage earned from supplying labor to productive activities, D_t corresponds to the dividends obtained from owning the production sector, and B_t represents cashflows from other financial assets such as corporate debt. The first-order conditions from the utility maximization problem provide us with intratemporal and intertemporal marginal rates of substitution. Specifically, optimality implies that the intertemporal marginal rate of substitution is given by

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left[\left(\frac{C_{t+1}}{C_t} \right)^{\alpha} \left(\frac{H - N_{t+1}}{H - N_t} \right)^{1-\alpha} \right]^{1-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathbb{E}_t [V_{t+1}^{1-\gamma}]^{1/(1-\gamma)}} \right)^{\frac{1}{\psi}-\gamma}, \tag{3}$$

[4]The elasticity of substitution and the coefficient of relative risk aversion when the utility function depends on consumption and labor differ from those when utility depends on consumption only. See Uhlig (2010).

a function of both consumption growth and leisure growth. The effects of leisure on the marginal rate of substitution depend on the relative weight of leisure in the utility function, α, the elasticity of intertemporal substitution (EIS), and the difference $\gamma - 1/\psi$.[5] Notice that the marginal rate of substitution reduces to the standard specification for $\alpha = 1$. Lower values for α increase the sensitivity of $M_{t,t+1}$ to the dynamics of leisure. When $\gamma = 1/\psi$, consumption is more valuable for households when leisure is high if the elasticity of intertemporal substitution is high ($\psi > 1$). On the other hand, if $\psi < 1$, households value consumption more when leisure is low. If $\gamma \neq 1/\psi$, leisure has additional effects on the marginal rate of substitution through its impact on the value V_t.

As discussed above, while investigation of the role of leisure in asset pricing and in a long run risks context is interesting in its own right, our focus is on the implications of the dynamics of labor income for asset pricing. We introduce labor income into the intertemporal marginal rate of substitution of consumption by using the intratemporal marginal rate of substitution between consumption and labor,

$$W_t = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{C_t}{H - N_t}\right), \tag{4}$$

to write the intertemporal marginal rate of substitution (3) directly as a function of wages,

$$M_{t,t+1} = \; = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{W_{t+1}}{W_t}\right)^{-(1-\alpha)\left(1-\frac{1}{\psi}\right)} \left(\frac{V_{t+1}}{\mathbb{E}_t[V_{t+1}^{1-\gamma}]^{1/(1-\gamma)}}\right)^{\frac{1}{\psi}-\gamma}. \tag{5}$$

The marginal rate of substitution of consumption decreases (increases) when wage growth is high if $\psi > 1$ ($\psi < 1$).

We also substitute equation (4) in the recursive equation (1) to obtain

$$V_t = \left\{ (1-\tilde{\beta}) \left[\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} C_t W_t^{-(1-\alpha)} \right]^{1-\frac{1}{\psi}} + \beta \mathbb{E}_t \left[V_{t+1}^{1-\gamma}\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

[5]In the presence of leisure in the utility function, the EIS is given by $(1 - \alpha(1 - 1/\psi))^{-1}$.

This equation can be written as

$$
\begin{aligned}
\left(1-\frac{1}{\psi}\right) v_t &= \log \left\{ (1-\tilde{\beta}) \left(\frac{1-\alpha}{\alpha}\right)^{(1-\alpha)(1-\frac{1}{\psi})} \right. \\
&+ \left. \beta \mathbb{E}_t \left[\exp\left((1-\gamma)(v_{t+1} + \Delta c_{t+1} - (1-\alpha)\Delta w_{t+1})\right)\right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}, \qquad (6)
\end{aligned}
$$

where $\Delta c_{t+1} \equiv \log C_{t+1} - \log C_t$ and $\Delta w_{t+1} \equiv \log W_{t+1} - \log W_t$, denote consumption growth and wage growth, respectively, and

$$
v_t \equiv \log \left(\frac{V_t}{C_t W_t^{-(1-\alpha)}} \right)
$$

is the optimal consumer's utility scaled down by consumption and wages. Thus, a solution to the consumer's optimization problem can be characterized by a solution for the process v_t. Since this process is determined by consumption and wage growth, a solution necessitates the definition of the dynamics of consumption and wage growth. We propose dynamics for these variables in the following section.

2.1 Consumption, Wages, and Dividends

We specify dynamics for consumption and wages that are similar to those hypothesized in Bansal and Yaron (2004). Specifically, we assume the following processes governing the evolution of consumption, wages, and dividends in the economy:

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_{c,t}\varepsilon_{c,t+1}, \\
x_{t+1} &= \phi_x x_t + \sigma_{x,t}\varepsilon_{x,t+1}, \\
\Delta w_{t+1} &= \mu_w + y_t + \sigma_{w,t}\varepsilon_{w,t+1} + \sigma_{wc}\sigma_{c,t}\varepsilon_{c,t+1}, \qquad (7) \\
y_{t+1} &= \phi_y y_t + \sigma_{y,t}\varepsilon_{y,t+1}, \\
\Delta d_{t+1} &= \mu_d + \phi_{dx} x_t + \phi_{dy} y_t + \sigma_{d,t}\varepsilon_{d,t+1} + \sigma_{dc}\sigma_{c,t}\varepsilon_{c,t+1} + \sigma_{dw}\sigma_{w,t}\varepsilon_{w,t+1},
\end{aligned}
$$

where x_t and y_t are the time-varying components of the conditional means of consumption growth and wage growth, respectively. As in Bansal and Yaron (2004), we allow for persistence in expected consumption growth through ϕ_x and persistence in wage growth through ϕ_y.

The existence of a time-varying mean of consumption growth, and the degree to which it persists is a matter of much controversy. As discussed in the introduction, autocorrelations in post-war quarterly consumption growth are low. Beeler and Campbell (2009) point out that autocorrelations in consumption growth are substantially lower than those implied by the parameterization of consumption dynamics in Bansal and Yaron (2004). Indeed, they point out that the autocorrelation in quarterly consumption growth is lower in the data than that implied by a time-averaged random walk (Working (1960)).[6] However, as discussed in the introduction, an ARMA(1,1) process for consumption growth implies a more persistent time-varying mean, yet matches the unconditional autocorrelation in consumption growth. This process is used in Bansal and Yaron (2000), and can be obtained by letting $\varepsilon_{c,t+1} = \varepsilon_{x,t+1}$. Further, the unconditional autocorrelation in the wage growth series discussed in the introduction is higher than that of consumption growth; in post-war quarterly data, the first order autocorrelation in wage growth is 0.33. Thus, it is potentially more plausible that this series deviates from random walk dynamics.

Our modeling of conditional volatility departs slightly from Bansal and Yaron (2004). All innovations $\varepsilon_{k,t}$ are IID$\mathcal{N}(0,1)$, and conditional volatilities are

$$\sigma_{k,t} = \sigma_k(1 - I_z + I_z z_t)^{1/2},$$

for $k = \{c, w, x, y\}$. The indicator I_z is 1 if the economy is affected by time varying uncertainty, and 0 otherwise. The parameters σ_{wc}, σ_{dc}, and σ_{dw}, allow non-zero conditional correlations between consumption growth, wage growth, and dividend growth. In the dynamics of wage, dividend, and consumption growth, the process z_t captures time variation in uncertainty in the economy. We assume that it follows a conditional autoregressive gamma process, representing the discrete counterpart of the square root process in Cox, Ingersoll, and Ross (1985). That is,

[6] The authors also point out that the second-order autocorrelation in annual consumption growth is negative, while the long-run risks model implies positive autocorrelation at all horizons. However, Uhlig (2007) notes that the higher order autocorrelations in consumption growth are difficult to statistically distinguish from zero.

for the set of parameters $(\delta_z, \phi_z, \varsigma_z)$, the conditional distribution of z_t is[7]

$$\frac{z_{t+1}}{\varsigma_z} \sim \text{Gamma}(\delta_z + \mathcal{P}), \quad \text{where} \quad \mathcal{P} \sim \text{Poisson}\left(\frac{\phi_z z_t}{\varsigma_z}\right).$$

The characteristics of this process are studied in Jasiak and Gourieroux (2006). The process has been recently used by Le, Singleton, and Dai (2010) to study the term structure of interest rates.

We approach modeling volatility in this way for two reasons. First, Beeler and Campbell (2009) note that the process for volatility in Bansal, Kiku, and Yaron (2007), which addresses some of the issues in consumption autocorrelation, frequently implies negative variances. The square root process we utilize here cannot take on negative values, and thus allows more easily for highly persistent volatility. Further, the process is more easily extended to consideration of the term structure of interest rates. Heteroskedastic volatility is a robust feature of interest rate data, and seems to be necessary to help explain violations of the expectations hypothesis.

