THE BOND RATING EXPERIENCE OF SOVEREIGN ISSUERS : AN APPLICATION OF MARKOV CHAIN ANALYSIS

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1. The Bond Rating experience

Most of the recent financial literature is mainly involved in the definition and measurement of different kinds of financial risks, from those originated by movements in interest rates and currencies to broad market and systemic fluctuations.

However, much less attention has been paid to a deep analysis of credit risk.

Complex and often incompatible international regulations, together with the difficulties related to the direct observation of the price of credit risk in the credit spreads quoted by the market, prevented more effective progresses in the theory and practice of credit risk analysis.

Nonetheless, major financial institutions are currently devoting more and more attention and resources to a more complete understanding of the evolution through time of the credit quality of private and public issuers of notes and bonds in the fixed income market.

"True" fixed income securities markets are becoming increasingly efficient, as a result of increasing competition among market participants and deeper knowledge of the structure of the market itself. Hence, as time goes by, bid-ask spreads, margins and profit opportunities available for brokers, intermediaries and speculators are tightening.

On the other side, the assessment of the current and future credit quality of an issuer is still a dramatically uncertain, but also fundamental step in determining the correct market price of non-default free securities.

The creditworthiness of an issuer, whether private or public, domestic or sovereign, does significantly contribute to the relative and absolute price paid for the funds borrowed. Market prices of credit-securities tend to result from the interaction of demand and supply factors with more fundamental evaluations of the relative risk the market itself attributes to different issuers.

The accurate pricing of total return and default swaps, as well as of other credit derivatives depends critically upon statistical distributions of future potential credit-quality drifts.

The ability of counterparts in structured transactions, like interest rates swaps, caps and floors and swaptions, to meet their contractual payments, and thus the price itself of these transactions is strictly dependent on the credit quality of the parties involved.

Finally, for a "total return"-oriented fixed income investor, the movements of credit qualities over time represent a critical factor in understanding the credit risk characteristics of his or her portfolio and the effects of perspective market fluctuations and in affecting his or her decisions regarding the composition of their portfolios.

The correct identification of the likelihood that a change in credit quality will occur, i.e. of a complete transition probability matrix, is a first necessary step for understanding the distribution of the future value of a credit-sensitive fixed income security.

One of the most important indicators of an issuer's credit quality is the bond rating assigned to its outstanding, publicly traded debt by independent rating agencies.

Bond ratings are usually first assigned to public debt at the time of issuance and then periodically reviewed by the rating companies. A rating drift reflects the agency's assessment that the issuer's credit quality has improved (upgrade) or deteriorated (downgrade). As a result, in the proximity of the date of the change, fluctuations in the price of the outstanding rated securities tend to reflect the decrease or the increase in the yield required by investors for securities of comparable risk.

This paper focuses its attention on the foreign currency credit risk related to sovereign issuers as provided by Standard & Poor's (S&P) in its monthly reports.

Standard Statistics, which merged with Poor's Publishing to form S&P in 1941, first assigned credit ratings to sovereigns during World War I. In most cases, however, the ratings were suspended after 1940. The postwar history of S&P sovereign ratings began two decades later, in 1960, when Canada became the second rated sovereign alongside the United States.

Thereafter, the number of rated sovereigns grew steadily, reaching 14 in 1981 (all in the "AAA" category) and 31 in 1990 (with ratings ranging between "AAA" and "B+").

With S&P introduction of sovereign local currency credit ratings in 1992, the number of rated sovereigns accelerated, to 49 at the end of 1994, to 62 at the end of 1996, and to 72 as of November of 1997.

The main rating distinction is between investment-grade securities – those rated BBB and above – and speculative securities – those rated BB and below. The lowest rating currently attributable to a sovereign issuer is B-. Table 1 summarizes the official interpretation S&P gives of its rating categories.

The purpose of this paper is to describe the rating drift experience of countries through the adoption of a well-specified Stochastic Markov Chain model.

It is opinion of the authors that a fully developed fundamentals-based econometric model would not be effective in capturing the ultimate, and somehow intangible factors that explain the rating process and attribution.

Those factors are embedded in the observed data. Hence, they come to be implicitly included in a Stochastic analysis of the process itself.

In the next section, transition probability matrixes will be derived from the actual data. Most of the studies available on similar issues estimate the probability that a change in credit quality will occur through observed proportions, i.e. conclude their analysis at this stage.

Interpretation	Standard & Poor’s
Investment-Grade Ratings	
Highest Quality	**AAA**
High Quality	**AA+** **AA** **AA-**
Strong Payment Capacity	**A+** **A** **A-**
Adequate Payment Capacity	**BBB+** **BBB** **BBB-**
Speculative-Grade Ratings	
Likely to Fulfill Obligations, ongoing uncertainty	**BB+** **BB** **BB-**
High-Risk Obligations	**B+** **B** **B-**

Table 1:Long-Term Debt Rating Symbols, Standard & Poor’s.

Section #3 will develop a Markovian interpretation of the bond-rating experience and propose the adoption of specific Stochastic processes to describe it.

Section #4 will be devoted to estimate the one-step transition matrixes consistent with the models adopted and to test some of the assumptions on which those models rely.

In Section #5 we will amend the discrete-time Markov Chain model to include some peculiar characteristics of the bond rating drift phenomenon.

In the final sections, we will interpret the results of our research and propose potential fields of application.

2. Observed Transition Matrixes

The available data, on a monthly basis, cover an interval of almost 23 years, from January 1975 to November 1997. Few of the countries under observation have a full credit history, i.e. cover the entire sample interval. The greatest majority of them received a rating in the last six years.

As a consequence of this limitation, we looked at the rating experience of each country as independent from the initial year when the experience itself started.

