Addendum

Addendum to 'Imperfect Competition, Information Heterogeneity, and Financial Contagion'
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This Addendum to Pasquariello (2007) discusses in greater detail the stylized representation of the notion of *information heterogeneity* leading to equilibrium financial contagion, as well as reports the step-by-step proof of the ensuing Proposition 1, which is only sketched in Appendix A for economy of space.

1 Information Heterogeneity

It is assumed in Section 1.2 of Pasquariello (2007, p. 399) that, for any two speculators k and i and their informational advantage vectors $\delta_k \equiv E(v|S_{uk}, S_{\vartheta k}) - \overline{v} = E_1^k(v) - \overline{v}$ (for $E(v|S_{uk}, S_{\vartheta k}) \equiv E_1^k(v)$ of Eq. (3)) and $\delta_i = E_1^i(v) - \overline{v}$ about asset payoffs $v = u + \beta\vartheta$ of Eq. (1), speculator k's conditional expectation of δ_i is taken with respect to his δ_k, i.e., is given by

$$E(\delta_i|\delta_k) \equiv E_1^k(\delta_i) = \Sigma_c \Sigma_\delta^{-1} \delta_k, \tag{5'}$$

because of properties of multivariate normally distributed (MND) random variables (e.g., Greene, 1997, pp. 89-90) as $E(\delta_k) = \underline{0}$, $var(\delta_k) \equiv \Sigma_\delta$ of Eq. (4), and $cov(\delta_k, \delta_i) \equiv \Sigma_c$ of Eq. (5), whereby $var[E(\delta_i|\delta_k)] = \Sigma_c \Sigma_\delta^{-1} \Sigma_c$ and $cov[\delta_k, E(\delta_i|\delta_k)] = \Sigma_c$ (so that $corr(\delta_k, \delta_i) = diag(\Sigma_\delta)^{-\frac{1}{2}} \Sigma_c diag(\Sigma_\delta)^{-\frac{1}{2}}$, where $diag(Y)$ is a diagonal matrix of the diagonal elements of the

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matrix Y, and $corr\left[\delta_k, E\left(\delta_i|\delta_k\right)\right] = diag\left(\Sigma_\delta\right)^{-\frac{1}{2}}\Sigma_c diag\left(\Sigma_c\Sigma_\delta^{-1}\Sigma_c\right)^{-\frac{1}{2}}$), rather than taken with respect to his signals $S_{uk} = u + \varepsilon_{uk}$ and $S_{\vartheta k} = \vartheta + \varepsilon_{\vartheta k}$ (as for $E_1^k(v)$ of Eq. (3)), i.e., given by

$$E\left(\delta_i|S_{uk}, S_{\vartheta k}\right) = \Sigma_u\Sigma_{S_u}^{-1}\delta_{uk} + \beta\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\delta_{\vartheta k}, \tag{5''}$$

for $\delta_{uk} \equiv E\left(u|S_{uk}, S_{\vartheta k}\right) - \overline{u} = \Sigma_u\Sigma_{S_u}^{-1}\left(S_{uk} - \overline{u}\right)$ and $\delta_{\vartheta k} \equiv E\left(\vartheta|S_{uk}, S_{\vartheta k}\right) - \overline{\vartheta} = \Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\left(S_{\vartheta k} - \overline{\vartheta}\right)$, whereby $\delta_k = \delta_{uk} + \beta\delta_{\vartheta k}$,

$$var\left[E\left(\delta_i|S_{uk}, S_{\vartheta k}\right)\right] \equiv \Sigma_{\delta\delta} = \Sigma_u\Sigma_{S_u}^{-1}\Sigma_u\Sigma_{S_u}^{-1}\Sigma_u\Sigma_{S_u}^{-1}\Sigma_u + \beta\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\Sigma_\vartheta\beta', \tag{5'''}$$

and $cov\left[\delta_k, E\left(\delta_i|S_{uk}, S_{\vartheta k}\right)\right] = \Sigma_c$ (so that $corr\left[\delta_k, E\left(\delta_i|S_{uk}, S_{\vartheta k}\right)\right] = diag\left(\Sigma_\delta\right)^{-\frac{1}{2}}\Sigma_c diag\left(\Sigma_{\delta\delta}\right)^{-\frac{1}{2}}$).

