

# Prospect Theory and Market Liquidity

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## **Abstract**

We study equilibrium trading strategies and market liquidity in an economy in which a better-informed speculator displays preferences consistent with Prospect Theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992), i.e., loss aversion and mild risk seeking over losses. Loss aversion (risk seeking) induces that speculator to trade more (less) cautiously with her private information — but also to purchase private information more (less) often when it is costly — in order to mitigate (enhance) her perceived risk of a trading loss. We demonstrate that these forces have novel, nontrivial effects on equilibrium market depth. Empirical analysis yields evidence in line with our model’s predictions and suggestive of the potentially important role of Prospect Theory in explaining the cross-sectional and time-series properties of U.S. stock market liquidity.

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# 1 Introduction

Over the past two decades, a large and long-standing body of experimental evidence on human behavior has provided support to the notion, first formulated by Kahneman and Tversky (1979) as *Prospect Theory*, that the decision-making process of economic agents may depart from the predictions of standard expected utility theory. In Tversky and Kahneman (1992)'s version, Prospect Theory postulates that those agents assess gambles with a value function defined over gains and losses relative to a reference point (instead of the absolute level of financial wealth or consumption), concave over gains (*risk aversion*), but convex (*risk seeking*) and steeper (*loss aversion*) over losses. Recent work employs modified versions of this theory to study the pricing of financial securities. Prospect Theory arguments have been proposed to explain such known asset pricing puzzles as the magnitude of the equity premium, excess stock return volatility, momentum and the disposition effect, the value premium, or stock return predictability and its implications for portfolio selection.<sup>1</sup>

The past two decades have also been characterized by an increasing interest in the study of the process of price formation in financial markets. Market microstructure research has studied (both theoretically and empirically) such issues as the mechanisms through which information is impounded into prices, agents' reasons for trade and optimal trading strategies, and liquidity.<sup>2</sup> Yet, to our knowledge, this literature has not examined any of these issues when investors make decisions according to Prospect Theory.

The main objective of this paper is to investigate the effects of Prospect Theory on market liquidity. Our novel theoretical analysis and suggestive empirical evidence indicate that these effects are nontrivial and may play an important role in explaining the cross-sectional and time-

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<sup>1</sup>E.g., Benartzi and Thaler (1995), Ait-Sahalia and Brandt (2001), Barberis, Huang, and Santos (2001), Barberis and Huang (2001), Berkelaar, Kouwenberg, and Post (2004), Gomes (2005), Grinblatt and Han (2005), Barberis, Huang, and Thaler (2006), Kyle, Ou-Yang, and Xiong (2006), Barberis and Huang (2008), and Barberis and Xiong (2009).

<sup>2</sup>E.g., see the surveys of O'Hara (1995) and Hasbrouck (2007).

series properties of U.S. stock market liquidity.

Our theory is based on a one-period model of trading in the spirit of Kyle (1985) and Subrahmanyam (1991) but also Grossman and Stiglitz (1980) and Vives (1995). The model is populated by a single informed trader (representing competitive speculation) submitting demand schedules (i.e., generalized limit orders), noise traders submitting market orders, and competitive market making (MM). If the speculator’s preferences are described by an exponential utility function (MV speculator), the model’s implications for trading strategies and market depth (the inverse of Kyle’s (1985) “lambda,” or price impact) are well-known in the literature (e.g., Vives, 2008). We depart from this standard setting by assuming that the (PT) speculator displays preferences capturing parsimoniously *all* of the aforementioned main features of Kahneman and Tversky (1979)’s Prospect Theory — as well as their *relative* importance (as assessed by Tversky and Kahneman, 1992).<sup>3</sup>

The model’s ensuing noisy rational expectations equilibrium yields several novel predictions for market liquidity. First, we show that the presence of a PT speculator *lowers* a traded risky asset’s equilibrium price impact. Intuitively, loss aversion induces a PT speculator either to trade more cautiously or not to trade at all with her private signal. Risk seeking induces a PT speculator to trade more aggressively with her private signal. According to Tversky and Kahneman (1992), agents’ risk seeking is *mild* relative to loss aversion.<sup>4</sup> Thus, for preference parameters consistent with their assessment, the former effect dominates the latter in equilibrium, making the PT speculator’s demand schedule less aggressive, the order flow less informative about the asset’s

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<sup>3</sup>Previous research (e.g., Barberis, Huang, and Santos, 2001; Barberis and Huang, 2001) concentrates on agents’ greater sensitivity to reductions of their financial wealth. Kyle, Ou-Yang, and Xiong (2006) develop an exponential version of Kahneman and Tversky’s (1979) Prospect Theory to examine an agent’s decision whether to liquidate an asset before its natural payoff. Tversky and Kahneman (1992) also suggest that, in assessing gambles, economic agents employ subjective nonlinear transformations of the objective cumulative probability distribution of payoffs overweighting its tails. Barberis and Huang (2008) study the asset pricing implications of the resulting utility model, known as Cumulative Prospect Theory.

<sup>4</sup>E.g., see Tversky and Kahneman (1992)’s descriptive utility function in Section 2.2 (Eq. (1)) and Figure 1a. Tversky and Kahneman (1992) base this observation on experimental evidence in which agents select among gambles that can lead to both gains and losses. Consistently, Barberis, Huang, and Santos (2001, p. 17) observe that “[for those gambles] — such as the one-year investment in stocks [...] — loss aversion at the kink is far more important than the degree of curvature away from the kink.”

fundamentals, and the MM's adverse selection problem less severe. We also show that the presence of a PT speculator makes equilibrium price impact *state-dependent* and on average *U-shaped*, i.e., higher during “good” or “bad” times (large absolute private signals and equilibrium price changes) than during “normal” times (small private signals and equilibrium prices changes). An intuitive explanation for this result is that *ceteris paribus*, the PT speculator's perceived risk of a trading loss is lower the greater is the private signal she observes. In those circumstances, risk aversion plays a more substantial role in the PT speculator's limit orders than loss aversion and risk seeking, leading to more aggressive informed trading and lower equilibrium market liquidity.

In most financial markets private information about asset payoffs is costly. This motivates us to extend our model to consider whether Prospect Theory preferences affect a speculator's endogenous information acquisition. The amended model generates a rich set of additional, novel implications. In particular, we show that when private signals are sufficiently expensive, the presence of a PT speculator *worsens* market liquidity and *attenuates* its state-dependency. The intuition for this result is that the availability of private information mitigates the speculator's perceived risk of a trading loss, yet at a certain cost. Faced with this trade-off, risk aversion often induces a MV speculator not to acquire private information during “normal” times (when a trading loss is most likely). Loss aversion induces a PT speculator to more aggressive information production to mitigate that risk, while risk seeking motivates her to purchase private signals less often to magnify it. As above, prevalence of the former effect on the latter in equilibrium leads on average to greater adverse selection risk and higher price impact during most times. These insights are important for they suggest that the extent to which Prospect Theory preferences affect market liquidity is sensitive to the traded asset's information environment.

An important contribution of our theory is that its predictions are testable, thus possibly refutable, rather than aimed at matching extant features of the data. As such, they provide an unbiased, albeit more challenging opportunity to assess the empirical relevance of unconventional utility models. In this study we document preliminary evidence in support of our theory's

predictions in a comprehensive sample of probably the two closest available annual proxies to the notion of market depth — the effective cost of trading and Amihud’s (2002) illiquidity — for U.S. stocks between 1926 and 2005.<sup>5</sup> Similarly direct proxies for the preferences of sophisticated investors trading in a stock and for the cost of private information about its fundamentals are unfortunately unavailable. We propose that the presence of PT speculation in a stock is more *likely* the more *active* that stock’s institutional investors are deemed to be according to the measure of Cremers and Petajisto (2009). Those investors’ preferences are arguably less likely to affect their portfolio allocation decisions the closer their holdings are to their benchmarks. We also propose that superior information about a stock is less likely to be expensive the greater is that stock’s analyst coverage and/or market capitalization. Both sets of proxies are intuitive, yet admittedly indirect. Thus, we rely on estimates of both their level and interaction effects in panel regressions to assess the empirical viability of our model.

We find that on average a firm’s effective cost of trading and illiquidity in the U.S. stock market are lower and U-shaped — i.e., greater in correspondence with greater absolute annual stock returns — the more so the more active are its mutual fund managers (i.e., the more likely is PT speculation in its stocks), the greater is the median number of analysts covering it, and the greater is its size (i.e., the cheaper superior information about it is likely to be). These findings are novel, consistent with the model’s implications, and both economically and statistically significant. For instance, we estimate that a one standard deviation increase in a firm’s measure of active portfolio management lowers the sensitivity of its effective cost of trading to absolute stock returns — normally about 6.3% of its sample median per unit return shock — by 50% on average, but by less than 25% when accompanied by a one standard deviation increase in its analyst coverage.

Our work is related to Subrahmanyam (1991) and Foster and Viswanathan (1993). Sub-

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<sup>5</sup>A stock’s effective cost of trading is the extent to which the execution price deviates from the prevailing midquote to accommodate a trade. A stock’s illiquidity is its average absolute price movement per daily trading volume. Further details on these measures are in Section 4.1.1.

rahmanyam (1991) allows for noncompetitive (risk-averse) speculators submitting market orders in the one-period noisy rational expectations model of Kyle (1985). In this paper we abstract from those features to concentrate on the implications of Prospect Theory preferences for market liquidity. Unreported analysis indicates that our intuition would be basically immune from such an extension. In a similar setting, Foster and Viswanathan (1993) demonstrate that representing speculators' beliefs with nonnormal, elliptically contoured distributions makes price impact state-dependent. However, there is little or no evidence guiding such modeling choice for those unobservable beliefs. In our model state-dependent market liquidity ensues from a speculator's Prospect Theory preferences even when all random variables are normally distributed. Another related literature explores asset pricing implications of investors exhibiting either irrationality or bounded rationality.<sup>6</sup> All agents in our model, including the better-informed speculator displaying nonconventional preferences, are instead fully rational.

We proceed as follows. In Section 2, we construct a model of trading with a PT speculator and discuss the implications of her presence for market liquidity. In Section 3, we enrich the model by endogenizing the PT speculator's decision to become informed. In Section 4, we present the empirical results. We conclude in Section 5.

## 2 A Model of Trading with Prospect Theory

In their seminal work, Kahneman and Tversky (1979) and Tversky and Kahneman (1992) introduce Prospect Theory as a model of decision-making under uncertainty based on experimental evidence of violations of the standard Morgenstern-von Neumann utility theory. The main features of Prospect Theory are a value function *i*) defined on changes in financial wealth; *ii*) displaying concavity in the domain of gains (risk aversion) and *mild* convexity in the domain of losses (*risk seeking*); and *iii*) steeper for losses than for gains (*loss aversion*). This theory is

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<sup>6</sup>E.g., Barberis, Schleifer, and Vishny (1998), Hong and Stein (1999), Daniel, Hirshleifer, and Subrahmanyam (2001), and Kogan, Ross, Wang, and Westerfield (2006).

supported by numerous experimental studies of human behavior in the psychology literature.<sup>7</sup>

In this section we describe a noisy rational expectations model of trading in the presence of a better-informed speculator with Prospect Theory-inspired preferences. The model's structure is similar to Kyle (1985) and Subrahmanyam (1991); yet, we assume that the speculator is competitive (instead of strategic) and submits demand schedules (i.e., limit orders instead of market orders), in the spirit of Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), Verrecchia (1982), and Vives (1995). These assumptions are made solely for simplicity. Allowing for multiple agents, imperfect competition, and informed market orders complicates the analysis that follows considerably without significantly affecting its main intuition.<sup>8</sup> All proofs are in the Appendix.

## 2.1 The Basic Economy

The model is a two-date, one-period economy in which a single risky asset is exchanged. Trading occurs only at the end of the period ( $t = 1$ ), after which the asset payoff ( $v$ ) is realized. The economy is populated by three types of traders: A single informed trader (labeled *speculator*) representing a competitive “speculative sector,” liquidity traders, and a competitive, risk-neutral market maker (MM).<sup>9</sup> All traders know the structure of the economy and the decision process leading to order flow and prices.

At time  $t = 0$  there is neither information asymmetry about  $v$  nor trading. Sometime between  $t = 0$  and  $t = 1$ , the speculator receives a private and noisy signal of  $v$ ,  $S = v + u$ . The random variables  $v$  and  $u$  are assumed to be mutually independent and normally distributed with

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<sup>7</sup>E.g., see the surveys in Camerer (2000), Barberis, Huang, and Santos (2001), and Nofsinger (2005).

<sup>8</sup>For instance, in the presence of imperfectly competitive, risk-averse informed agents submitting market orders, a closed-form expression for equilibrium price impact cannot be obtained (e.g., see Subrahmanyam, 1991). Further, Kovalenkov and Vives (2007) and Vives (2008) show that in the presence of risk-averse informed traders submitting limit orders, a competitive rational expectations equilibrium provides a reasonably close approximation to the corresponding strategic equilibrium.

<sup>9</sup>For an analysis of risk-averse market making in an adverse selection model based on Kyle (1985), see Subrahmanyam (1991).

mean zero and variance  $\sigma_v^2$  and  $\sigma_u^2$ , respectively. It then ensues that  $\text{var}[S] = \sigma_s^2 = \sigma_v^2 + \sigma_u^2$  and  $\text{cov}[v, S] = \sigma_v^2$ . At time  $t = 1$ , liquidity traders submit *market orders* and the speculator submits a *demand schedule* (i.e., *generalized limit order*) to the MM, before the equilibrium price  $P$  has been set. Liquidity traders generate a random, normally distributed demand  $z$ , with mean zero and variance  $\sigma_z^2$ . For simplicity, we assume that  $z$  is independent from all other random variables. As in Vives (1995), we denote the speculator's demand schedule by  $x(S, \cdot)$ ; thus, when the price is  $P$ , her desired trade is  $x = x(S, P)$  and her profits from trading are given by  $\pi = x(v - P)$ .

## 2.2 Prospect Theory Speculator

The speculator makes trading decisions under Prospect Theory (PT). To represent PT preferences, Tversky and Kahneman (1992) propose a specific power utility function over trading gains and losses  $\pi$  based on experimental evidence:

$$U_{TK} = \begin{cases} \pi^{0.88} & \text{if } \pi \geq 0 \\ -2.25(-\pi)^{0.88} & \text{if } \pi < 0. \end{cases} \quad (1)$$

This functional form, plotted in Figure 1a (dashed line), captures the main features of Prospect Theory for it is concave over gains, and *mildly* convex and steeper over losses. Yet, it makes the PT speculator's problem intractable in our setting. Therefore, in the spirit of Barberis, Huang, and Santos (2001), Barberis and Huang (2001), and Kyle, Ou-Yang, and Xiong (2006), we assume that the PT speculator chooses the optimal trading strategy  $x$  that maximizes the following piecewise value function:

$$E[U_{PT}] = E[U_{MV}] + E[V(\pi)], \quad (2)$$

where

$$E[U_{MV}] = E[\pi] - \frac{1}{2}\alpha var[\pi], \quad (3)$$

$$E[V(\pi)] = \begin{cases} 0 & \text{if } \pi \geq 0 \\ \gamma E[\pi] + \frac{1}{2}\beta var[\pi] & \text{if } \pi < 0 \end{cases}, \quad (4)$$

and  $\alpha > 0$ ,  $\gamma \geq 0$ , and  $\beta \geq 0$ .

