UNDERSTANDING RISK – ESTIMATING THE CONTRIBUTION TO RISK OF INDIVIDUAL BETS

BY

KEMAL ASAD-SYED

(INVESTMENT OFFICER – THE WORLD BANK INVESTMENT DEPARTMENT)

1818H Street NW, Washington DC 20433

kasadsyed@worldbank.org

(202-473-0798)

ARUN S. MURALIDHAR

(VICE PRESIDENT & HEAD OF CURRENCY RESEARCH – JPMorgan Investment Management)

320 East 46th Street, Apt. 14-H, New York, NY 10017

arunmura@hotmail.com

(212-837-2733)

PAOLO PASQUARIELLO

(Ph-D CANDIDATE – NEW YORK UNIVERSITY – STERN SCHOOL OF BUSINESS; contributed to this paper during an internship at J.P. Morgan Investment Management)

40W 4th Street, 9th Floor, Room 9-180, New York, NY 10012

ppasquar@stern.nyu.edu

(212-998-0369)

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Abstract

Portfolio managers may take many bets to outperform a benchmark. This note provides two simple methodologies to calculate the contribution to total risk of a specific bet, whether the risk measure is an absolute or a relative one, and demonstrates how investors can develop simple in-house models to measure such risk. In addition, we demonstrate how other measures, that are not derived from finance theory, but are used as first approximations are incorrect. These simple tools can allow investors to measure, monitor and hence manage the risks in their portfolios more effectively.

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1. INTRODUCTION

The issue of risk management is becoming more important for institutional investors, especially pension funds, and a number of working groups have been formed to evaluate what risk standards should be adopted by oversight committees for the management of such plans. However, one of the present shortcomings in the industry is that there is no uniform model that has been adopted to measure risks, which would then allow management to manage them. In addition, many software providers have focused only on the absolute risk of a portfolio in measuring the value-at-risk of a portfolio. Most risk systems have often not captured the largest risk that most pension plans are exposed to, namely asset-liability risk, and this was highlighted in Kemal Asad-Syed and Muralidhar (1998).[1] Further, when the performance of an institutional investor is measured relative to a passive benchmark, it is imperative to measure not only the absolute risk of the benchmark and the actual portfolio, but also the risk relative to a benchmark.[2] Even those software packages that have focused on relative risk have not adequately captured the contribution to total risk of any specific bet taken by portfolio managers.

Litterman (1996) highlights the usefulness of this measure and gives an indication of how this measure may be computed; however, the paper does not provide the methodology for the calculation. This note provides two simple methodologies to calculate the contribution to total risk of a specific bet, whether the risk measure is an absolute or a relative one, and demonstrates how plan sponsors can develop simple in-house models to

[1] Pensionmetrics® is one of the few software products that is targeted specifically to pension funds and evaluates the risk of assets vis-à-vis liabilites.

measure plan risk. The first approach develops the mathematical technique suggested by Litterman (1996); the second provides a more intuitive approach that is derived from asset pricing theory. In addition, we demonstrate how other measures, that are not derived from finance theory, but are used as first approximations are incorrect.[3] This approach also provides valuable insight into the correlation of bets with the entire portfolio of bets, thereby enhancing the evaluation of risk-taking activities. These simple tools can allow sponsors to measure, monitor and hence manage the risks in their portfolios more effectively. This paper will also look at the feasibility of using such statistics for the allocation of capital. The paper is developed in the context of a pension plan, but the concepts and conclusions apply more generally to any investor, whether a portfolio manager or an institutional investor with investment advisers.

2. PENSION PLAN RISKS

Prior to discussing how one estimates the contribution to risk of a specific bet, we detail the different risks that a plan is exposed to. Risk is generated in pension plans at different levels. At the highest level, selecting a benchmark for the asset portfolio creates the possibility for risks from asset-liability mismatches (or asset-liability risk). Alternatively, selecting an asset benchmark for purely asset reasons implies targeting an absolute risk point or a target variability of returns. At the next level, once target asset class allocations and benchmarks have been determined, a plan sponsor may create additional risk by investing tactically in the actual portfolio away from these target levels (or tactical

[2] Ambarish and Seigel (1996).

risk).[4] At the simplest level, tactical risk is created by under or over weighting individual asset classes.