Given Eqs. (5') and (5''), it can be shown that in general $E_1^k\left(\delta_i\right)(n) \neq E\left(\delta_i|S_{uk}, S_{\vartheta k}\right)(n)$ (and $corr\left[\delta_k, E\left(\delta_i|\delta_k\right)\right](n,j) \neq corr\left[\delta_k, E\left(\delta_i|S_{uk}, S_{\vartheta k}\right)\right](n,j)$) except in any of the following circumstances labeled in Pasquariello (2007) as *information homogeneity*: $K = 1$, $S_{uk} = S_u$ and $S_{\vartheta k} = S_\vartheta$ (so that $\Sigma_c = \Sigma_\delta = \Sigma_{\delta\delta}$ and $corr\left(\delta_k, \delta_i\right) = corr\left[\delta_k, E\left(\delta_i|\delta_k\right)\right] = corr\left[\delta_k, E\left(\delta_i|S_{uk}, S_{\vartheta k}\right)\right] = I$), $S_{uk} = u$ and $S_{\vartheta k} = \vartheta$ (so that $\Sigma_c = \Sigma_\delta = \Sigma_{\delta\delta} = \Sigma_v$ and $corr\left(\delta_k, \delta_i\right) = corr\left[\delta_k, E\left(\delta_i|\delta_k\right)\right] = corr\left[\delta_k, E\left(\delta_i|S_{uk}, S_{\vartheta k}\right)\right] = I$), or $\Sigma_u\Sigma_{S_u}^{-1} = \rho I$ and $\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1} = \rho I$ (with $\rho \in (0,1)$; so that $\Sigma_c = \rho\Sigma_\delta = \rho^2\Sigma_v$, $\Sigma_{\delta\delta} = \Sigma_c\Sigma_\delta^{-1}\Sigma_c = \rho^3\Sigma_v$, and $corr\left(\delta_k, \delta_i\right) = \rho I$ but $corr\left[\delta_k, E\left(\delta_i|\delta_k\right)\right] = I$; see Pasquariello, 2007, Footnote 12, p. 399).

Eq. (5') imposes Remark 2.1 of Caballé and Krishnan (1994) in the economy of Pasquariello (2007). Thus, Proposition 1 shows that a linear equilibrium of such an economy can be derived in closed form as an application of Caballé and Krishnan's (1994) Proposition 3.1 — where, accordingly, market clearing prices P_1 and each speculator's optimal trading strategy X_k take the following linear functional forms: $P_1 = A_0 + A_1\omega_1$ of Eq. (A1) and $X_k = B_0 + B_1\delta_k$ of Eq. (A2).[2] The derivation of a linear equilibrium in closed form given Eq. (5'') — where, accordingly, $P_1 = A_0 + A_1\omega_1$ and $X_k = B_0 + B_1^u\delta_{uk} + B_1^\vartheta\beta\delta_{\vartheta k}$ — mimics the proof of Proposition 1 (detailed

[2] See also Section 1.4 and Appendix A of Pasquariello (2007), as well as the detailed proof of Proposition 1 below.

below), but is analytically more involved and can be shown to yield the following price function:

$$P_1 = P_0 + \frac{\sqrt{K}}{2}\Lambda\left(\omega_1 - \overline{z}\right) = P_0 + H_u \sum\nolimits_{i=1}^{K} \delta_{ui} + H_\vartheta \beta \sum\nolimits_{i=1}^{K} \delta_{\vartheta i} + \frac{\sqrt{K}}{2}\Lambda\left(z - \overline{z}\right), \qquad (9")$$

and each speculator's demand strategy:

$$X_k = C_u \delta_{uk} + C_\vartheta \beta \delta_{\vartheta k}, \qquad (10")$$

where $C_u = \frac{2}{\sqrt{K}}\Lambda^{-1}H_u$, $C_\vartheta = \frac{2}{\sqrt{K}}\Lambda^{-1}H_\vartheta$, $\Lambda = \Sigma_z^{-1/2}\Psi^{1/2}\Sigma_z^{-1/2}$ is a symmetric positive definite (SPD) matrix (see Pasquariello, 2007, Definition A2, p. 421), $\Psi = \Sigma_z^{1/2}\Gamma\Sigma_z^{1/2}$, Γ is the following SPD matrix:

$$\begin{aligned} \Gamma \;=\; & 4\left\{\left\{\Sigma_{\delta_u} - H_u\left[\Sigma_{\delta_u} + (K-1)\,\Sigma_{c_u}\right]\right\} H_u' \right. \qquad \text{(A3')} \\ & \left. + \left\{\beta\Sigma_{\delta_\vartheta}\beta' - H_\vartheta\beta\left[\Sigma_{\delta_\vartheta} + (K-1)\,\Sigma_{c_\vartheta}\right]\beta'\right\} H_\vartheta'\right\}, \end{aligned}$$

in which $H_u = \left[2I + (K-1)\,\Sigma_{c_u}\Sigma_{\delta_u}^{-1}\right]^{-1}$, $H_\vartheta = \left[2I + (K-1)\,\beta\Sigma_{c_\vartheta}\beta'\left(\beta\Sigma_{\delta_\vartheta}\beta'\right)^{-1}\right]^{-1}$, $var\left(\delta_{uk}\right) \equiv \Sigma_{\delta_u} = \Sigma_u\Sigma_{S_u}^{-1}\Sigma_u$, $cov\left(\delta_{ui}, \delta_{uk}\right) \equiv \Sigma_{c_u} = \Sigma_u\Sigma_{S_u}^{-1}\Sigma_u\Sigma_{S_u}^{-1}\Sigma_u$, $var\left(\delta_{\vartheta k}\right) \equiv \Sigma_{\delta_\vartheta} = \Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\Sigma_\vartheta$, and $cov\left(\delta_{\vartheta i}, \delta_{\vartheta k}\right) \equiv \Sigma_{c_\vartheta} = \Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\Sigma_\vartheta$ (whereby $\Sigma_\delta = \Sigma_{\delta_u} + \beta\Sigma_{\delta_\vartheta}\beta'$ and $\Sigma_c = \Sigma_{c_u} + \beta\Sigma_{c_\vartheta}\beta'$).

Noteworthily, given the fundamental linear factor structure of Eq. (1) and the accompanying informational and distributional assumptions in Pasquariello (2007) — which imply that there are no redundant idiosyncratic sources of risk, systematic factors, assets, or signals in the economy — the ensuing nonsingularity of $\Sigma_v = \Sigma_u + \beta\Sigma_\vartheta\beta'$ does not impose any restriction on either the vector of systematic sources of risk ϑ or the matrix of factor loadings β for the equilibrium of Proposition 1 to exist (since both Σ_u and Σ_ϑ are nonsingular, e.g., Maddala, 1987, p. 446; see also Pasquariello, 2007, Footnote 8, p. 396, as well as the detailed proof of Proposition 1 below). When information is homogeneous so that $E_1^k\left(\delta_i\right) = E\left(\delta_i | S_{uk}, S_{\vartheta k}\right)$, the linear equilibrium of Eqs. (9") and (10") reduces to the one in Proposition 1. In all other circumstances, however, the existence of this equilibrium in the economy of Eq. (1) remains an open question unless ϑ

and β are such that the matrix $\beta\Sigma_{\vartheta}\beta'$ is nonsingular, too (see also Caballé and Krishnan, 1994).

More importantly, Eq. (5') is a computationally convenient specification of the conditional relationship between speculators' information endowments (i.e., between each δ_k and any other δ_i) such that each speculator may draw "incorrect" inference about the origins of shocks to the information endowments of other speculators (δ_i) while continuing to draw "correct" inference about the origins of shocks to his own information endowment (δ_k). For instance, given the three-asset economy implied by Eq. (2) and Appendix B in which $cov\left[v\left(1\right),v\left(3\right)\right]=0$, it ensues from Eq. (5') that $\frac{\partial E_1^k[\delta_i(3)]}{\partial u(1)}=\Sigma_c\Sigma_{\delta}^{-1}\Sigma_u\Sigma_{S_u}^{-1}\left(3,1\right)=-0.0507$ while $\frac{\partial \delta_k(3)}{\partial u(1)}=\Sigma_u\Sigma_{S_u}^{-1}\left(3,1\right)=0$ (see also Pasquariello, 2007, Footnote 10, p. 398).

