

Buenas Notches: Lines and Notches in Tax System Design

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1. Introduction

A wide range of tax and other policies create discontinuous jumps—*notches*—in the choice set of individuals or firms, because incremental changes in behavior cause discrete changes in net tax liability. But, in spite of their ubiquity in practice, tax notches have attracted little attention from public finance economists.¹ This is in clear contrast to the attention paid to tax-induced *kinks* in budget sets, a common feature of graduated income tax schedules, which feature discontinuous marginal tax rates but not discontinuous jumps in the choice set itself; indeed, a recent literature (Chetty, 2009; Chetty et al, 2009; Saez 2009) has argued that analyzing taxpayer behavior near kinks in budget sets provides a unique opportunity to identify tax-price elasticities.

Notch-like policies appear, and are more widely studied, in other areas of economic policy. For example, certain regulations apply only to firms above a certain size.² Eligibility for Medicaid, which provides a basic set of free or subsidized medical services, is entirely lost once earned income goes beyond a certain level.³ Notches are also common in non-government policies. For example, non-profit organizations often publicize the names of donors and assign “titles” (e.g., “leader,” “founder”) according to brackets of gifts.⁴ Businesses often offer quantity discounts that create a notch.

¹ Blinder and Rosen (1985), discussed below, is a notable exception.

² See, for example, Hahn, Todd, and Van der Klaauw (1999), who use the fact that Title VII of the Civil Rights Act of 1964 applies only to firms with at least 15 employees. The bread and butter of regression discontinuity analysis is situations where assignment to a treatment is based on a covariate lying on either side of a fixed threshold that is *not* subject to choice or manipulation; see Imbens and Lemieux (2008) for a review; public finance economists are drawn to precisely those cases where the variable *is* chosen.

³ See, for example, Yelowitz (1995). Another infamous policy known popularly as a notch, but structurally unrelated to what this paper addresses, has to do with Social Security. When, in 1972, the U.S. Congress decided to link Social Security benefits to the consumer price index to preserve recipients’ purchasing power, it did so with an error in the formula that provided benefits higher than intended. Congress corrected the formula in 1977, so the windfall affected only retirees born roughly between 1910 and 1916. But when the error was corrected, Congress granted a special break to the next group of retirees, who arguably had counted on getting similar benefits. Instead of restoring a lower level of benefits all at once, they restored it slowly, over five years. This transition formula covered people born roughly from 1917 to 1921--a group known as the “notch babies.”

⁴ Harbaugh (1998a, 1998b) shows that donations bunch on the low side of the brackets, and characterizes the notch design that maximizes contributions, and Lacetera and Macis (2009) show the same kind of pattern for the frequency of blood donations in Italy, where there is public recognition for the frequency of donations above a threshold.

Incentive contracts often include bonuses for reaching particular targets, and/or penalties for failing to reach certain quotas.⁵

In this paper I examine notches in tax policy. In Section 2, I present a taxonomy of the wide range of policies that create notches. Sections 3 and 4 address some analytical and normative issues, respectively, raised by notches. Section 5 concludes.

2. A Taxonomy of Tax Notches

Tax notches come in many varieties. Perhaps the most important distinction is between what I will refer to as *quantity notches* and *characteristic notches*. I discuss each in turn.

2.1. Quantity Notches

The simplest example of a notch arises when tax liability is a discontinuous, or step, function of the size of the base, conditional on the rules that determine the size of the base. I will refer to this type of notch as a *quantity notch*.

Many readers may be surprised by the assertion that the U.S. federal income tax schedule provides an example of a quantity notch. At first blush, it features kinks—discontinuous jumps in marginal tax rates—but no notches. In fact a step function, with admittedly small steps, is embedded in the system. Persons with taxable income under \$100,000⁶ may use a “tax table” to calculate tax liability that provides cells of tax liability for ranges of taxable income for each of four marital status categories. For taxable incomes above \$3,000, the bracket widths are \$50, and are smaller below \$3,000 in taxable income. The size of the steps in tax liability is at most \$17, and generally is either \$7-\$8 or \$12-\$13 in the regime of the 15% and 25% marginal tax regimes, respectively.⁷

⁵ See Oyer (1998).

⁶ There are some other minor restrictions regarding which taxpayers are eligible to use the tax table.

⁷ Slemrod (1985) provides evidence of bunching within these brackets and argues that bunching is a symptom of tax evasion. Taxpayers have the option of using continuous schedules to compute their tax liability, where the continuous schedules intersect the steps about midway, so that to minimize tax liability (and therefore ignoring any saving in calculation cost), one would be better off using the table if one’s income fell in the upper range of the (usually) \$50 bracket, and using the continuous schedule if taxable income fell in the lower range of the \$50 bracket. If everyone behaves this way, then the effective budget set features both kinks and notches. At the midpoint of a \$50 bracket, the rational taxpayers switches over from the schedule to the table, with no discrete change in tax liability but a discrete decline from a positive

The tax table notches in the U.S. income tax are not substantively important, but do nicely illustrate one of the common justifications for notches—that they facilitate calculation of a tax liability. Of course, the piecewise-linear (i.e., kinked) nature of the U.S. income tax schedule, ignoring the \$50 notches, is itself a simplification compared to a general non-linear function. Note, though, that once most people use a tax table to calculate tax liability given taxable income, there is no compelling simplification reason to stick to a piecewise linear function: the complexity of the look-up process is not affected by the complexity of the underlying tax function.