The observed transition probabilities Pij(t) for the t-step matrix have been derived as a simple proportion of the number of countries in state j at time t and in state i at time zero over the total number of countries rated for a period of t years and in state i at time zero.

Transition probability matrixes have been derived also for a three-state world in which, given a certain starting rating, a country can be simply upgraded, downgraded, or unchanged at the end of the t-th year.

The following matrixes describe the observed rating drift experience one, three, seven and ten years after the initial rating.

1 YEAR MATRIX

	UPGRADE	STEADY	DOWNGRADE
AAA	0%	100%	0%
AA+	0%	100%	0%
AA	0%	100%	0%
AA-	0%	100%	0%
A+	33%	67%	0%
A	0%	100%	0%
A-	0%	100%	0%
BBB+	0%	100%	0%
BBB	14%	71%	14%
BBB-	0%	100%	0%
BB+	0%	100%	0%
BB	67%	33%	0%
BB-	14%	86%	0%
B+	50%	50%	0%
B	100%	0%	0%
B-	0%	100%	0%

1 YEAR MATRIX

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	**100%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA+	0%	**100%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA	0%	0%	**100%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA-	0%	0%	0%	**100%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	0%	0%	0%	33%	**67%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	0%	0%	0%	0%	**100%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A-	0%	0%	0%	0%	0%	0%	**100%**	0%	0%	0%	0%	0%	0%	0%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	**100%**	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	0%	0%	0%	0%	0%	0%	14%	**71%**	0%	14%	0%	0%	0%	0%	0%
BBB-	0%	0%	0%	0%	0%	0%	0%	0%	0%	**100%**	0%	0%	0%	0%	0%	0%
BB+	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**100%**	0%	0%	0%	0%	0%
BB	0%	0%	0%	0%	0%	0%	0%	0%	0%	33%	33%	**33%**	0%	0%	0%	0%
BB-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	14%	0%	**86%**	0%	0%	0%
B+	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	50%	**50%**	0%	0%
B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%	**0%**	0%
B-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**100%**

3 YEAR MATRIX

	UPGRADE	STEADY	DOWNGRADE
AAA	0%	94%	6%
AA+	0%	100%	0%
AA	67%	33%	0%
AA-	0%	100%	0%
A+	33%	33%	33%
A	67%	33%	0%
A-	50%	50%	0%
BBB+	0%	0%	0%
BBB	33%	17%	50%
BBB-	33%	67%	0%
BB+	33%	33%	33%
BB	100%	0%	0%
BB-	33%	67%	0%
B+	0%	100%	0%
B	100%	0%	0%
B-	0%	0%	0%

3 YEAR MATRIX

AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
94%	6%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0%	**100%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0%	67%	**33%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0%	0%	0%	**100%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0%	0%	0%	33%	**33%**	33%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0%	0%	0%	0%	67%	**33%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
0%	0%	0%	0%	0%	50%	**50%**	0%	0%	0%	0%	0%	0%	0%	0%	0%
0%	0%	0%	0%	0%	0%	0%	**0%**	0%	0%	0%	0%	0%	0%	0%	0%
0%	0%	0%	0%	0%	17%	17%	0%	**17%**	17%	17%	0%	0%	17%	0%	0%
0%	0%	0%	0%	0%	0%	0%	0%	33%	**67%**	0%	0%	0%	0%	0%	0%
0%	0%	0%	0%	0%	0%	0%	0%	0%	33%	**33%**	33%	0%	0%	0%	0%
0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%	**0%**	0%	0%	0%	0%
0%	0%	0%	0%	0%	0%	0%	0%	0%	33%	0%	0%	**67%**	0%	0%	0%
0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**100%**	0%	0%
0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%	0%	**0%**	0%
0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**0%**

7 YEAR MATRIX

	UPGRADE	STEADY	DOWNGRADE
AAA	0%	81%	19%
AA+	0%	50%	50%
AA	67%	33%	0%
AA-	0%	0%	0%
A+	67%	0%	33%
A	100%	0%	0%
A-	100%	0%	0%
BBB+	0%	0%	0%
BBB	0%	0%	100%
BBB-	100%	0%	0%
BB+	0%	0%	0%
BB	0%	0%	0%
BB-	0%	0%	0%
B+	0%	0%	0%
B	0%	0%	0%
B-	0%	0%	0%

7 YEAR MATRIX

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	**81%**	6%	6%	0%	0%	0%	0%	0%	0%	0%	0%	6%	0%	0%	0%	0%
AA+	0%	**50%**	50%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA	33%	33%	**33%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA-	0%	0%	0%	**0%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	0%	0%	33%	33%	**0%**	33%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	0%	0%	50%	50%	**0%**	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A-	0%	0%	0%	0%	50%	50%	**0%**	0%	0%	0%	0%	0%	0%	0%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	**0%**	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	0%	0%	0%	0%	0%	0%	0%	**0%**	50%	50%	0%	0%	0%	0%	0%
BBB-	0%	0%	0%	0%	0%	0%	100%	0%	0%	**0%**	0%	0%	0%	0%	0%	0%
BB+	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**0%**	0%	0%	0%	0%	0%
BB	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**0%**	0%	0%	0%	0%
BB-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**0%**	0%	0%	0%
B+	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**0%**	0%	0%
B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**0%**	0%
B-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**0%**

Note that the presence of total probabilities summing up to zero in some of the rows of the three-state matrixes and of the complete-states matrixes is simply a consequence of the fact that for the ratings corresponding to those rows none of the underlying countries has such a long rating history.