In this note, we will focus only on (i) the absolute risk of the benchmark portfolio; (ii) the absolute risk of the actual portfolio on any given day (which if tactical bets are permitted could be quite different from that of the benchmark); and (iii) the relative risk implied by the actual portfolio vis-à-vis the benchmark or tactical risk. Thereafter, it is possible to demonstrate the contribution to the total risk or variability of returns of each asset class in which the plan has made either a target allocation or a tactical deviation.[5] The concept of the "marginal" is very well developed in economics in determining optimal consumption, pricing etc. and in an analogous fashion we will attempt to demonstrate whether the marginal risk measure can be used in the optimal utilization of a risk budget.

3. DEFINITION OF TERMS.

For convenience, we define two portfolios, the benchmark and the actual portfolio, and three risk measures, the absolute risk of the benchmark, the absolute risk of the actual portfolio, and the relative risk of the actual portfolio.

(a) Benchmark Portfolio: This is the strategic long-term asset allocation of the plan that is described by listing the various asset classes in which the plan is invested and the

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[3] Litterman (1996) makes a similar point in a footnote for one of these methods.
[4] Mashayekhi Beschloss and Muralidhar (1996).
[5] For the purpose of this article we will demonstrate how asset class allocations at a target or tactical level can be used to determine contribution to risk. The extension of determining the contribution of any deviation from a benchmark (*e.g.*, security, country or currency selection) is trivial.

long-term target allocations. A hypothetical benchmark portfolio is provided in Table 1.

(b) Actual Portfolio: This is the investor's portfolio on any measurement day. As a consequence of portfolio managers overweighting or underweighting asset classes, the live portfolio can and will differ from the benchmark. For illustrative purposes, we provide a hypothetical actual portfolio in Table 1, which is relative to the benchmark. In the last column in Table 1, is the percentage deviation of each asset class from its benchmark; the sum of these deviations is zero.[6]

(c) Absolute Risk of the Benchmark: In asset space, the risk of the benchmark portfolio is described by the variance or standard deviation of the expected return of this portfolio.[7] Mathematically, the absolute risk is estimated by taking the benchmark or target weights and multiplying them through a variance-covariance matrix i.e.,

$$\sigma^2(\text{benchmark}) = (v^T \Gamma v), \quad \text{(1a)}$$

where v = matrix of benchmark asset class weights (v^T being the transpose of v) and Γ is the assumed variance-covariance matrix, and v_i is the target weight of the ith asset class. The square root or the standard deviation is also a risk measure, as it captures the dispersion of the portfolio return around its mean. Using the hypothetical benchmark portfolio in Table 1 and the assumed variance-covariance matrix in Appendix 1, the standard deviation (*i.e.*, risk) of this portfolio is provided in Table 1.[8]

[6] We are assuming unleveraged deviations from the benchmark, but the results would be unaffected if leverage is appropriately captured.
[7] See for example Markowitz (1952).
[8] Since numerical simulations are provided to illuminate the key points of this article, we provide a variance-covariance matrix of the various asset classes. Every institutional investor can select their own matrix; these values were based on estimates from historical data.

(d) The Absolute Risk of the Actual Portfolio: The variance of this portfolio is calculated in a fashion identical to that of the benchmark, i.e.,

σ^2(actual) = ($w^T \Gamma w$) ....(1b)

where w = matrix of actual asset class weights, and w_i is the actual weight of the ith asset class. The square root or standard deviation is an alternative expression of this risk measure and is provided in Table 1.