The expression for  $E[U_{PT}]$  has several desirable properties. Figures 1a and 1b plot realizations of Eq. (2) over the domain of  $\pi$ . For  $\gamma = 0$  and  $\beta = 0$ ,  $E[U_{PT}]$  reduces to  $E[U_{MV}]$  of Eq. (3), the mean-variance value function of a risk-averse (MV) speculator with CARA preferences (e.g., Huang and Litzenberger, 1988, pp. 265-266). As such,  $E[U_{MV}]$  is strictly concave over both gains and losses (e.g., Figure 1b, thin line, for  $\alpha = 1$ ). For  $\gamma > 0$  and  $\beta = 0$ ,  $E[U_{PT}]$  is kinked at the origin (where trading gains are zero) and steeper over losses, making the speculator *loss-averse* (e.g., Figure 1b, dashed line, for  $\gamma = 2$ ). For  $\gamma = 0$  and  $\beta > \alpha$ , the expression for  $E[U_{PT}]$  is concave over gains but convex over losses, making the speculator *risk-seeking* over losses (e.g., Figure 1b, dotted line, for  $\beta = 2$ ). Thus, for  $\gamma = 1$  and  $\beta = 1.05$  (Figure 1a, solid line),  $E[U_{PT}]$  of Eq. (2) captures parsimoniously both the main features of Kahneman and Tversky (1979)'s Prospect Theory and Tversky and Kahneman (1992)'s assessment of their *relative* strength in Eq. (1): *Mild* risk seeking in losses relative to loss aversion.

### 2.3 Prospect Theory Trading

At time  $t = 1$ , the speculator submits her demand schedule  $x(S, \cdot)$  maximizing  $E[U_{PT}]$  of Eq. (2) conditional upon her private and noisy signal  $S$  — the best information available about the risky asset's payoff  $v$ . As such, the speculator neither learns from market prices nor internalizes the impact of her trades on market prices (e.g., Vives, 1995, 2008). Standard formulas for the

moments of a truncated normal distribution (e.g., Greene, 1997, pp. 950-952) imply that

$$E[\pi|S, \pi < 0] = x(\phi S - P)\Phi(\text{sgn}(x)\chi) + x\text{sgn}(x)\sqrt{\sigma_v^2(1-\phi)}\Lambda^-(\text{sgn}(x)\chi), \quad (5)$$

$$\text{var}[\pi|S, \pi < 0] = x^2\sigma_v^2(1-\phi)[1 - \Delta^-(\text{sgn}(x)\chi)]\Phi(\text{sgn}(x)\chi), \quad (6)$$

where  $\text{sgn}(\cdot)$  is the sign function,  $\Phi(\cdot)$  and  $\psi(\cdot)$  are the standard normal cdf and pdf,  $\chi = \frac{P-\phi S}{\sqrt{\sigma_v^2(1-\phi)}}$ ,  $\Lambda^-(\cdot) = -\frac{\psi(\cdot)}{\Phi(\cdot)}$ , and  $\Delta^-(\cdot) = \Lambda^-(\cdot)[\Lambda^-(\cdot) - (\cdot)]$ . In light of the aforementioned properties of  $E[U_{PT}|S]$ , both  $\Phi(\text{sgn}(x)\chi)$  and  $\psi(\text{sgn}(x)\chi)$  — the conditional *cumulative* and *marginal* probability of a trading loss, respectively — play an important role in the speculator's trading strategy. Eq. (2) then becomes

$$\begin{aligned} E[U_{PT}|S] &= E[\pi|S] - \frac{1}{2}\alpha\text{var}[\pi|S] + \gamma E[\pi|S, \pi < 0]\Phi(\text{sgn}(x)\chi) \\ &\quad + \frac{1}{2}\beta\text{var}[\pi|S, \pi < 0]\Phi(\text{sgn}(x)\chi). \end{aligned} \quad (7)$$

Substituting Eqs. (5) and (6) into Eq. (7) and differentiating with respect to  $x$  yields

$$x_{PT} = \begin{cases} \frac{[1+\gamma\Phi(\chi)]}{\alpha^*(\chi)\sigma_v^2(1-\phi)}(\phi S - P) - \frac{\gamma\psi(\chi)}{\alpha^*(\chi)\sigma_v\sqrt{1-\phi}} > 0 & \text{if } S > S_H \\ \frac{[1+\gamma\Phi(-\chi)]}{\alpha^*(-\chi)\sigma_v^2(1-\phi)}(\phi S - P) + \frac{\gamma\psi(-\chi)}{\alpha^*(-\chi)\sigma_v\sqrt{1-\phi}} < 0 & \text{if } S < S_L \\ 0 & \text{if } S_L \leq S \leq S_H \end{cases}, \quad (8)$$

where  $\phi = \frac{\sigma_v^2}{\sigma_s^2}$  is the relative precision of the speculator's private signal  $S$ ,  $S_H = \frac{P}{\phi} + \frac{\gamma\psi(\chi)\sigma_v\sqrt{1-\phi}}{\phi[1+\gamma\Phi(\chi)]}$ ,  $S_L = \frac{P}{\phi} - \frac{\gamma\psi(-\chi)\sigma_v\sqrt{1-\phi}}{\phi[1+\gamma\Phi(-\chi)]}$ , and  $\alpha^*(\cdot) = \alpha - \beta[1 - \Delta^-(\cdot)]\Phi(\cdot) > 0$ .<sup>10</sup>

The optimal demand schedule of Eq. (8) has standard and many novel features. For  $\gamma = 0$  and  $\beta = 0$ ,  $x_{PT}$  reduces to the optimal generalized limit order of a speculator with CARA

<sup>10</sup>The s.o.c. is satisfied if risk seeking is not "too high," i.e., if  $\beta < \frac{\alpha}{[1-\Delta^-(\cdot)]\Phi(\cdot)}$  such that  $\alpha^*(\cdot) > 0$ .

preferences,

$$x_{MV} = \frac{1}{\alpha \sigma_v^2 (1 - \phi)} (\phi S - P) \quad (9)$$

(e.g., Vives, 1995). Figure 2a plots  $x_{MV}$  (dashed line) for  $\alpha = 1$ ,  $\sigma_v^2 = 1$ , and  $\sigma_u^2 = 1$  over the domain of  $P$  for a private signal  $S = 0$ . In Eq. (9), risk aversion  $\alpha$  induces the MV speculator, even if better informed, to submit cautious limit orders ( $|x_{MV}| < \infty$ ) to the MM. The stylized Prospect Theory preferences of Eq. (2) — i.e.,  $\gamma > 0$  and  $\beta > \alpha$  in Eq. (8) — have two additional effects of the opposite sign on the optimal trading activity of the speculator. For any given signal  $S$  and price  $P$ , risk seeking in losses induces the speculator to more aggressive trading. In Eq. (9), this effect is captured by the *effective* risk aversion coefficient  $\alpha^*(\pm\chi)$  being lower than  $\alpha$ , the more so the greater is  $\Phi(\pm\chi)$ , the conditional cumulative probability of a trading loss. In Figure 2b, for  $\gamma = 0$  and  $\beta = 2$ , cumulative loss probability is relatively high at low prices (i.e., for  $|P|$  close to  $S = 0$ ), leading that speculator to larger limit orders than if she had MV preferences ( $|x_{PT}^{\beta=2}| > |x_{MV}|$ , dotted line). Loss aversion instead induces the speculator to either more cautious trading or no trading at all (e.g.,  $x_{PT}^{\gamma=2}$  in Figure 2b, dashed line) in order to minimize the expected loss of Eq. (5). Consistently, the extent of cautious trading (e.g.,  $|x_{PT}^{\gamma=2}| < |x_{MV}|$ ) and the width of the no-trade interval  $[S_L, S_H]$  are increasing in the conditional marginal probability of a trading loss  $\psi(\pm\chi)$ . Only when both  $\Phi(\pm\chi)$  and  $\psi(\pm\chi)$  are small (e.g., at high  $|P|$  relative to  $S = 0$  in Figures 2a and 2b), does the speculator trade as if driven by risk aversion alone. The PT speculator's optimal demand schedule  $x_{PT}$  (Figure 2a, solid line, for  $\gamma = 1$  and  $\beta = 1.05$ ) reflects the tension between these forces.

## 2.4 Equilibrium

The MM does not receive any information, but observes the aggregate order flow (i.e., the noisy limit-order book schedule)  $\omega = x_{PT} + z$  before setting the market clearing price  $P = P(\omega)$ .

Dealership competition and risk neutrality then imply semi-strong market efficiency (e.g., Kyle, 1985; Hirshleifer, Subrahmanyam, and Titman, 1994; Vives, 1995):

$$P(\omega) = E[v|\omega]. \quad (10)$$

The expression for  $x_{PT}$  of Eq. (8) makes clear that the order flow's informativeness about the asset payoff  $v$  depends on the net effect of risk aversion, loss aversion, and risk seeking on the PT speculator's trading activity. That effect also depends, in complex fashion, on the market clearing price. For instance if, at a given price  $P$ , the speculator's noisy signal  $S$  falls within the no-trade interval  $[S_L, S_H]$ , the resulting aggregate order flow is uninformative about  $v$  (i.e.,  $\omega = z$ ). Thus, the MM must also conjecture the speculator's trading status. In the spirit of Yuan (2005), the MM's inference problem can be expressed as

$$\begin{aligned} E[v|\omega] &= E[v|\omega, S > S_H][1 - \Phi(H)] + E[v|\omega, S < S_L]\Phi(L) \\ &\quad + E[v|\omega, S_L \leq S \leq S_H][\Phi(H) - \Phi(L)], \end{aligned} \quad (11)$$

where  $H = \frac{S_H}{\sigma_S}$ ,  $L = \frac{S_L}{\sigma_S}$ , and  $[1 - \Phi(H)]$ ,  $[\Phi(L)]$ , and  $[\Phi(H) - \Phi(L)]$  are the probability of the order flow being informative ( $S > S_H$  or  $S < S_L$ ) or uninformative ( $S_L \leq S \leq S_H$ ) about  $v$ , respectively.

Unfortunately,  $x_{PT}$  of Eq. (8) makes  $\omega$  a nonlinear function of the normally distributed private signal  $S$ , thus the MM's inference problem in Eq. (11) in the trade region outside of  $[S_L, S_H]$  analytically intractable. There are several approaches in the literature for approximating nonlinear rational expectation models.<sup>11</sup> In this paper we employ a numerical technique to express both conditional first moments  $E[v|\omega, S > S_H]$  and  $E[v|\omega, S < S_L]$  as explicit functions

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<sup>11</sup>E.g., see Blanchard and Kahn (1980), Bernardo and Judd (2000), Klein (2000), and Sims (2000), as well as the discussion in Lombardo and Sutherland (2007).

of  $\omega$  and  $P$ .<sup>12</sup> This approach, described in the Appendix, yields the following lemma.

**Lemma 1** *The MM estimates the conditional means of  $v$  given the order flow  $\omega$  as*

$$E[v|\omega, S > S_H] = a_H + b_H\omega + c_HP, \quad (12)$$

$$E[v|\omega, S < S_L] = a_L + b_L\omega + c_LP, \quad (13)$$

$$E[v|\omega, S_L \leq S \leq S_H] = \sigma_s\Lambda(L, H), \quad (14)$$

where the constants  $a_H$ ,  $a_L$ ,  $b_H$ ,  $b_L$ ,  $c_H$ , and  $c_L$  are defined in the Appendix, and  $\Lambda(L, H) = \frac{\psi(L) - \psi(H)}{\Phi(H) - \Phi(L)}$ .

Given Lemma 1, Proposition 1 accomplishes the task of solving for the equilibrium of this economy.

**Proposition 1** *The rational expectations equilibrium price function of the model described by Eqs. (8) and (10) is the unique fixed point of the implicit function*

$$P_{PT} = P_{PT}^0 + \lambda_{PT}\omega, \quad (15)$$

where

$$P_{PT}^0 = \frac{a_H [1 - \Phi(H)] + a_L \Phi(L) + \sigma_s \Lambda(L, H) [\Phi(H) - \Phi(L)]}{1 - c_H [1 - \Phi(H)] - c_L \Phi(L)}, \quad (16)$$

$$\lambda_{PT} = \frac{b_H [1 - \Phi(H)] + b_L \Phi(L)}{1 - c_H [1 - \Phi(H)] - c_L \Phi(L)}. \quad (17)$$

To characterize the properties of this equilibrium, we begin by observing that if the speculator displays CARA preferences (i.e., if  $\gamma = 0$  and  $\beta = 0$ ), there exists a closed-form solution.

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<sup>12</sup>In unreported analysis, we find an alternative approach, employing first-order approximations of  $x_{PT}$  around the private signal's conditional means in either trade interval of Eq. (8) — i.e.,  $E[S|S > S_H] = \sigma_S\Lambda(H)$  and  $E[S|S < S_L] = \sigma_S\Lambda^-(L)$ , where  $\Lambda(\cdot) = \frac{\psi(\cdot)}{[1 - \Phi(\cdot)]}$  (e.g., see Greene, 1997, pp. 951-952) — to yield similar insights, while being less accurate.

**Remark 1** *In the presence of a MV speculator ( $\gamma = 0$  and  $\beta = 0$ ), the unique rational expectations equilibrium price  $P_{MV}$  of the model described by Eqs. (9) and (10) is*

$$P_{MV} = \lambda_{MV}\omega, \tag{18}$$

where

$$\lambda_{MV} = \frac{\sigma_v^2}{\alpha\sigma_u^2\sigma_z^2} > 0. \tag{19}$$

Equilibrium market liquidity (depth) in  $P_{MV}$  — the inverse of price impact  $\lambda_{MV}$  of Eq. (19) — reflects the MM’s attempt to be compensated for the losses he anticipates from trading with the better-informed speculator, as it affects her profits from liquidity trading. Consistent with Kyle (1985), market liquidity deteriorates ( $\lambda_{MV}$  increases) the more uncertain is the asset payoff  $v$  (i.e., the greater is  $\sigma_v^2$ ) since the more valuable is the speculator’s private information and the more vulnerable is the MM to adverse selection. Accordingly, market liquidity improves if  $S$  is less precise (i.e., the greater is  $\sigma_u^2$ ) or in the presence of more intense noise trading (i.e., the greater is  $\sigma_z^2$ ). Consistent with Subrahmanyam (1991) and Vives (1995), market liquidity deteriorates the less risk-averse the speculator is (i.e., the lower is  $\alpha$ ) for she trade more aggressively with  $S$  (see  $x_{MV}$  of Eq. (9)).<sup>13</sup>

When the speculator displays Prospect Theory-inspired preferences (i.e., if  $\gamma = 1$  and  $\beta = 1.05$ ), additional forces affect equilibrium market liquidity. We illustrate these forces and the intuition behind them via numerical analysis of an economy in which  $\alpha = 1$ ,  $\sigma_v^2 = 1$ ,  $\sigma_u^2 = 1$  (as in Figure 2), and  $\sigma_z^2 = 1$ . Specifically, in Figure 3a we plot the average price impact  $\lambda_{PT}$  of Eq. (17) over the domain of  $S$  (by virtue of numerical integration, with respect to all possible

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<sup>13</sup>In particular, Remark 1 is a special case of the linear equilibrium in Vives (1995, Proposition 1.1; 2008, Proposition 4.2) when a continuum of risk-averse speculators receives identical private signals. As the discussion in Vives (1995, 2008) suggests, allowing for multiple speculators with heterogeneous information would complicate the model’s analysis without affecting its main intuition. We examine the effects of Prospect Theory preferences on a speculator’s endogenous information acquisition in Section 3.

information and noise trading shocks  $u$  and  $z$ ) when  $\gamma = 1$  and  $\beta = 1.05$  ( $E[\lambda_{PT}|S]$ , solid line), as well as  $\lambda_{MV}$  of Eq. (19) (dashed line). Figure 3a motivates two noteworthy conclusions, novel to the literature, about a risky asset’s equilibrium market liquidity in the presence of a PT speculator.