A possible intuition for $E_1^k\left(\delta_i\right)$ of Eq. (5') is that each speculator uses only his informational advantage vector δ_k to form conditional expectations of other speculators' δ_i rather than its components δ_{uk} for δ_{ui} and $\delta_{\vartheta k}$ for $\delta_{\vartheta i}$, as he would do, if constrained by computational ability (e.g., in the spirit of bounded rationality; Simon, 1982), by first simulating a large number of realizations of pairs of MND vectors δ_i and δ_k in the economy and then estimating a linear relation between them via ordinary least squares (e.g., Hayashi, 2000, pp. 138-140) — like when approximating nonlinear rational expectations equilibrium (REE) models (see also Bernardo and Judd, 2000; Pasquariello, 2014, 2024). Alternatively, Eq. (5') may stem from each speculator receiving only a specific information endowment δ_k about asset payoffs v of Eq. (1) rather than both sets of private and noisy signals S_{uk} and $S_{\vartheta k}$ about shocks u and ϑ in v, respectively, from which δ_k ensues. In either of those circumstances, given $E_1^k\left(v\right)=\delta_k+\overline{v}$ and Definition 1 in Pasquariello (2007, Eq. (7), p. 400), the speculator would then use exclusively his information advantage vector δ_k to determine his trading strategy X_k maximizing $E_1^k\left(U_k\right)=NAV_{0k}+X_k'\left(\delta_k+\overline{v}\right)-X_k'E\left\{P_1\left[X_k+\sum_{\substack{i=1\\i\neq k}}^{K}X_i\left(\delta_i\right)+z\right]|\delta_k\right\}$ from Eqs. (6), (7), (A1), and (A2), i.e., exactly as implied by $E_1^k\left(\delta_i\right)$ of Eq. (5'); see also the detailed proof of Proposition 1 below.

In any case, Eq. (5') is meant to characterize parsimoniously a plausible, nontrivial form of information heterogeneity whereby each speculator's informational advantage about the same asset fundamentals is perceived by other speculators as less than perfectly correlated with (hence

different from) their own — fueling their cautious, "quasi-monopolistic," less than perfectly correlated strategic trading activity across many assets (see, e.g., the discussion in Pasquariello, 2007, p. 402). These trades ultimately induce financial contagion because the otherwise uninformed market makers (MMs) are unable to disentangle the "correct" sources of shocks to the economy when revising their priors about asset fundamentals v from the observed aggregate order flow in each asset ($E(v|\omega_1)$). Instead, MMs can at most only incorporate speculators' aforementioned "incorrect" inference into their market-clearing, semi-strong efficient prices $P_1(\omega_1) = E(v|\omega_1)$ of Eq. (9) via the matrix $H = \left[2I + (K-1)\Sigma_c\Sigma_\delta^{-1}\right]^{-1}$ defined in Proposition 1 since speculators internalize the price impact of their trades ($X_k = C\delta_k$ and $C = \frac{2}{\sqrt{K}}\Lambda^{-1}H$ in Eq. (10)) — relative to when private information is homogeneous. Therefore, $var(P_1) = KH\Sigma_\delta$ in Eq. (11), and contagion from real and information noise shocks (as defined in Eq. (12)) can occur in equilibrium: $\frac{\partial P_1}{\partial u'} = KH\Sigma_u\Sigma_{S_u}^{-1}$, $\frac{\partial P_1}{\partial \vartheta'} = KH\beta\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}$, $\frac{\partial P_1}{\partial \varepsilon'_{uk}} = H\Sigma_u\Sigma_{S_u}^{-1}$, and $\frac{\partial P_1}{\partial \varepsilon'_{\vartheta k}} = H\beta\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}$ in Eqs. (15) to (18).

However, Eq. (5") implies no such information heterogeneity among speculators about the fundamental sources of risk in the economy of Eq. (1): $cov(\delta_{uk}, \delta_{ui}) = cov[\delta_{uk}, E(\delta_{ui}|S_{uk}, S_{\vartheta k})] = \Sigma_u\Sigma_{S_u}^{-1}\Sigma_u\Sigma_{S_u}^{-1}\Sigma_u$ and $cov(\delta_{\vartheta k}, \delta_{\vartheta i}) = cov[\delta_{\vartheta k}, E(\delta_{\vartheta i}|S_{uk}, S_{\vartheta k})] = \Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}\Sigma_\vartheta$ are diagonal matrices because of the distributional assumptions in Sections 1.1 and 1.2 of Pasquariello (2007) such that not only both $corr(\delta_{uk}, \delta_{ui}) = \Sigma_u\Sigma_{S_u}^{-1}$ and $corr(\delta_{\vartheta k}, \delta_{\vartheta i}) = \Sigma_\vartheta\Sigma_{S_\vartheta}^{-1}$ are diagonal matrices, but also $corr[\delta_{uk}, E(\delta_{ui}|S_{uk}, S_{\vartheta k})] = I$ and $corr[\delta_{\vartheta k}, E(\delta_{\vartheta i}|S_{uk}, S_{\vartheta k})] = I$, i.e., rather like those aforementioned circumstances labeled in Pasquariello (2007) as information homogeneity whereby each speculator's informational advantage about the same asset fundamentals is perceived by other speculators as perfectly correlated with their own — fueling their aggressive, "quasi-competitive," perfectly correlated strategic trading activity across many assets (Pasquariello, 2007, p. 402).