(e) Relative Risk of the Actual Portfolio: This is the risk engendered by off-benchmark positions and Ambarish and Seigel (1996) demonstrate why this measure should be used when a portfolio is measured relative to a benchmark. The relative risk or variance of the active portfolio is calculated in a fashion identical to those above, i.e.,

σ^2(relative) = ($z^T \Gamma z$)..……...................................(1c)

where z = matrix of the differences between the actual and target asset class weights, and z_i is the deviation from benchmark in the ith asset class. Any component of the z matrix can be positive or negative, as the investment team could have chosen to underweight or overweight a particular asset class. The square root of σ^2(relative) per unit of time is referred to as the tracking error of a portfolio. Mathematically,

tracking error = $\sqrt{z^T \Gamma z} = \dfrac{z^T \Gamma z}{\sqrt{z^T \Gamma z}}$ ……………………………..…………….(1d)

The tracking error measures the amount by which the performance of the actual portfolio can deviate from the benchmark and is provided in Table 1.

4. THE MATHEMATICAL SOLUTION FOR MARGINAL CONTRIBUTION

The marginal contribution to total risk from an individual bet is nothing but a function of the first derivative of the risk measure vis-à-vis the bet under consideration. Litterman (1996) defines it loosely as, "the marginal rate of change in risk per unit change in the position (at the current position size) times the position size itself, can be thought of as the rate of change in risk with respect to a small percentage change in the size of the position."

For simplicity, we will use the tracking error for this estimation.

Marginal contribution of the bet in the ith asset class (z_i) to tracking error :

$$= z_i * \partial(\text{tracking error}) / \partial z_i$$

$$(\text{such that } {}_{i} \; z_i * \partial(\text{tracking error}) / \partial z_i = \text{total tracking error})$$[9]

$$= z_i * \frac{\partial \sqrt{z^T \Gamma z}}{\partial z_i}$$

$$= [z_i * (\frac{z^T \Gamma}{\sqrt{z^T \Gamma z}})] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1e)}$$

where $(\frac{z^T \Gamma}{\sqrt{z^T \Gamma z}})$ is a 1xN matrix measuring the marginal risk per unit of deviation. Notice that the denominator in the second term is nothing but the tracking error, thereby normalizing the calculation.

[9] See also Litterman (1996).

The same approach can be followed to measure the marginal contribution of each individual position to the total absolute risk of the portfolio. In this case, the marginal contribution of the position in the ith asset class to the portfolio's risk is given by:

$$= w_i *\partial(stdev)/ \partial w_i$$

$$= [w_i * (\frac{w^T \Gamma}{\sqrt{w^T \Gamma w}})] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1f)$$

where $\{(w^T \Gamma)/ \sqrt{(w^T \Gamma z)}\}$ is a 1xN matrix measuring the marginal risk per unit of the positions.

Finally, the marginal contribution of the position in the ith asset class to the benchmark's risk is given by:

$$= v_i *\partial(stdev)/ \partial v_i$$

$$= [v_i * (\frac{v^T \Gamma}{\sqrt{v^T \Gamma v}})] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1g)$$

where $\{(v^T \Gamma)/ \sqrt{(v^T \Gamma v)}\}$ is a 1xN matrix measuring the marginal risk per unit of the positions.

5. THE INTUITIVE APPROACH

There is another approach to estimating the contribution of an allocation to total risk that is derived from asset pricing theory. For simplicity, we call this the intuitive approach. Define the contribution of a stock I to the total risk of a portfolio of N stocks (P) as r_i. Define the contribution of an asset class I to the total risk of a portfolio of N asset classes (P) as c_i. From the basics in finance, we know that the contribution to total risk of a stock I to a total portfolio of N stocks (P) or r_i is equal to

$$r_i = s_i * \text{covariance}(I,P) \qquad \text{.......(2a)}$$

Mathematically, this is equivalent to

$$r_i = s_i*\sigma(I,P) = s_i*\rho_{I,P} *\sigma(I)* \sigma(P), \qquad \text{.......(2b)}$$

where s_i is the weight of stock i in portfolio P, $\rho_{I,P}$ is the correlation between I and P, $\sigma(I,P)$ is the covariance between I and P and $\sigma(P)$ and $\sigma(I)$ represent the standard deviations of P and I respectively. Usually, the correlation among stocks is known and stable, while that of an individual stock to a specific portfolio is uncertain. Where the correlation factor is unknown *ex-ante*, the contribution to risk can be calculated by the following:

$$r_i = s_i* \quad s_j* \sigma(i,j), \qquad \text{.......(2c)}$$

where   is the summation operator for j = 1 through N stocks and $\sigma(i,j)$ is the covariance of stocks i and j. The sum of all r_i in the portfolio must equal $\sigma^2(P)$ and hence in percentage terms, the contribution of stock I to the variance of portfolio P would be $r_i / \sigma^2(P)$.