The process for dividend growth in equation (7) can be motivated from the link between consumption, labor, and dividends. Following Bansal and Yaron (2004) and Abel (1999), we interpret the parameter ϕ_{dx} as the leverage ratio on consumption growth, since dividends represent a levered claim to consumption growth. Ignoring interest paid on debt, aggregate consumption is equal to labor income and dividends.[8] That is, $C_t = W_t N_t + D_t$. The marginal rate of substitution between consumption and labor allows us to express dividend growth as

$$\Delta d_{t+1} = \Delta c_{t+1} + \log\left[1 - \kappa_t\left(e^{\Delta w_{t+1} - \Delta c_{t+1}} - 1\right)\right],$$

[7]The autoregressive gamma process has conditional mean and variance are, respectively,

$$\mathbb{E}_t[z_{t+1}] = \delta_z \varsigma_z + \phi_z z_t, \quad \text{and} \quad \text{var}_t(z_{t+1}) = \delta_z \varsigma_z^2 + 2\varsigma_z \phi_z z_t.$$

It follows that the unconditional mean and variance are, respectively,

$$\mathbb{E}[z] = \frac{\delta_z \varsigma_z}{1 - \phi_z}, \quad \text{and} \quad \text{var}(z) = \frac{\delta_z \varsigma_z^2}{(1 - \phi_z)^2}.$$

Its conditional moment generating function is

$$\mathbb{E}_t\left[e^{u z_{t+1}}\right] = e^{-\delta_z \log(1 - u\varsigma_z) + \frac{u\phi_z}{1 - u\varsigma_z} z_t}.$$

[8]Including interest income, and therefore the role of leverage in these dynamics is an interesting topic in its own right. An example that follows a framework similar to this paper is Chen (2009), who investigates the endogenous choice of leverage in a long run risks model.

where

$$\kappa_t = \frac{H\frac{W_t}{C_t}}{\frac{1}{\alpha} - H\frac{W_t}{C_t}}.$$

This equation can be approximated around $\bar{\delta} = \mathbb{E}[\Delta w_t - \Delta c_t]$ to obtain

$$\Delta d_{t+1} \approx \bar{\eta}_{d,t} + (1 + \eta_{d,t})\Delta c_{t+1} - \eta_{d,t}\Delta w_{t+1},$$

where $\eta_{d,t} = \frac{\kappa_t \exp(\bar{\delta})}{1-\kappa_t(\exp(\bar{\delta})-1)}$.

2.2 Solution for the Process v_t

Given the dynamics of wages, consumption, and dividends specified in the previous section, we now turn to the solution for the process v_t. Equation (6) can be approximated as

$$\left(1 - \frac{1}{\psi}\right) v_t = \bar{\eta}_v + \eta_v \log \mathbb{E}_t \left[\exp\left((1-\gamma)(v_{t+1} + \Delta c_{t+1} - (1-\alpha)\Delta w_{t+1})\right)\right], \tag{8}$$

where

$$\eta_v = \frac{\beta}{D_v}\left(\frac{1-\frac{1}{\psi}}{1-\gamma}\right) e^{\frac{1-\frac{1}{\psi}}{1-\gamma}\bar{\vartheta}}, \quad \bar{\eta}_v = \log D_v - \bar{\vartheta}\eta_v,$$

$$D_v = (1-\tilde{\beta})\left(\frac{1-\alpha}{\alpha}\right)^{(1-\alpha)\left(1-\frac{1}{\psi}\right)} + \beta e^{\frac{1-\frac{1}{\psi}}{1-\gamma}\bar{\vartheta}}.$$

The linearization point $\bar{\vartheta}$ can be obtained by solving the fixed-point problem

$$\bar{\vartheta} = \mathbb{E}\left\{\log \mathbb{E}_t \left[\exp\left((1-\gamma)(v_{t+1} + \Delta c_{t+1} - (1-\alpha)\Delta w_{t+1})\right)\right]\right\}.$$

We solve for v_t using the guess-and-verify method. We hypothesize a solution of the form

$$v_t = \bar{v} + v_x x_t + v_y y_t + v_z z_t.$$

Replacing the guessed solution for v_t above in equation (8), using equation (7), and matching coefficients, we obtain a system of equations to find $\bar{v}$, v_x, v_y, and v_z. The solutions are

$$\begin{aligned}
\bar{v} &= \frac{\bar{\eta}_v + (1-\gamma)\eta_v(\mu_c - (1-\alpha)\mu_w) + \frac{1}{2}\eta_v(1-\gamma)^2(1-I_z)\sigma_v^2 - \eta_v\delta_z \log(1-(1-\gamma)\varsigma_z v_z)}{1-\frac{1}{\psi} - \eta_v(1-\gamma)}, \\
v_x &= \frac{(1-\gamma)\eta_v}{1-\frac{1}{\psi} - (1-\gamma)\eta_v\phi_x}, \\
v_y &= -\frac{(1-\alpha)(1-\gamma)\eta_v}{1-\frac{1}{\psi} - (1-\gamma)\eta_v\phi_y}, \\
v_z &= \frac{1}{1-\frac{1}{\psi}}\left(\frac{1}{2}\eta_v(1-\gamma)^2\sigma_v^2 I_z + \frac{\eta_v(1-\gamma)\phi_z v_z}{1-(1-\gamma)\varsigma_z v_z}\right),
\end{aligned}$$

for $\sigma_v^2 = [(1-(1-\alpha)\sigma_{wc})^2\sigma_c^2 + (1-\alpha)^2\sigma_w^2 + v_x^2\sigma_x^2 + v_y^2\sigma_y^2]$. Notice that the equation for v_z is a quadratic equation with up to two possible solutions. We follow McCallum (1983) and take the solution that makes $v_z = 0$ when $I_z = 0$.[9]

2.3 Prices of Risk

The log-pricing kernel $m_{t,t+1} \equiv \log M_{t,t+1}$ can be expressed from equations (3) and (8) as

$$\begin{aligned}
m_{t,t+1} &= \log\beta + \left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)\frac{\bar{\eta}_v}{\eta_v} - \left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)\left(\frac{1-\frac{1}{\psi}}{\eta_v}\right)v_t \\
&- \gamma\Delta c_{t+1} - (1-\alpha)(1-\gamma)\Delta w_{t+1} + \left(\frac{1}{\psi}-\gamma\right)v_{t+1},
\end{aligned}$$

where the solution for v_t is given in the preceding section. The solution to v_t is expressed in the form of the deep parameters in the economy, governing preferences and the dynamics of consumption and wages. Hence, the specification of the log stochastic discount factor above is in terms of economic primitives.

The stochastic components of the pricing kernel contain the prices of the different types of risk affecting the economy. The solutions for v_t above imply expected and stochastic components

[9] We follow this approach to find all endogenous coefficients with more than one possible solution.

for the pricing kernel given by

$$
\begin{aligned}
-\mathbb{E}_t[m_{t,t+1}] &= \Gamma_0 + \frac{1}{\psi}x_t + (1-\alpha)\left(1-\frac{1}{\psi}\right)y_t + \left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)\left(\frac{1-\frac{1}{\psi}}{\eta_v}\right)v_z z_t, \\
-\left(m_{t,t+1} - \mathbb{E}_t[m_{t,t+1}]\right) &= \lambda_c\sigma_{c,t}\varepsilon_{c,t+1} + \lambda_w\sigma_{w,t}\varepsilon_{w,t+1} + \lambda_x\sigma_{x,t}\varepsilon_{x,t+1} + \lambda_y\sigma_{y,t}\varepsilon_{y,t+1} + \lambda_z z_{t+1},
\end{aligned}
$$

respectively, where

$$
\begin{aligned}
\Gamma_0 &= -\log\beta + \frac{1}{\psi}\mu_c + \left(1-\frac{1}{\psi}\right)(1-\alpha)\mu_w + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)\,\sigma_v^2(1-I_z) \\
&- \left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)\delta_z \log(1-(1-\gamma)\varsigma_z v_z),
\end{aligned}
$$

and the prices of risk are

$$
\begin{aligned}
\lambda_c &= \gamma + (1-\gamma)(1-\alpha)\sigma_{wc}, && (9)\\
\lambda_w &= (1-\alpha)(1-\gamma), && (10)\\
\lambda_x &= -\frac{\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)\eta_v}{1-\frac{1}{\psi}-(1-\gamma)\eta_v\phi_x}, && (11)\\
\lambda_y &= \frac{(1-\alpha)(\frac{1}{\psi}-\gamma)(1-\gamma)\eta_v}{1-\frac{1}{\psi}-(1-\gamma)\eta_v\phi_y}, && (12)\\
\text{and} \quad \lambda_z &= -\left(\frac{1}{\psi}-\gamma\right)v_z. && (13)
\end{aligned}
$$

The prices of risk λ_c, λ_w, λ_x, and λ_y capture the pricing kernel's exposure to consumption and wage shocks and shocks to expected growth rates in consumption and wages. The premium λ_z captures exposure to time-varying uncertainty.