Accordingly, and consistent with Eq. (11) and Propositions 2 and 3, it can be shown that no contagion from real and information noise shocks can occur if each speculator draws such "correct" inference about the origins of shocks to other speculators' information endowments

via Eq. (5") — e.g., $\frac{\partial E[\delta_i(3)|S_{uk},S_{\vartheta k}]}{\partial u(1)} = \Sigma_u \Sigma_{S_u}^{-1} \Sigma_u \Sigma_{S_u}^{-1}(3,1) = 0$ in the aforementioned three-asset economy of Eq. (2) and Appendix B where $cov[v(1), v(3)] = 0$ — since linear pricing (Eq. (A1)) and trading (Eq. (A2)) in their first order conditions (see Eq. (A4') below) imply that MMs must incorporate those "correct" inferences into their market-clearing prices for these conditions to be satisfied (e.g., see Eq. (A13')). In particular, in those circumstances Eq. (9") implies that $var(P_1) = K(H_u \Sigma_{\delta_u} + H_\vartheta \beta \Sigma_{\delta_\vartheta} \beta')$, $\frac{\partial P_1}{\partial u'} = K H_u \Sigma_u \Sigma_{S_u}^{-1}$, $\frac{\partial P_1}{\partial \vartheta'} = K H_\vartheta \beta \Sigma_\vartheta \Sigma_{S_\vartheta}^{-1}$, $\frac{\partial P_1}{\partial \varepsilon'_{uk}} = H_u \Sigma_u \Sigma_{S_u}^{-1}$, and $\frac{\partial P_1}{\partial \varepsilon'_{\vartheta k}} = H_\vartheta \beta \Sigma_\vartheta \Sigma_{S_\vartheta}^{-1}$, where the matrices H_u and H_ϑ defined above "correctly" reflect the fundamental structure of asset payoffs v in the economy of Eq. (1). Further investigation of properties of equilibrium price formation given Eq. (5") is left for future work.

Additional intuition for these observations comes from Figures 1 and 3 where — after calibrating the information setting of Section 1.2 by defining $\Sigma_c^* \equiv \alpha \Sigma_c + (1-\alpha)\rho \Sigma_\delta$ in Section 2.1, assuming that $cov(\delta_k, \delta_i) = \Sigma_c^*$, and finally substituting Σ_c with Σ_c^* in Proposition 1 — the greater (smaller) is α the more (less) heterogeneous are speculators' information endowments, the more (less) "incorrect" is each speculator's inference about shocks to other speculators' information endowments in Eq. (5'), leading to more (less) intense financial contagion in equilibrium.[3] However, Proposition 4 and Figure 4 show that MMs' own "incorrect" cross-inference from the aggregate order flow vector ω_1 (e.g., as reflected by the cross-price impact $\Lambda(3,1) = 0.0122$ for $K = 15$ although $cov[v(1), v(3)] = 0$) suffices to induce financial contagion in equilibrium when order flow shocks stem from liquidity trading (i.e., from noise traders not internalizing the impact of their behavior on prices; e.g., $\frac{\partial P_1(3)}{\partial z(1)} = 0.0236$), regardless of whether speculators are homogeneously or heterogeneously informed — hence, regardless of whether each speculator formulates "correct" or "incorrect" inference about other speculators' information endowments (see also Pasquariello and Vega, 2015).[4]