A quantity notch can also occur when an incremental change in income triggers a discrete change in, for example, the value of a credit. Consider the U.S. Saver's Credit, enacted in 2001, that provides for a non-refundable credit equal to a percentage of (capped) contributions to retirement savings accounts. The Saver's Credit design features a notch because the percentage credit rate is a discontinuous function of adjusted gross income. For example, a married couple filing jointly with income of \$30,000 receives a 50% tax credit on up to \$2,000 of deposits to a retirement account, but receives only a 20% credit if income is \$30,001 or more. Thus, reporting an extra dollar of adjusted gross income can cause a loss in tax credits of as much as \$600: there is a notch in tax liability net of the Saver's Credit as a function of income.⁸ The original version of the American Recovery and Reinvestment Act of 2009 contained a notable notch, an \$8,000 tax credit for first-time purchasers of a primary residence whose income does not exceed \$75,000 for singles and \$150,000 for married couples; when extended the notch was replaced by an income-related phase-out, creating kinks. The child care credit has a phase-out range with several notches, so that within this range the percentage of expenses allowed as a credit falls by 1% for every \$2,000 of adjusted gross income above a threshold. The phase-out of tuition deductions features two notches in adjusted gross income.

to a zero marginal tax rate. At the end of each table bracket, there is a discrete increase in tax liability of about half the jump between table brackets and a discrete increase in marginal tax rate from zero to the implied marginal tax rates in the schedule.

⁸ Ramnath (2009) provides evidence of significant bunching of reported taxable income around the taxable income notches created by the Saver's Credit, especially for those returns with business income, whose net value is subject to more taxpayer discretion.

Quantity notches can be triggered by incremental changes in tax bases other than income. The Israeli municipal property tax, known as the *arnona*, has separate tax rates per square meter for different size categories. For example, in 2010 in Zone C of Jerusalem, the annual rate of tax is NIS 40.68 for apartments of up to 120 square meters and NIS 54.70 for apartments of more than 120 square meters, thus creating a notch equal to NIS 1682.40 at 120 square meters.⁹ The same feature applies to other property tax systems, both in the U.S. and outside of it; often the stated objective is to exempt low-value properties, for equity or administrative-cost-saving reasons, but this is accomplished not by exempting a certain amount of tax base but rather by exempting properties of below a certain size and subjecting larger properties to the tax rate without deduction of the exempt size threshold.

2.2 Characteristic Notches

The other kind of notch, which I will call a *characteristic notch*, concerns what determines whether a given action or event expands a tax base, and thus triggers additional tax liability. To discuss the wide variety of characteristic notches I appeal to the venerable framework in journalism for information gathering known as the Five Ws. This framework holds that, in order for a report to be complete, it must address five questions: Who (was involved)? What (happened)? When (did it take place)? Where (did it take place)? Why (did it happen)?¹⁰

Similar questions generally apply to the determination of the tax base under any given tax levy. A natural example is a retail sales tax. In general the tax liability of a retail firm depends on the volume of sales (how much?), but also on whether the sales are taxable or tax-exempt (what?), in which state the sales were made (where?), in which tax year the sales were made (when?), and which firm made the sales (who?).

⁹ The arnona rates are taken from http://www.jerusalem.muni.il/jer_main/defaultnew.asp?lng=2, accessed on 1/4/2010. Anecdotally one hears that the arnona notch induces some people to buy two adjacent apartments of less than 120 square meters and knock down the separating walls to create one dwelling for living, but not property tax, purposes.

¹⁰ Barzel (1973) stresses that tax statutes cannot cover all of the multiple dimensions of commodities, thus inducing substitution away from taxed attributes and into the others.

In principle, most¹¹ of these arguments of the tax liability function are continuous variables. This is obvious for the volume of sales, where a discontinuous relationship between tax liability and the volume of sales would constitute a quantity notch, but is also generally true for the other arguments. The location of a retail sale can be represented continuously with latitude and longitude (and, I suppose, altitude). Firms may be characterized by size. The time of sale has a day, hour, and so on. The multiple characteristics of a good or service can also be measured continuously—how much salt is in a can of soup, what color is a pair of trousers, etc. This is not to say that these aspects are easily measurable or even always conceptually clear, as evidenced by the ongoing controversies about where an Internet sale takes place.

For reasons discussed later, the who, what, where, and when of tax base determination are generally subject to notches and lines. In what follows I offer a classification of characteristic notches based on these questions about a tax base. Economic analysis of the policy issues that arise in this area is scarce, although under the moniker “line drawing” it is a major theme in the legal tax literature.¹²

Income Determination and Classification

Whether a particular transaction or other aspect of taxpayer behavior generates taxable income or loss (or, more generally, whether a separate tax rate applies) is subject to scores, indeed hundreds, of categorizing lines that create notches. Whether a transaction triggers ordinary income or preferentially taxed capital gains, whether a form of compensation is an untaxed fringe benefit or taxable salary, whether a contribution is deductible or not, whether the cost of raising capital is (deductible) debt or (non-deductible) equity are just a few examples. All of these categories create lines that are generally based on characteristics, and therefore create notches in choice sets because

¹¹ But not all. In some cases the tax treatment depends on a characterization that is an artifact of law and is essentially discrete. The classification of business entities is an example; crossing a characteristic line between a partnership and a corporation triggers a discrete change in tax treatment, but it is difficult to think of a meaningful sense in which the tax treatment could be made continuous. I thank Mitchell Kane for raising this point with me.