Our specification of utility slightly alters the prices of consumption risk relative to Bansal and Yaron (2004) and introduces two new risk premia. First, note that the price of consumption shocks is now affected by the covariance between wage growth and consumption growth. If the weight of labor in the utility function is zero ($\alpha = 1$) or the wage and consumption processes are uncorrelated, then the price of consumption risk will collapse to the standard $\lambda_c = \gamma$. If the conditional correlation between consumption and wage growth is positive and $\gamma > 1$, the price of consumption shocks declines relative to the standard case. Wage growth acts as a hedging instrument for consumption risk. The price of risk for innovation in the conditional mean, λ_x, is the same as in Bansal and Yaron (2004) since the innovations in the mean of consumption growth is uncorrelated with the wage process. Two new risks emerge in comparison to Bansal and Yaron (2004) from our specification. The shock to wage growth, $\varepsilon_{w,t}$, bears a price of risk, as does the shock to wage growth's conditional mean, $\varepsilon_{y,t}$. The price of the shock of wage growth risk, λ_w is similar to that of the shock in consumption; again, this risk depends on the parameter of substitution between leisure and consumption, α. When $\gamma > 1$, the price of wage growth risk is negative since a positive shock to wage growth reduces the marginal utility of consumption. The price of exposure to risks in the conditional mean of wage growth, λ_y, are isomorphic to the price of exposure to risks in the conditional mean of consumption growth, λ_x, up to a scale factor of $1-\alpha$. The size of this price of risk increases as the persistence parameter ϕ_y increases. Finally, notice that shocks to volatility are only priced if $\gamma \neq 1/\psi$.

2.4 Equity Premium

In this section, we discuss the pricing of an equity claim and the associated risk premium. A claim on all future dividends has a price

$$S_t = \mathbb{E}_t[M_{t,t+1}(D_{t+1} + S_{t+1})],$$

and return

$$e^{r_{m,t+1}} = \left(1 + \frac{S_{t+1}}{D_{t+1}}\right)\left(\frac{D_{t+1}}{D_t}\right)\left(\frac{D_t}{S_t}\right).$$

Denote the price-dividend ratio by $p_t \equiv \log S_t - \log D_t$. Following Campbell and Shiller (1988), we approximate the equation above around $\bar{\rho} = \mathbb{E}[p_{t+1}]$ to obtain

$$r_{m,t+1} = \bar{\eta}_p + \eta_p p_{t+1} + \Delta d_{t+1} - p_t,$$

where $\eta_p = \frac{\exp(\bar{\rho})}{1+\exp(\bar{\rho})}$, and $\bar{\eta}_p = \log(1 + \exp(\bar{\rho})) - \bar{\rho}\eta_p$. Notice that the solution for $\bar{\rho}$ involves a fixed point problem.

Following Bansal and Yaron (2004), we assume that the price-dividend ratio is linear in the state variables,

$$p_t = \bar{p} + p_x x_t + p_y y_t + p_z z_t,$$

and use this specification to solve for the undetermined coefficients $\bar{p}$, p_x, p_y, and p_z. The approximate coefficients are given by

$$\bar{p} = \frac{\bar{\eta}_p - \Gamma_0 + \mu_d + \frac{1}{2}\sigma_p^2(1 - I_z) - \delta_z \log(1 - (\eta_p p_z - \lambda_z)\varsigma_z)}{1 - \eta_p} \tag{14}$$

$$p_x = \frac{\phi_{dx} - \frac{1}{\psi}}{1 - \eta_p \phi_x}, \tag{15}$$

$$p_y = \frac{\phi_{dy} - (1 - \alpha)\left(1 - \frac{1}{\psi}\right)}{1 - \eta_p \phi_y} \tag{16}$$

$$p_z = \frac{1}{2}\sigma_p^2 I_z - \left(\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\right)\left(\frac{1 - \frac{1}{\psi}}{\eta_v}\right) v_z + \frac{(\eta_p p_z - \lambda_z)\phi_z}{1 - (\eta_p p_z - \lambda_z)\varsigma_z}, \tag{17}$$

where $\sigma_p^2 = (\sigma_{dc} - \lambda_c)^2\sigma_c^2 + (\sigma_{dw} - \lambda_w)^2\sigma_w^2 + (\eta_p p_x - \lambda_x)^2\sigma_x^2 + (\eta_p p_y - \lambda_y)^2\sigma_y^2 + \sigma_d^2$. Notice that the coefficient on the volatility factor solves a quadratic equation. We choose the root that makes $p_z = 0$ if $I_z = 0$; that is, there is no price of time-varying uncertainty risk if there is no time-varying uncertainty risk.

The coefficients deserve some additional discussion. The coefficient of the price-dividend exposure to the conditional mean of consumption growth, p_x is identical in form to that in Bansal and Yaron (2004). Again, since innovations to the conditional mean of consumption growth are uncorrelated with innovations to the conditional mean of wage growth, this is not surprising. The coefficient of the price-dividend exposure to the conditional mean of wage growth has a similar form, except that it also involves the coefficient of substitution between labor and leisure. Larger magnitudes for ϕ_{dy} or ϕ_y increase the sensitivity of the price-dividend ratio to shocks to expected wage growth.

The solution shows that even if leisure, and by corollary, wages do not affect utility directly ($\alpha = 1$), the wage growth dynamics will still affect price-dividend ratios. The source of this effect is ϕ_{dy}, the sensitivity of dividends to labor. This parameter emerges simply because wage claims affect the amount of total cash flows available to distribute to shareholders. Moreover, the wage process will affect the sensitivity of price-dividend ratios to time-varying uncertainty risk, p_z through the parameter ϕ_{dy}. However, when $\alpha = 1$, the price of risk in the conditional mean of labor income, $\lambda_y = 0$. Consequently, wages will not impact the equity premium.

Given the solution to the price-dividend ratio, it follows that the conditional equity premium is[10]

$$-\operatorname{cov}_t(m_{t,t+1}, r_{m,t+1}) = \lambda_c B_c \sigma_{c,t}^2 + \lambda_w B_w \sigma_{w,t}^2 + \lambda_x B_x \sigma_{x,t}^2 + \lambda_y B_y \sigma_{y,t}^2 + \lambda_z B_{z,t} \qquad (18)$$

where the prices of risk, λ_c, λ_w, λ_x, λ_y, and λ_z, are given in equations (9)-(13). The coefficients B_c, B_w, B_x, B_y, and B_z represent the return sensitivity to unconditional consumption and wage risk, conditional consumption and wage mean risk, and conditional economic uncertainty risk, respectively. These sensitivities, in turn, are given by $B_c = \sigma_{dc}$, $B_w = \sigma_{dw}$, $B_x = p_x$, $B_y = p_y$, and $B_{z,t} = \eta_p p_z \left(\delta_z \varsigma_z^2 + 2\varsigma_z \phi_z z_t\right)$. Non separability of labor and consumption in the utility function adds two terms to the equity premium, which depend on $\lambda_y p_y$ and $\lambda_w \sigma_{wd}$. Labor also has effects on the price of volatility risk. Additionally, the expression for the equity premium shows that it depends on the persistence of the consumption and wage process. The coefficients of the price-dividend ratio

$$B_x = p_x = \frac{\phi_{dx} - \frac{1}{\psi}}{1 - \eta_p \phi_x}, \qquad B_y = p_y = \frac{\phi_{dy} - (1-\alpha)\left(1 - \frac{1}{\psi}\right)}{1 - \eta_p \phi_y}.$$

are the sensitivities of the equity premium to consumption mean risk exposure and wage mean risk exposure, respectively. Larger values for ϕ_x and ϕ_y increase the sensitivity of equity returns to shocks to expected consumption and wage growth, respectively. The sensitivity of the equity premium to long run consumption risk is explained in Bansal and Yaron (2004). The sensitivity of the equity premium to long run wage risk depends on wealth and substitution effects. A negative dependence of dividends on expected wage growth, $\phi_{dy} < 0$, generates a negative wealth effect for equity holders. Good news for wage growth represent bad news for equity cashflows, increasing the equity premium. Preferences on leisure generate a substitution effect. If the elasticity of intertemporal substitution is large enough ($\psi > 1$), a positive shock to expected wage growth induces a larger demand for stocks which decreases the equity premium. The opposite occurs for lower elasticities of intertemporal substitution ($\psi < 1$).