[3] Consistently, unreported plots of $\frac{\partial P_1(3)}{\partial u(1)} = K H_u^* \Sigma_u \Sigma_{S_u}^{-1}(3,1)$ and $\frac{\partial P_1(3)}{\partial \varepsilon_{uk}(1)} = H_u^* \Sigma_u \Sigma_{S_u}^{-1}(3,1)$ with respect to the number of better informed speculators (K) for different values of α, where $H_u^* \equiv \left\{2I + (K-1)\left[\alpha \Sigma_c \Sigma_\delta^{-1} + (1-\alpha)\Sigma_u \Sigma_{S_u}^{-1}\right]\right\}^{-1}$, within an economy both suitable for equilibrium in Eqs. (9"), (10"), and (A3') and similar to the one implied by Eq. (2) and Appendix B yield the same insights as those of $\frac{\partial P_1(3)}{\partial u(1)} = K H \Sigma_u \Sigma_{S_u}^{-1}(3,1)$ and $\frac{\partial P_1(3)}{\partial \varepsilon_{uk}(1)} = H \Sigma_u \Sigma_{S_u}^{-1}(3,1)$ in Figures 1 and 3, respectively.

[4] For instance, Pasquariello and Vega (2015) amend the economy of Pasquariello (2007) by assuming that each speculator receives a signal vector S_{vk} of v, where $\Sigma_{S_v} = \frac{1}{\rho}\Sigma_v$ so that $\delta_k \equiv E(v|S_{vk}) - \overline{v} = \Sigma_v \Sigma_{S_v}^{-1}(S_{vk} - \overline{v}) =$

2 Detailed Proof of Proposition 1

The proof, which mimics Caballé and Krishnan (1994) and Pasquariello (2003, Chapter 1), is by construction. We first specify general linear functionals for the pricing rule and speculators' demands, and then show that those functionals indeed represent a rational expectations equilibrium when their parameters are the ones in Eqs. (9) and (10).

We start by guessing that the equilibrium price vector (P_1) and speculators' market orders (X_k) are given by

$$P_1(\omega_1) = A_0 + A_1\omega_1 \tag{A1}$$

$$X_k(\delta_k) = B_0 + B_1\delta_k, \tag{A2}$$

where the matrix A_1 is SPD and the matrix B_1 is nonsingular.

The definition of ω_1 and Eqs. (A1) and (A2) imply that each speculator's expected equilibrium prices before trading occurs, $E_1^k(P_1)$, are equal to

$$E_1^k(P_1) = A_0 + A_1\left[X_k + (K-1)B_0 + B_1\sum\nolimits_{\substack{i=1\\ i\neq k}}^{K} E_1^k(\delta_i)\right], \tag{A4'}$$

where $E_1^k(\delta_i) = \Sigma_c\Sigma_\delta^{-1}\delta_k$ in Eq. (5') above. From Eq. (A4'), the symmetry of A_1 ($A_1'X_k = A_1X_k$), the fact that $E_1^k(v) = \delta_k + \overline{v}$ (by definition of δ_k), and each speculator's utility function U_k of Eq. (6), we then derive the first order condition of the maximization of $E_1^k(U_k) = NAV_{0k} + X_k'\left[E_1^k(v) - E_1^k(P_1)\right]$ implied by Definition 1 in Pasquariello (2007, Eq. (7), p. 400) as

$$\underline{0} = \delta_k + \overline{v} - A_0 - (K-1)A_1B_0 - A_1\overline{z} - (K-1)A_1B_1\Sigma_c\Sigma_\delta^{-1}\delta_k - 2A_1X_k. \tag{A5'}$$

$\rho(S_{vk} - \overline{v})$, $\Sigma_c = \rho\Sigma_\delta$, $E(\delta_i|S_{vk}) = E_1^k(\delta_i) = \rho\delta_k$, and $corr(\delta_k, \delta_i) = \rho I$ but $corr[\delta_k, E(\delta_i|\delta_k)] = I$ (as for the aforementioned special case in Pasquariello, 2007, Footnote 12, p. 399). In that setting, unlike the example in Pasquariello (2007, Footnote 10, p. 398) where also $E(\delta_i|S_{vk}) = E_1^k(\delta_i)$ but $\Sigma_v\Sigma_{S_v}^{-1}$ is a nondiagonal matrix, each speculator draws "correct" inference about the origins of shocks to both his own information endowment (e.g., $\frac{\partial\delta_k(3)}{\partial S_{vk}(1)} = 0$ as $cov[v(1), v(3)] = 0$) and the information endowments of other speculators (e.g., $\frac{\partial E_1^k[\delta_i(3)]}{\partial S_{vk}(1)} = 0$), while MMs continue to draw "incorrect" inference from the aggregate order flow (e.g., $\Lambda(3,1) \neq 0$), yielding financial contagion in equilibrium yet exclusively from noise trading shocks (e.g., $\frac{\partial P_1(3)}{\partial z(1)} \neq 0$ but $\frac{\partial P_1(3)}{\partial S_{vk}(1)} = 0$).