¹² Much of this literature is normative, concerning the appropriate placement of the distinguishing line. With some exceptions (e.g., Weisbach 1998, 2000), the criterion is not explicitly social welfare, but how closely the line reflects existing law and regulations, which may be instrumentally related to welfare.

close to a line a small change in a characteristic discretely changes the tax treatment. The same type of issue arises in the presence of different rates of taxation.

In almost all such cases the tax code and/or regulations establish a series of tests that determine on which side of a tax/tax-exempt or regular-tax/preferred-tax line a case lies, which determines whether it is taxable or not. These tests are invariably multi-dimensional. In almost all cases, the ruling is either-or; for example, a corporate liability is, for tax purposes, either debt or equity. There are exceptions, though; under the U.S. income tax only 50 percent of business-related meals and entertainment expenses are income-tax-deductible.

Commodity Characteristics

Commodity tax regimes that feature exempt kinds of consumption and/or levy more than one different rate inevitably feature lines and notches. A non-capricious tax system must have procedures for distinguishing among goods subject to different tax rates, and real-world consumption tax systems do that by appealing to the characteristics of the commodities. This implies that, although characteristics are conceptually continuous, in characteristics space there are lines that determine where the discontinuous changes in tax status occur—that is, where the notches lie.

For example, the retail sales taxes of U.S. states often exempt food but not restaurant meals, requiring the tax law to draw a line between the two categories. This is done by appealing to a set of characteristics of a restaurant meal, and the line must be precise when, for example, grocery stores sell pre-prepared meals that may or may not be eaten on the premises set up in-store salad bars, or provide nearby tables, silverware, and napkins. This issue was recognized, but for the most part not pursued, in the early optimal taxation literature. For example, Stiglitz and Dasgupta (1971, p. 165) note that it is administratively difficult to have separate tax rates for every commodity, although in general an optimal tax structure would require good-specific tax rates; thus, they note that “almost all tax systems group commodities into fairly wide classes.”¹³

¹³ Stiglitz and Dasgupta (1971, p. 165) also mention that it is often impossible for tax authorities to differentiate between different kinds of income such as, in unincorporated enterprises, differentiating between the labor of the owner, returns to his capital, and pure profits, and thus they are generally taxed at the same rate, “even though the optimal tax structure almost certainly would instruct us to tax them

In some cases tax treatment is differentiated on the basis of one quantifiable characteristic of a commodity. An example of this is the U.S. Gas Guzzler Tax, under which high-performance cars are subject upon initial sale to a per-vehicle tax that is higher, the lower is the fuel economy of the car. For cars (but not light trucks or SUVs) that get less than 22.4 miles per gallon, the tax levy rises discontinuously as the miles-per-gallon rating crosses downward from a (rounded) 0.5 decimal ending to a 0.4 decimal ending, with the change in the tax amounting to as much as \$1300 and averaging about \$800. Note that this tax schedule is discontinuous in miles-per-gallon even though this variable is continuous and fairly easy to measure, and that the presumed social benefit of more fuel-efficient cars is certainly not a step function. In this case basing the tax on a single characteristic is facilitated by the transparent motivation for the tax—to increase the fuel efficiency of new cars—so that the tax can be related to a measure of that one aspect of a vehicle. In several countries notched taxes apply to vehicles whose engine exceeds a given size. However, as the commodity tax example illustrates, in the more common scenario the line depends on multiple, difficult-to-quantify underlying characteristics. In all commodity notch cases, though, a marginal change in some characteristic can change the classification so as to produce a discrete change in the tax consequences.

Border Notches

Physical borders that divide jurisdictions are lines that create discontinuous tax treatment depending on the location of, for example, retail sales. These discontinuities create notches in budget sets where the location of the tax-triggering event matters. People may cross borders to buy lower-taxed items.¹⁴ Corporations choose the location of production, inventories, employees, sales, and other aspects of their operations with an eye on the tax consequences of non-harmonized tax rules.

Where a good or service is purchased can be thought of as one of its characteristics. The characteristic is of heterogeneous importance to consumers

differentially.” They note this to motivate their result that production efficiency is not necessarily part of an optimal tax system when there are constraints on levying differentiated taxes on goods or factors.

¹⁴ The incentive to do so is diminished to the extent that the retail sales tax systems are residence-based. Many U.S. states levy “use” taxes at the same rate as their retail sales taxes that are triggered by out-of-state goods consumed in-state, but these are notorious for being poorly enforced.

depending on where they live (and/or work or otherwise visit), because this determines the transportation cost of obtaining the item. Under some conditions each consumer will buy in one place or the other depending on whether the transportation costs exceed the saving from the tax differential. We would expect few cigarettes purchases just on the high-tax side of the border, and a mass of purchases just on the low-tax side of the border.

Clearly where a jurisdictional border lies is not a policy choice, at least not a choice made by the tax authorities. It does, though, raise the question of why jurisdictions do not levy continuous tax rates at borders so that the closer to a low-tax neighboring jurisdiction, the lower the tax. For example, why doesn't high-alcohol-tax Massachusetts, which borders low-tax New Hampshire, levy lower excise taxes the closer one gets to the New Hampshire border? This policy would just codify what is effectively true when the full price includes transportation costs—a lower price for those who live close to New Hampshire—but keeps more revenue for Massachusetts. If not everyone drives to New Hampshire, there are horizontal equity and efficiency issues, but these issues arise even with no geographical differentiation. The welfare economics of border notches is unique because each government jurisdiction presumably cares only about its own residents' welfare and there may be fiscal externalities across jurisdictions.