Notice that the unconditional equity premium is given by

$$\begin{aligned} -\mathbb{E}[\operatorname{cov}_t(m_{t,t+1}, r_{m,t+1})] &= \left(\lambda_c B_c \sigma_c^2 + \lambda_w B_w \sigma_w^2 + \lambda_x B_x \sigma_x^2 + \lambda_y B_y \sigma_y^2\right)\left(1 - I_z + I_z \frac{\varsigma_z \delta_z}{1-\phi_z}\right) \\ &+ \lambda_z \eta_p p_z (1+\phi_z) \varsigma_z^2 \delta_z, \end{aligned}$$

[10] In the presence of autoregressive gamma shocks, the negative of the conditional covariance between log stock returns and the log pricing kernel is an approximation to the equity premium, $\log R_{m,t} - \log R_{f,t}$, where $R_{m,t}$ and $R_{f,t}$ are the simple return on equity and the risk-free rate, respectively. If all shocks are Gaussian, the covariance term becomes the familiar term $\log R_{m,t} - \log R_{f,t} = E_t\left[r_{m,t+1}\right] - r_{f,t} + \frac{1}{2}\sigma_t^2(r_{m,t+1})$.

and the unconditional volatility of the equity premium is

$$\sigma(\text{cov}_t(m_{t,t+1}, r_{m,t+1})) \quad = \quad I_z|\lambda_c B_c \sigma_c^2 + \lambda_w B_w \sigma_w^2 + \lambda_x B_x \sigma_x^2 + \lambda_y B_y \sigma_y^2 + 2\lambda_z \eta_p p_z| \frac{\varsigma_z \sqrt{\delta_z}}{(1-\phi_z)},$$

2.5 Risk-Free Rate

The level and volatility of the risk-free rate are affected by the non-separability of consumption and leisure in the utility function. The risk-free rate is

$$\begin{aligned} r_{f,t} &= \Gamma_0 + \delta_z \log(1+\lambda_z \varsigma_z) - \frac{1}{2}\left(\lambda_c^2 \sigma_{c,t}^2 + \lambda_w^2 \sigma_{w,t}^2 + \lambda_x^2 \sigma_{x,t}^2 + \lambda_y^2 \sigma_{y,t}^2\right) \\ &+ \frac{1}{\psi} x_t + (1-\alpha)\left(1 - \frac{1}{\psi}\right) y_t + \left[\left(\frac{\frac{1}{\psi} - \gamma}{1-\gamma}\right)\left(\frac{1-\frac{1}{\psi}}{\eta_v}\right) v_z + \frac{\lambda_z \phi_z}{1+\lambda_z \varsigma_z}\right] z_t. \end{aligned}$$

Shocks affecting expected wage growth have effects on the risk-free rate that depend on the elasticity of intertemporal substitution. If $\psi > 1$, a positive shock ε_y increases the risk-free. Good news for expected wage growth combined with high elasticity of intertemporal substitution of consumption increase the risk-free rate to induce savings in equilibrium. In addition, shocks to wage growth and expected wage growth generate potentially time-varying precautionary savings motives that decrease the level of the risk-free rate. The unconditional mean and unconditional volatility of the risk-free rate are

$$\begin{aligned} \mathbb{E}[r_{f,t}] &= -\log\beta + \frac{1}{\psi}\mu_c + \left(1 - \frac{1}{\psi}\right)(1-\alpha)\mu_w + \frac{1}{2}\left[\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\sigma_v^2 - \sigma_\lambda^2\right](1-I_z) \\ &+ \delta_z \log\left(\frac{1+\lambda_z \varsigma_z}{(1-(1-\gamma)v_z \varsigma_z)^{\frac{1/\psi - \gamma}{1-\gamma}}}\right) \\ &+ \left[\frac{\left(\frac{1}{\psi} - \gamma\right)}{1-\gamma}\left(\frac{1-\frac{1}{\psi}}{\eta_v}\right) v_z + \frac{\lambda_z \phi_z}{1+\lambda_z \varsigma_z} - \frac{1}{2}\sigma_\lambda^2 I_z\right] \frac{c_z \delta_z}{1-\phi_z}. \end{aligned}$$

and

$$\begin{aligned} \sigma(r_{f,t}) &= \frac{1}{\psi^2}\text{var}(x_t) + \left(1 - \frac{1}{\psi}\right)^2 (1-\alpha)^2 \text{var}(y_t) \\ &+ \left[\frac{\left(\frac{1}{\psi} - \gamma\right)}{1-\gamma}\left(\frac{1-\frac{1}{\psi}}{\eta_v}\right) v_z + \frac{\lambda_z \phi_z}{1+\lambda_z \varsigma_z} - \frac{1}{2}\sigma_\lambda^2 I_z\right]^2 \text{var}(z_t) \end{aligned}$$

respectively, where $\sigma_\lambda^2 = \lambda_c^2\sigma_c^2 + \lambda_w^2\sigma_w^2 + \lambda_x^2\sigma_x^2 + \lambda_y^2\sigma_y^2$, $\text{var}(k_t) = \frac{\sigma_k^2}{1-\phi_k^2}\frac{\varsigma_z\delta_z}{1-\phi_z}$ for $k = \{x, y\}$, and $\text{var}(z_t) = \frac{\varsigma_z^2\delta_z}{(1-\phi_z)^2}$. Notice that variation in the conditional mean of wage growth increases the variability of the risk-free rate. Also, time-varying uncertainty affects this variability. Higher persistence in shocks to volatility increases the variability of this rate.

3 Model Calibration and Implications

We calibrate the parameters for the consumption and wage processes to match the mean, volatility, first, and second order autocorrelations in the data. Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007) demonstrate the importance of including time-varying economic uncertainty, so we set $I_z = 1$ in all of our calibrations. Consumption and wage data are constructed from the NIPA tables at the Bureau of Economic analysis. Consumption is per capita real consumption of nondurable goods and services, and wages are real per capita wage and salary disbursements. We also compare moments for the dividend process to those observed in the data. Dividends are calculated as the sum over the year of the monthly dividends per share paid on the CRSP value weighted index. All sets of data are converted to real using the personal consumption expenditure deflator from the NIPA tables. Parameter values are presented in Table 2.

Data and model-implied moments for wages, consumption, and dividends are presented in Table 3. The column labeled 'Data' provides point estimates for data on log annual real consumption, wage, and dividend growth. The columns $\phi_{dy} = 0$ and $\phi_{dy} = -0.5$ present the means of 5,000 simulations of 972 monthly observations under the cases of zero and non-zero exposure of dividends to the conditional mean of wage growth, respectively. A few points are worth noting. First, in order to capture the volatility, first, and second order autocorrelation in consumption growth, we lower the persistence in the conditional mean of the consumption growth process and the volatility of consumption itself relative to the calibrations in Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007). Second, while we are able to match the first order autocorrelation of the wage process well, we have more difficulty matching the second order autocorrelation. The large gap in the first and second order autocorrelation in wage growth may suggest a source of mean reversion, as noted in Beeler and Campbell (2009) that we fail to account for in this specification. On a related note, the calibration somewhat overstates dividend growth volatility and second order autocorrelation.

3.1 Asset Return Moments without Leisure

Under the calibrated parameters above, we first examine the performance of the model in explaining asset return moments when $\alpha = 1$, that is, agents have preference only over consumption and not leisure. Thus, we consider scenarios in which we vary the impact of the sensitivity of dividends to the conditional mean of wage growth. Following Bansal and Yaron (2004), we consider a relatively high intertemporal elasticity of substitution parameter, $\psi = 1.5$. Bansal and Yaron argue that high IES is potentially consistent with a consumption process that exhibits conditional volatility and is necessary to capture the equity premium. Throughout, we set the rate of time preference, $\beta = 0.9995$.