In an analogous fashion to 2(a), 2(b) and 2(c), the contribution to total risk of an asset class for either absolute or relative risk can be defined as above. However, in the case of asset class structuring, the correlation between that of a specific asset class and the total portfolio (or those of asset class bets with the portfolio of bets) is difficult to determine *ex-ante* and probably changing as the portfolio composition changes. The correlation between two asset classes is easier to estimate. Hence an adaptation of equation (2c) is applied to estimate the contribution of an asset class to portfolio risk. Thus, we have:

c_i (actual) = $w_i * \quad w_j * \sigma(i,j)$,..(3a)

in the case of the actual risk of the portfolio and where $\sigma(i,j)$ is the covariance between the ith and the jth asset class and   is the summation operator for j = 1 through N asset classes. For the absolute risk of the benchmark portfolio, we define:

c_i (benchmark) = $v_i * \quad v_j * \sigma(i,j)$,..(3b)

In the case of the relative risk calculations, we will be concerned with the correlation of a tactical bet in an asset class with the portfolio of tactical bets.

c_i (tactical deviation) = $z_i * z_j * \sigma(i,j)$,..(3c)

It is clear that the c_i are calculated using variance as a measure of risk. To normalize for the standard deviation being the measure of risk and using (1d) we define:

$$c'_i \text{ (actual)} = c_i(\text{absolute})/\sqrt{(w^T \Gamma w)} = \frac{w_i * \Sigma w_j * \sigma(i,j)}{\sqrt{w^T \Gamma w}} \text{ ....(3d)}$$

$$c'_i \text{ (benchmark)} = c_i(\text{benchmark})/\sqrt{(v^T \Gamma v)} = \frac{v_i * \Sigma v_j * \sigma(i,j)}{\sqrt{v^T \Gamma v}} \text{ ....(3e)}$$

$$c'_i \text{ (tactical)} = c_i(\text{tactical})/\sqrt{(z^T \Gamma z)} = \frac{z_i * \Sigma z_j * \sigma(i,j)}{\sqrt{z^T \Gamma z}} \text{ ....(3f)}$$

Note that the last equation describes the marginal risk of a single bet to total tracking error. For the portfolios in Table 1 (benchmark, actual and deviation), we provide in Tables 2 (a) and 2 (b) the marginal contribution to total risk (in percentage points) and the percentage contribution of each asset class or asset class bet to total risk.

6. CORRELATIONS OF ASSET ALLOCATIONS TO PORTFOLIO ALLOCATIONS

An interesting statistic that can be derived from the above is the correlation of an asset class allocation to the overall allocation (as differentiated from the asset class correlations in Appendix I) or a specific asset class bet to a portfolio of bets. In this section we develop the analytical solutions for estimating these correlations. This statistic is useful as it allows the portfolio managers to determine whether bets are positively, negatively or uncorrelated with other bets – something that is not obvious at the time of constructing portfolios.

Measures of correlation of a single position with the total portfolio and of a single bet with the total portfolio of bets can be explicitly obtained from the following definitions. First, for the absolute portfolio:

$$\operatorname{cov}(\omega_i y_i, y_P) = \operatorname{cov}\left(\omega_i y_i, \sum_{j=1}^{N} \omega_j y_j\right) = \omega_i^2 \operatorname{var}(y_i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \omega_i \omega_j \operatorname{cov}(y_i, y_j) = \sum_{j=1}^{N} \omega_i \omega_j \operatorname{cov}(y_i, y_j)$$

$$\rho_{iP} = \frac{\operatorname{cov}(\omega_i y_i, y_P)}{\sigma_{\omega_i y_i} \sigma_P} = \frac{\sum_{j=1}^{N} \omega_i \omega_j \operatorname{cov}(y_i, y_j)}{\omega_i \sigma_i * \sqrt{\omega^T \Gamma \omega}} = \frac{c_i(actual)}{\omega_i \sigma_i * \sqrt{\omega^T \Gamma \omega}} \text{..................................}(4a)$$

where y_i is the return from the ith asset class.