Two main facts can be observed from the analysis of the “effective” transition matrixes described above:

- AAA ratings tend to perpetuate themselves over time;
- Low Investment-grade and Speculative-grade ratings are more likely to be upgraded than downgraded over time. This phenomenon may be interpreted as a result of independent rating agencies adopting a generally more conservative approach in assessing the credit quality of potentially low investment-grade countries.

3. The Markov Processes

The simple adoption of observed transition probability matrixes as a structure for a distribution of future potential rating drifts is not fully satisfactory.

This section analyzes whether the long and short-term tendencies of credit rating drift for countries observed in the matrixes above are empirically compatible with a series of specific stochastic processes. The estimation of a one-step transition matrix from a series of observed one-step transition matrixes will permit us to calculate n-step matrixes, i.e. n-step transition probabilities by applying the properties of the assumed model and then to compare those probabilities with those resulting from the sample, and to evaluate the “fit” of the model.

A stochastic process may be described as a collection of random variables and a probability distribution for the values of each of them. In the framework of our analysis, the rating of a country at a specific period t is a random variable; thus, the sequence of ratings for a country, as a function of an indicator n, is a stochastic process.

We will adopt a specific stochastic process in describing statistically the rating drift experience of sovereign issuers, i.e. a discrete time Markov Chain.

The stationary discrete-time Markov Chain (MC) process is a stochastic model that, given some stringent assumptions, is completely determined by an initial distribution for the states and a one-step transition matrix.

The hypothesis underlying the MC process are the following:

- The population is homogeneous;
- The Markov Property: future transitions are uniquely determined by the present state and not by the way in which that present state has been reached, i.e. are completely independent from past transitions;
- Time-Homogeneity: one-step transition matrixes are assumed to be constant over time.

Empirical processes like occupational mobility, employment-unemployment status and, for our purposes, bond rating drift history can be viewed as Markov Chains.

Given the characteristics of rating attribution and credit-risk assessment over time, it is natural to believe that the past rating history, i.e. the pattern that brings to a current rating valuation, does not have explanatory power in predicting the future rating drift from the present grade.

It is also natural to think that, for example,

$$P\left(R_{t+s} = AAA / R_s = AA+\right) = P\left(R_t = AAA / R_0 = AA+\right)$$

Thus, it seems intuitive to consider the rating process as time-homogeneous, i.e. dependent on the relative time-distance among present and future values of the process, and not on where those variables are located.

The adoption of discrete time MC was a necessary step, given the way the available information has been organized, as an artificial sequence of year-by-year rating conditions rather than a sequence of months spent in each of the states.

The state-space S of this process is a countable set of 16 states (AAA, AA+, AA, AA-, A+, A, A-, BBB+, BBB, BBB-, BB+, BB, BB-, B+, B, B-). Pij is the one-step probability for the process to be in state j after one "period" (one-year), given that in the preceding period the process was in state i.

4. Estimation and test

We define Rn as the rating of a country at time n. {Rn, n ≥0} is our discrete-time MC on S. We are interested in the estimation of the one-step transition matrix P of {Rn, n ≥0}based on a sample of 72 countries, where for the l'th country we observe:

$$\left\{R^l(0), R^l(\Delta), R^l(2\Delta), \ldots., R^l(n\Delta)\right\}$$

Where Δ = 1 year is the time-interval between two rating records and $1 \leq l \leq 72$.

A paper by Anderson and Goodman[1]suggests the adoption of a Maximum Likelihood estimator for the one-step probability matrix:

$$\hat{P}_{ij} = \left| \frac{n_{ij}}{n_i} \right|$$

Where (nij) is the total number of i-to-j transitions in the sample and (ni) is the total number of visits to state i. The proposed estimator is simply a sort of total proportion of i-to-j transitions over the entire sample.

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[1] T.W. Anderson and Leo A. Goodman, *Statistical Inference about Markov Chains*, Annuals of Mathematical Statistics, 1957, pp. 89-110.

What follows is the estimated one-step probability matrix for {Rn, n ≥0}:

MAXIMUM LIKELIHOOD ESTIMATOR OF P, ONE-STEP TRANSITION MATRIX OF A TIME HOMOGENEOUS DISCRETE MC

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	77%	9%	7%	2%	0%	0%	0%	0%	0%	0%	0%	3%	0%	2%	0%	0%
AA+	0%	78%	22%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA	11%	41%	48%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA-	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	0%	0%	11%	33%	26%	26%	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	0%	0%	24%	29%	48%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A-	0%	0%	0%	0%	25%	38%	31%	0%	6%	0%	0%	0%	0%	0%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	0%	0%	0%	0%	8%	8%	8%	22%	22%	19%	0%	0%	6%	6%	0%
BBB-	0%	0%	0%	0%	0%	0%	16%	11%	11%	63%	0%	0%	0%	0%	0%	0%
BB+	0%	0%	0%	0%	0%	0%	0%	0%	0%	21%	57%	21%	0%	0%	0%	0%
BB	0%	0%	0%	0%	0%	0%	0%	0%	0%	33%	50%	17%	0%	0%	0%	0%
BB-	0%	0%	0%	0%	0%	0%	0%	0%	0%	13%	13%	6%	69%	0%	0%	0%
B+	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	40%	60%	0%	0%
B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	33%	67%	0%	0%
B-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	50%	50%

The application of the Chapman-Kolmogorov equations permits us to estimate n-th step probability matrixes, as a result of the one-step matrix raised to the power of n:

$$P^{(n)} = P^n$$

If n is big, it is possible to find the estimated long-term stationary transition matrix for Rn, n ≥0:

$$\lim_{n \to \infty} P^{(n)} = P^{(\infty)}$$

In the following pages, selected n-th step estimated probability matrixes and the derived stationary matrix are presented.