We begin by investigating the parameterization preferred in Bansal and Yaron (2004), with $\phi_{dx} = 3.0$ and $\gamma = 10.0$. Results of this parameterization are shown in Table 4. Note that, given the relatively low level of persistence in conditional mean of consumption growth, the model is expected to face some difficulty in generating an equity premium. Indeed, the table suggests that the lower persistence generates a substantially lower unconditional equity premium, 1.66%, than exhibited in the data, 7.02%. On other dimensions, the model continues to perform reasonably well, capturing low autocorrelation of market returns, a low risk-free rate, and comes close to generating the autocorrelation of the risk-free rate. However, the level, volatility, and persistence of the risk premium are substantially lower than in the data.

In the next column, we increase the parameter of risk aversion, $\gamma = 15.0$, and note that this increases the risk premium without raising the risk-free rate. Increasing the risk aversion parameter reduces the risk-free rate, as the precautionary savings motive becomes dominant. However, this result now comes at the cost of lowering the persistence of the price-dividend ratio, which now has a first order autocorrelation of 0.27, in contrast to 0.88 in the data. As shown in the third column, the persistence can be substantially increased by letting dividends depend on the conditional mean of wage growth. This is because the price-dividend ratio becomes a function of the persistence in the conditional mean of wage growth; as shown in equation (16), even when $\alpha = 1$, the sensitivity of the price-dividend ratio to the conditional mean of wage growth, $p_y = \phi_{dy}/(1 - \eta_p \phi_y)$.

The main result in this section is that with persistence in consumption growth calibrated to match that shown in the data (up to the second autocorrelation), the long run risks model has considerably more difficulty in matching the level of the equity premium. In the subsequent section, we examine the ability of the model to capture asset return moments when leisure is incorporated into the utility function.

3.2 Asset Return Moments with Leisure

We next examine the performance of the model under the assumption that investors have preference for leisure and thus disutility for labor. As above, we set the IES parameter $\psi = 1.5$. The literature examining nonseparable consumption and labor in the utility function utilizes values of α between 0.33 and 0.50. We follow Uhlig (2010) and let $\alpha = 0.33$. We examine the sensitivity of asset return moments to different values of risk aversion, γ, and sensitivity of dividends to the conditional mean of wage growth, ϕ_{dy}.

Results of these parameterizations are exhibited in Table 4. The first column of the table again presents the moments in the data. The second column presents a parameterization to complement the results above, setting $\phi_{dy} = 0$ and $\gamma = 10$. That is, in this calibration, dividends are unaffected by labor income, but investors derive utility over leisure. As shown in the table, even though there is no mechanism for wage growth to impact dividends, the equity premium is substantially larger than in the case where $\alpha = 1$; the unconditional premium is 5.10%, and the model captures equity volatility relatively well at 18.52%. The risk-free rate is substantially lower than in the data, with a mean of 0.40.

The model generates a risk premium, despite the lack of sensitivity of dividends to the conditional mean of wage growth, because investors derive utility over leisure. Since wages represent an incentive to shift time away from leisure, wages affect marginal utility, and thus asset prices. Specifically, the sensitivity of asset returns to the conditional mean of wage growth, B_y, remains negative even if dividends do not respond to the conditional mean. Coupled with a negative price of conditional mean risk, λ_y, this sensitivity generates a positive risk premium in asset prices. However there is nothing asset-specific in this exposure; when $\phi_{dy} = 0$, exposure to the conditional mean of wage growth generates no cross-sectional variation in returns. Rather, the effect simply raises the overall required risk premium.

In the next column, we let dividends be exposed to the conditional mean of wage growth, and set $\phi_{dy} = -0.5$. As shown in the table, this specification generates too large of an equity premium; the unconditional risk premium implied by the model rises to 9.62% per annum. The large risk premium arises from two sources. First, the volatility of wage growth is roughly twice as large as the volatility of consumption growth. Second, the persistence in the conditional mean of wage growth necessary to match the first order autocorrelation in wage growth results in a price of conditional mean risk, λ_y, that is very large in magnitude, as is evident from equation (12). This effect by which the persistence in the conditional mean of wage growth is levered into a large price of risk follows essentially the same mechanism as Bansal and Yaron

(2004). A shock to wage growth is near-permanent, and as such generates a large risk premium.

In the final column, we reduce the risk aversion to $\gamma = 7.5$. Under this parameterization, the asset return moments are substantially more reasonable. The unconditional equity premium, at 7.46%, nearly matches that observed in the data, as does the volatility, at 20.12%. The risk-free rate continues to be low, but close to the unconditional mean at 0.89, with autocorrelation of 0.72, comparable to the data. The level of the price-dividend ratio, 3.09, is similar to that exhibited in the data, and its autocorrelation, 0.62, is comparable to, albeit lower than that exhibited in the data. Like the original Bansal and Yaron (2004) model, the model with labor income has its most difficulty capturing the volatility of the price-dividend ratio; the model generates only about half of the price-dividend ratio volatility exhibited in the data.

3.3 Sensitivity of Asset Return Moments to Persistence

In the preceding sections, we allow the conditional mean of wage growth to exhibit high persistence, while reducing the persistence of the conditional mean of consumption growth. As stated above, our motivation is to match the second order autocorrelation of consumption growth, which suggests lower persistence than calibrated in either Bansal and Yaron (2004) or Bansal, Kiku, and Yaron (2007). However, we are unable to simultaneously match the first and second order autocorrelation of wage growth, and the standard errors on the second order autocorrelation are sufficiently large that it is difficult to ascertain whether calibrations with higher persistence are inconsistent with the data. Hence, in this section, we examine the sensitivity of the results to different levels of persistence in the conditional mean of consumption growth and wage growth.

Our first specification parameterizes the consumption process as in Bansal and Yaron (2004). Specifically, we increase the volatility of consumption parameter, $\sigma_c = 0.0078$ and the persistence in the mean of consumption growth, $\phi_x = 0.979$. The volatility parameter for wage growth, $\sigma_w = 0.0128$ and persistence in the conditional mean of wage growth, $\phi_y = 0.996$, are also increased. The impact of these changes on consumption, wage, and dividend dynamics are shown in Table 6 in the second column. As shown, the parameterization matches the mean and first order autocorrelation of consumption growth, the mean, volatility, and first order autocorrelation of wage growth, and the mean and volatility of dividend growth. The increased volatility parameter for consumption growth results in a higher level of consumption volatility than that observed in the data, and the dividend series is substantially more positively autocorrelated.

The second specification parameterizes the consumption and wage growth dynamics to match the mean and volatility of consumption and wage growth, as well as their second order autocorrelations. The parameterization for consumption growth is similar to our baseline calibration, with $\phi_x = 0.947$ and $\sigma_c = 0.0042$. We reduce the persistence in the conditional mean of wage growth, $\phi_y = 0.977$, and increase the volatility parameter $\delta_z = 30.5875$. The model-implied moments are in the final column of the table. The model matches most moments of the data fairly well, with the exception of the first order autocorrelation in wage growth, which at 0.37 is substantially below that in the data, at 0.52.

We examine the asset pricing implications of these parameterizations in Table 7. As shown in the table, under the first specification, with high persistence in both the conditional mean of consumption growth and wage growth, the model is able to match the asset pricing moments quite well. Of particular note are the volatility and autocorrelation of the price-dividend ratio. As shown in the table, increasing the persistence in the conditional means results in volatility of the price dividend ratio ($\sigma(p) = 0.34$) that very nearly matches that observed in the data of 0.45. The persistence is very close as well ($AC(p, 1) = 0.81$). Also notable is the low level of risk aversion; these moments are all matched with $\gamma = 3$.

The specification with lower persistence is less successful. As shown in the final column, the model replicates the moments of the equity return fairly well. However, it falls far short on the volatility of the price-dividend ratio and its autocorrelation. The suggestion from these results is that in order to capture the dynamics of the price-dividend ratio, it is necessary to introduce fairly strong persistence in conditional means in the context of a model of long run risks.

4 Return Predictability

In this section, we explore the impact that incorporating labor income into the analysis has for the predictability of aggregate quantities by model ratios. In particular, we focus on the predictability of consumption growth and asset returns by the price-dividend and wealth-consumption ratios. We are particularly interested in predictability of consumption growth by these ratios. Beeler and Campbell (2009) note that, under the long run risks model, price-dividend ratios should have strong predictive power for consumption growth, which is not found in the data.

4.1 Price-Dividend Ratio

Table 8 presents statistics for predictive regressions at 1, 3, and 5 year horizons for equity excess returns, consumption growth, wage growth, and dividend growth on price-dividend ratios. The table allows a comparison of two specifications for the sensitivity of dividends to the time-varying conditional mean of wage growth: a specification where dividends do not react to this mean ($\phi_{dy} = 0$), and a specification where a positive innovation to expected wage growth have a negative effect on dividends ($\phi_{dy} = -0.5$).