**Conclusion 1** *The presence of a better-informed speculator with Prospect Theory preferences ( $\gamma = 1$  and  $\beta = 1.05$ ) lowers equilibrium price impact.*

The intuition for this conclusion is as follows. First, risk seeking in losses leads the speculator to trade more aggressively with her private signal  $S$  when the conditional cumulative probability of a loss ( $\Phi(\pm\chi)$ ) is high (and her effective risk aversion coefficient  $\alpha^*(\pm\chi)$  is low), so worsening the MM’s adverse selection risk. E.g.,  $|x_{PT}^{\beta=2}| > |x_{MV}|$  in Figure 2b, leading to  $E[\lambda_{PT}^{\beta=2}|S] > \lambda_{MV}$  in Figure 3b (dashed line). Second, loss aversion induces the speculator either to trade more cautiously or not to trade at all with  $S$  when the conditional marginal probability of a loss ( $\psi(\pm\chi)$ ) is high, thus making the MM’s adverse selection risk less severe. E.g.,  $|x_{PT}^{\gamma=2}| < |x_{MV}|$  and  $x_{PT}^{\gamma=2} = 0$ , respectively, in Figure 2b, leading to  $E[\lambda_{PT}^{\gamma=2}|S] < \lambda_{MV}$  in Figure 3b (dotted line). Since the PT speculator’s risk seeking is mild relative to loss aversion — see Figure 1a for  $\gamma = 1$  and  $\beta = 1.05$ , consistent with Tversky and Kahneman (1992)’s Eq. (1) — the second effect dominates upon the first in equilibrium, leading to greater market depth:  $E[\lambda_{PT}|S] < \lambda_{MV}$  in Figure 3a.

**Conclusion 2** *The presence of a better-informed speculator with Prospect Theory preferences ( $\gamma = 1$  and  $\beta = 1.05$ ) makes equilibrium price impact state-dependent and on average U-shaped — higher during “good” and “bad” times, lower during “normal” times.*

As Figures 2 and 3 make clear, the extent to which these forces affect the PT speculator’s optimal demand schedule and the MM’s perceived adverse selection risk is state-dependent. When conditional cumulative and marginal loss probability ( $\Phi(\pm\chi)$  and  $\psi(\pm\chi)$ ) are low — e.g.,

if the private signal  $S$  is “large” relative to its mean — risk aversion explains exhaustively the trading activity of the PT speculator, yielding worse equilibrium market liquidity (e.g., higher  $E[\lambda_{PT}|S]$  in Figure 3a), the more so the greater is  $|S|$ . When conditional cumulative and marginal loss probability are instead high — e.g., if  $S$  is “closer” to its mean — especially loss aversion but also risk seeking in losses play a more substantial role in the PT speculator’s limit orders (as discussed for Conclusion 1), yielding greater equilibrium market liquidity (e.g., lower  $E[\lambda_{PT}|S]$  in Figure 3a), the more so the smaller is  $|S|$ . Identical inference ensues from plotting  $E[\lambda_{PT}|S]$  with respect to either equilibrium prices (i.e., price changes)  $E[P_{PT}|S]$  or asset payoffs  $E[v|S]$ . In particular, equilibrium price impact is higher (lower) in correspondence with large (small) equilibrium price changes — the closest (observable) proxy for its (possibly unobservable) fundamentals.

### 3 Endogenous Information Acquisition

An important characteristic of most financial markets is that information about the fundamentals of traded securities is available only at a cost. There is a vast literature showing that endogenous information acquisition affects both the dynamics of asset prices and the liquidity of their trading venues.<sup>14</sup> In this section, we investigate whether Prospect Theory affects a speculator’s production of information and accompanying trading decisions by studying an extension of the economy of Section 2 in which the private signal is costly.

#### 3.1 The Amended Economy

We consider a market that is identical to that of Section 2. We further assume that at time  $t = 1$ , the speculator simultaneously decides whether to pay a fixed cost  $c > 0$  to observe

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<sup>14</sup>An incomplete list includes Verrecchia (1982), Li, McKelvey, and Page (1987), Admati and Pfleiderer (1988), Vives (1988), Foster and Viswanathan (1993), Burguet and Vives (2000), and Hellwig and Veldkamp (2009). See also Vives (2008) and Veldkamp (2011) for a review.

the private signal  $S$  of the asset payoff  $v$  and submits her optimal demand schedule contingent upon her information set. Thus, the speculator's optimal strategy is a pair determining the information purchased and her limit order. The speculator purchases  $S$  if her expected utility when becoming informed, net of the cost of acquiring information, is greater than her expected utility when remaining uninformed (e.g., Foster and Viswanathan, 1993), i.e., if

$$B_{PT} = E \{E [U_{PT}|S]\} - E [U_{PT}] \geq c. \quad (20)$$

$B_{PT}$  can be interpreted as the maximum price the speculator is willing to pay to purchase the private signal  $S$ .

If the speculator purchases  $S$ , her optimal demand schedule — maximizing her value function conditional upon  $S$  ( $E [U_{PT}|S]$  of Eq. (7)) — is the one described by  $x_{PT}$  of Eq. (8). If the speculator does not purchase  $S$ , her optimal demand schedule, labeled  $x_{PT}^0$ , is obtained by differentiating her unconditional value function ( $E [U_{PT}]$  of Eq. (2)), in which

$$E [\pi|\pi < 0] = -xP\Phi(\text{sgn}(x)\chi_0) + x\text{sgn}(x)\sigma_v\Lambda^-(\text{sgn}(x)\chi_0), \quad (21)$$

$$\text{var} [\pi|\pi < 0] = x^2\sigma_v^2 [1 - \Delta^-(\text{sgn}(x)\chi_0)] \Phi(\text{sgn}(x)\chi_0), \quad (22)$$

and  $\chi_0 = \frac{P}{\sigma_v}$ , with respect to  $x$ . We then have

$$x_{PT}^0 = \begin{cases} -\frac{[1+\gamma\Phi(\chi_0)]P - \frac{\gamma\psi(\chi_0)}{\alpha^*(\chi_0)\sigma_v}}{\alpha^*(\chi_0)\sigma_v^2} > 0 & \text{if } P < P_L^0 \\ -\frac{[1+\gamma\Phi(-\chi_0)]P + \frac{\gamma\psi(-\chi_0)}{\alpha^*(-\chi_0)\sigma_v}}{\alpha^*(-\chi_0)\sigma_v^2} < 0 & \text{if } P > P_H^0 \\ 0 & \text{if } P_L^0 \leq P \leq P_H^0 \end{cases}, \quad (23)$$

where  $P_L^0 = -\frac{\gamma\psi(\chi_0)\sigma_v}{[1+\gamma\Phi(\chi_0)]}$  and  $P_H^0 = \frac{\gamma\psi(-\chi_0)\sigma_v}{[1+\gamma\Phi(-\chi_0)]}$ . Given  $x_{PT}$  and  $x_{PT}^0$ , the following lemma shows  $B_{PT}$  of Eq. (20) to be a nonlinear function of the equilibrium price  $P$ .

**Lemma 2** *The speculator acquires the private signal  $S$  iff*

$$B_{PT} = \begin{cases} F_H^0 P^2 + F_H^1 P + F_H^2 \geq c & \text{if } P < P_L^0 \\ F_L^0 P^2 + F_L^1 P + F_L^2 \geq c & \text{if } P > P_H^0 \\ E^0 P^2 + E^1 P + E^2 \geq c & \text{if } P_L^0 \leq P \leq P_H^0 \end{cases} \quad (24)$$

where the variables  $F_H^0, F_L^0, E^0, F_H^1, F_L^1, E^1, F_H^2, F_L^2,$  and  $E^2$  are functions of  $P$  defined in the Appendix. For a MV speculator ( $\gamma = 0$  and  $\beta = 0$ ),  $B_{PT}$  reduces to

$$B_{MV} = \frac{P^2 + \sigma_v^2}{2\alpha}. \quad (25)$$

The resulting optimal demand schedule of the speculator in the presence of information costs,  $x_{PT}^c$ , is given by

$$x_{PT}^c = 1_{B_{PT}} x_{PT} + (1 - 1_{B_{PT}}) x_{PT}^0, \quad (26)$$

where  $1_{B_{PT}} = 1$  if  $B_{PT} \geq c$  and zero otherwise.

We illustrate the intuition for Lemma 2 and Eq. (26) by plotting  $x_{PT}^c$  over the domain of  $P$  (within the same economy as in Figure 2) for a private signal  $S = 0$  costing  $c = 0.60$ , in Figure 4. Not surprisingly, the speculator trades *more aggressively* when conditioning on  $S$  (e.g.,  $|x_{PT}^c| \geq |x_{PT}^0|$  in Figure 4), since her expected trading profits are higher and her perceived risk from trading is lower.<sup>15</sup> The trade-off between these expected benefits and the signal's certain cost  $c$  drives the speculator's information acquisition decision. When the speculator is risk-averse ( $\gamma = 0$  and  $\beta = 0$ ), Eq. (25) implies a price interval with no information production:  $|P| < \sqrt{2\alpha c - \sigma_v^2}$ . At low prices, risk aversion alone ( $\alpha > 0$  in Eq. (25)) induces the MV speculator not to purchase (and learn about)  $S$  when setting her trading strategy  $x_{MV}^c$  (Figure

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<sup>15</sup>E.g., it can be shown that ceteris paribus,  $E[\pi|S] > E[\pi]$  and  $E\{var[\pi|S]\} < var[\pi]$  (see Greene, 1997, p. 83).

4a, dashed line). This effect weakens the more valuable is private information about the asset payoff  $v$  (i.e., the smaller is  $\sigma_v^2$ ).<sup>16</sup>

Two additional forces of opposite sign also contribute to determine the PT speculator's optimal limit orders  $x_{PT}^c$  when  $S$  is costly (Figure 4a, solid line, for  $\gamma = 1$  and  $\beta = 1.05$ ). Risk seeking in losses induces the speculator to *more cautious* information production and trading. E.g., see  $x_{PT}^{c,\beta=2}$  in Figure 4b (dotted line) relative to  $x_{MV}^c$  (in Figure 4a). As mentioned above, availability of  $S$  mitigates the speculator's perceived risk of trading in losses.<sup>17</sup> Thus, when the conditional cumulative probability of a trading loss is high (e.g., when  $P$  is large), the speculator seeks risk by trading without the costly signal  $S$ . Loss aversion instead induces the speculator to *more aggressive* information production and trading. E.g., see  $x_{PT}^{c,\gamma=2}$  in Figure 4b (dashed line) relative to  $x_{MV}^c$  (in Figure 4a). Acquiring  $S$  allows a loss-averse speculator to lower her odds of experiencing a trading loss at most prices  $P$ , except when  $P$  is large enough to make her perceived marginal loss probability if trading ( $\psi(\pm\chi)$ ) too high relative to the signal's cost  $c$ .

### 3.2 Equilibrium

As in Section 2.4, the uninformed MM observes the noisy limit-order book  $\omega^c = x_{PT}^c + z$  before setting the semi-strong efficient price  $P^c = P(\omega^c)$ . Eq. (26) indicates that when private information  $S$  is costly, the informativeness of  $\omega^c$  about the asset payoff  $v$  depends first on whether the speculator decides to purchase  $S$  (if  $B_{PT} \geq c$ ) and, if so, on whether she decides to trade with it afterwards (if  $S$  falls outside of the no-trade interval  $[S_L, S_H]$ ). While the MM can only conjecture about the latter, the former ensues deterministically from Eq. (24). The MM's inference

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<sup>16</sup>Specifically,  $x_{MV}^c = x_{MV}^0 = -\frac{P}{\alpha\sigma_v^2}$  for  $|P| < \sqrt{2\alpha c - \sigma_v^2}$ , and  $x_{MV}^c = x_{MV}$  of Eq. (9) otherwise. Yet, the MV speculator always purchases the private signal  $S$  if  $\sigma_v^2 > 2\alpha c$ .

<sup>17</sup>E.g., Theorem 3.4 in Greene (1997, p. 83) and Eqs. (6) and (22) imply that ceteris paribus,  $E\{var[\pi|S, \pi < 0]\} < var[\pi|\pi < 0]$ .

about  $v$  from  $\omega^c$  accounts for both, as follows:

$$\begin{aligned} E[v|\omega^c] &= E[v|\omega^c, S > S_H] 1_{B_{PT}} [1 - \Phi(H)] + E[v|\omega^c, S < S_L] 1_{B_{PT}} \Phi(L) \\ &\quad + E[v|\omega^c, S_L \leq S \leq S_H] 1_{B_{PT}} [\Phi(H) - \Phi(L)]. \end{aligned} \quad (27)$$

We use Lemma 1 to solve for the equilibrium of this amended economy.

**Proposition 2** *The rational expectations equilibrium price function of the model described by Eqs. (10) and (26) is the unique fixed point of the implicit function*

$$P_{PT}^c = P_{PT}^{c,0} + \lambda_{PT}^c \omega^c, \quad (28)$$

where

$$P_{PT}^{c,0} = \frac{\{a_H [1 - \Phi(H)] + a_L \Phi(L) + \sigma_s \Lambda(L, H) [\Phi(H) - \Phi(L)]\} 1_{B_{PT}}}{1 - \{c_H [1 - \Phi(H)] + c_L \Phi(L)\} 1_{B_{PT}}} = P_{PT}^0 1_{B_{PT}}, \quad (29)$$

$$\lambda_{PT}^c = \frac{\{b_H [1 - \Phi(H)] + b_L \Phi(L)\} 1_{B_{PT}}}{1 - \{c_H [1 - \Phi(H)] + c_L \Phi(L)\} 1_{B_{PT}}} = \lambda_{PT} 1_{B_{PT}}. \quad (30)$$

As discussed in Section 3.1, the speculator does not observe the private signal  $S$  unless she decides to purchase it ( $1_{B_{PT}} = 1$ ); hence, her optimal strategy is a pair  $\{1_{B_{PT}}, x_{PT}^c\}$ . The MM, being rational, accounts for the effect of the price  $P$  on both  $1_{B_{PT}}$  and  $x_{PT}^c$  when clearing the market. It is immediate from Propositions 1 and 2 that the only price such that  $1_{B_{PT}} = 0$  ( $1_{B_{PT}} = 1$ ) is  $P_{PT}^c = 0$  ( $P_{PT}^c = P_{PT} \neq 0$ ). If the MM sets  $P_{PT}^c = 0$ , there is no information acquisition, speculative trading, or adverse selection risk in the economy; thus, equilibrium market depth is the highest ( $\lambda_{PT}^c = 0$ ). If the MM sets  $P_{PT}^c = P_{PT} \neq 0$ , equilibrium informed speculative trading and market depth in the presence of adverse selection risk are those of Proposition 1.

To illustrate the intuition behind Proposition 2, we begin by analyzing the economy's equilibrium in the presence of a risk-averse speculator.

**Remark 2** *In the presence of a MV speculator ( $\gamma = 0$  and  $\beta = 0$ ), the rational expectations equilibrium price of the model described by Eqs. (10) and (26) is the unique fixed point of the implicit function*

$$P_{MV}^c = \lambda_{MV}^c \omega^c, \quad (31)$$

where

$$\lambda_{MV}^c = \frac{\alpha \sigma_u^2 \sigma_v^2 1_{B_{MV}}}{\sigma_s^2 (1 - 1_{B_{MV}}) + \alpha^2 \sigma_u^4 \sigma_z^2} = \lambda_{MV}^c 1_{B_{MV}}, \quad (32)$$

and  $1_{B_{MV}} = 1$  if  $B_{MV} \geq c$  and zero otherwise.