MAXIMUM LIKELIHOOD ESTIMATOR OF P3, THREE-STEP TRANSITION MATRIX OF A TIME HOMOGENEOUS DISCRETE MC

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	48%	23%	13%	5%	0%	0%	0%	0%	0%	2%	2%	3%	2%	3%	0%	0%
AA+	5%	66%	29%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA	14%	55%	28%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA-	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	2%	7%	7%	58%	10%	14%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	1%	4%	59%	14%	20%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A-	0%	1%	4%	34%	20%	28%	5%	1%	2%	2%	1%	0%	0%	1%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	0%	0%	6%	6%	9%	7%	16%	5%	22%	13%	4%	7%	6%	0%	0%
BBB-	0%	0%	0%	3%	7%	11%	13%	23%	8%	29%	3%	0%	0%	1%	1%	0%
BB+	0%	0%	0%	0%	1%	1%	6%	6%	4%	36%	33%	12%	0%	0%	0%	0%
BB	0%	0%	0%	0%	1%	2%	8%	8%	5%	37%	29%	10%	0%	0%	0%	0%
BB-	0%	0%	0%	0%	0%	1%	4%	4%	3%	26%	21%	8%	32%	0%	0%	0%
B+	0%	0%	0%	0%	0%	0%	1%	1%	1%	12%	11%	5%	50%	22%	0%	0%
B	0%	0%	0%	0%	0%	0%	1%	0%	0%	10%	10%	4%	50%	24%	0%	0%
B-	0%	0%	0%	0%	0%	0%	0%	0%	0%	2%	2%	1%	33%	37%	13%	13%

MAXIMUM LIKELIHOOD ESTIMATOR OF P7, SEVEN-STEP TRANSITION MATRIX OF A TIME HOMOGENEOUS DISCRETE MC

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	22%	35%	17%	8%	0%	1%	1%	1%	1%	4%	3%	2%	3%	2%	0%	0%
AA+	11%	59%	28%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA	14%	55%	26%	2%	0%	0%	0%	0%	0%	1%	1%	1%	1%	1%	0%	0%
AA-	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	3%	10%	5%	75%	2%	3%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A	1%	5%	3%	83%	3%	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A-	1%	6%	4%	69%	5%	7%	1%	2%	0%	2%	1%	0%	1%	0%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	1%	1%	21%	5%	7%	5%	25%	3%	15%	8%	3%	5%	2%	0%	0%
BBB-	0%	2%	2%	24%	6%	9%	4%	33%	2%	10%	3%	1%	2%	1%	0%	0%
BB+	0%	0%	1%	8%	5%	8%	8%	21%	5%	24%	13%	4%	1%	1%	0%	0%
BB	0%	0%	1%	10%	6%	9%	7%	23%	4%	22%	12%	4%	1%	1%	0%	0%
BB-	0%	0%	1%	6%	4%	6%	7%	17%	4%	26%	15%	5%	8%	1%	0%	0%
B+	0%	0%	0%	2%	2%	3%	6%	9%	4%	26%	18%	7%	21%	3%	0%	0%
B	0%	0%	0%	1%	2%	3%	6%	9%	3%	26%	18%	7%	22%	3%	0%	0%
B-	0%	0%	0%	0%	1%	1%	3%	4%	2%	20%	16%	6%	34%	10%	1%	1%

MAXIMUM LIKELIHOOD ESTIMATOR Of the long-term TRANSITION MATRIX OF A TIME HOMOGENEOUS DISCRETE MC																
	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	0%	0%	0%	69%	0%	0%	0%	31%	0%	0%	0%	0%	0%	0%	0%	0%
AA+	0%	0%	0%	69%	0%	0%	0%	31%	0%	0%	0%	0%	0%	0%	0%	0%
AA	0%	0%	0%	69%	0%	0%	0%	31%	0%	0%	0%	0%	0%	0%	0%	0%
AA-	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	0%	0%	0%	94%	0%	0%	0%	6%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	0%	0%	97%	0%	0%	0%	3%	0%	0%	0%	0%	0%	0%	0%	0%
A-	0%	0%	0%	92%	0%	0%	0%	8%	0%	0%	0%	0%	0%	0%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	0%	0%	58%	0%	0%	0%	42%	0%	0%	0%	0%	0%	0%	0%	0%
BBB-	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
BB+	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
BB	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
BB-	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
B+	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
B	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
B-	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%

The one-step probability matrix resulting from the Anderson-Goodman estimation procedure is reducible finite. AA- and BBB+ are absorption states. Hence, for the number of positive recurrent classes being at least equal to two, there are infinitely many stationary distributions, i.e. the limiting behavior of the chain depends on the initial distribution[2].

Is the time-homogeneity assumption compatible with the data? Anderson and Goodman also propose a chi-squared test for the following homogeneity hypothesis:

$$H_0 = P\left(\left(k-1\right)\Delta, k\Delta\right) = P \qquad k = 1,2,\ldots.,n$$

For the chi-squared test, for (m-1)*(n-1) degrees of freedom, we calculate:

$$\chi_i^2 = \sum_{t,j} \left[n_i\left(t-1\right)\cdot\left(\hat{P}_{ij}\left(t\right)-\hat{p}_{ij}\right)\right]^2 \Big/ \hat{p}_{ij}$$

[2] This is also evident from the observation of the long-term transition matrix above.

Where:

$$\hat{p}_{ij}(t) = n_{ij}(t) / n_i(t-1)$$

Nij(t) is the number of countries in state i at (t-1) and in j at (t), ni(t) is the number of countries in state i at time (t-1) and P^ij is our MLE estimator.

The result of the test is that:

$$\sum_{i=1}^{16} \chi_i^2 = 630.936 \quad \chi_{315}^2 = 376.32$$

i.e. that the hypothesis of time-homogeneity cannot be statistically rejected.