Time Notches

In any tax system in which tax liability is not simply a constant proportion of the tax base, the use of accounting periods, generally years, implies that there will often be discrete changes in tax treatment, i.e. notches, at year-end versus year-start. This may occur because of anticipated legislated changes in the tax rules from one year to the next or because, with a graduated tax schedule, a given taxpayer's marginal tax rate is expected to change because of expected changes in the tax base.

Just as administrative considerations limit the number of distinct commodity tax rates, they limit the number of number of distinct tax accounting periods. The income tax accounting period is typically one year, although this is arbitrary. In some transfer systems, the accounting period is considerably shorter because of concerns that income support must be delivered quickly to households with temporarily low income.

Examples of the sensitivity of behavior to time notches abound. When the U.S. top income tax rate increased from 1992 to 1993, Wall Street bonuses shifted from 1/3 end-of-year, 2/3 beginning-of-year, to the reverse.¹⁵ When the U.S. top capital gains tax rate increased from 20% in 1986 to 28% in 1987, there was an extraordinary amount of realizations at year-end 1986.¹⁶ This is particularly notable because, under a realizations-based capital gains tax system like that of the U.S., the tax obligation depends on the date of sale, which is generally unrelated to, e.g., the date of consumption of the proceeds of the sale, which in any event is not well-defined. So the line is drawn in a space different from the arguments of individuals' utility functions.¹⁷

Taxpayer/Remitter Notches

This brings us to the “who” of tax base determination.¹⁸ The same tax base may trigger different tax liabilities depending on some characteristics of the taxpayer or remitter of the tax. Recall that, in the U.S. federal income tax, there are separate schedules for four different categories of taxpayer marital status. When tax is based on family income, marriage penalties and bonuses arise where the sum of two individuals' tax liability depends on whether they are married. Under an individual-based system, the total tax liability of a couple depends on the division of earnings between the spouses. These sharp distinctions obtain in spite of the fact that there are many dimensions to relationships that are, in principle, continuous.

Remitter notches arise when firm characteristics, often size-related, trigger discrete changes in tax treatment. Onji (2009) discusses the Japanese VAT, where firms below a certain size threshold may opt for a favorable regime, and documents the presence of bunching in firm size right below the size threshold. Many countries' VATs feature thresholds, usually in terms on turnover, below which a firm need not register for

¹⁵ Note that, under cash accounting for tax purposes, within limits the payment for labor income can be recorded in either year, regardless of when work was “done.” Thus, we can expect that the substitutability of dated payments to exceed even than the substitutability between dated consumption.

¹⁶ See Burman, Clausing, and O'Hare (1994).

¹⁷ Cole (2009) shows a large time sensitivity of purchases of goods—especially computers—subject to retail sales tax “holidays” that have become widespread in the U.S.

¹⁸ There are also “how” notches; for example, in the United States and other countries a given tax understatement is subject to discretely different penalties depending on the judged intent of the taxpayer.

the VAT.¹⁹ Differentiation of tax liability based on firm size apparently violates production efficiency, which Diamond and Mirrlees (1971) showed characterized an optimal tax system under some conditions. But, as Dharmapala, Slemrod, and Wilson (2009) argue, firm-size differentiation can indeed be part of an optimal tax system when there are fixed-per-firm cost elements in the administrative costs of running a tax system.

Permitting firms below a size threshold the option of a simplified VAT is an example of a case where the consequence of moving from one side of the notch to the other cannot be naturally continuous; i.e., it is difficult to imagine a continuous gradation of regular VAT rules and simplified VAT rules. Several examples in the U.S. tax and tax accounting systems come to mind. Only corporations with greater than \$10 million of assets must file the Schedule M-3 as part of their corporate tax return, which requires a complete reconciliation from financial accounting net income to taxable income in a standardized and detailed format. Corporations with less than \$5 million of gross receipts averaged over the three previous years may use the cash method of accounting, and are exempt from the corporate alternative minimum tax.

3. Some Notch Analytics

3.1 What Bunching Around Quantity Notches Reveals About Price Elasticities

Recently, Saez (2009) and Chetty et al (2009) have argued that the behavioral responses to kinks can provide an estimate of price elasticity that is not subject to the concerns about identification that plague other methods. This is because individuals whose consumption and labor supply choices put them in the neighborhood of a kink are likely to be similar except for the local slope of their budget set.²⁰

¹⁹ VAT thresholds are discussed in Ebrill et al (2001, pp. 113-124).

²⁰ At first blush, this sounds like a regression discontinuity (RD) design, where the treatment is a discretely different relative price. In RD, one is interested in the causal effect of a binary intervention or treatment where assignment to the treatment is determined, either completely or partly, by the value of a predictor being on either side of a fixed threshold. Assuming that the association of the predictor to the outcome is smooth, then any discontinuity of the conditional distribution of the outcome at the threshold value can be interpreted as evidence of the causal effect of the treatment. In some settings, a notch provides a nice setting for an RD design. In others, such as the one that Saez investigates, it does not because the outcome variable is the forcing variable, and so any behavioral response invalidates the key RD assumption of no manipulability of the forcing variable. Nevertheless, it retains the essence of the RD advantage in that it compares the behavior of individuals who are arguably similar except for the relative prices they face.