Figure 5a and 6a plot (by virtue of numerical integration) the average equilibrium information acquisition decision  $E[1_{B_{MV}}|S]$  and price impact  $E[\lambda_{MV}^c|S]$  (dashed lines) in Remark 2 over the domain of  $S$ , when  $S$  costs  $c = 0.60$ , within the same economy of Figures 2 to 4 ( $\alpha = 1$ ,  $\sigma_v^2 = 1$ ,  $\sigma_u^2 = 1$ , and  $\sigma_z^2 = 1$ ). Displaying our variables of interest with respect to  $S$  allows comparison with the basic equilibrium in Figure 3; plots with respect to the corresponding equilibrium prices (i.e., price changes)  $E[P_{MV}^c|S]$  or asset payoffs  $E[v|S]$  are identical.

Upon observing the aggregate demand schedule  $\omega^c$ , the MM is uncertain about both the private signal the MV speculator may observe (if she sets a price at which the MV speculator purchases it) and the extent of noise trading  $z$  available in  $\omega^c$  to offset her expected losses from trading with the informed MV speculator (via  $\lambda_{MV} > 0$ ). When conjecturing  $S$  to be “small,” the MM most often does not expect her profits from noise trading in  $\omega^c$  to be sufficient at the relatively “low” semi-strong efficient price  $P_{MV}$  of Eq. (18). Thus, the MM most often sets  $P_{MV}^c = 0$  (and  $\lambda_{MV}^c = 0$ ) such that  $1_{B_{MV}} = 0$  and  $x_{MV}^c = 0$ , leading on average to greater market liquidity (low  $E[\lambda_{MV}^c|S]$  in Figure 6a). However, when the MM conjectures  $S$  to be “large,” she most often expects her profits from noise trading in  $\omega^c$  to be sufficient (i.e.,  $\lambda_{MV} > 0$  to be high

enough) at the relatively “high” semi-strong efficient price  $P_{MV}$  of Eq. (18). Thus, the MM most often sets “high”  $P_{MV}^c = P_{MV} \neq 0$  such that  $1_{B_{MV}} = 1$  and  $x_{MV}^c = x_{MV} \neq 0$  of Eq. (9), leading on average to worse market liquidity (high  $E[\lambda_{MV}^c|S]$  in Figure 6a).

**Conclusion 3** *When private information is costly, equilibrium price impact in the presence of a MV speculator ( $\gamma = 0$  and  $\beta = 0$ ) is state-dependent and U-shaped.*

When the speculator displays PT preferences ( $\gamma = 1$  and  $\beta = 1.05$ ), additional forces affect her endogenous information acquisition and the MM’s market clearing. We plot these average equilibrium outcomes from Proposition 2,  $E[1_{B_{PT}}|S]$  and  $E[\lambda_{PT}^c|S]$ , in Figures 5 and 6, respectively. As discussed in Section 3.1, availability of  $S$  lowers the speculator’s perceived risk of trading in her piecewise value function of Eq. (7). Therefore, risk seeking in losses yields a wider price interval over which the speculator magnifies her trading risk by deciding to remain uninformed. E.g., see  $E[1_{B_{PT}^{\beta=2}}|S]$  in Figure 5b (dotted line). Vice versa, loss aversion induces the speculator to lower her trading risk by purchasing  $S$  at most prices, except when expected trading losses are low (at “large”  $S$  and  $P_{PT}^c$ ). E.g., see the reverse U-shape for  $E[1_{B_{PT}^{\gamma=2}}|S]$  in Figure 5b (dashed line). Accordingly, the former leads to lower adverse selection risk and greater market depth than the latter. E.g., see  $E[\lambda_{PT}^{c,\beta=2}|S] < E[\lambda_{PT}^{c,\gamma=2}|S]$  in Figure 6b, reflecting  $x_{PT}^{c,\beta=2}$  and  $x_{PT}^{c,\gamma=2}$  of Figure 4b.

As in Conclusion 1, in equilibrium the effect of loss aversion on the optimal strategy  $\{1_{B_{PT}}, x_{PT}^c\}$  of a PT speculator ( $\gamma = 1$  and  $\beta = 1.05$ ) dominates those stemming from risk aversion and mild risk seeking, leading to more frequent information acquisition (see  $E[1_{B_{PT}}|S]$  in Figure 5a, solid line) but often worse market liquidity (see  $E[\lambda_{PT}^c|S]$  in Figure 6a, solid line) in correspondence with most fundamental shocks and price changes.

**Conclusion 4** *When private information is costly, the presence of a speculator with Prospect Theory preferences ( $\gamma = 1$  and  $\beta = 1.05$ ) generally increases equilibrium price impact and attenuates its state-dependency.*

This conclusion provides an important qualification to the insights in Conclusions 1 and 2. In particular, it suggests that the extent to which PT preferences affect equilibrium market liquidity is crucially related to speculation's endogenous information acquisition and the traded asset's information environment.

## 4 Empirical Analysis

The economies of Sections 2 and 3 generate a rich set of implications of the presence of better-informed speculation with Prospect Theory-inspired preferences for the liquidity of traded securities. In particular, according to Conclusions 1 to 4 the presence of PT speculation yields the following time-series and cross-sectional predictions for a stock's liquidity:

- H1** *A stock's liquidity is worse in correspondence with large absolute price movements.*
- H2** *When private information is expensive, the presence of PT speculation worsens a stock's liquidity.*
- H3** *When private information is cheap, the presence of PT speculation improves a stock's liquidity.*
- H4** *When private information is expensive, the presence of PT speculation attenuates the state-dependency of a stock's liquidity (i.e., weakens H1).*
- H5** *When private information is cheap, the presence of PT speculation enhances the state-dependency of a stock's liquidity (i.e., strengthens H1).*

In this section, we test these implications in the U.S. stock market using a comprehensive sample of firm-level liquidity measures.

## 4.1 Data Description

### 4.1.1 Liquidity

Over the last two decades, market microstructure has studied the problem of measuring and explaining the liquidity of traded securities. This is a challenging task, for two reasons. First, liquidity is an elusive concept, commonly associated to several related (but only jointly exhaustive) measures capturing such notions as cost of trading, elasticity, resiliency, and immediacy (e.g., O’Hara, 1995; Hasbrouck, 2007). Second, the relative scarcity of high-frequency trade and quote data (e.g., TAQ) limits the scope of analyses of both the time-series and cross-sectional properties of these measures (e.g., Hasbrouck, 2009). In light of these issues, in this study we employ two measures of liquidity that most closely capture the notion of adverse selection-driven price impact in our model (i.e., “lambda”) while being available for the largest panel of U.S. stocks.

The first one is the *effective cost of trading*,  $EFFCOST_i$ , the extent to which the execution price deviates from the prevailing midquote to accommodate a trade. To compute it, Hasbrouck (2004, 2009) proposes an extension of the model of Roll (1984) accounting for the effect of market returns on transaction price changes, and advocates its estimation from daily closing prices via a Bayesian Gibbs approach. The second one is the *illiquidity measure* of Amihud (2002),  $ILLIQ_i$ , defined as the average ratio of daily absolute stock returns and their corresponding nonzero daily dollar volume. The intuition behind both measures is that when a firm’s stock is liquid, trading activity is accompanied by small price changes, i.e., a low  $EFFCOST_i$  and/or  $ILLIQ_i$ . Amihud (2002) and Hasbrouck (2009) find both  $EFFCOST_i$  and  $ILLIQ_i$  to be closely positively related to estimates of lambda from intraday TAQ data.

Hasbrouck (2009) provides annual estimates of  $EFFCOST_i$  and  $ILLIQ_i$  from a subset of the CRSP daily stock and index database made of 17,607 ordinary common shares (CRSP codes

10 and 11) between 1926 and 2005.<sup>18</sup> Summary statistics are in Table 1, over 190,780 firm-years. Gibbs estimates for  $EFFCOST_{i,t}$  are expressed as fraction of transaction prices, and can be interpreted as the average percentage effective cost for trades in firm  $i$ 's stocks on year  $t$ . Annual averages for  $ILLIQ_{i,t}$  are in basis points (bps), and can be interpreted as firm  $i$ 's average price movement in correspondence with \$10,000 of daily volume in its stock over year  $t$ . Median effective costs in the U.S. stock market are less than 1% and have been nearly steadily declining over the sample period (e.g., see Figure 3 in Hasbrouck, 2009). Median illiquidity is also low, roughly half a bps, but displays significant sample variability. Consistent with the aforementioned literature, Pearson correlation between  $EFFCOST_{i,t}$  and  $ILLIQ_{i,t}$  is positive (0.35) yet not large, strongly statistically significant, and increasing over time (e.g., about 0.55 from 1993 onward). This motivates the use of both measures to characterize the relation between time-series and cross-sectional properties of firm-level lambdas and speculation's preferences.

#### 4.1.2 Preferences and Information

Testing for the empirical relevance of many of our model's implications for market liquidity requires the assessment of the extent of better-informed speculation by agents displaying Prospect Theory preferences. In our model, a PT speculator can be interpreted as a sophisticated market participant — such as a mutual fund or a professional money manager — endowed with (or purchasing) superior information (or superior interpretation of public information) about firm-level or market-level factors affecting the future resale value of a stock (e.g., see the discussion in Pasquariello and Vega, 2009). Unfortunately, no direct measure of speculators' attitude toward risk is available in the literature.

In this paper, we propose *Active Share*,  $AS_j$  — a measure of *active portfolio management* developed by Cremers and Petajisto (2009) and Petajisto (2010) — as an indirect proxy for the

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<sup>18</sup>This data and an exhaustive description of the estimation procedure is available on Hasbrouck's website, at <http://pages.stern.nyu.edu/~jhasbrou/Research/GibbsEstimates2006/Liquidity%20estimates%202006.htm>.

*likelihood* that the trading activity in a stock may be driven, at least partly, by PT speculation. This proxy is motivated by the observation that professional investors, like individual agents in experimental settings, may display PT preferences in their portfolio allocations (e.g., see Barberis and Huang, 2008). Hence, we conjecture that the more a manager’s portfolio holdings differ from those in her benchmark index — i.e., the greater is her  $AS_j$  (e.g., because of active strategic and/or tactical asset allocation decisions) — the greater is the likelihood that those decisions may have been driven by PT preferences.

Cremers and Petajisto (2009) provide quarterly estimates of  $AS_j$  of U.S. equity mutual funds between 1990 and 2005.<sup>19</sup> We augment this sample with those funds’ quarterly U.S. stock holdings,  $N_{i,j}$  — the number of shares held in stock  $i$  by mutual fund  $j$  — from the Thompson/CDA/Spectrum database. We then compute, for each firm  $i$ , a weighted average of year-end  $t$  Active Share of all the mutual funds in our sample holding firm  $i$ ’s stocks at  $t$  (when available):

$$PT_{i,t} = \frac{\sum_j N_{i,j,t} AS_{j,t}}{N_{i,t}}, \quad (33)$$

where  $AS_{j,t}$  is in percentage (and multiplied by 100) and  $N_{i,t}$  is the total number of firm  $i$ ’s shares outstanding at year-end  $t$ . We scale  $PT_{i,t}$  by  $N_{i,t}$  to adjust for the relative importance of mutual fund holdings for a firm’s shares. Scaling  $PT_{i,t}$  by the total number of firm  $i$ ’s shares held by mutual funds at year-end  $t$  (i.e., by  $\sum_j N_{i,j,t}$ ) yields nearly identical inference. Consistent with Cremers and Petajisto (2009),  $PT_{i,t}$  can be interpreted as a measure of the average extent of active management among the mutual funds holding stock  $i$ . In light of the above discussion, in this paper we interpret a higher  $PT_{i,t}$  as indirectly indicating a greater likelihood of sophisticated investors in that stock displaying PT preferences.

According to Section 3, the impact of the presence of PT speculation on equity market liquidity crucially depends on whether superior information about stocks’ future payoffs is cheaply

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<sup>19</sup>This data is available on Petajisto’s website, at <http://www.petajisto.net/data.html>.

available. Superior information is arguably expensive in most circumstances. Thus, we employ two proxies for the less common circumstances in which superior information may instead be cheap. The first one is  $ANALYST_{i,t}$ , the median number of EPS forecasts for stock  $i$  in year  $t$  (e.g., Brennan and Subrahmanyam, 1995; Chang, Dasgupta, and Hilary, 2006), from the I/B/E/S database over the interval 1976-2005. The second one is  $MKTCAP_{i,t}$ , the market capitalization of firm  $i$  at year-end  $t$ . Greater analyst coverage of a firm (higher  $ANALYST_{i,t}$ ) may imply not only greater availability of (superior) information but also greater competition among information providers, hence cheaper information. (Superior) information production is likely easier and cheaper for larger, more transparent firms (higher  $MKTCAP_{i,t}$ ) as well (e.g., Bharath, Pasquariello, and Wu, 2009, and references therein).

Table 1 shows that both  $PT_{i,t}$  and  $ANALYST_{i,t}$  are available for just a fraction of the firm-years of our liquidity sample. Neither displays a discernible time trend. The (unreported) median weighted Active Share among mutual funds is roughly 75%, suggesting that active management is prevalent in our sample.<sup>20</sup> The modestly positive correlation between  $PT_{i,t}$  and either  $EFFCOST_{i,t}$  or  $ILLIQ_{i,t}$  suggests that more liquid firms are only weakly more likely to be held by active mutual funds, despite their lower turnover costs. Consistent with the aforementioned literature, larger firms tend to be more liquid and to have greater analyst coverage.

Of course, these proxies are only suggestive of the unobservable nature of speculators' preferences and information environment. Thus, in the analysis that follows, we perform a set of tests to ascertain whether not only their *level* but also their *interaction* effects can explain equity market liquidity in a manner consistent with our model's predictions.

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<sup>20</sup>Cremers and Petajisto (2009) label mutual funds as displaying low (high) Active Share if their  $AS_i$  is below (above) 60%.

## 4.2 Panel Regressions

We test our model’s predictions, listed in hypotheses H1 to H5, by estimating various versions of the following panel regressions (with and without firm fixed effects):

$$LAMBDA_{i,t} = \alpha + \alpha_t + \beta_1 * ABSRET_{i,t} + \beta_2 * PT_{i,t} + \varepsilon_{i,t}, \quad (34)$$

where  $LAMBDA_{i,t}$  is either  $EFFCOST_{i,t}$  or  $ILLIQ_{i,t}$ ,  $ABSRET_{i,t}$  is firm  $i$ ’s annual absolute return, and  $a_t$  are calendar year fixed effects (to control for the aforementioned time trends in U.S. equity market liquidity). We report estimates of the coefficients in Eq. (34) in Tables 2 and 3, respectively. Slope estimates are in units of the dependent variable — multiplied by one standard deviation shocks to the corresponding independent variable (from Table 1) — to ease their comparison and interpretation.

Tables 2 and 3 provide some support for hypothesis H1, namely that the depth of U.S. stocks is lower in correspondence with larger stock price movements: Coefficients for  $ABSRET_{i,t}$  are mostly positive and statistically significant, especially for stocks’ effective cost of trading (columns (1) and (2)).<sup>21</sup> Yet, such state-dependency may stem from frictions to liquidity provision absent from our model by construction and unrelated to speculators’ unconventional preferences.<sup>22</sup> For instance, Remark 2 suggests that equilibrium market depth may be lower in “good” or “bad” times when superior information is not cheaply available, even if speculators are risk-averse. Hence, these average effects may depend primarily on the role of information costs. Accordingly, and consistent with hypothesis H2, when superior information is costly a firm’s stock liquid-

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<sup>21</sup>Recent empirical studies also suggest that marketwide and stock-level liquidity may be state-dependent: E.g., see Chordia, Roll, and Subrahmanyam (2001), Chordia, Shivakumar, and Subrahmanyam (2004), Griffin, Nardari, and Stulz (2007), and Kamara, Lou, and Sadka (2008).