The next necessary step is to test the Markov property of the model, i.e. to ask whether the discrete-time MC fits the data, thus whether the estimated n'th step probability matrixes fit the observed n'th step probabilities. Formal testing procedures of this hypothesis are not currently available. As a consequence, an informal testing of the Markovian Hypothesis can be implemented through the comparison of the diagonal entries of the n'th step observed probability matrixes with the corresponding diagonal entries on the estimated MC n'th step matrixes.

"Small" differences will lead us to an acceptance of the Markovian Hypothesis.

Very well known studies in the literature[3] observe positive "diagonal" spreads for matrixes of order ≥ 2, thus suggesting that the discrete time MC model tends to overestimate the probability of movement, i.e. that the data give less movement than what the model predicts.

To test the presence of this empirical regularity, we calculate the following average diagonal spread term:

$$\frac{\sum_{i=1}^{16} \left[\tilde{P}_{ii}(0, m\Delta) - \hat{P}_{ii}(0, m\Delta) \right]}{16}$$

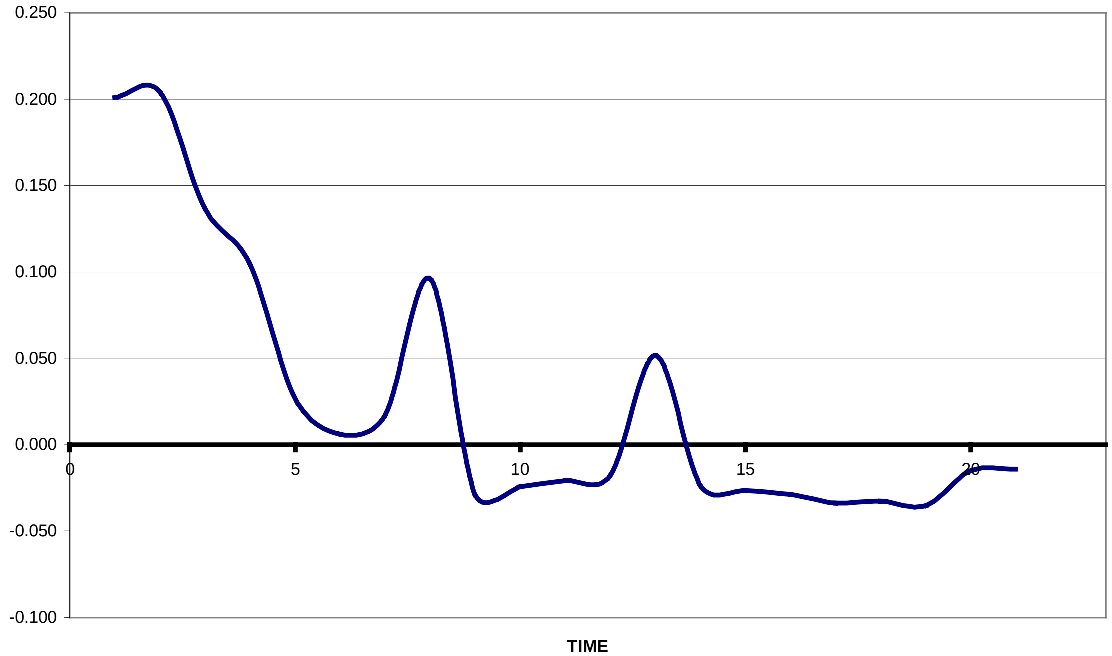
And then we plot this indicator for m going from 2 to 22 (years ahead), i.e. from 2-step to 22-step transition matrix.

---

[3] Blumen, Kogan and McCharty, *The industrial mobility of Labor as a Probability Process*, Cornell University Press, 1955.

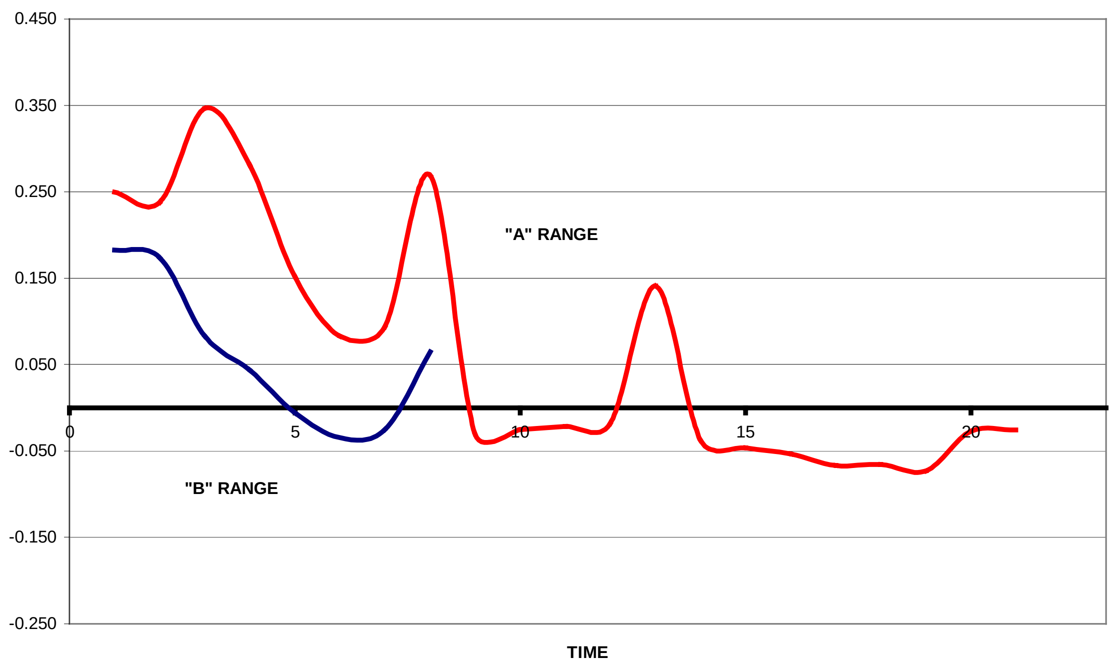
The following chart shows the pattern of the diagonal spread over successive n’th step matrixes,