Analysis of the behavioral response to notches offers the same promise, with some additional considerations. What all of the foregoing notch examples share is that there is a discontinuous break—a notch—in the budget set. The space over which the budget set is defined differs, though. For some quantity notches the budget space has familiar coordinates—leisure and consumption. For characteristic notches the coordinates are not familiar in tax analysis—the characteristics that determine into which discrete tax category a potentially taxable activity is placed.

Here I consider behavior in the presence of quantity notches. When consumers have heterogeneous preferences,²¹ one would expect to see bunching at the more tax-attractive side of the notch. One would also expect to observe bunching in the presence of kinks in convex budget sets. Unlike the case of kinks, with notches we would also expect to see *no one* choosing to be just on the “wrong” side of the notch—the density distribution should feature a “hole” on the tax-disfavored side of the notch. For example, if the government provided a \$1000 tax credit to everyone who made charitable donations of at least \$2000, no one (who understood the program and who was not otherwise constrained) would donate between \$1000 and \$1999, because increasing their contribution to \$2000 would enable them to donate more and retain more money for non-charitable consumption. But in general some people who would have given less than \$1000 will also be persuaded to give \$2000 because the price in terms of foregone consumption is very low; observing how much of this happens reveal information about the price elasticity of charitable giving. Thus, even though a notch does not change the slope of budget sets on either side of the notch, it does produce behavioral response that depends on price sensitivity.

One can then back out the implied price elasticity from the extent of bunching, using a methodology similar to that used for kinks by Saez (2009). The key difference is that, with kinks, one infers the elasticity from the change in behavior to two discretely different and observed relative prices (on the budget segments on either side of the kink), in particular by seeing the maximum change in the consumption basket toward the kink a

²¹ When the choice variables are specified as after-tax income and before-tax income, heterogeneous preferences can arise from underlying taste differences as between leisure and consumption, or because the wage rate differs for given leisure-consumption preferences. Note that preferences can vary while the elasticity of response is uniform.

price change can induce; the larger is that change, the bigger is the price elasticity. With notches, there is no explicit change in relative price at different parts of the budget set, but there is an implicit relative price for any choice between the tax-favored notch point and an alternative on the tax-disfavored part of the budget set. One infers the price elasticity by observing the minimum implicit price change that is required to induce an individual to stay on the tax-disfavored side of the budget set. The larger is the required price change, the smaller is the elasticity. But, because for a given notch size the implicit price change is larger the smaller is the distance between the notch point and the best point on the tax-disfavored part of the budget line, a small pothole corresponds to a small price elasticity. As with kinks, more bunching corresponds to a higher price elasticity.

3.2 Beyond Elasticities: Issues with Characteristic Responsiveness

Economists are accustomed to using price elasticities (the percentage change in the outcome with respect to a percentage change in price) to measure responsiveness. Elasticities are unit-free, and so facilitate comparisons of responsiveness across variables. In their compensated form (i.e., ignoring income effects), they figure prominently in critical optimal tax formulas characterizing, for example, tax progressivity and commodity taxation. Some limitations of elasticities are well known; for example, an elasticity may not be an informative measure of responsiveness when the variable of interest may be either positive or negative.

An elasticity is often not helpful in summarizing the price responsiveness of characteristics because the outcome measure does not have a natural unit. Color is an obvious example. This problem does not apply to all characteristics; the elasticity of a vehicle's fuel efficiency, measured by miles-per-gallon or its inverse gallons-per-mile, is meaningful.

Summarizing the price responsiveness of the timing of a transaction or activity, a characteristic that often has notch-like tax implications, raises a set of interesting issues. An elasticity cannot well portray how responsive a date is to incentives to change the date, because there is no obvious denominator against which to calculate a percentage change in date.

A deeper question concerns the relationship between the pattern of bunching at year-end to fundamental parameters of interest such as the intertemporal substitutability of consumption. One problem with the standard specification of intertemporal preferences, where preferences are separable over time (i.e., the amount of consumption in time t does not affect the relative valuation of consumption in time u relative to consumption in time v), is that it does not allow for the intuitive notion that consumption at dates local to time t should be relatively more substitutable for consumption at time t : “local substitution.” One way to generalize the standard model is to allow that purchases of consumption goods augment “stocks,” which generate consumption services proportional to stocks in current and future time periods, and which depreciate. As first noted by Eichenbaum and Hansen (1990), this mapping from consumption good purchases into consumption services can be viewed as a dynamic version of the household technology suggested by Gorman (1980) and Lancaster (1966, 1975), in which consumption *goods* produce the ultimate arguments of utility functions--consumption *services* in current and future periods. The dynamic Gorman-Lancaster technology introduces time nonseparabilities into consumer’s indirect preferences for goods, even though preferences for consumption services are time-separable. Perhaps the most common interpretation of this model arises from considering durability (and storability). In this interpretation consumption goods are durable and are purchased to augment the stocks of household capital. Because the marginal utility of goods purchased is lower the higher is the carried-over stock, purchases that are closely spaced in time are relatively more substitutable.²² This implies that the pattern of behavior around a time notch does not directly provide evidence about intertemporal substitutability, but about the combination of intertemporal substantiality and the durability (or storability) of the goods in question.