<sup>22</sup>Some of these frictions may have asymmetric effects on market liquidity. For instance, adverse selection risk may be more severe, dealers’ inventory carrying costs higher, or their risk-bearing capacity lower during “bad” times than during “good” times. In unreported analysis, we find our liquidity proxies to be state-dependent during both “good” and “bad” times (i.e., in correspondence with both positive and negative returns), albeit unsurprisingly more intensely so during the latter than during the former. Quadratic terms for  $ABSRET_{i,t}$  in Eq. (34) are only rarely significant and do not affect our inference, so are omitted.

ity may deteriorate if the presence of PT speculation in its shares is more likely (i.e., greater  $LAMBDA_{i,t}$  if  $PT_{i,t}$  is higher; columns (3) to (6)).

In light of these observations, we amend Eq. (34) to include the cross-products of  $PT_{i,t}$  with either of our proxies for the cost of a firm’s superior information, its analyst coverage ( $ANALYST_{i,t}$ ) and size ( $MKTCAP_{i,t}$ ).<sup>23</sup> Consistent with hypothesis H3, their estimated coefficients in Tables 2 and 3 (columns (7) to (14)) are nearly always *negative*, large, and statistically significant. Hence, equity liquidity tends to deteriorate when speculators are more likely to display PT preferences, if private information at their disposal is more likely to be cheaper. For instance, column (7) of Table 3 indicates that on average, a firm’s mean percentage price change per \$10,000 of daily volume in its stock ( $ILLIQ_{i,t}$ ) in correspondence with a one standard deviation increase in its  $PT_{i,t}$  is 0.10 bps lower (i.e., 24% lower than its sample median, in Table 1) if accompanied by a one standard deviation increase in its median analyst coverage ( $ANALYST_{i,t}$ ).

Further analysis reveals that not only the extent of a stock’s liquidity but also its state-dependency — as measured in Eq. (34) with respect to  $ABSRET_{i,t}$  — may be sensitive to both the presence of PT speculators and their endogenous information acquisition decisions. In particular, Tables 2 and 3 report that coefficient estimates for the cross-products of  $ABSRET_{i,t}$  and  $PT_{i,t}$  (columns (15) to (18)) are generally *negative* (as postulated in H4) — while estimated cross-product coefficients of  $ABSRET_{i,t}$ ,  $PT_{i,t}$ , and either  $ANALYST_{i,t}$  or  $MKTCAP_{i,t}$  (columns (19) to (22)) are *positive* (consistent with H5) — and generally statistically significant, especially for  $EFFCOST_{i,t}$ . For example, column (19) of Table 2 shows that a one standard deviation increase in  $PT_{i,t}$  lowers the sensitivity of the effective cost of stock trading to absolute stock returns — 0.044% versus a sample median of 0.70% in Table 1 — by 50% on average, but by less than 25% when accompanied by cheaper private information (a one standard deviation increase in  $ANALYST_{i,t}$ ).

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<sup>23</sup>The ensuing inference is virtually unaffected by the inclusion of these state variables as additional regressors in Eq. (34).

Overall, the evidence in Tables 2 and 3 provides support for our model, for it suggests that Prospect Theory preferences may be an important determinant of the cross-sectional and time-series properties of U.S. equity market liquidity.

## 5 Conclusions

This paper studies the implications of the main features of Tversky and Kahneman’s (1979) Prospect Theory — loss aversion and risk seeking over losses — for market liquidity. Our theoretical analysis demonstrates that introducing a better-informed speculator with these preferences in an otherwise standard economy (with or without information costs) yields novel, important predictions for the cross-section and time-series properties of equilibrium market depth. Intuitively, risk seeking induces a speculator to trade more aggressively with her private information (but to purchase it less often when expensive), while loss aversion induces her to use private information more cautiously (but to invest in it more often). These forces affect both a market maker’s perceived adverse selection risk and her liquidity provision in a complex fashion. Our preliminary empirical analysis of firm-level estimates of the effective cost of equity trading and illiquidity in a comprehensive sample of U.S. stocks between 1926 and 2005 yields evidence that is consistent with the predictions of our theory.

In absence of direct measures of the preferences of sophisticated speculators (and the information costs they face), our evidence is only suggestive and warrants further investigation. Our study also raises several unexplored questions. In particular, we note that for simplicity’s sake, our model allows for a single round of trading over a single risky asset. Risk seeking and loss aversion are likely to provide further insights for speculators’ multiperiod, multiasset trading strategies, especially if combined with that particular form of *mental accounting* (Thaler, 1980), often displayed by individuals in experimental settings, known as *narrow framing*.<sup>24</sup> Accord-

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<sup>24</sup>For instance, these are the circumstances when agents get direct utility from gains and losses of individual assets rather than portfolio fluctuations, as in Barberis and Huang (2001).

ingly, the portfolio decisions of such agents and their implications for asset pricing have received increasing attention in the literature (e.g., Levy, De Giorgi, and Hens, 2003; Levy and Levy, 2007; Barberis and Huang, 2008). We look forward to future research on the implications of so-motivated dynamic portfolio decisions for market liquidity.

## 6 Appendix

**Numerical Approach.** Semi-strong price efficiency (Eq. (10)) requires the computation of the MM’s estimates of the conditional first moments of the asset payoff  $v$  when the PT speculator is conjectured to trade (i.e., if either her private signal  $S > S_H$  or  $S < S_L$ ) in Eq. (11). Because the PT speculator’s optimal demand schedule  $x_{PT}$  (Eq. (8)) is nonlinear, these moments cannot be computed analytically, except when the speculator has CARA preferences ( $\gamma = 0$  and  $\beta = 0$ , see Remark 1). We compute these moments numerically, as explicit functions of  $\omega$  and  $P$ . Conceptually, a rational MM would use knowledge of the economy’s structure and the PT speculator’s trading rule in Eq. (8) — based on  $S$  and  $P$  — to learn about  $v$  from the order flow  $\omega$ . We capture the spirit of this inference problem by allowing the MM to form conditional expectations about  $v$  from simulating a large number of realizations of the economy and estimating a linear relation between  $v$  and both  $\omega$  and  $P$  — as the MM would do analytically in the presence of a MV speculator — as follows.

**Step 1:** We specify a price grid  $P(n)$  made of  $N$  points between  $P_L$  and  $P_H$ . Our procedure is robust to sensible grid choices. We set  $P_L$  ( $P_H$ ) as the equilibrium price in the basic economy of Section 2 under the assumptions that the speculator has MV preferences (i.e.,  $\gamma = 0$ ,  $\beta = 0$ ), noise trading and private signal noise are at their unconditional means of zero, and  $v = -3\sigma_v$  ( $v = 3\sigma_v$ ). As shown in Remark 1, closed-form solutions for these economies exist.

**Step 2:** We draw  $M$  triplets  $\{v(m), u(m), z(m)\}$  from their independent normal distribu-

tions, and compute the corresponding realization of the private signal,  $S(m) = v(m) + u(m)$ .

**Step 3:** For each triplet  $\{v(m), u(m), z(m)\}$ , at each price  $P(n)$  we compute

$$\chi(m, n) = \frac{P(n) - \phi S(m)}{\sqrt{\sigma_v^2(1 - \phi)}}, \quad (\text{A-1})$$

$$S_H(m, n) = \frac{P(n)}{\phi} + \frac{\gamma\psi(\chi(m, n))\sigma_v\sqrt{1 - \phi}}{\phi[1 + \gamma\Phi(\chi(m, n))]}, \quad (\text{A-2})$$

$$S_L(m, n) = \frac{P(n)}{\phi} - \frac{\gamma\psi(-\chi(m, n))\sigma_v\sqrt{1 - \phi}}{\phi[1 + \gamma\Phi(-\chi(m, n))]}, \quad (\text{A-3})$$

as well as  $x_{PT}(m, n)$  of Eq. (8), and  $\omega(m, n) = x_{PT}(m, n) + z(m)$ , and populate either of two matrices  $PT_H = \{v(m), \omega(m, n), P(n)\}$  and  $PT_L = \{v(m), \omega(m, n), P(n)\}$  depending on whether  $S(m) > S_H(m, n)$  or  $S(m) < S_H(m, n)$ , respectively.

**Step 4:** Based on the observation that any random variable  $y$  can be expressed as

$$y = E[y|x] + (y - E[y|x]) = E[y|x] + \varepsilon, \quad (\text{A-4})$$

(e.g., Greene, 1997, p. 80), we use  $PT_H$  and  $PT_L$  to estimate  $E[v|\omega, S > S_H]$  and  $E[v|\omega, S < S_L]$  from the following linear regressions for  $v(m)$ :

$$v(m) = a_H + b_H\omega(m, n) + c_HP(m) + \varepsilon(m, n), \quad (\text{A-5})$$

$$v(m) = a_L + b_L\omega(m, n) + c_LP(m) + \varepsilon(m, n). \quad (\text{A-6})$$

Specifically, OLS estimation of Eqs. (A-5) and (A-6) over  $PT_H$  and  $PT_L$ , respectively, yields

$$E[v|\omega, S > S_H] \approx \hat{a}_H + \hat{b}_H\omega + \hat{c}_HP, \quad (\text{A-7})$$

$$E[v|\omega, S < S_L] \approx \hat{a}_L + \hat{b}_L\omega + \hat{c}_LP. \quad (\text{A-8})$$

Further investigation reveals the linear functional forms in Eqs. (A-7) and (A-8) to be very accurate, and the ensuing inference to be insensitive to either using higher-order polynomials of  $\omega$  and/or  $P$  in Eqs. (A-5) and (A-6) or imposing exact symmetry to their coefficients (i.e., such that  $a_H = -a_L$ ,  $b_H = b_L$ , and  $c_H = c_L$ ). ■

**Proof of Lemma 1.** The expressions for  $E[v|\omega, S > S_H]$  and  $E[v|\omega, S < S_L]$  in Eqs. (12) and (13) stem from those in Eqs. (A-7) and (A-8), respectively, where we omit the estimation superscripts for economy of notation. The conditional mean  $E[v|\omega, S_L \leq S \leq S_H]$  can be expressed in closed form, as Eq. (14), using well-known properties of incidentally truncated bivariate normal variables (see, e.g., Johnson and Katz, 1974; Greene, 1997, p. 975), since  $x_{PT} = 0$  and  $\omega = z$  over the no-trade interval  $[S_L, S_H]$ . ■

**Proof of Proposition 1.** To prove this statement, we substitute Eqs. (12) to (14) from Lemma 1 in Eq. (11) and the resulting expression for  $E[v|\omega]$  in Eq. (10) to get:

$$\begin{aligned}
 P &= (a_H + b_H\omega + c_HP)[1 - \Phi(H)] + (a_L + b_L\omega + c_LP)\Phi(L) & \text{(A-9)} \\
 &+ \sigma_s\Lambda(L, H)[\Phi(H) - \Phi(L)].
 \end{aligned}$$

Solving Eq. (A-9) for  $P$  leads to the implicit function of Eq. (15). The equilibrium price  $P_{PT}$  is a fixed point of Eq. (15). Let  $f(P)$  be the right side of Eq. (15) and  $g(P) = f(P) - P$ . Because of properties of  $\Phi(\cdot)$  and  $\psi(\cdot)$  (in particular,  $\lim_{y \rightarrow +\infty} \Phi(y) = 1$ ,  $\lim_{y \rightarrow -\infty} \Phi(y) = 0$ , and  $\lim_{y \rightarrow \pm\infty} \psi(y) = 0$ ), it is immediate that  $\lim_{P \rightarrow +\infty} g(P) < 0$  and  $\lim_{P \rightarrow -\infty} g(P) > 0$ . Existence of a solution to  $g(P) = 0$  follows from the Intermediate Value Theorem, since  $g(P)$  is a continuous function of  $P$ . As  $g(P)$  is decreasing in  $P$ , such solution is therefore unique. ■

**Proof of Remark 1.** We prove this statement by using Eq. (9) and properties of conditional normal distributions to solve for  $P_{MV}$  from Eq. (10). The distributional assumptions of Section 2.1 imply that the order flow  $\omega = x_{MV} + z$  is normally distributed with mean  $E[\omega] = -C_{MV}P$

and variance  $var[\omega] = C_{MV}^2 \phi^2 \sigma_s^2 + \sigma_z^2$ , where  $C_{MV} = \frac{1}{\alpha \sigma_v^2 (1-\phi)}$ . Since  $cov[v, \omega] = C_{MV} \phi \sigma_v^2$ , it then follows (e.g., Greene, 1997, p. 90) that:

$$P = \frac{cov[v, \omega]}{var[\omega]} \{\omega - E[\omega]\} = \frac{C_{MV} \phi \sigma_v^2}{C_{MV}^2 \phi^2 \sigma_s^2 + \sigma_z^2 - C_{MV}^2 \phi \sigma_v^2} \omega. \quad (\text{A-10})$$

Substituting the expression for  $C_{MV}$  into Eq. (A-10) and observing that  $\phi = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2}$  leads to  $P_{MV}$  of Eq. (18) with  $\lambda_{MV}$  of Eq. (19). ■