In the first panel, we present results for the predictive power of price-dividend ratios for excess equity returns. The literature (e.g. Campbell and Shiller (1988)) documents small, negative predictive power of price-dividend ratios for excess equity returns at short horizons with low explanatory power (R^2). At longer horizons, the predictive and explanatory power increases. This pattern is documented in the first column of the table, with the point estimate increasing in magnitude from -0.09 at the one year horizon to -0.40 at the five year horizon, with an accompanying increase in R^2 from 4% to 26%. Both of the model parameterizations reproduce the broad patterns in predictability; the magnitude of the coefficients increases across the horizon, as does the predictive power. The model with $\phi_{dy} = -0.5$ produces more predictability; the coefficients and explanatory power are roughly twice as large as for $\phi_{dy} = 0$. While these fall short of the magnitudes exhibited in the data, the evidence suggests that allowing for negative sensitivity of dividends to the conditional mean of wage growth improves the model's ability to capture predictability in the data.

In the final three panels, we consider the predictive power of price-dividend ratios for exogenous variables; consumption, wage, and dividend growth. The first point to note is that, as suggested in Beeler and Campbell (2009), when $\phi_{dy} = 0$, the price-dividend ratio has too much predictive power for consumption and dividend growth. The point estimate for consumption growth at the one-year horizon (0.08) is four times as large as that in the data and eight times as large (0.16) at the five-year horizon. The point estimate for dividend growth at the one-year horizon (0.82) is ten times as large as that in the data, and remains large at the 5-year horizon (0.99). Further, the price-dividend ratio exhibits strong explanatory power, explaining 44% of the variation in one-year and 10% of the variation in 5-year consumption growth.

In contrast, when $\phi_{dy} = -0.5$, the model shows reduced predictive power for consumption and wage growth. The point estimates for consumption growth are close to that exhibited in the data at the one-year horizon (0.03) and while higher at the 5-year horizon (0.06), are less than half that exhibited in the case where $\phi_{dy} = 0$. The explanatory power of the regressions is also reduced to a level closer to that shown in the data. The model continues to have too much predictive power for dividend growth; the point estimate (0.37) is considerably larger than in the data, but less than half that exhibited in the case where $\phi_{dy} = 0$.

The principal weakness of the model where $\phi_{dy} = -0.5$ emerges in the predictive power for wage growth. Similar to the result that the model with $\phi_{dy} = 0$ produces too much predictability of consumption growth, negative exposure results in too much predictability of wage growth. At short horizons, the price-dividend ratio exhibits virtually no predictive power for growth in wages. However, consistent with the model, the predictive power increases in magnitude with horizon, and is negative at longer horizons. Therefore, although the model generates too much predictability, the evidence is broadly consistent with predictability of wage growth as a function of horizon.

In summary, the evidence shows that the model ability to explain the predictive regressions for equity excess returns and growth rates can be improved by allowing for a negative feedback between dividend growth and expected wage growth. Allowing for negative exposure of dividend growth to the conditional mean of wage growth generates improved return predictability and predictability of consumption and dividend growth that are more consistent with that observed in the data. While the model generates too much predictability in wage growth, the broad patterns are consistent with those exhibited in the data.

4.2 Consumption-Wealth Ratio

A claim on on all future consumption can be priced recursively using the definition of wealth and the budget constraint. Wealth, $\tilde{V}_t$, can be written as

$$\tilde{V}_t = C_t + \mathbb{E}_t[M_{t,t+1}\tilde{V}_{t+1}].$$

It follows that the return on a claim on consumption is

$$r_{a,t+1} = \log\left(\frac{\tilde{V}_{t+1}}{\tilde{V}_{t+1} - C_{t+1}}\right) \approx wc_{t+1} + \Delta c_{t+1} - \bar{\eta}_{wc} - \eta_{wc} wc_t,$$

where $wc_t \equiv \log \tilde{V}_t - \log C_t$ is the wealth-consumption ratio, $\eta_{wc} = \frac{e^{\bar{\omega}}}{e^{\bar{\omega}}-1}$, and $\bar{\eta}_{wc} = -\log(e^{\bar{\omega}} - 1) - \eta_{wc}\bar{\omega}$. The linearization point $\bar{\omega}$ is found recursively as $\bar{\omega} = \mathbb{E}[wc_t]$. Following a procedure similar to the one we used to characterize the price-dividend ratio, we guess a solution for the wealth-consumption ratio of the form

$$wc_t = \bar{wc} + wc_x x_t + wc_y y_t + wc_z z_t,$$

and solve for the undetermined coefficients $\bar{wc}$, wc_x, wc_y, and wc_z to obtain

$$\bar{wc} = \frac{\bar{\eta}_{wc} + \Gamma_0 - \mu_c - \frac{1}{2}\sigma_{wc}^2(1 - I_z) + \delta_z \log(1 - (\eta_{wc} wc_z - \lambda_z)\varsigma_z)}{1 - \eta_{wc}} \tag{19}$$

$$wc_x = \frac{1 - \frac{1}{\psi}}{1 - \eta_{wc}\phi_x}, \tag{20}$$

$$wc_y = \frac{-(1-\alpha)\left(1 - \frac{1}{\psi}\right)}{1 - \eta_{wc}\phi_y} \tag{21}$$

$$wc_z = \frac{1}{2}\sigma_{wc}^2 I_z - \left(\frac{\frac{1}{\psi} - \gamma}{1 - \gamma}\right)\left(\frac{1 - \frac{1}{\psi}}{\eta_v}\right) v_z + \frac{(\eta_{wc} wc_z - \lambda_z)\phi_z}{1 - (\eta_{wc} wc_z - \lambda_z)\varsigma_z}, \tag{22}$$

where $\sigma_{wc}^2 = (1 - \lambda_c)^2\sigma_c^2 + \lambda_w^2\sigma_w^2 + (\eta_{wc} wc_x - \lambda_x)^2\sigma_x^2 + (\eta_{wc} wc_y - \lambda_y)^2\sigma_y^2$. Notice that the coefficient on the volatility factor solves a quadratic equation. We choose the root that makes $wc_z = 0$ if $I_z = 0$.

Table 9 presents statistics for predictive regressions at 1, 3, and 5 year horizons for equity excess returns, consumption growth, wage growth, and dividend growth on price-dividend ratios. The table shows that high wealth-consumption ratios in the model predict low equity excess returns, high consumption growth and high dividend growth. The sensitivity of dividend growth on expected wage growth does not play a role in this predictability.

5 Conclusion

We present a framework in which the dynamics of wage growth impact equity prices, augmenting the impact of the dynamics of consumption growth on equity prices. We incorporate long run risks in a model of utility over consumption and leisure, as in Eichenbaum, Hansen, and Singleton (1988), using the preferences in Epstein and Zin (1989) and Weil (1989). Our investigation is motivated by several concerns. Researchers have documented that labor income is important for explaining cross-sectional and time series variation in equity returns. Further, with the persistence in the conditional mean of consumption growth needed to generate equity return moments, the model with long run risks only in consumption generates dynamics that seem inconsistent with the data along several dimensions.

Our results suggest that with more moderate persistence in the conditional mean of consumption growth and low risk aversion, that we can capture a broad array of both asset price and aggregate macroeconomic moments. The mechanism by which we obtain this result is the inclusion of labor income dynamics, which exhibit more persistence than consumption growth and thus allow us to capture the moments observed in the data with less reliance on consumption. We also show that incorporating labor income dynamics improves upon the long run risk model's ability to capture predictability in asset returns, consumption growth, and dividend growth.

In future versions of this work, we will extend consideration of the wealth-consumption ratio to predictability, and examine the returns to human capital in order to provide comparison for research that explicitly models the returns to human capital. Our framework also allows for estimation of the model either by simulated method of moments or GMM. In particular, a GMM estimation can be undertaken assuming ARMA (1,1) dynamics for consumption and wage growth; these dynamics are special cases of the dynamics we assume in this paper. We leave this empirical estimation for future research.