EMPIRICAL AVERAGE DIAGONAL SPREAD



and for Investment and Speculative-grade countries:

EMPIRICAL AVERAGE DIAGONAL SPREAD by category



It is evident that the over-estimation of movement-probabilities deriving from the adoption of the discrete time MC model is positive but declining. Limiting the analysis of the data to a smaller range of n'th step diagonal average spreads, thus excluding long-term n'th step matrixes, estimated on few data, the results are still somehow puzzling.

The charts above do not strongly push toward a rejection of the MC as a model for the description of the rating-drift experience of a country. However, the better performance of the model for B-rated sovereign issuers, i.e. for issuers characterized by a stronger tendency to movement, suggests that the initial hypothesis of homogeneous population has to be relaxed and that a possible amendment to the model we adopted in this section may improve the final fit.

5. The Mover-Stayer Model

A careful analysis of the available data strongly suggests that some initial conditions tend to perpetuate themselves over time, i.e. that countries starting with specific ratings tend to "stay" in that initial position over the entire sample.

90 % of the countries rated originally as AAA kept their highest-credit quality assessment up to the last observed period.

Hence, we propose the adoption of a discrete time MC model that distinguishes among two different categories of countries, Stayers and Movers.

Stayers are countries that have never moved from their initial rating over the sample time-frame.

Movers are countries that moved or are likely to move over the sample time-frame.

Naturally, just countries that have a very long rating history and whose rating history has been stationary over the entire time-frame are considered to be truly stayers in the context of our amended model. For example, France has kept its AAA rating from 1975 to 1997; thus, we will consider it as a stayer. On the contrary, Pakistan maintained its original BB+ rating from 1994 to the end of 1997, but we will still consider it as a potential mover, since the corresponding time-interval of 4 years is not significant, as a portion of the entire time-frame.

We compute the estimated one-step mover-transition matrix as the solution of the following equation:

$$\hat{P}_{ij} = \begin{cases} m_{ij} \cdot (1 - s_i) & \text{if } i \neq j \\ m_{ii} \cdot (1 - s_i) + s_i & \text{if } i = j \end{cases}$$

Where mij is the one-step probability, for a mover, to evolve from state i to state j. Si is the proportion of stayers among countries whose original rating was i.

Hence, the resulting model, in matrix form, is:

$$P = S + (I - S) \cdot M$$

$$P^{(n)} = S + (I - S) \cdot M^n$$

Given the fact that the time-frame may be considered "large", the maximum likelihood estimator of Si is simply the fraction of individuals who stay continuously in state i.

This estimator is intuitively appealing when the time-frame is large, since it is then reasonable to assume that all countries that stay continuously in their initial states are stayers.

A simple observation of the data suggests that just originally AAA-rated countries satisfy the requirements for a country to be a true stayer. Thus, the estimated Stayer-matrix in our amended model will have a unique non-zero term, 13/18, on the first cell on the left.

The estimating Mover Matrix and the resulting matrixes for three, seven and long-term step transition probabilities follow:

MOVER MATRIX M1

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	18%	33%	24%	7%	0%	0%	0%	0%	0%	0%	0%	10%	0%	6%	1%	0%
AA+	0%	78%	22%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA	11%	41%	48%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA-	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	0%	0%	11%	33%	26%	26%	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	0%	0%	24%	29%	48%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A-	0%	0%	0%	0%	25%	38%	31%	0%	6%	0%	0%	0%	0%	0%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	0%	0%	0%	0%	8%	8%	8%	22%	22%	19%	0%	0%	6%	6%	0%
BBB-	0%	0%	0%	0%	0%	0%	16%	11%	11%	63%	0%	0%	0%	0%	0%	0%
BB+	0%	0%	0%	0%	0%	0%	0%	0%	0%	21%	57%	21%	0%	0%	0%	0%
BB	0%	0%	0%	0%	0%	0%	0%	0%	0%	33%	50%	17%	0%	0%	0%	0%
BB-	0%	0%	0%	0%	0%	0%	0%	0%	0%	13%	13%	6%	69%	0%	0%	0%
B+	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	40%	60%	0%	0%
B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	33%	67%	0%	0%
B-	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	50%	50%

MOVER-STAYER ESTIMATOR OF P 3 — 3 STEP TRANSITION MATRIX OF A TIME HOMOGENEOUS DISCRETE MC

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	73%	12%	6%	2%	0%	0%	0%	0%	0%	1%	1%	1%	1%	1%	0%	0%
AA+	4%	66%	30%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
AA	5%	59%	31%	1%	0%	0%	0%	0%	0%	0%	1%	1%	0%	1%	0%	0%
AA-	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	1%	7%	7%	58%	10%	14%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	1%	4%	59%	14%	20%	1%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A-	0%	1%	4%	34%	20%	28%	5%	1%	2%	2%	1%	0%	0%	1%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	0%	0%	6%	6%	9%	7%	16%	5%	22%	13%	4%	7%	6%	0%	0%
BBB-	0%	0%	0%	3%	7%	11%	13%	23%	8%	29%	3%	0%	0%	1%	1%	0%
BB+	0%	0%	0%	0%	1%	1%	6%	6%	4%	36%	33%	12%	0%	0%	0%	0%
BB	0%	0%	0%	0%	1%	2%	8%	8%	5%	37%	29%	10%	0%	0%	0%	0%
BB-	0%	0%	0%	0%	0%	1%	4%	4%	3%	26%	21%	8%	32%	0%	0%	0%
B+	0%	0%	0%	0%	0%	0%	1%	1%	1%	12%	11%	5%	50%	22%	0%	0%
B	0%	0%	0%	0%	0%	0%	1%	0%	0%	10%	10%	4%	50%	24%	0%	0%
B-	0%	0%	0%	0%	0%	0%	0%	0%	0%	2%	2%	1%	33%	37%	13%	13%