The second order condition is satisfied, for the matrix $2A_1$ is positive definite by assumption.

By replacing X_k with the conjecture of Eq. (A2), and equating the resulting coefficients, we obtain

$$(K+1)\,A_1B_0 = \overline{v} - A_0 - A_1\overline{z} \tag{A6'}$$

$$2A_1B_1 = I - (K-1)\,A_1B_1\Sigma_c\Sigma_\delta^{-1}. \tag{A7'}$$

The conjecture of Eq. (A2) is consistent with the assumption in Eq. (5') about each speculator's inference about the information endowments of other speculators given his own. A more general conjecture for $X_k = B_0 + B_1^u\delta_{uk} + B_1^\vartheta\beta\delta_{\vartheta k}$ must reduce to Eq. (A2) to satisfy the first order condition in Eq. (A5') given Eq. (5'). Specifically, $E(\delta_i|\delta_k) = E(\delta_{ui} + \beta\delta_{\vartheta i}|\delta_k) = E(\delta_{ui}|\delta_k) + E(\beta\delta_{\vartheta i}|\delta_k)$ and $E_1^k(\delta_i) = E_1^k(\delta_{ui} + \beta\delta_{\vartheta i}) = E_1^k(\delta_{ui}) + E_1^k(\beta\delta_{\vartheta i})$ by Eq. (3), the aforementioned definitions of δ_{uk} and $\delta_{\vartheta k}$, and linearity of conditional expectation (e.g., Siegrist, 2017), while $E(\delta_{ui} + \beta\delta_{\vartheta i}|\delta_k) \equiv E_1^k(\delta_{ui} + \beta\delta_{\vartheta i}) = \Sigma_c\Sigma_\delta^{-1}(\delta_{uk} + \beta\delta_{\vartheta k}) = \Sigma_c\Sigma_\delta^{-1}\delta_{uk} + \Sigma_c\Sigma_\delta^{-1}\beta\delta_{\vartheta k}$ by the assumption in Eq. (5'), such that $E(\delta_{ui}|\delta_k) \equiv E_1^k(\delta_{ui}) = \Sigma_c\Sigma_\delta^{-1}\delta_{uk}$ and $E(\beta\delta_{\vartheta i}|\delta_k) \equiv E_1^k(\beta\delta_{\vartheta i}) = \Sigma_c\Sigma_\delta^{-1}\beta\delta_{\vartheta k}$ by implication of information homogeneity or for internal consistency given information heterogeneity. Substitution of these expressions into Eq. (A5') then yields

$$\begin{aligned}\underline{0} = {} & \delta_{uk} + \beta\delta_{\vartheta k} + \overline{v} - A_0 - (K-1)\,A_1B_0 - A_1\overline{z} - (K-1)\,A_1B_1^u\Sigma_c\Sigma_\delta^{-1}\delta_{uk} \\ & - (K-1)\,A_1B_1^\vartheta\Sigma_c\Sigma_\delta^{-1}\beta\delta_{\vartheta k} - 2A_1B_0 - 2A_1B_1^u\Sigma_c\Sigma_\delta^{-1}\delta_{uk} - A_1B_1^\vartheta\Sigma_c\Sigma_\delta^{-1}\beta\delta_{\vartheta k},\end{aligned} \tag{A5''}$$

whereby equating coefficients for δ_{uk} and $\beta\delta_{\vartheta k}$ implies that $2A_1B_1^u = I - (K-1)\,A_1B_1^u\Sigma_c\Sigma_\delta^{-1}$ and $2A_1B_1^\vartheta = I - (K-1)\,A_1B_1^\vartheta\Sigma_c\Sigma_\delta^{-1}$, which can both be satisfied iff $B_1^u = B_1^\vartheta$.