Then, for the correlation of an individual bet with the portfolio of bets:

$$\operatorname{cov}(y_{Zi}, y_{Zp}) = \operatorname{cov}\left(z_i y_i, \sum_{j=1}^{N} z_j y_j\right) = z_i^2 \operatorname{var}(y_i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} z_i z_j \operatorname{cov}(y_i, y_j) = \sum_{j=1}^{N} z_i z_j \operatorname{cov}(y_i, y_j)$$

$$\rho_{ZiZP} = \frac{\operatorname{cov}(y_{Zi}, y_{ZP})}{\sigma_{Zi} \sigma_{ZP}} = \frac{\sum_{j=1}^{N} z_i z_j \operatorname{cov}(y_i, y_j)}{z_i \sigma_i * \sqrt{z^T \Gamma z}} = \frac{c'_i(tactical)}{z_i \sigma_i} \text{.....................................}(4b)$$

where y_{zi} is the spread expected return from the ith asset class.

Alternatively, using the intuitive approach, since the correlation between an asset class and the portfolio is unknown ex-ante, from 3(a), 3(b) and 2(b), the correlation coefficient of each asset class to the benchmark portfolio (or $\rho_{i,B}$) can also be implied by the following:

$$\rho_{i,B} = \frac{c_i(benchmark)}{\sigma(P)*\sigma(I)*v_i} \quad \text{...(4c)}$$

The correlation coefficient of each asset class to the actual portfolio (or $\rho_{i,P}$)

$$= \frac{c_i(actual)}{\sigma(Actual)*\sigma(I)*w_i} \quad \text{...(4d)}$$

or the correlation of the bet in asset class I to the portfolio of bets ($\rho_{Zi,ZP}$)

$$= \frac{c_i(tactical)}{\sigma(TE)*\sigma(I)*z_i} \quad \text{...(4e)}$$

I n Table 3, we provide the implied correlation coefficient of each asset class bet to portfolio of bets based on their respective allocations.[10] This table shows that the bets in four asset classes are negatively correlated with the portfolio of bets, at the current position.

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[10] As the allocation weights change, the total risk of a portfolio and hence the implied correlation will also change.

7. RESULTS

There are a number of useful insights from these diagnostics. First, notice that while the portfolio of Table 1 is overweight US equities, overweighting this asset class reduces the tracking error, as this bet is negatively correlated with other bets in the portfolio thereby lowering total relative risk (Tables 2 (a) and (b)). Second, the absolute or relative size of a bet may mask the true contribution to total risk. For example, while the 2% overweight in US Equities actually lowers tracking error, the same absolute bet in High Yield (+2%) contributes positively to relative risk (3.3%).[11] In addition, the 2% underweight in Non-US Fixed Income has a negligible impact on tracking error, while the same absolute and relative deviation in Private Equities contributes 20% of total tracking error. While Private Equities are more volatile, there is a more complex relationship at work which includes the relationship with other bets in the portfolio.[12]

Third, in evaluating the correlation of bets with the overall portfolio of bets, it is revealing to notice that the bets in U.S. Equities, Non-U.S. Equities, Non-U.S. Fixed Income and Private Equities are all negatively correlated with the portfolio of bets. One could ask if all these are therefore risk reducing by offsetting other bets in the portfolio. However, where the portfolio is long with respect to the benchmark and is negatively correlated, the contribution to tracking error is negative (as in U.S. Equities). On the other hand, where the portfolio is short with respect to the benchmark (Non-U.S.