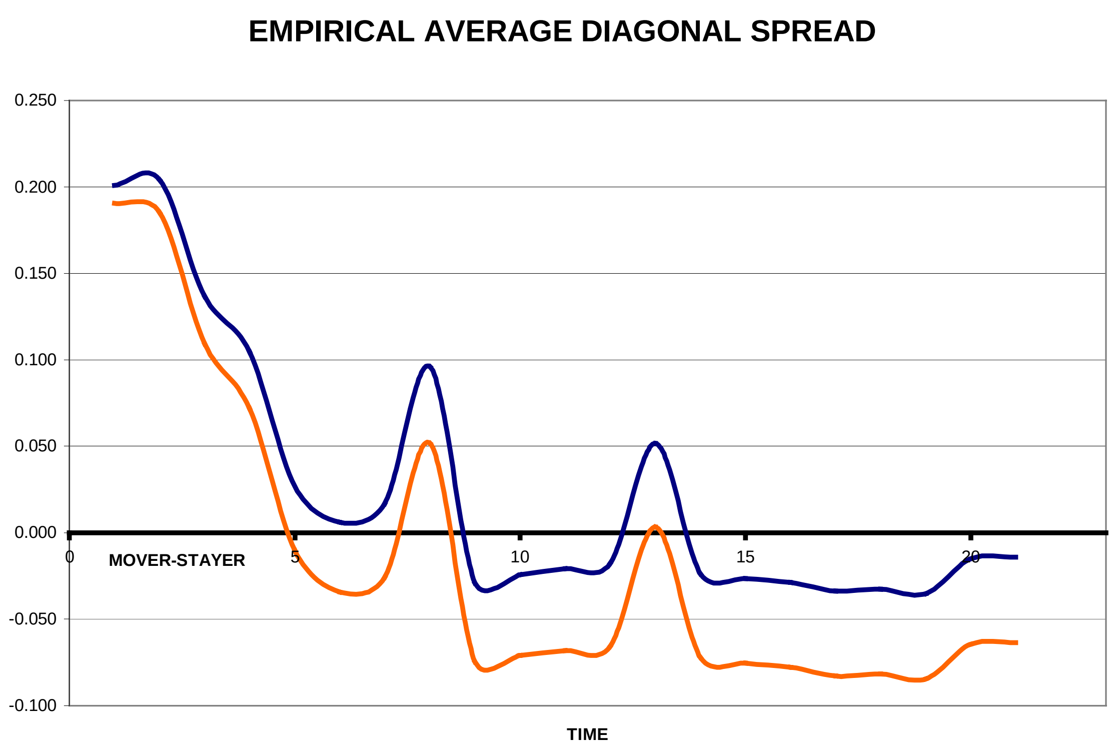
MOVER-STAYER ESTIMATOR OF P 7 — 7 STEP TRANSITION MATRIX OF A TIME HOMOGENEOUS DISCRETE MC

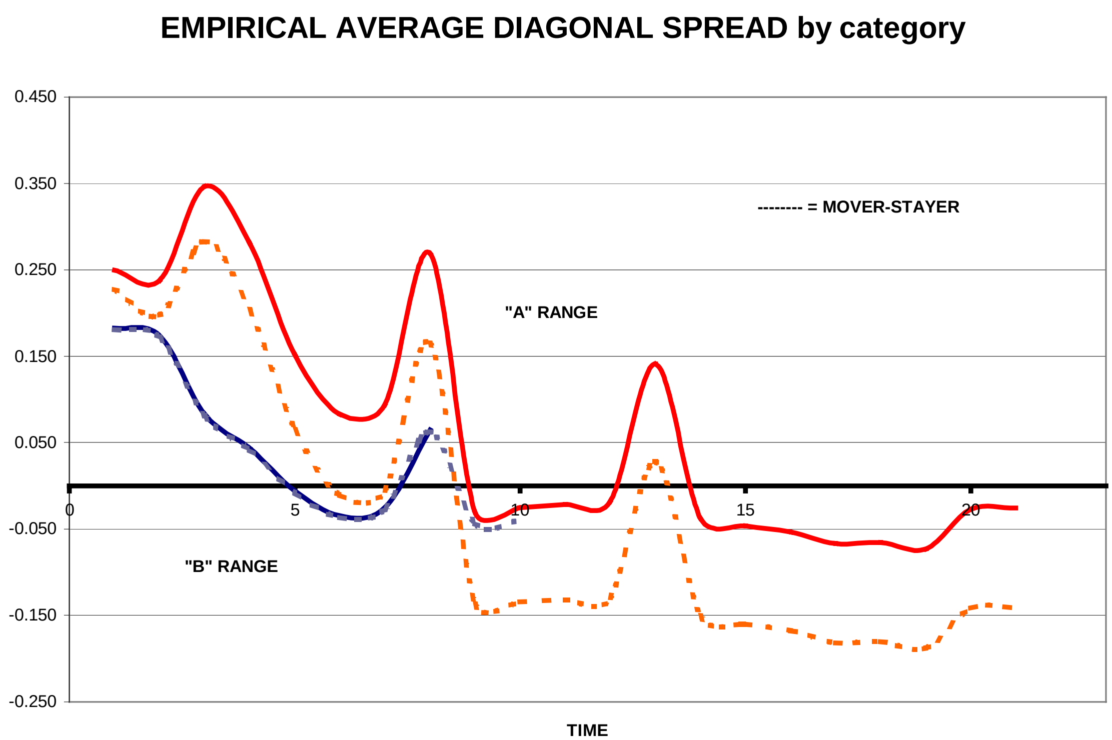
	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	73%	12%	6%	3%	0%	0%	0%	1%	0%	2%	1%	0%	1%	0%	0%	0%
AA+	4%	62%	29%	1%	0%	0%	0%	0%	0%	1%	1%	1%	0%	1%	0%	0%
AA	4%	60%	28%	3%	0%	0%	0%	0%	0%	1%	1%	1%	1%	1%	0%	0%
AA-	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	1%	11%	6%	76%	2%	3%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	5%	3%	83%	3%	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A-	1%	6%	4%	69%	5%	7%	1%	2%	0%	2%	1%	0%	1%	0%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	1%	1%	21%	5%	7%	5%	25%	3%	15%	8%	3%	5%	2%	0%	0%
BBB-	0%	2%	2%	24%	6%	9%	4%	33%	2%	10%	3%	1%	2%	1%	0%	0%
BB+	0%	0%	1%	8%	5%	8%	8%	21%	5%	24%	13%	4%	1%	1%	0%	0%
BB	0%	0%	1%	10%	6%	9%	7%	23%	4%	22%	12%	4%	1%	1%	0%	0%
BB-	0%	0%	1%	6%	4%	6%	7%	17%	4%	26%	15%	5%	8%	1%	0%	0%
B+	0%	0%	0%	2%	2%	3%	6%	9%	4%	26%	18%	7%	21%	3%	0%	0%
B	0%	0%	0%	1%	2%	3%	6%	9%	3%	26%	18%	7%	22%	3%	0%	0%
B-	0%	0%	0%	0%	1%	1%	3%	4%	2%	20%	16%	6%	34%	10%	1%	1%