²² Under the habit formation interpretation, past consumption produces a “stock of habits” that depreciates over time and has a *positive* effect on the marginal utility of consumption. This introduces complementarity of purchases across periods but, because habits fade away over time, the net substitutability is greater for longer horizons, and local consumption tends to be complementary. Translating observed behavior around a time notch into conclusions about intertemporal substitutability would also have to confront the issue of temporal aggregation; Heaton (1993) shows that time non-separability and temporal aggregation can interact in important ways, so that assumptions about one can affect the conclusions drawn from data about the other.

Because time is an integral part of the model, and time moves in only one direction, this modeling approach does not readily carry over to notches in other characteristics. There are a number of approaches to modeling environments where the characteristics of products are central. One is Gorman-Lancaster approach where characteristics generated by goods, not the goods themselves, are the direct objects of utility, and there exists a mapping of each good into characteristics space. Another approach is based on the Hotelling (1929) spatial model.²³

A common tax thread among these issues is that what triggers tax in practice is often different than what triggers tax liability in stylized models. For example, retail *purchases* rather than *consumption* trigger retail sales tax liability, receipt of labor income rather than the physical labor itself often determines the timing of tax liability. Sales of appreciated capital assets trigger tax liability rather than accrual of gain or consumption itself. Operational definitions of taxable income differ on many dimensions from the Haig-Simons definition of income. These tax bases, which we might call *surrogate* tax bases, may be part of an optimal tax system because of the difficulty of measuring or monitoring the otherwise optimal tax base.²⁴ I argue that most, if not all, actual tax systems have elements of surrogate tax bases.

The combination of a surrogate tax system featuring lines with *no* substantive distinction between what is on one side or the other of a tax line gives rise to a large amount of bunching on the tax-favored side; this may accurately portray the case of corporate debt versus equity finance. When short holdings are possible, simultaneous long and short holdings of securities that are essentially identical but are on opposite sides of a tax line produce pure tax arbitrage gains. Incoherent tax treatment of capital income implies that the same characteristic outcome can be generated in many different ways by combining component securities to construct derivative securities with the most

²³ Another approach, due to Dixit and Stiglitz (1977), places commodities in groups, so that goods are excellent substitutes within the group, but poor substitutes for the other commodities not in the group; there are intra- and inter-group elasticities of substitution.

²⁴ The concept of a surrogate tax base also helps to clarify the semantic distinctions among real behavioral response, (illegal) evasion and (legal) avoidance—and in particular to offer a useful definition of avoidance. In Slemrod and Yitzhaki (2002), avoidance is defined as “taxpayer efforts to reduce their tax liability that do not alter their consumption basket other than due to income effects.” Substitution across elements of a surrogate tax base does not directly alter one’s consumption basket although, through the function linking the surrogate tax base to the consumption basket, may alter the effective relative prices of the latter and thereby change consumption choices.

advantageous tax treatment. This is possible because there is essentially perfect substitutability among the surrogate tax bases.

The presence of notches in surrogate tax bases sheds light on the hierarchy of behavioral responses proposed by Slemrod (1990, 1992), which asserts that of behavioral responses, timing responses are the most elastic, followed by avoidance/accounting responses, with the least responsive being real responses such as labor supply and saving. Although much evidence is broadly consistent with the hierarchy hypothesis, a satisfactory explanation has not yet been offered. But now consider that the evidence cited in favor of a high elasticity of response, exemplified by the striking increase in capital gains realizations in advance of known increases in the capital gains tax, is response of a surrogate tax base (capital gains do not enter utility functions directly) around a notch, the notch in time at the end of a year. This largely reflects the response to effectively very high tax rates per day of postponement near the year-end notch, plus the fact that sale itself does not constrain the time pattern of consumption. Thus the reduced-form estimates of capital gains realization elasticities do not provide direct evidence about any fundamental, or structural, parameters. The same is true for the high observed elasticity of response to sales tax holidays or expiring investment incentive provision²⁵, where the durability of the consumer or investment good comes into play.

3.3. The Welfare Cost of Notches

A tax notch creates a discontinuity in budget sets and, in its pure form, does not change relative prices within segments. However, for local choices between consumption baskets on different segments, a notch creates widely varying effective relative prices.

The welfare cost, or gain, of a notched policy depends on the alternative policy instruments available relative to the second-best optimum unconstrained by functional form.²⁶ If the unconstrained optimal income tax rates schedule were highly non-linear,

²⁵ See House and Shapiro (2008) for an analysis of the response of investment to a time-notched bonus depreciation scheme.

²⁶ The same statement holds for, say, kinks. If the optimal income tax schedule is irregularly non-linear (i.e., with continuously changing marginal tax rates), then in general a kinked schedule where the marginal tax rate is constant within segments and changes abruptly at bracket points will be valuable relative to a

then compared to a linear system, having notches available as an additional policy instrument may help (and can't hurt). General statements about their potential value cannot be made.

We can, though, say more regarding some particular situations. Consider the case where the optimal tax is linear, but that instead a notched tax is levied—what is the welfare outcome of the notched system relative to the optimum? This is the case addressed by Sallee and Slemrod (2009) in the context of subsidies to fuel-efficient cars. Both Canada and the United States levy taxes on fuel-inefficient cars that are notched, and Canada also provides notched rebates to fuel-efficient cars as part of a “feebate” program. For the U.S. tax, the Gas Guzzler Tax, the notches are at each .5 decimal of MPG under 22.4; increasing a vehicle’s miles-per-gallon (MPG) by .1 can reduce the tax liability by as much as \$1700, and on average does by \$800.