**Proof of Lemma 2.** We begin by rewriting the expressions for  $x_{PT}$  (Eq. (8)) and  $x_{PT}^0$  (Eq. (23)) in a more compact form as

$$x_{PT} = \begin{cases} A_H (\phi S - P) - B_H > 0 & \text{if } S > S_H \\ A_L (\phi S - P) + B_L < 0 & \text{if } S < S_L \\ 0 & \text{if } S_L \leq S \leq S_H \end{cases}, \quad (\text{A-11})$$

where  $A_H = \frac{[1+\gamma\Phi(\chi)]}{\alpha^*(\chi)\sigma_v^2(1-\phi)}$ ,  $B_H = \frac{\gamma\psi(\chi)}{\alpha^*(\chi)\sigma_v\sqrt{1-\phi}}$ ,  $A_L = \frac{[1+\gamma\Phi(-\chi)]}{\alpha^*(-\chi)\sigma_v^2(1-\phi)}$ , and  $B_L = \frac{\gamma\psi(-\chi)}{\alpha^*(-\chi)\sigma_v\sqrt{1-\phi}}$ , and

$$x_{PT}^0 = \begin{cases} -A_H^0 P - B_H^0 > 0 & \text{if } P < P_L^0 \\ -A_L^0 P + B_L^0 < 0 & \text{if } P > P_H^0 \\ 0 & \text{if } P_L^0 \leq P \leq P_H^0 \end{cases}, \quad (\text{A-12})$$

where  $A_H^0 = \frac{[1+\gamma\Phi(\chi_0)]}{\alpha^*(\chi_0)\sigma_v^2}$ ,  $B_H^0 = \frac{\gamma\psi(\chi_0)}{\alpha^*(\chi_0)\sigma_v}$ ,  $A_L^0 = \frac{[1+\gamma\Phi(-\chi_0)]}{\alpha^*(-\chi_0)\sigma_v^2}$ , and  $B_L^0 = \frac{\gamma\psi(-\chi_0)}{\alpha^*(-\chi_0)\sigma_v}$ . Substituting Eqs. (A-11) and (A-12) in the expressions for  $E[U_{PT}|S]$  (Eq. (7)) and  $E[U_{PT}]$  (Eq. (2)) yields

$$E[U_{PT}|S] = \begin{cases} D_H^0 (\phi S - P)^2 - D_H^1 (\phi S - P) - D_H^2 & \text{if } S > S_H \\ D_L^0 (\phi S - P)^2 + D_L^1 (\phi S - P) - D_L^2 & \text{if } S < S_L \\ 0 & \text{if } S_L \leq S \leq S_H \end{cases}, \quad (\text{A-13})$$

where

$$D_H^0 = A_H [1 + \gamma\Phi(\chi)] - \frac{1}{2} (A_H)^2 \alpha^*(\chi) \sigma_v^2 (1 - \phi), \quad (\text{A-14})$$

$$D_H^1 = B_H [1 + \gamma\Phi(\chi)] - \gamma A_H \psi(\chi) \sigma_v \sqrt{1 - \phi} - A_H B_H \alpha^*(\chi) \sigma_v^2 (1 - \phi), \quad (\text{A-15})$$

$$D_H^2 = \gamma B_H \psi(\chi) \sigma_v \sqrt{1 - \phi} + \frac{1}{2} (B_H)^2 \alpha^*(\chi) \sigma_v^2 (1 - \phi), \quad (\text{A-16})$$

$$D_L^0 = A_L [1 + \gamma\Phi(-\chi)] - \frac{1}{2} (A_L)^2 \alpha^*(-\chi) \sigma_v^2 (1 - \phi), \quad (\text{A-17})$$

$$D_L^1 = B_L [1 + \gamma\Phi(-\chi)] - \gamma A_L \psi(-\chi) \sigma_v \sqrt{1 - \phi} - A_L B_L \alpha^*(-\chi) \sigma_v^2 (1 - \phi), \quad (\text{A-18})$$

$$D_L^2 = \gamma B_L \psi(-\chi) \sigma_v \sqrt{1 - \phi} + \frac{1}{2} (B_L)^2 \alpha^*(-\chi) \sigma_v^2 (1 - \phi), \quad (\text{A-19})$$

and

$$E[U_{PT}] = \begin{cases} D_H^{0,0} P^2 + D_H^{0,1} P - D_H^{0,2} & \text{if } P < P_L^0 \\ D_L^{0,0} P^2 - D_L^{0,1} P - D_L^{0,2} & \text{if } P > P_H^0 \\ 0 & \text{if } P_L^0 \leq P \leq P_H^0 \end{cases}, \quad (\text{A-20})$$

where

$$D_H^{0,0} = A_H^0 [1 + \gamma\Phi(\chi_0)] - \frac{1}{2} (A_H^0)^2 \alpha^*(\chi_0) \sigma_v^2, \quad (\text{A-21})$$

$$D_H^{0,1} = B_H^0 [1 + \gamma\Phi(\chi_0)] - \gamma A_H^0 \psi(\chi_0) \sigma_v - A_H^0 B_H^0 \alpha^*(\chi_0) \sigma_v^2, \quad (\text{A-22})$$

$$D_H^{0,2} = \gamma B_H^0 \psi(\chi_0) \sigma_v + \frac{1}{2} (B_H^0)^2 \alpha^*(\chi_0) \sigma_v^2, \quad (\text{A-23})$$

$$D_L^{0,0} = A_L^0 [1 + \gamma\Phi(-\chi_0)] - \frac{1}{2} (A_L^0)^2 \alpha^*(-\chi_0) \sigma_v^2, \quad (\text{A-24})$$

$$D_L^{0,1} = B_L^0 [1 + \gamma\Phi(-\chi_0)] - \gamma A_L^0 \psi(-\chi_0) \sigma_v - A_L^0 B_L^0 \alpha^*(-\chi_0) \sigma_v^2, \quad (\text{A-25})$$

$$D_L^{0,2} = \gamma B_L^0 \psi(-\chi_0) \sigma_v + \frac{1}{2} (B_L^0)^2 \alpha^*(-\chi_0) \sigma_v^2. \quad (\text{A-26})$$

After substituting Eq. (A-20) in Eq. (20), we are left with the task of computing

$$E \{E [U_{PT}|S]\} = E [U_{PT}|S > S_H] [1 - \Phi(H)] + E [U_{PT}|S < S_L] \Phi(L). \quad (\text{A-27})$$

Closed-form expressions for  $E [U_{PT}|S > S_H]$  and  $E [U_{PT}|S < S_L]$  do not exist. We rely on the observation that, given a random variable  $y$  with mean  $\bar{y}$  and variance  $\sigma_y^2$  and a nonlinear function  $g(y)$ , a natural approximation of that function's first moment is

$$E [g(y)] \approx g(\bar{y}), \quad (\text{A-28})$$

(e.g., Greene, 1997, pp. 66-67). We also observe that, because of properties of truncated normal variables (e.g., Greene, 1997, pp. 951-952),  $E [S|S > S_H] = \sigma_s \Lambda(H)$ ,  $\text{var} [S|S > S_H] = \sigma_s^2 [1 - \Delta(H)]$  (where  $\Delta(\cdot) = \Lambda(\cdot) [\Lambda(\cdot) - (\cdot)]$ ),  $E [S^2|S > S_H] = \sigma_s^2 [1 - \Delta(H) + \Lambda(H)^2]$ , as well as that  $E [S|S < S_L] = \sigma_s \Lambda^-(L)$ ,  $\text{var} [S|S < S_L] = \sigma_s^2 [1 - \Delta^-(L)]$ ,  $E [S^2|S < S_L] = \sigma_s^2 [1 - \Delta^-(L) + \Lambda^-(L)^2]$ , implying that  $E [\chi|S > S_H] = \bar{\chi}_H = \frac{P - \phi \sigma_s \Lambda(H)}{\sigma_v \sqrt{1 - \phi}}$  and  $E [\chi|S < S_L] = \bar{\chi}_L = \frac{P - \phi \sigma_s \Lambda(L)}{\sigma_v \sqrt{1 - \phi}}$ . We then substitute  $\chi$  with  $\bar{\chi}_H$  in  $A_H$ ,  $B_H$ ,  $D_H^0$ ,  $D_H^1$ , and  $D_H^2$ , as well  $\chi$  with  $\bar{\chi}_L$  in  $A_L$ ,  $B_L$ ,  $D_L^0$ ,  $D_L^1$ , and  $D_L^2$ , and label the resulting variables as  $\bar{A}_H$ ,  $\bar{B}_H$ ,  $\bar{D}_H^0$ ,  $\bar{D}_H^1$ ,  $\bar{D}_H^2$ ,  $\bar{A}_L$ ,  $\bar{B}_L$ ,  $\bar{D}_L^0$ ,  $\bar{D}_L^1$ , and  $\bar{D}_L^2$ . These substitutions and Eqs. (A-13) and (A-28) yield the following piecewise approximations:

$$E [U_{PT}|S > S_H] \approx \bar{D}_H^0 P^2 + \left[ \bar{D}_H^1 - 2\bar{D}_H^0 \phi \sigma_s \Lambda(H) \right] P + \left[ \bar{D}_H^0 \phi^2 \sigma_s^2 \Lambda(H)^2 - \bar{D}_H^1 \phi \sigma_s \Lambda(H) - \bar{D}_H^2 \right], \quad (\text{A-29})$$

$$E [U_{PT}|S < S_L] \approx \bar{D}_L^0 P^2 - \left[ \bar{D}_L^1 + 2\bar{D}_L^0 \phi \sigma_s \Lambda^-(L) \right] P + \left[ \bar{D}_L^0 \phi^2 \sigma_s^2 \Lambda^-(L)^2 + \bar{D}_L^1 \phi \sigma_s \Lambda^-(L) - \bar{D}_L^2 \right]. \quad (\text{A-30})$$

Lastly, substituting Eqs. (A-29) and (A-30) in Eq. (A-28), and the resulting expression for

$E \{E [U_{PT}|S]\}$  in Eq. (20) yields the expression of Eq. (24), where

$$E^0 = \bar{D}_H^0 [1 - \Phi(H)] + \bar{D}_L^0 \Phi(L), \quad (\text{A-31})$$

$$E^1 = \left[ \bar{D}_H^1 - 2\bar{D}_H^0 \phi \sigma_s \Lambda(H) \right] [1 - \Phi(H)] - \left[ \bar{D}_L^1 + 2\bar{D}_H^0 \phi \sigma_s \Lambda^-(L) \right] \Phi(L) \quad (\text{A-32})$$

$$E^2 = \left[ \bar{D}_H^0 \phi^2 \sigma_s^2 \Lambda(H)^2 - \bar{D}_H^1 \phi \sigma_s \Lambda(H) - \bar{D}_H^2 \right] [1 - \Phi(H)] \quad (\text{A-33})$$

$$+ \left[ \bar{D}_L^0 \phi^2 \sigma_s^2 \Lambda^-(L)^2 + \bar{D}_L^1 \phi \sigma_s \Lambda^-(L) - \bar{D}_L^2 \right] \Phi(L),$$

$F_H^0 = E^0 - D_H^{0,0}$ ,  $F_H^1 = E^1 - D_H^{0,1}$ ,  $F_H^2 = E_H^2 + D_H^{0,0}$ ,  $F_L^0 = E^0 - D_L^{0,0}$ ,  $F_L^1 = E_H^1 + D_L^{0,1}$ , and  $F_L^2 = E^2 + D_L^{0,2}$ . In the presence of a MV speculator ( $\gamma = 0$  and  $\beta = 0$ ), it is immediate from Eqs. (3), (9), and (23) that  $x_{MV}^0 = -\frac{P}{\alpha \sigma_v^2}$  and

$$E \{E [U_{MV}|S]\} = \frac{\sigma_v^4 + \sigma_s^2 P^2}{2\alpha \sigma_v^2 \sigma_u^2}, \quad (\text{A-34})$$

$$E [U_{MV}] = \frac{P^2}{2\alpha \sigma_v^2}. \quad (\text{A-35})$$

Substituting Eqs. (A-34) and (A-35) in Eq. (20) yields Eq. (25). ■

**Proof of Proposition 2.** The proof of this statement mimics the proof of Proposition 1. Specifically, we use Lemma 1 and Eq. (27) to express the semi-strong price efficiency condition of Eq. (10) as:

$$P^c = (a_H + b_H \omega^c + c_H P^c) [1 - \Phi(H)] 1_{B_{PT}} + (a_L + b_L \omega^c + c_L P^c) \Phi(L) \quad (\text{A-36})$$

$$+ \sigma_s \Lambda(L, H) [\Phi(H) - \Phi(L)] 1_{B_{PT}},$$

and then solve Eq. (A-36) for  $P^c$  to obtain the implicit function of Eq. (28). The equilibrium price  $P_{PT}^c$  is the unique fixed point of Eq. (28). ■

**Proof of Remark 2.** As in the proof of Remark 1, we begin by observing that when a

MV speculator purchases the private signal  $S$  ( $1_{B_{MV}} = 1$ ), the ensuing order flow  $\omega^c = x_{MV}^c + z$  is normally distributed with  $E[\omega^c] = -C_{MV}P$ ,  $var[\omega^c] = C_{MV}^2\phi^2\sigma_s^2 + \sigma_z^2$ , and  $cov[v, \omega^c] = C_{MV}\phi\sigma_v^2$ , where  $C_{MV} = \frac{1}{\alpha\sigma_v^2(1-\phi)}$ . Semi-strong price efficiency (Eq. (10)) then implies that  $P^c = E[v|\omega^c]1_{B_{MV}}$ , where (as in Eq.(A-10))

$$E[v|\omega^c] = \frac{C_{MV}\phi\sigma_v^2}{C_{MV}^2\phi^2\sigma_s^2 + \sigma_z^2} (\omega^c + C_{MV}P^c). \quad (\text{A-37})$$

Substituting the expression for  $C_{MV}$  into Eq. (A-37) and solving for  $P^c$  yields  $P_{MV}^c$  of Eq. (31) with  $\lambda_{MV}^c$  of Eq. (32). ■

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Figure 1. Prospect Theory Preferences

In this figure we plot realizations of a speculator's utility functions described in Section 2.1. Specifically, we plot realizations of the power utility function of Tversky and Kahneman (1992) —  $U_{TK}$  of Eq. (1) (Figure 1a, dashed line) — as well as realizations of the Prospect Theory-inspired, piecewise value function of Eq. (2) — i.e.,  $U_{PT} = \pi - \frac{1}{2}\alpha\pi^2 + V(\pi)$ ,  $V(\pi) = \gamma\pi + \frac{1}{2}\beta\pi^2$  for  $\pi < 0$  and zero otherwise — for  $\alpha = 1$ ,  $\gamma = 1$ , and  $\beta = 1.05$  (Figure 1a, solid line), as well as for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 2$  (Figure 1b, thin line, labeled  $U_{MV}$ ), for  $\alpha = 1$ ,  $\gamma = 2$ , and  $\beta = 0$  (Figure 1b, dashed line, labeled  $U_{PT}^{\beta=2}$ ), and for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 2$  (Figure 1b, dotted line, labeled  $U_{PT}^{\beta=2}$ ), over the domain of trading profits  $\pi$ .

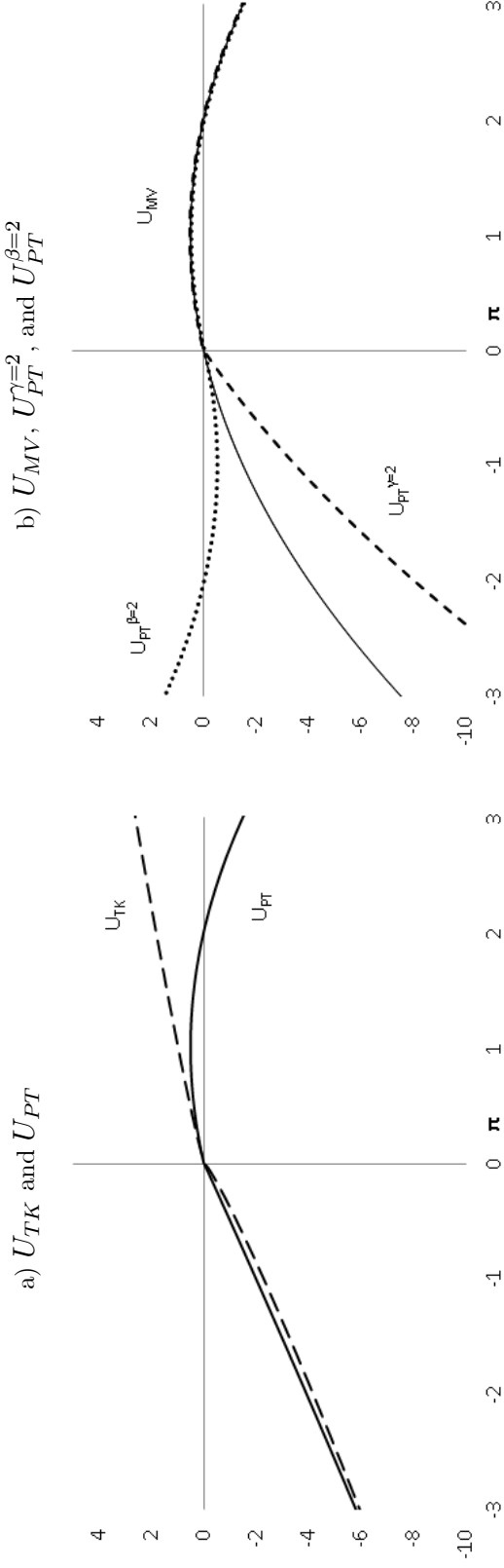


Figure 2. Prospect Theory Trading

In this figure we plot realizations of a speculator's optimal demand schedules described in Section 2.3. Specifically, we plot realizations of the optimal demand schedule of a MV speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 0$  ( $x_{MV}$  of Eq. (9) in Figure 2a, dashed line) — as well as the optimal demand schedule of a PT speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 1$ , and  $\beta = 1.05$  ( $x_{PT}$  of Eq. (8) in Figure 2a, solid line) — of a loss averse speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 2$ , and  $\beta = 0$  ( $x_{PT}^{\gamma=2}$  in Figure 2b, dotted line) — and of a speculator seeking risk in losses — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 2$  ( $x_{PT}^{\beta=2}$  in Figure 2b, dashed line) — over the domain of  $P$  for a private signal  $S = 0$  when  $\sigma_v^2 = 1$  and  $\sigma_u^2 = 1$ .