[11] This point has been made elsewhere, more specifically with respect to managing the risks of currency overlays. See Mashayekhi Beschloss and Muralidhar (1996).

Equities, Non-U.S. Fixed Income and Private Equities), the negative correlation, in conjunction with the short position contributes positively to tracking error.

8. USEFULNESS OF THIS MEASURE

Any ability to drill down into a total risk measure and attribute the value to its components is useful for portfolio managers. As highlighted in the results, when the marginal contribution is negative, all else equal, a marginal unit increase in the direction of the current bet lowers total tracking error.[13] Only the US Equity bet changes the risk posture by effectively being risk reducing. Therefore, this breakdown can be used to size bets more effectively and capture maximum alpha for a given risk tolerance.[14] In addition, the portfolio manager determines whether the bets are all correlated and is able to disaggregate how diversified their bets may be. For example, in Table 1 there are 8 asset class bets; however, the three bets in Non-US Equity, Emerging Markets and Private Equity contribute 99% of the risk exposure. If the marginal contribution is concentrated in a few bets even though a large number of bets may have been implemented suggests that risks are not diversified. Similarly, a negative correlation is insufficient information to know whether bets are risk increasing or risk reducing as demonstrated above.

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[12] Litterman (1996) makes a similar observation and terms the point where risk contribution is zero as a candidate for a "best hedge."

[13] Up to a point. If the bet size increases, this becomes the dominant bet in the portfolio and will contribute positively to tracking error.

[14] One cautionary note – any risk analysis depends on the correlations and variances remaining stable over time and a violation of this assumption would put any risk analysis into doubt. Also, once the positions are changed, the statistics will need to be recalculated for the new portfolio.

The marginal contribution is dependent on current allocations, hence a slight change to a position implies very different results. For example, by shifting 2% more to U.S. Equities from Non-U.S. Fixed Income (i.e., extending the previous bet), the contribution from U.S. Equities to tracking error turns positive and the correlation of U.S. Equities and Non-U.S. Fixed Income to other bets in the portfolio are now positive. However, now the contribution to tracking error from Non-U.S. Fixed Income turns mildly negative as the correlation has shifted sign. Therefore, the sensitivity of the marginal risk analysis to portfolio changes would make it very difficult to allocate risk capital on this basis, for it would require a constant fine tuning and each asset class manager will need to be cognizant not only of the view that they may have on their specific market, but also its impact on other views.

9. COMPARISON WITH OTHER METHODOLOGIES.

In this section, we demonstrate other methodologies that are applied in standard risk management software and explain the deficiencies of each. In Tables 4 a and b, we compare the contribution to total risk using these three methods to provide an estimate of the magnitude of the error of not capturing the diversification benefits of an asset class.

(a) Contribution Assuming an Identity Correlation Matrix: Under this method, it is assumed that diagonal elements are unity and off-diagonal elements in the correlation matrix are zero. This is done to make the calculation simple. Therefore, assuming independence between assets would provide a variance estimate whereby adding the weighted variance of each asset class equals the portfolio variance. The problem with assuming that off-diagonal elements is zero is that the true benefits of diversification are never captured in these analyses. In addition, as Table 4a demonstrates the total risk of the benchmark portfolio is mis-estimated and hence this approach is incorrect (7.73% versus 11.69%).

(b) "Marginal Contribution": Under this methodology, embedded in the most commonly available software, the user calculates the variance using all assets and then extracts one asset class at a time from the portfolio and recomputes the variance or standard deviation. This new standard deviation (excluding a particular asset class) is compared to the full portfolio risk to give an estimate of the "contribution" of that particular asset class.[15] The most important problem is that the contribution to risk to a total portfolio is to be computed when portfolios are complete (i.e., with all asset classes, including the one whose contribution we are trying to estimate) and not using subsets of portfolios. Therefore, even if correct, the sum of all "marginal estimates" should equal the true variance of the portfolio (i.e., in the case of the benchmark portfolio = 11.69%). As is evident from Table 4a, this is not the case and the marginal method overestimates the total risk of the portfolio (12.9%). The rebalancing approach is clearly incorrect as we obtain a negative variance which is an infeasible result for an asset portfolio.