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Table 1: **Dynamics of Consumption and Labor Income Growth**

Table 1 presents estimates of the dynamics of consumption (Panel A) and labor income (Panel B) growth. Both series are modeled as autoregressive processes. In the first row of each table, the series are modeled as an AR(1). In the second row we accommodate ARMA(1,1) dynamics,

$$\begin{aligned}\Delta c_{t+1} &= \mu_c + \rho_c \Delta c_t + (\rho_c - \omega_c)\,\eta_{c,t+1} \\ \Delta w_{t+1} &= \mu_w + \rho_w \Delta w_t + (\rho_w - \omega_w)\,\eta_{w,t+1}.\end{aligned}$$

In both cases, the volatility of the innovations, $\eta_{c,t+1}$ and $\eta_{w,t+1}$ follow a GARCH(1,1) process,

$$\begin{aligned}\sigma^2_{\eta_c,t+1} &= \pi_c + \nu_c \sigma^2_{\eta_c,t} + \zeta_c \eta^2_{c,t+1} + \xi_{c,t+1} \\ \sigma^2_{\eta_w,t+1} &= \pi_w + \nu_w \sigma^2_{\eta_w,t} + \zeta_c \eta^2_{w,t+1} + \xi_{w,t+1}.\end{aligned}$$

Data on consumption are quarterly observations of real per capita consumption of nondurable goods and services taken from the NIPA tables at the Bureau of Economic Analysis. Labor income is measured as real per capita wage and salary disbursements, also taken from the NIPA tables. Both series are deflated to real using the personal consumption expenditure deflator. Data are sampled from 1947:2 through 2009:4, for 243 observations.

Panel A: Consumption Growth

	μ_c	ρ_c	ω_c	π_c	ν_c	ζ_c	LLF
Est.	0.004	0.384		1.82e-6	0.758	0.175	959.245
SE	(0.001)	(0.069)		(8.42e-7)	(0.067)	(0.057)	
Est.	0.001	0.759	0.431	1.29e-6	0.802	0.145	963.505
SE	(0.001)	(0.102)	(0.126)	(6.52e-7)	(0.065)	(0.052)	

Panel B: Labor Income Growth

	μ_w	ρ_w	ω_w	π_w	ν_w	ζ_w	LLF
Est.	0.003	0.534		1.29e-5	0.482	0.518	792.734
SE	(0.001)	(0.068)		(5.14e-6)	(0.075)	(0.076)	
Est.	0.002	0.697	0.256	1.22e-5	0.508	0.482	795.023
SE	(0.001)	(0.084)	(0.127)	(4.61e-6)	(0.072)	(0.078)	

Table 2: **Baseline Parameter Values**

Parameter	Description	Value
μ_c	Average consumption growth	0.0015
σ_c	Volatility parameter for consumption growth	0.0040
ϕ_x	Autocorrelation parameter for x_t	0.949
$\sigma_x \times 10^{-4}$	Volatility parameter for x_t	3.432
μ_w	Average wage growth	0.0016
σ_w	Volatility parameter for wage growth	0.0098
σ_{wc}	Correlation parameter for wage and consumption growth	0
ϕ_y	Autocorrelation parameter for y_t	0.990
$\sigma_y \times 10^{-4}$	Volatility parameter for y_t	3.432
μ_d	Average dividend growth	0.0015
σ_d	Volatility parameter for dividend growth	0.0351
σ_{dc}	Correlation parameter for dividend and consumption growth	0
σ_{dw}	Correlation parameter for dividend and wage growth	0
ϕ_{dx}	Loading of dividend growth on x_t	2.5
$\varsigma_z \times 10^{-4}$	Parameter of time-varying volatility	7.1925
ϕ_z	Autocorrelation parameter of time-varying volatility	0.990
δ_z	Parameter of time-varying volatility	21.416

Table 3: **Annualized Time Average Growth Rates**

The model parameter values are presented in Table 2. The statistics for the data are based on annual observations from 1929 to 2009. Data on consumption are annual observations of real per capita consumption of nondurable goods and services taken from the NIPA tables at the Bureau of Economic Analysis. Labor income is measured as real per capita wage and salary disbursements, also taken from the NIPA tables. Dividends are dividends per share computed from the CRSP monthly database and aggregated annually. All series are deflated to real using the personal consumption expenditure deflator. The statistics for the model are mean values for 3,000 simulations each with 840 monthly observations that are aggregated to an annual frequency. $AC(u, i)$ denotes the i-th autocorrelation for variable u., and $corr$ denotes correlations. Statistics for models where dividends are not sensitive to wage growth ($\phi_{dy} = 0$), and sensitive to wage growth ($\phi_{dy} = -0.5$) are reported.

Variable	Data	Model	
		$\phi_{dy} = 0$	$\phi_{dy} = -0.5$
$\mathbb{E}[\Delta c]$	2.03	1.80	1.80
$\sigma(\Delta c)$	2.26	2.18	2.18
$AC(\Delta c, 1)$	0.46	0.46	0.46
$AC(\Delta c, 2)$	0.13	0.16	0.16
$\mathbb{E}[\Delta w]$	1.94	1.94	1.94
$\sigma(\Delta w)$	4.89	5.10	5.10
$AC(\Delta w, 1)$	0.52	0.52	0.52
$AC(\Delta w, 2)$	0.12	0.31	0.31
$\mathbb{E}[\Delta d]$	1.12	1.80	1.82
$\sigma(\Delta d)$	11.21	14.14	14.26
$AC(\Delta d, 1)$	0.20	0.26	0.27
$AC(\Delta d, 2)$	-0.21	0.01	0.02
$corr(\Delta c, \Delta w)$	0.60	0.00	0.00
$corr(\Delta c, \Delta d)$	0.60	0.19	0.19
$corr(\Delta w, \Delta d)$	0.40	0.00	-0.08

Table 4: **Asset Pricing Implications - No Leisure Preference**

The model baseline parameter values are presented in Table 2. The statistics for the model are mean values for 5,000 simulations each with 972 monthly observations that are aggregated to an annual frequency. The expression $\mathbb{E}[r_m - r] + J.I.$ is the annualized equity premium, where $J.I.$ is a Jensen's inequality term. The expressions $\mathbb{E}[r]$ and $\mathbb{E}[p]$ are, respectively, the annualized average risk-free rate and price-dividend ratio. The annual price-dividend ratio was constructed taking the price for the last month of the year and accumulating monthly dividends to be paid at the end of the year (assuming a reinvestment rate of 0). Investors are assumed to have no preference over leisure ($\alpha = 1$).

Variable	Data	$\gamma = 10.0$ $\phi_{dy} = 0.0$	$\gamma = 15.0$ $\phi_{dy} = 0.0$	$\gamma = 15.0$ $\phi_{dy} = -0.5$
$\mathbb{E}[r_m - r] + J.I.$	7.02	1.66	3.41	3.46
$\sigma(r_m - r)$	20.60	17.87	18.20	19.80
$AC(r_m - r, 1)$	-0.01	-0.01	-0.01	-0.01
$\mathbb{E}[r]$	1.11	1.34	1.10	1.10
$\sigma(r)$	3.90	1.05	1.06	1.06
$AC(r, 1)$	0.77	0.64	0.65	0.64
$\mathbb{E}[p]$	3.37	2.65	4.53	4.86
$\sigma(p)$	0.45	0.13	0.13	0.19
$AC(p, 1)$	0.88	0.36	0.27	0.59

Table 5: **Asset Pricing Implications - Preference over Leisure**

The model baseline parameter values are presented in Table 2. The statistics for the model are mean values for 5,000 simulations each with 972 monthly observations that are aggregated to an annual frequency. The expression $\mathbb{E}[r_m - r] + J.I.$ is the annualized equity premium, where $J.I.$ is a Jensen's inequality term. The expressions $\mathbb{E}[r]$ and $\mathbb{E}[p]$ are, respectively, the annualized average risk-free rate and price-dividend ratio. The annual price-dividend ratio was constructed taking the price for the last month of the year and accumulating monthly dividends to be paid at the end of the year (assuming a reinvestment rate of 0). The fraction of leisure preference in the utility function is set to 2/3 ($\alpha = 0.33$).