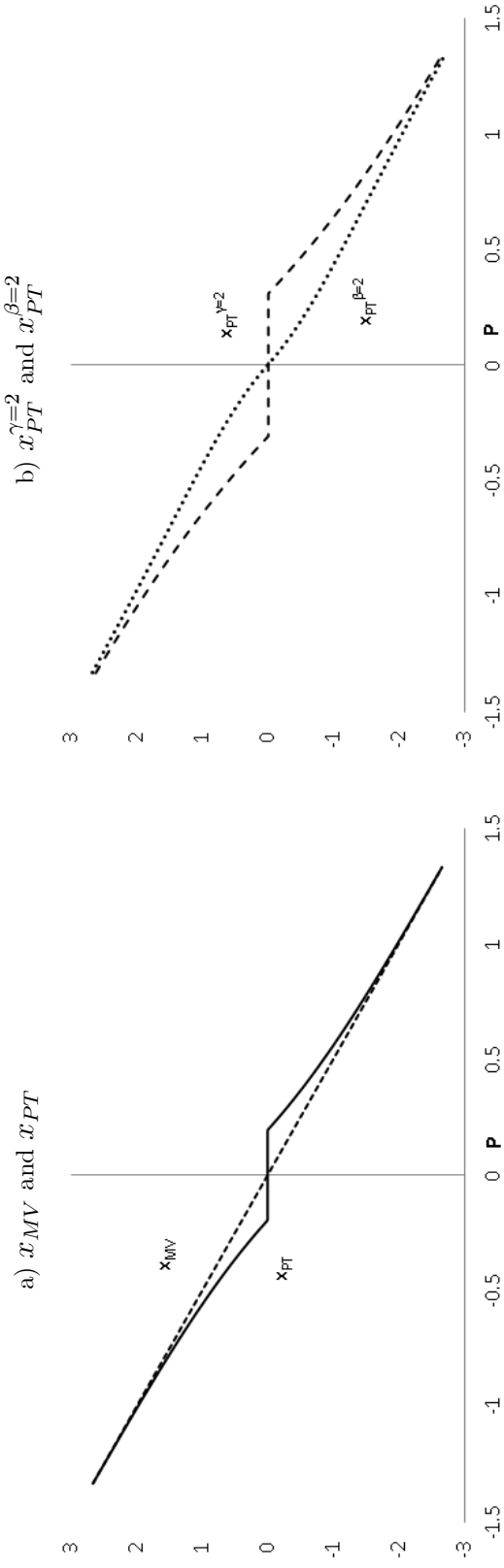


Figure 3. Prospect Theory and Market Depth

In this figure we plot, by virtue of numerical integration, the average equilibrium price impact  $\lambda_{PT}$  of Proposition 1 over the domain of a speculator's private signal  $S$ . Specifically, we plot the equilibrium price impact in the presence of a MV speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 0$  ( $\lambda_{MV}$  of Eq. (19) in Figure 3a, dashed line) — as well as in the presence of a PT speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 1$ , and  $\beta = 1.05$  ( $E[\lambda_{PT}|S]$  from Eq. (17) in Figure 3a, solid line) — of a loss averse speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 2$ , and  $\beta = 0$  ( $E[\lambda_{PT}^{\gamma=2}|S]$  in Figure 3b, dashed line) — and of a speculator seeking risk in losses — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 2$  ( $E[\lambda_{PT}^{\beta=2}|S]$  in Figure 3b, dotted line) — over the domain of  $S$  when  $\sigma_v^2 = 1$ ,  $\sigma_u^2 = 1$ , and  $\sigma_z^2 = 1$ . Both figures also display  $N(0, \sigma_s^2)$ , the normal pdf of  $S$  (thin line, reverse right axis).

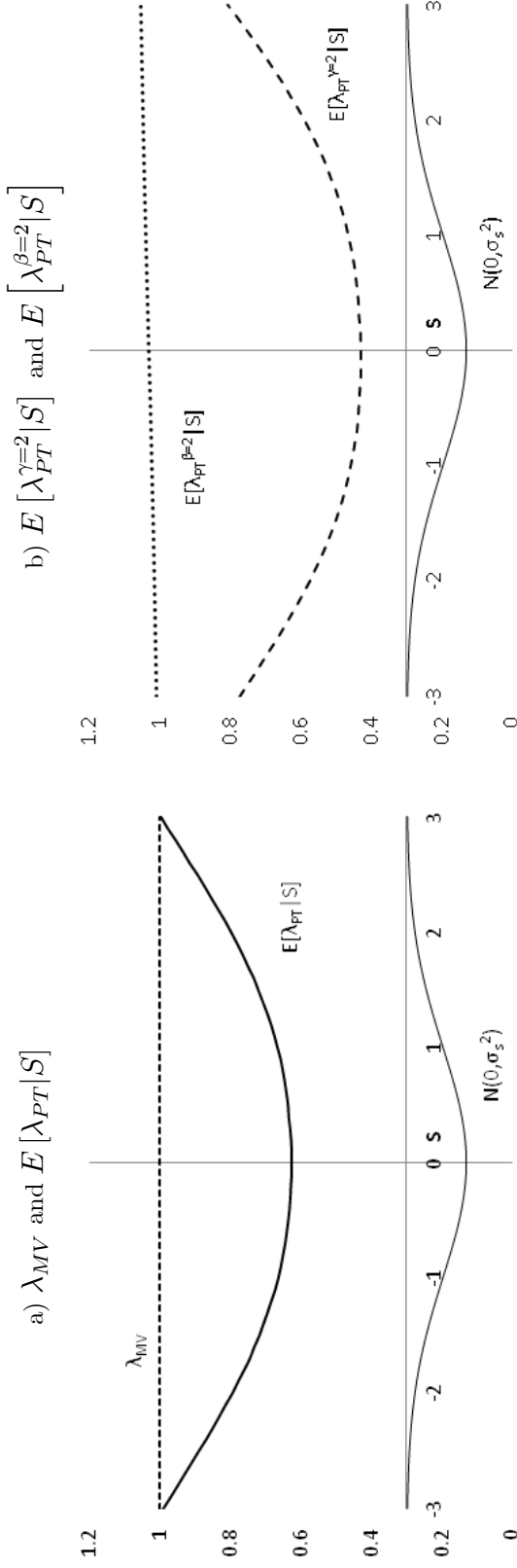


Figure 4. Prospect Theory Trading when Information is Costly

In this figure we plot realizations of a speculator's optimal demand schedules described in Section 3.1, when  $S$  is costly. Specifically, we plot realizations of the optimal demand schedule of a MV speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 0$  ( $x_{MV}^c$  of Eq. (9) in Figure 4a, dashed line) — as well as the optimal demand schedule of a PT speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 1$ , and  $\beta = 1.05$  ( $x_{PT}^c$  of Eq. (26) in Figure 4a, solid line) — of a loss averse speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 2$ , and  $\beta = 0$  ( $x_{PT}^{c,\gamma=2}$  in Figure 4b, dashed line) — and of a speculator seeking risk in losses — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 2$  ( $x_{PT}^{c,\beta=2}$  in Figure 4b, dotted line) — over the domain of  $P$  for a private signal  $S = 0$  with cost  $c = 0.60$  when  $\sigma_v^2 = 1$  and  $\sigma_u^2 = 1$ .

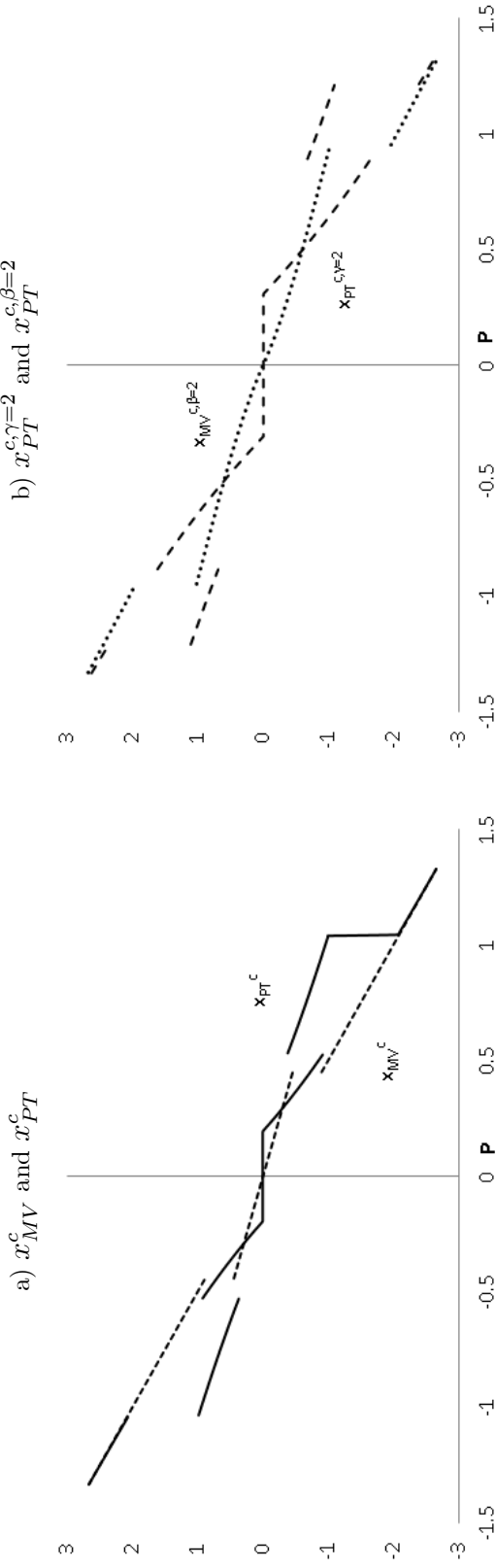


Figure 5. Prospect Theory and Endogenous Information Acquisition

In this figure we plot, by virtue of numerical integration, the average equilibrium information acquisition decision  $1_{B_{PT}}$  from Proposition 2 over the domain of a speculator's private signal  $S$ , when  $S$  is costly. Specifically, we plot the equilibrium information acquisition decision of a MV speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 0$  ( $E[1_{B_{MV}}|S]$  in Figure 5a, dashed line) — as well as that of a PT speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 1$ , and  $\beta = 1.05$  ( $E[1_{B_{PT}}|S]$  in Figure 5a, solid line) — of a loss averse speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 2$ , and  $\beta = 0$  ( $E[1_{B_{PT}^{\gamma=2}}|S]$  in Figure 5b, dashed line) — and of a speculator seeking risk in losses — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 2$  ( $E[1_{B_{PT}^{\beta=2}}|S]$  in Figure 5b, dotted line) — over the domain of  $S$  when  $c = 0.60$ ,  $\sigma_v^2 = 1$ ,  $\sigma_u^2 = 1$ , and  $\sigma_z^2 = 1$ .

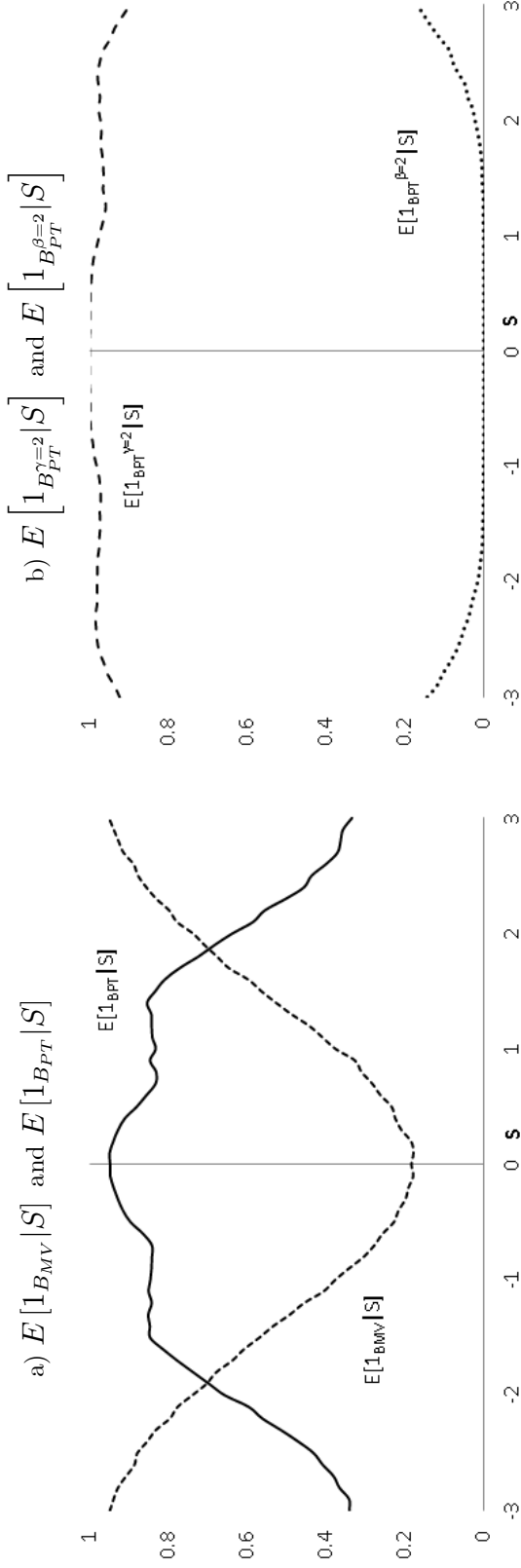


Figure 6. Prospect Theory and Market Depth when Information is Costly

In this figure we plot, by virtue of numerical integration, the average equilibrium price impact  $\lambda_{PT}$  of Proposition 2 over the domain of a speculator's private signal  $S$ , when  $S$  is costly. Specifically, we plot the equilibrium price impact in the presence of a MV speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 0$  ( $E[\lambda_{MV}^c | S]$  from Eq. (32) in Figure 6a, dashed line) — as well as in the presence of a PT speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 1$ , and  $\beta = 0$  ( $E[\lambda_{PT}^{c,\gamma=2} | S]$  from Eq. (30) in Figure 6a, solid line) — of a loss averse speculator — i.e., for  $\alpha = 1$ ,  $\gamma = 2$ , and  $\beta = 0$  ( $E[\lambda_{PT}^{c,\beta=2} | S]$  in Figure 6b, dashed line) — and of a speculator seeking risk in losses — i.e., for  $\alpha = 1$ ,  $\gamma = 0$ , and  $\beta = 2$  ( $E[\lambda_{PT}^{c,\beta=2} | S]$  in Figure 6b, dotted line) — over the domain of  $S$  when  $c = 0.60$ ,  $\sigma_u^2 = 1$ ,  $\sigma_v^2 = 1$ , and  $\sigma_z^2 = 1$ .