[15] There are two ways to perform this calculation; namely, to not rebalance the remaining asset class weights (i.e., so that the sum of the assets need not total 100%) and to rebalance the remaining assets. I would like to thank Mr. P.S. Srinivas for pointing this out. The rebalancing method is clearly incorrect as it excludes the possibility of the asset class ever being in the portfolio.

(c) Tracking Error of each bet is isolation: Under this method one assumes that the bet being evaluated is the only bet in the portfolio and looks at its risk in isolation. This assumes that the bets are independent and is identical to (a).

Clearly, the marginal method assuming rebalancing and the method assuming independence of assets is incorrect in estimating either the total variance or the percentage contribution of an asset class or asset class bet. Similarly, we show numerically that these alternative methods are inadequate to estimate the percentage contribution to relative risk. For completeness, we show the results of the Marginal-No Rebalancing calculation vis-à-vis the proposed method for the tracking error calculation in Table 4 (b). Not only are the resulting totals wrong, but also the magnitude and often the signs are incorrect for the portfolio bets hence providing the user with incorrect statistics about the contribution of bets to the risk of the overall portfolio.

11. CAVEATS

In the case of the two absolute measures of benchmark risk and actual risk, the implied correlations are meaningful. However, the implied correlations of the asset class bet to the portfolio of bets are based on the assumption that the variance-covariance matrix for asset classes applied also for asset class bets (which need not always be true), but this is an acceptable first approximation and assumes no bias in the bets away from the respective benchmarks.

12. CONCLUSIONS

This article set out to demonstrate two simple methods by which the contribution of an asset class allocation or asset class bet to the total absolute or relative risk of the portfolio could be determined. In addition, this methodology is superior to other methodologies that may be implemented by risk management software companies that are available to institutional investors. More important, this analysis has shown that the size of bet need not be a good indicator of contribution to total risk. It is possible to be overweight an asset class and have that bet contribute negatively to total risk. This follows as the correlation of an asset class bet to the total portfolio of bets could be negative even where the correlation of that asset class to others is positive. With respect to correlations of positions, rather than assume static correlations between asset classes and portfolios, we have demonstrated how these can be implied and examined *ex-post*. Finally, we have only demonstrated how contributions of asset class allocations to total risk are determined; the extensions to estimating the contribution of a selection of a benchmark of a manager or an individual manager's security selection (in either equities, bonds or currencies) to an entire portfolio is a simple extension of this methodology.

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Appendix 1 – Data on Asset Classes

Asset Classes	Standard Deviation	Correlations							
		USEQ	NUSEQ	EMEQ	USFI	NUSFI	HY	PE	Cash
US Equties	15.0%	1.0	0.5	0.3	0.4	0.4	0.5	0.4	-0.08
Non-US Equities	19.5%	0.5	1.0	0.3	0.2	0.3	0.2	0.1	-0.13
Emerging Equities	23.3%	0.3	0.3	1.0	0.3	0.3	0.2	0.1	-0.10
US Fixed Income	5.2%	0.4	0.2	0.3	1.0	0.4	0.3	0.0	-0.02
Non-US Fixed Inc.	4.5%	0.4	0.3	0.3	0.4	1.0	0.3	0.0	-0.05
High Yield	9.8%	0.5	0.2	0.2	0.3	0.3	1.0	0.0	-0.07
Private Equities	27.0%	0.4	0.1	0.1	0.0	0.0	0.0	1.0	0.00
Cash	1.0%	-0.08	-0.13	-0.10	-0.02	-0.05	-0.07	0.00	1.00