Variable	Data	$\gamma = 10$	$\gamma = 10$	$\gamma = 7.5$
		$\phi_{dy} = 0$	$\phi_{dy} = -0.5$	$\phi_{dy} = -0.5$
$\mathbb{E}[r_m - r] + J.I.$	7.02	5.10	9.62	7.46
$\sigma(r_m - r)$	20.60	18.52	19.83	20.12
$AC(r_m - r, 1)$	-0.01	-0.00	-0.01	-0.02
$\mathbb{E}[r]$	1.11	0.40	0.41	0.89
$\sigma(r)$	3.99	1.33	1.33	1.31
$AC(r, 1)$	0.77	0.72	0.73	0.72
$\mathbb{E}[p]$	3.37	3.90	2.77	3.09
$\sigma(p)$	0.45	0.14	0.20	0.21
$AC(p, 1)$	0.88	0.36	0.61	0.62

Table 6: **Annualized Time Average Growth Rates: Alternative Persistence**

The table presents results with higher and lower persistence in the conditional mean of wage and consumption growth rates. Other model parameter values are presented in Table 2. The statistics for the data are based on annual observations from 1929 to 2009. Data on consumption are annual observations of real per capita consumption of nondurable goods and services taken from the NIPA tables at the Bureau of Economic Analysis. Labor income is measured as real per capita wage and salary disbursements, also taken from the NIPA tables. Dividends are dividends per share computed from the CRSP monthly database and aggregated annually. All series are deflated to real using the personal consumption expenditure deflator. The statistics for the model are mean values for 5,000 simulations each with 972 monthly observations that are aggregated to an annual frequency. $AC(u,i)$ denotes the i-th autocorrelation for variable u., and *corr* denotes correlations. We consider two specifications:

1. $\gamma = 3.5$, $\sigma_c = 0.0078$, $\sigma_w = 0.0128$, $\phi_x = 0.979$, $\phi_y = 0.996$, $\delta_z = 15.294$
2. $\gamma = 15$, $\sigma_c = 0.0042$, $\sigma_w = 0.0108$, $\phi_x = 0.947$, $\phi_y = 0.977$, $\delta_z = 30.588$

Sensitivity of dividends to the conditional mean of wage growth is set to $\phi_{dy} = -0.5$.

Variable	Data	Model	
Specification:		1.	2.
$\mathbb{E}[\Delta c]$	2.03	1.78	1.81
$\sigma(\Delta c)$	2.26	2.83	2.27
$AC(\Delta c, 1)$	0.46	0.47	0.44
$AC(\Delta c, 2)$	0.13	0.23	0.14
$\mathbb{E}[\Delta w]$	1.94	1.84	1.91
$\sigma(\Delta w)$	4.89	4.86	4.92
$AC(\Delta w, 1)$	0.52	0.53	0.37
$AC(\Delta w, 2)$	0.12	0.35	0.12
$\mathbb{E}[\Delta d]$	1.12	1.77	1.82
$\sigma(\Delta d)$	11.21	11.35	14.77
$AC(\Delta d, 1)$	0.20	0.38	0.27
$AC(\Delta d, 2)$	-0.21	0.14	0.02

Table 7: **Asset Pricing Implications - Alternative Persistence**

The model baseline parameter values are presented in Table 2. The statistics for the model are mean values for 3,000 simulations each with 840 monthly observations that are aggregated to an annual frequency. The expression $\mathbb{E}[r_m - r] + J.I.$ is the annualized equity premium, where $J.I.$ is a Jensen's inequality term. The expressions $\mathbb{E}[r]$ and $\mathbb{E}[p]$ are, respectively, the annualized average risk-free rate and price-dividend ratio. The annual price-dividend ratio was constructed taking the price for the last month of the year and accumulating monthly dividends to be paid at the end of the year (assuming a reinvestment rate of 0). We consider two specifications:

1. $\gamma = 3.5$, $\sigma_c = 0.0078$, $\sigma_w = 0.0128$, $\phi_x = 0.979$, $\phi_y = 0.996$, $\delta_z = 15.294$
2. $\gamma = 15$, $\sigma_c = 0.0042$, $\sigma_w = 0.0108$, $\phi_x = 0.947$, $\phi_y = 0.977$, $\delta_z =$

Sensitivity of dividends to the conditional mean of wage growth is set to $\phi_{dy} = -0.5$.

Variable	Data	Model	
		1.	2.
$\mathbb{E}[r_m - r] + J.I.$	7.02	6.91	6.77
$\sigma(r_m - r)$	20.60	20.54	19.17
$AC(r_m - r, 1)$	-0.01	-0.01	-0.01
$\mathbb{E}[r]$	1.11	0.82	1.15
$\sigma(r)$	3.99	1.44	1.20
$AC(r, 1)$	0.77	0.84	0.67
$\mathbb{E}[p]$	3.37	3.26	3.16
$\sigma(p)$	0.45	0.34	0.14
$AC(p, 1)$	0.88	0.81	0.37

Table 8: **Predictability of Returns, Growth Rates, and the Price-Dividend Ratio**

The table displays regression coefficients $\hat{\beta}$, R-squared and t-statistics for predictive regressions of equity excess returns, and consumption, wage and dividend growth on log price-dividend ratios. The model baseline parameter values are presented in Table 2. The statistics for the model are mean values for 5,000 simulations each with 912 monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with $2(j-1)$ lags. The superscript "a" denotes annual. The annual price-dividend ratio was constructed taking the price for the last month of the year and accumulating monthly dividends to be paid at the end of the year (assuming a reinvestment rate of 0). Model 1 and 2 correspond to specifications for the sensitivity of dividends to the conditional mean of wage growth of $\phi_{dy} = 0$ and $\phi_{dy} = -0.5$, respectively.

Variable	Data			Model 1			Model 2		
$\sum_{j=1}^{J}(r^a_{m,t+j} - r^a_{f,t+j})$									
Years (J)	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t
1	-0.09	0.04	1.81	-0.03	0.01	0.98	-0.05	0.01	0.82
3	-0.28	0.20	4.39	-0.07	0.02	1.73	-0.13	0.03	1.71
5	-0.40	0.26	5.05	-0.10	0.02	2.18	-0.20	0.05	2.34
$\sum_{j=1}^{J} \Delta c^a_{t+j}$									
Years (J)	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t
1	0.02	0.12	3.23	0.08	0.18	0.11	0.03	0.08	0.09
3	0.02	0.04	2.09	0.15	0.13	0.29	0.06	0.08	0.27
5	0.02	0.04	1.77	0.16	0.09	0.40	0.06	0.07	0.41
$\sum_{j=1}^{J} \Delta w^a_{t+j}$									
Years (J)	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t
1	0.01	0.00	0.42	0.00	0.02	0.28	-0.14	0.28	0.18
3	-0.02	0.01	0.91	0.00	0.02	0.76	-0.36	0.33	0.55
5	-0.04	0.02	1.26	0.00	0.02	1.13	-0.51	0.31	0.89
$\sum_{j=1}^{J} \Delta d^a_{t+j}$									
Years (J)	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t
1	0.08	0.10	2.86	0.82	0.44	0.60	0.37	0.24	0.52
3	0.11	0.06	2.27	0.97	0.16	1.51	0.52	0.14	1.42
5	0.10	0.05	1.99	0.99	0.10	2.06	0.59	0.12	2.07

Table 9: **Predictability of Returns, Growth Rates, and the Wealth-Consumption Ratio**

The table displays regression coefficients $\hat{\beta}$, R-squared and t-statistics for predictive regressions of equity excess returns, and consumption, wage and dividend growth on log wealth-consumption ratios. The model baseline parameter values are presented in Table 2. The statistics for the model are mean values for 5,000 simulations each with 912 monthly observations that are aggregated to an annual frequency. Standard errors are Newey-West with $2(j-1)$ lags. The superscript "a" denotes annual. The annual wealth-consumption ratio was constructed taking the implied wealth for the last month of the year and accumulating monthly consumption to the end of the year. Model 1 and 2 correspond to specifications for the sensitivity of dividends to the conditional mean of wage growth of $\phi_{dy} = 0$ and $\phi_{dy} = -0.5$, respectively.

Variable	Data			Model 1			Model 2		
$\sum_{j=1}^{J}(r^a_{m,t+j} - r^a_{f,t+j})$									
Years (J)	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t
1				-0.09	0.01	5.00	-0.10	0.01	5.47
3				-0.22	0.02	9.85	-0.28	0.02	10.73
5				-0.36	0.03	13.09	-0.44	0.03	14.24
$\sum_{j=1}^{J}\Delta c^a_{t+j}$									
Years (J)	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t
1				0.91	0.66	0.35	0.91	0.66	0.35
3				1.46	0.33	1.24	1.46	0.33	1.24
5				1.55	0.20	1.93	1.55	0.20	1.95
$\sum_{j=1}^{J}\Delta w^a_{t+j}$									
Years (J)	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t
1				0.01	0.02	1.42	-0.01	0.02	1.41
3				0.02	0.03	4.32	-0.01	0.03	4.30
5				0.04	0.03	6.57	-0.02	0.03	6.58
$\sum_{j=1}^{J}\Delta d^a_{t+j}$									
Years (J)	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t	$\hat{\beta}$	R^2	t
1				1.59	0.06	3.89	1.57	0.06	3.89
3				3.02	0.06	9.35	2.95	0.06	9.36
5				3.27	0.05	12.88	3.17	0.05	12.98