MOVER-STAYER ESTIMATOR OF THE LONG-TERM TRANSITION MATRIX OF A DISCRETE-TIME MARKOV CHAIN

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-
AAA	72%	0%	0%	19%	0%	0%	0%	9%	0%	0%	0%	0%	0%	0%	0%	0%
AA+	0%	0%	0%	69%	0%	0%	0%	31%	0%	0%	0%	0%	0%	0%	0%	0%
AA	0%	0%	0%	69%	0%	0%	0%	31%	0%	0%	0%	0%	0%	0%	0%	0%
AA-	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
A+	0%	0%	0%	94%	0%	0%	0%	6%	0%	0%	0%	0%	0%	0%	0%	0%
A	0%	0%	0%	97%	0%	0%	0%	3%	0%	0%	0%	0%	0%	0%	0%	0%
A-	0%	0%	0%	92%	0%	0%	0%	8%	0%	0%	0%	0%	0%	0%	0%	0%
BBB+	0%	0%	0%	0%	0%	0%	0%	100%	0%	0%	0%	0%	0%	0%	0%	0%
BBB	0%	0%	0%	58%	0%	0%	0%	42%	0%	0%	0%	0%	0%	0%	0%	0%
BBB-	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
BB+	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
BB	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
BB-	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
B+	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
B	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%
B-	0%	0%	0%	56%	0%	0%	0%	44%	0%	0%	0%	0%	0%	0%	0%	0%

The following chart shows the pattern of the diagonal spread over successive n'th step matrixes estimated with the Mover-Stayer model as compared to the original MC. The Mover-Stayer model generates a parallel shift downward in the diagonal spread. The improvement is evident.





6. Time-period potential bias

The time–frame of our analysis covers a period of more than 22 years. Different phases of the business cycle and different events and shocks in the financial markets are embedded in the sample data we used in this paper.

Therefore, the results we produced may be subject to the issue of time-period sensitivity.

We reject the issue of time-sensitivity for three main reasons:

- The sample of data covers subsequent regime shifts. Hence, the estimated parameters of our model are not regime-biased, since they incorporate information resulting from the most various economic and financial contexts.
- The adoption of a shorter, but more actual sample would limit the statistical power of the adopted estimators.
- One could argue that the most recent experience on bond rating changes would reflect rating drift more accurately an that, consequently, the results for the entire sample period may not be indicative of the future. Thus, we replicated the entire analysis for a smaller and more recent time-frame, from January 1992 to November 1997. The resulting estimated parameters

did not change significantly, as compared to the estimates from the models based on the entire sample.

7. Potential applications

We suggest three potential applications of the results of our analysis:

- Valuation of swaps and interest rate derivatives when credit –risk is involved for sovereign issuers. Countries tend to issue structured debentures on the Eurobond markets with characteristics that satisfy the needs and the demand of the investors, in the attempt to reduce their cost of borrowing. However, they simultaneously prefer to hold obligations that fit their currency and rate-view preferences. For example, Republic of Argentina will issue a floating-rate bond denominated in Italian lira to satisfy European institutional investors demand, but will immediately swap its obligation to a more preferred exposure to a fixed rate bond denominated in pesos. The probability distribution resulting from our estimated transition probability for BB-rated countries may help in assessing the credit-risk related to Republic of Argentina.
- Country-allocation decisions in the context of international fixed-income portfolio analysis may be implemented on the basis of expected downgrades and upgrades, as they result from the transition probabilities we estimated for each of the sovereign bonds held.
- Risk-management and risk-assessment for international fixed-income portfolios may be greatly facilitated by the adoption of a probability distribution for upgrades and downgrades.