TABLES

TABLE 1 – ABSOLUTE AND RELATIVE PORTFOLIOS

Asset Classes	Benchmark Portfolio (v)	Actual Portfolio (w)	Deviation Portfolio (z)
US Equties	30.0%	32.0%	2.0%
Non-US Equities	35.0%	29.0%	-6.0%
Emerging Equities	5.0%	8.0%	3.0%
US Fixed Income	7.0%	9.0%	2.0%
Non-US Fixed Inc.	10.0%	8.0%	-2.0%
High Yield	2.0%	4.0%	2.0%
Private Equities	10.0%	8.0%	-2.0%
Cash	1.0%	2.0%	1.0%
Total	100.0%	100.0%	0.0%
Standard Deviation	11.69%	11.37%	1.24%

TABLE 2 (a) - CONTRIBUTION TO PORTFOLIO STANDARD DEVIATION

Asset Classes	Absolute Risk Benchmark (%)	Absolute Risk Actual (%)	Relative Portfolio (%)
US Equties	3.8%	4.1%	-0.045%
Non-US Equities	5.8%	4.9%	0.783%
Emerging Equities	0.5%	0.8%	0.241%
US Fixed Income	0.1%	0.2%	0.019%
Non-US Fixed Inc.	0.2%	0.2%	0.001%
High Yield	0.1%	0.1%	0.040%
Private Equities	1.2%	1.0%	0.203%
Cash	0.0%	0.0%	0.000%
Total Standard Deviation	11.69%	11.37%	1.24%

TABLE 2 (b) - PERCENTAGE CONTRIBUTION TO PORTFOLIO STANDARD DEVIATION

Asset Classes	Absolute Risk Benchmark (% of Total)	Absolute Risk Actual (% of Total)	Relative Portfolio (% of Total)
US Equties	32.2%	36.2%	-3.7%
Non-US Equities	49.7%	43.4%	63.1%
Emerging Equities	4.4%	7.4%	19.4%
US Fixed Income	1.1%	1.5%	1.5%
Non-US Fixed Inc.	1.6%	1.3%	0.1%
High Yield	0.6%	1.3%	3.3%
Private Equities	10.5%	8.8%	16.4%
Cash	0.0%	0.0%	0.0%
Total	100.0%	100.0%	100.0%

TABLE 3 – IMPLIED CORRELATION OF ASSET CLASS BET TO PORTFOLIO OF BETS

Asset Classes	Relative Portfolio
US Equties	(0.152)
Non-US Equities	(0.670)
Emerging Equities	0.343
US Fixed Income	0.179
Non-US Fixed Inc.	(0.009)
High Yield	0.206
Private Equities	(0.376)
Cash	0.000

TABLE 4 (a) - COMPARING THE METHODS – BENCHMARK RISK

Asset Classes	Assuming Independence (% Contribution)	Marginal No-Rebalancing (% Contribution)	Marginal Rebalanced (% Contribution)	Proposed Method (% Contribution)
US Equties	71.5%	40.67%	-61.2%	32.2%
Non-US Equities	21.5%	37.7%	-70.5%	49.7%
Emerging Equities	1.6%	5.9%	4.2%	4.4%
US Fixed Income	1.8%	2.7%	92.7%	1.1%
Non-US Fixed Inc.	0.1%	2.1%	70.8%	1.6%
High Yield	0.5%	2.4%	27.8%	0.6%
Private Equities	3.0%	8.5%	8.8%	10.5%
Cash	0.0%	0.0%	27.4%	0.0%
Total	100.0%	100.0%	100.0%	100.0%
Total Variance of Portfolio	0.60%	1.66%	-0.26%	1.37%
Standard Deviation	7.73%	12.90%	N/A	11.69%

TABLE 4 (b) - COMPARING THE METHODS – RELATIVE RISK

Asset Classes	Marginal No-Rebalancing (% Contribution)	Proposed Method (% Contribution)
US Equties	45.3%	-3.7%
Non-US Equities	-34.6%	63.1%
Emerging Equities	35.3%	19.4%
US Fixed Income	-3.3%	1.5%
Non-US Fixed Inc.	-1.0%	0.1%
High Yield	6.4%	3.3%
Private Equities	51.9%	16.4%
Cash	0.0%	0.0%
Total	100.0%	100.0%
Total Tracking Error	1.06%	1.24%