Think of the tax scheme as a Pigouvian correction for a positive externality related to fuel efficiency. Because it seems reasonable to presume that this externality would be a smooth, not notched, function of MPG, so too would the optimal Pigouvian tax, equal to the marginal social benefit of increased fuel efficiency. Assume further that the marginal social cost per MPG is a uniform value of e . The total social gain from such a tax can be approximated as:

$$SG = (1/2)(2e-t)\Delta X,$$

where t is the tax rate and ΔX is the change in vehicle-MPGs (a function of t). Some straightforward algebra can show that the social gain is maximized when $t=e$, which is the appropriate Pigouvian tax. But if $t \neq e$, the social gain is lower, and (e.g., if $t > 2e$) it can even be negative because it induces changes in behavior whose private cost exceeds the social gain. Because of its notches, the Gas Guzzler Tax does not feature a uniform value of the per-MPG subsidy. For those vehicles that otherwise would have a MPG ending in .4, a .1 increase in MPG saves an average of \$800, so the per-MPG tax/subsidy is \$8000, ten times the assumed positive externality. More generally, depending on how close to the next notch the initial MPG is, the effective average tax/subsidy varies widely; for vehicles whose MPG absent the program would be far from a notch, there is no local

linear system, but will not provide as much social welfare as the optimal non-linear system unless one considers some cost of the more flexible schedule, say in terms of administrative cost.

incentive to improve fuel efficiency. Moreover, the MPG re-engineering will happen precisely where the per-MPG subsidy is highest; in these instances, manufacturers are willing to expend up to \$8000 per-MPG to make a .1 improvement, even though the social benefit is just \$800. Based on observations of the distribution of vehicles' decimal MPGs, which are clearly bunched on the low-tax side of MPGs, Sallee and Slemrod (2009) calculate the local response-weighted-average subsidy per MPG to be \$4720, compared to the appropriate Pigouvian subsidy per MPG of \$800. Furthermore, the social gain, net of the private cost of changing MPG, is *negative*, and is about 5 times the absolute value of the net social gain from the Pigouvian incentive to improve MPG.

4. Why Notches?

4.1 Quantity Notches

Would quantity notches be part of an optimal income tax system, if there were no particular administrative cost associated with them? As suggested above, the answer depends on how flexible the income tax schedule can be. When it can be completely flexible, the result of Mirrlees (1971) suggest that the answer is no. He shows that, in an optimal nonlinear income tax, the marginal tax rate always lies between zero and one precludes either a discrete drop or increase in after-tax income as pre-tax income increases. As Diamond (1998, p. 84) discusses, the reason for the two proscriptions is different. Marginal tax rates should not be greater than 100 percent because “Assuming that labor supply can be continually adjusted, there is no gain from having marginal tax rates above 100 percent since no one will have such a tax at the margin. That is, the same outcome can be achieved with taxes no greater than 100 percent.” He goes on to explain that marginal tax rates should not be less than 0 percent because: “It is usually presumed that preferences are such that consumption is an increasing function of the wage. Then, earnings will be non-decreasing in skill. It follows that the optimal tax structure has nonnegative marginal rates...”

However, no theorem rules out the possibility that instituting a downward notch can be part of an optimal schedule when the flexibility of the income tax schedule is constrained, say, to be linear. This possibility is in the same spirit as the argument made by Blinder and Rosen (1985) that, in cases where the government wants to encourage

consumption of a particular activity (say charitable giving), notch schemes may be better than per-unit subsidies. The intuition behind this result is that, compared to a constant per-unit subsidy that applies to all charitable donations, a notch grant that kicks in only for those whose consumption exceeds a certain amount limits the amount of subsidy for inframarginal giving. In principle, when revenue is costly to raise, the ideal subsidy scheme would provide a subsidy only at the margin of favored consumption but, in the absence of personalized incentive schemes or other non-linear consumption taxes or subsidies, a notch may increase welfare.²⁷ Whether a nonlinear consumption tax, and indeed an extreme version of a nonlinear consumption tax with a notch, could be part of an optimal tax system would depend on how flexible the income tax schedule can be.

4.2. Characteristic Notches

Because of the infeasibility of a large number of distinct tax rates and the continuous change in the set of available commodities, commodity tax systems inevitably feature a small number of distinct tax rates and thus create notches in tax base definition, and therefore liability, as a function of characteristics. A small number of tax rates means that lines must be drawn in characteristic space, and lines create notches. Notches, in turn, generate *tax-driven product innovation*, as new goods are created just on the low-tax side of the line.²⁸ To address the issues this raises, Kleven and Slemrod (2009) reformulate optimal commodity tax theory in the language of characteristics using the Gorman-Lancaster notion that is the quantities of the characteristics generated by goods, not the goods themselves, which are the direct objects of utility, and there exists a mapping of each good into characteristics space. They establish that, the closer two goods are in characteristics space, the smaller the optimal tax rate differential. Second, they present an optimal tax analysis of line drawing, and show that, under some conditions, an

²⁷ Blinder and Rosen do not, though, pose this question within a formal optimal taxation problem, nor would this be easy in their framework, in which there is no explicit reason to subsidize consumption of the “favored” good, nor any other (e.g., Ramsey) reason to differentiate the tax on the two goods (there is no valued leisure in the individuals’ utility functions). Blinder and Rosen do not investigate another alternative, a per-unit subsidy that kicks in only at a given level of consumption, which would create a kink, rather than a notch, in the budget set. They mention in a footnote that they did investigate schemes that had both a per-unit subsidy and a notch but note that, to their apparent surprise, the simulations suggested that the optimum always features a notch subsidy plus a positive flat *tax* on charitable giving.