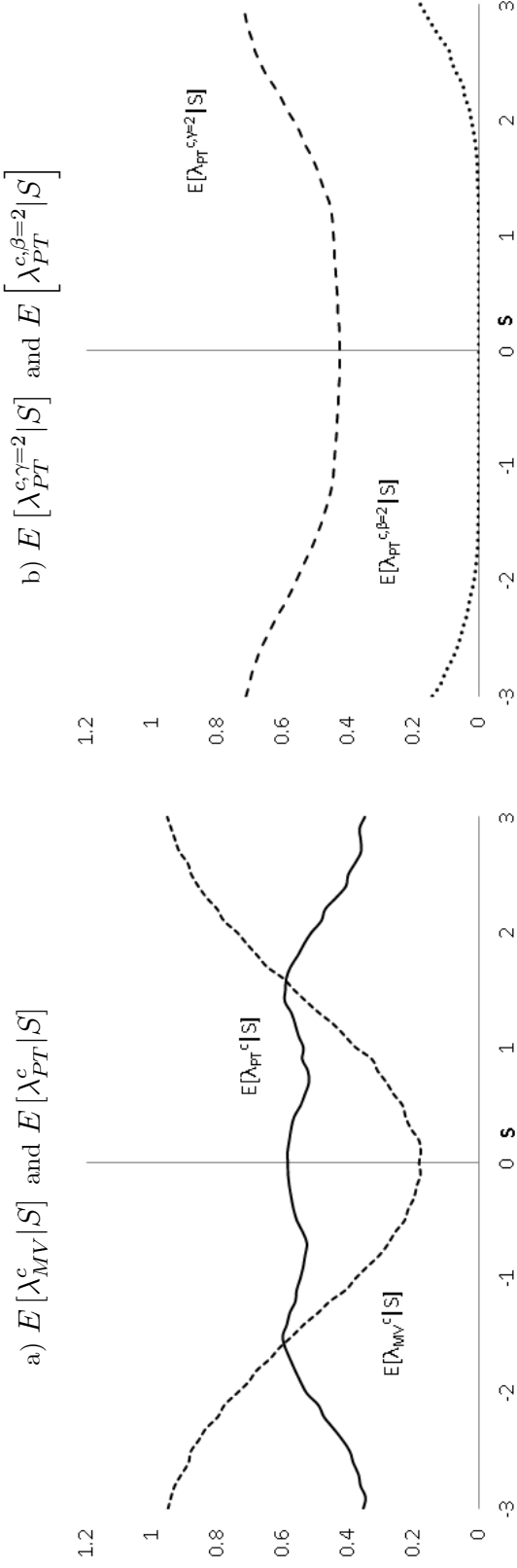


Table 1. Summary Statistics

This table reports summary statistics for the measures of firm-level liquidity, presence of speculation with Prospect Theory preferences, and private information cost described in Section 4. Specifically,  $EFFCOST_{i,t}$  is the average percentage effective cost for trades in firm  $i$ 's stocks on year  $t$ ;  $ILLIQ_{i,t}$  is the annual average of the ratio of firm  $i$ 's daily absolute stock returns and its corresponding nonzero daily dollar volume, in bps per \$10,000 of trading volume;  $PT_{i,t}$  is the weighted average of year-end  $t$ 's Active Share (multiplied by 100) of all the mutual funds holding firm  $i$ 's stocks (see Eq. (33));  $ANALYST_{i,t}$  is the median number of EPS forecasts for stock  $i$  over year  $t$ ;  $MKTCAP_{i,t}$  is the market capitalization of firm  $i$  at year-end  $t$  (in millions);  $ABSRET_{i,t}$  is firm  $i$ 's absolute annual return, in percentage (multiplied by 100). All Pearson correlations are significant at the 1% level.

Measure	Firm-Year	Obs	Sample	Mean	Median	Stdev	Pearson Correlation Matrix						
							EFFCOST	ILLIQ	PT	ANALYST	MKTCAP	ABSRET	
EFFCOST	190,780		1926-2005	1.26%	0.70%	1.60%	1						
ILLIQ	190,780		1926-2005	9.69	0.44	121.65	0.349	1					
PT	30,572		1990-2005	0.62	0.23	1.05	0.099	0.039	1				
ANALYST	51,377		1976-2005	6.16	4.00	6.51	-0.334	-0.088	-0.074	1			
MKTCAP	190,682		1926-2005	\$927.4	\$65.96	\$7,075	-0.076	-0.010	-0.061	0.390	1		
ABSRET	190,780		1926-2005	41.98	28.13	64.76	0.075	0.025	0.010	-0.088	-0.014	1	

Table 2. Panel Regressions: Effective Cost

This table reports estimates for the intercept and slope coefficients from the panel regressions of Eq. (34) in which the dependent variable  $LAMBDA_{i,t}$  is  $EFFCOST_{i,t}$ , the average percentage effective cost for trades in firm  $i$ 's stocks on year  $t$ . Independent variables are  $ABSRET_{i,t}$ , firm  $i$ 's absolute annual return, in percentage times 100;  $PT_{i,t}$ , the weighted average of year-end  $t$ 's Active Share of all the mutual funds holding firm  $i$ 's stocks (see Eq. (33));  $ANALYST_{i,t}$ , the median number of EPS forecasts for stock  $i$  over year  $t$ ;  $MKT CAP_{i,t}$ , the market capitalization of firm  $i$  at year-end  $t$  (in millions). Estimates for the slope coefficients are reported in the same unit as  $EFFCOST_{i,t}$  for a one standard deviation shock to the corresponding independent variable (from Table 1). The superscripts "a," "b," and "c" denote statistical significance at the 1%, 5%, and 10% level, respectively, assessed with (heteroskedasticity) robust standard errors adjusted for firm-level clustering; the resulting t-statistics are reported below each estimate.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	0.92% <sup>a</sup>	0.87% <sup>a</sup>	0.55% <sup>a</sup>	0.76% <sup>a</sup>	0.53% <sup>a</sup>	0.75% <sup>a</sup>	0.56% <sup>a</sup>	0.72% <sup>a</sup>	0.75% <sup>a</sup>	0.71% <sup>a</sup>	0.56% <sup>a</sup>	0.76% <sup>a</sup>
	14.31	13.09	34.67	33.64	33.34	33.25	37.82	34.34	28.34	33.87	35.58	36.17
ABSRET	0.099% <sup>a</sup>	-0.009% <sup>a</sup>			0.034% <sup>a</sup>	0.008% <sup>a</sup>			0.036% <sup>a</sup>	0.009% <sup>a</sup>		
	15.4	-2.80			10.50	3.32			10.60	3.41		
PT			0.045% <sup>a</sup>	0.010% <sup>a</sup>	0.045% <sup>a</sup>	0.010% <sup>a</sup>	0.125% <sup>a</sup>	0.033% <sup>a</sup>	0.123% <sup>a</sup>	0.033% <sup>a</sup>	0.057% <sup>a</sup>	0.007% <sup>c</sup>
			7.76	2.65	7.74	2.68	12.20	4.75	12.13	4.72	9.43	1.89
PT*ANALYST							-0.079% <sup>a</sup>	-0.019% <sup>a</sup>	-0.078% <sup>a</sup>	-0.019% <sup>a</sup>		
							-13.82	-5.91	-13.72	-5.83		
PT*MKT CAP											-0.105% <sup>a</sup>	0.017% <sup>a</sup>
											-8.99	5.13
Year F.E.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm F.E.	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Firm-Year Obs	190,780	190,780	30,572	30,572	30,572	30,572	28,088	28,088	28,088	28,088	30,572	30,572
Overall R <sup>2</sup>	12.42%	6.64%	10.81%	9.98%	11.20%	10.16%	14.60%	12.20%	15.16%	12.47%	12.08	10.18%

Table 2. (Continued)

	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
Intercept	0.54% <sup>a</sup> 34.26	0.75% <sup>a</sup> 35.86	0.75% <sup>a</sup> 28.17	0.71% <sup>a</sup> 33.73	0.54% <sup>a</sup> 34.18	0.75% <sup>a</sup> 35.74	0.75% <sup>a</sup> 28.16	0.71% <sup>a</sup> 33.74	0.54% <sup>a</sup> 34.2	0.75% <sup>a</sup> 35.75
ABSRET	0.033% <sup>a</sup> 10.24	0.008% <sup>a</sup> 3.40	0.044% <sup>a</sup> 9.72	0.012% <sup>a</sup> 3.85	0.036% <sup>a</sup> 9.01	0.009% <sup>a</sup> 3.39	0.044% <sup>a</sup> 9.77	0.011% <sup>a</sup> 3.86	0.035% <sup>a</sup> 8.79	0.009% <sup>a</sup> 3.36
PT	0.057% <sup>a</sup> 9.39	0.007% <sup>c</sup> 1.94	0.134% <sup>a</sup> 12.21	0.037% <sup>a</sup> 4.82	0.060% <sup>a</sup> 8.82	0.009% <sup>b</sup> 2.02	0.138% <sup>a</sup> 12.29	0.037% <sup>a</sup> 4.74	0.061% <sup>a</sup> 8.92	0.009% <sup>b</sup> 2.00
PT*ANALYST			-0.079% <sup>a</sup> -13.80	-0.020% <sup>a</sup> -5.89			-0.084% <sup>a</sup> -13.16	-0.020% <sup>a</sup> -5.55		
PT*MKTCAP	-0.105% <sup>a</sup> -8.94	0.017% <sup>a</sup> 5.10			-0.105% <sup>a</sup> -8.93	0.017% <sup>a</sup> 5.07			-0.113% <sup>a</sup> -8.59	0.016% <sup>a</sup> 3.61
ABSRET*PT			-0.016% <sup>a</sup> -3.96	-0.005% -1.53	-0.006% -1.44	-0.003% -0.78	-0.022% <sup>a</sup> -4.47	-0.006% -1.31	-0.007% <sup>c</sup> -1.91	-0.003% -0.79
ABSRET*PT*ANALYST							0.010% <sup>b</sup> 2.51	0.001% 0.34		
ABSRET*PT*MKTCAP									0.015% <sup>b</sup> 2.42	0.001% 0.22
Year F.E.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm F.E.	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Firm-Year Obs	30,572	30,572	28,088	28,088	30,572	30,572	28,088	28,088	30,572	30,572
Overall R <sup>2</sup>	12.43%	9.63%	12.83%	12.53%	12.44%	10.94%	15.25%	12.53%	12.45%	9.64%

Table 3. Panel Regressions: Illiquidity

This table reports estimates for the intercept and slope coefficients from the panel regressions of Eq. (34) in which the dependent variable  $LAMBDA_{i,t}$  is  $ILLIQ_{i,t}$ , the annual average of the ratio of firm  $i$ 's daily absolute stock returns and its corresponding nonzero daily dollar volume, in bps per \$10,000 of trading volume. Independent variables are  $ABSRET_{i,t}$ , firm  $i$ 's absolute annual return, in percentage times 100;  $PT_{i,t}$ , the weighted average of year-end  $t$ 's Active Share of all the mutual funds holding firm  $i$ 's stocks (see Eq. (33));  $ANALYST_{i,t}$ , the median number of EPS forecasts for stock  $i$  over year  $t$ ;  $MKTCAP_{i,t}$ , the market capitalization of firm  $i$  at year-end  $t$  (in millions). Estimates for the slope coefficients are reported in the same unit as  $ILLIQ_{i,t}$  for a one standard deviation shock to the corresponding independent variable (from Table 1). The superscripts "a," "b," and "c" denote statistical significance at the 1%, 5%, and 10% level, respectively, assessed with (heteroskedasticity) robust standard errors adjusted for firm-level clustering; the resulting t-statistics are reported below each estimate.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Intercept	14.57 <sup>b</sup>	-56.05 <sup>a</sup>	0.10 <sup>a</sup>	0.54 <sup>a</sup>	0.10 <sup>a</sup>	0.54 <sup>a</sup>	0.12 <sup>a</sup>	0.57 <sup>a</sup>	0.68 <sup>a</sup>	0.57 <sup>a</sup>	0.11 <sup>a</sup>	0.55 <sup>a</sup>
	2.11	-4.40	2.73	3.20	2.86	3.24	7.32	7.77	8.42	7.71	3.04	3.59
ABSRET	2.831 <sup>a</sup>	1.347 <sup>b</sup>			-0.012	-0.003			-0.006	-0.002		
	4.87	2.30			-1.39	-0.36			-1.49	-0.60		
PT			0.081 <sup>a</sup>	0.033 <sup>a</sup>	0.081 <sup>a</sup>	0.032 <sup>a</sup>	0.155 <sup>a</sup>	0.064 <sup>a</sup>	0.155 <sup>a</sup>	0.064 <sup>a</sup>	0.096 <sup>a</sup>	0.030 <sup>b</sup>
			2.60	2.99	2.60	2.99	8.08	3.05	8.08	3.05	2.98	2.58
PT*ANALYST							-0.104 <sup>a</sup>	-0.028 <sup>a</sup>	-0.104 <sup>a</sup>	-0.028 <sup>a</sup>		
							-10.58	-3.23	-10.56	-3.24		
PT*MKTCAP											-0.134 <sup>a</sup>	0.012 <sup>b</sup>
											-6.60	2.13
Year F.E.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm F.E.	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Firm-Year Obs	190,780	190,780	30,572	30,572	30,572	30,572	28,088	28,088	28,088	28,088	30,572	30,572
Overall R <sup>2</sup>		2.16%	1.71%	1.44%	1.71%	1.44%	5.62%	5.02%	5.62%	5.03%	1.83%	1.38%

Table 3. (Continued)

	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
Intercept	0.11 <sup>a</sup>	0.55 <sup>a</sup>	0.68 <sup>a</sup>	0.57 <sup>a</sup>	0.11 <sup>a</sup>	0.55 <sup>a</sup>	0.68 <sup>a</sup>	0.57 <sup>a</sup>	0.11 <sup>a</sup>	0.55 <sup>a</sup>
	3.19	3.64	8.37	7.69	3.24	3.63	8.37	7.68	3.24	3.63
ABSRET	-0.014	-0.003	0.006	0.002	-0.013	-0.003	0.006	0.002	-0.013	-0.002
	-1.57	-0.42	1.23	0.43	-1.33	-0.32	1.29	0.49	-1.36	-0.30
PT	0.096 <sup>a</sup>	0.030 <sup>b</sup>	0.171 <sup>a</sup>	0.070 <sup>a</sup>	0.097 <sup>a</sup>	0.031 <sup>b</sup>	0.169 <sup>a</sup>	0.068 <sup>a</sup>	0.097 <sup>a</sup>	0.031 <sup>b</sup>
	2.98	2.57	7.50	2.98	3.53	2.12	6.99	2.83	3.56	2.04
PT*ANALYST			-0.106 <sup>a</sup>	-0.029 <sup>a</sup>			-0.104 <sup>a</sup>	-0.027 <sup>a</sup>		
			-10.47	-3.22			-8.60	-2.77		
PT*MKTCAP	-0.134 <sup>a</sup>	0.012 <sup>b</sup>			-0.134 <sup>a</sup>	0.012 <sup>b</sup>			-0.136 <sup>a</sup>	0.013
	-6.61	2.15			-6.64	2.11			-6.18	1.51
ABSRET*PT			-0.023 <sup>b</sup>	-0.007	-0.001	-0.001	-0.020	-0.005	-0.002	-0.001
			-2.48	-1.14	-0.07	-0.09	-1.54	-0.53	-0.09	-0.06
ABSRET*PT*ANALYST							-0.004	-0.004		
							-0.53	-0.85		
ABSRET*PT*MKTCAP									0.004	-0.002
									0.24	-0.18
Year F.E.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm F.E.	No	Yes	No	Yes	No	Yes	No	Yes	No	Yes
Firm-Year Obs	30,572	30,572	28,088	28,088	30,572	30,572	28,088	28,088	30,572	30,572
Overall R <sup>2</sup>	1.83%	1.38%	5.66%	5.06%	1.83%	1.38%	5.66%	5.07%	1.83%	1.38%