²⁸ Belan and Gauthier (2006) and Belan, Gauthier, and Laroque (2008) investigate the optimal grouping of goods when only a limited number of commodity tax rates can be levied.

optimal policy will place the line so as to eliminate tax-driven product innovation, a production efficiency result.

The answer to why the 5 Ws often feature lines depends in part on the feasibility of the alternative of a smoothly changing tax base definition, and that depends on which W one is talking about. Before addressing the distinctions across Ws, it is worth noting that standard optimal tax theory (meaning theory that ignores the cost of administering and enforcing tax laws) prescribes mind-bogglingly complex tax features such as non-linear income taxes that are age-dependent, non-linear consumption taxes that treat each of the hundreds of thousands of goods and services separately, and tax liability that is a function of every available tag (correlate of ability) such as height and genomic information. Policy does, and ought to, forego many such features.

The feasibility of more granular definitions of tax bases varies depending on the characteristic space. Consider When. The exact time of an event that triggers tax liability is continuous and generally knowable at relatively small cost. But under an annual system of accounting the date, other than the year, has no at consequences, so having to keep track of that would be an added burden, as would enforcing it.²⁹ Discrete accounting periods, generally annual, have many advantages. Daily income, as measured by current means, would be a highly variable measure of ability to pay. Even absent policy changes from year to year, though, the graduated income tax system provides incentives for cross-year movement of taxable income. The realization system plus deferral limited loss offset provides incentives for capital gains transactions at year-end, and there are rules to limit his kind of behavior.

Similar arguments apply to Where. Precise location is cheaply knowable, but is not now an argument to tax liability functions. There are advantages to the decentralization of political and economic authority that are beyond the scope of this paper. Once in place, though, decentralization provides incentives for movement of economic activity across borders, including but not limited to local at borders.

The hardest issue is What, which arises in all tax systems. Although standard optimal tax theory prescribes it, it is practically infeasible to levy as many tax rates as

²⁹ Even in an annual accounting system, dates of transaction may matter, as to distinguish short-term from long-term holding periods for capital gains tax. The holding period distinctions are, of course, themselves notched. Thanks to Leandra Lederman for alerting me to this set of issues.

there are separate goods. So it is natural to think of grouping goods that are close substitutes with each other. The infeasibility is even clearer when one considers that new goods are constantly being created. Occasionally what it is about a good that justifies tax differentiation is easily measurable and of low dimension: the Gas Guzzler Tax is an example. More common is the distinction in the U.S. (and other) income tax systems between an employee and an independent contractor, which depends on a twenty-factor test where many of the factors are themselves difficult or impossible to measure. Over time regulations and rulings clarify what combinations of characteristics are on one side of the line, and which combinations are on the other. Once that becomes clear, bunching will follow.

4.3 Other Justifications

It may be that notches get people's attention in ways that smooth or kinked programs do not, so that they may be more effective in influencing behavior. It may be that they are more easily understood, an issue that is related but not identical to attention. Also possible is that notches are widely *misunderstood*, and so induce people to behave in ways that are not in their self-interest.³⁰ As of now these reactions to notches are a matter of speculation, as there is no evidence about the salience or related properties of notches, relative to either kinked or smooth policies. It may also be that policy makers are subject to the same type of cognitive bounds in formulating policy.

To the extent that disputes arise about the arguments of the tax base, be they quantities or characteristics, a notch system limits the scope of the disputes while raising the stakes of the disputes that arise. This is parallel to the point that notches create capricious³¹ and widely varying local incentives. The cost of the adjudication system may vary across these dimensions.³²

³⁰ Based on my personal observation, about half of undergraduates beginning a public finance class believe that the kinks in the income tax structure are in fact notches; about a quarter of those completing the class do, too.

³¹ I am presuming that the exact placement of a notch is usually arbitrary. That is certainly true for the case of the Gas Guzzler Tax (at .5 decimals of miles-per-gallon), but may not be true in all cases. Knowledge of local areas where response elasticities are relatively high would be a factor in the optimal placement of notches.

³² Consider the adjudications costs of alternative class grading systems of 0-to-100 number grades versus (a small number of) letter grades. Under the former system all students have an incentive to complain, while

As discussed by Blinder and Rosen (1985), quantity notches, like kinks, are susceptible to bunching across time (intertemporal substitution), and Who notches are subject to cheating (interpersonal substitution). When the reward or penalty is indivisible, notches may be unavoidable

5. Conclusions

The ubiquity of tax policy notches calls for further inquiry into their consequences for behavior and their role in an optimal tax system. The taxonomy of notches proposed here is a first step. The demonstration of their welfare inferiority absent considerations of administrative cost or salience suggests that the latter issues need more attention. As long as they persist, though, taxpayer behavior in their presence has the potential to provide information about preferences and technologies. However, this agenda is made difficult by the need to separate out preferences and technologies on the one hand from mitigating salience factors on the other. Finally, the widespread use of surrogate tax bases, where the base comprises items that do not enter utility (or production) functions, implies that behavioral response depends both on the structural parameters of utility and production functions and on the relationship between the surrogate tax base and the variables that directly affect utility or profit.

in the latter only those near letter grade notches have the incentive to complain, but will do so more vigorously. Professors may have the incentive to not reveal who is close to a notch.

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