Since ω_1 is MND with mean $E(\omega_1) = KB_0 + \overline{z}$ and variance

$$var(\omega_1) = KB_1\Sigma_\delta B_1' + \Sigma_z + K(K-1)\,B_1\Sigma_c B_1' \tag{A8'}$$

(as $cov\left(\delta_k, \delta_i\right) = \Sigma_c$), and $cov\left(v, \omega_1\right) = K\Sigma_\delta B_1'$, then

$$E\left(v|\omega_1\right) = \overline{v} + K\Sigma_\delta B_1' \left[KB_1\Sigma_\delta B_1' + \Sigma_z + K\left(K-1\right)B_1\Sigma_c B_1'\right]^{-1}\left[\omega_1 - KB_0 - \overline{z}\right]. \tag{A9'}$$

According to Definition 1 in Pasquariello (2007, Eq. (8), p. 400), $P_1 = E\left(v|\omega_1\right)$ in equilibrium. Therefore, the conjecture of Eq. (A1) implies that, given the invertibility of B_1,

$$A_1 = \left[B_1 + \left(K-1\right)B_1\Sigma_c\Sigma_\delta^{-1} + \frac{1}{K}\Sigma_z\left(B_1'\right)^{-1}\Sigma_\delta^{-1}\right]^{-1} \tag{A10'}$$

$$A_0 = \overline{v} - A_1\overline{z} - KA_1B_0. \tag{A11'}$$

The expressions for A_0, A_1, B_0, and B_1 implied by Eq. (9) for P_1 and by Eq. (10) for X_k must solve the system made of Eqs. (A6'), (A7'), (A10'), and (A11') to represent a linear equilibrium of the economy of Section 1 of Pasquariello (2007). Defining A_1B_0 from Eq. (A6') and substituting it into Eq. (A11') leads to $A_0 = \overline{v} - A_1\overline{z}$. Plugging this expression into Eq. (A6'), we obtain $B_0 = \underline{0}$. We are left with the task of finding A_1 and B_1. Solving Eq. (A7') for A_1, we get

$$A_1 = \left[2B_1 + \left(K-1\right)B_1\Sigma_c\Sigma_\delta^{-1}\right]^{-1}. \tag{A12'}$$

Equating Eq. (A12') to Eq. (A10'), it follows that $B_1 = \frac{1}{K}\Sigma_z\left(B_1'\right)^{-1}\Sigma_\delta^{-1}$. Substituting this expression for B_1 back into Eq. (A10') gives us

$$A_1 = \left[\frac{2}{K}\Sigma_z\left(B_1'\right)^{-1}\Sigma_\delta^{-1} + \left(K-1\right)B_1\Sigma_c\Sigma_\delta^{-1}\right]^{-1}. \tag{A13'}$$

From Eq. (A12'), using the definition of H in Proposition 1 ($H = \left[2I + \left(K-1\right)\Sigma_c\Sigma_\delta^{-1}\right]^{-1}$) and the invertibility of A_1 (see Pasquariello, 2007, Theorem A1, p. 421), we derive

$$B_1 = A_1^{-1}H \tag{A14'}$$

$$\left(B_1'\right)^{-1} = A_1\left[H^{-1} + \left(K-1\right)\left(\Sigma_\delta^{-1}\Sigma_c - \Sigma_c\Sigma_\delta^{-1}\right)\right]. \tag{A15'}$$

We then insert Eqs. (A14') and (A15') into Eq. (A13') and rearrange terms to obtain

$$\frac{K}{4} A_1^{-1} \Gamma = \Sigma_z A_1, \tag{A16'}$$

where the matrix Γ, defined as

$$\Gamma \equiv 2 \left[\Sigma_\delta - (K-1) H \Sigma_c \right] \left[H^{-1} + (K-1) \left(\Sigma_\delta^{-1} \Sigma_c - \Sigma_c \Sigma_\delta^{-1} \right) \right]^{-1}, \tag{A3}$$

in Pasquariello (2007, p. 421), is SPD by the *Rayleigh's principle* (e.g., Bodewig, 1959, p. 283) and Theorem A1, since Γ reduces to the matrix G in Eq. (6) of Caballé and Krishnan (1994) and, as noted earlier, the distributional assumptions in Pasquariello (2007) imply that there are no redundant assets or signals in the economy such that the ensuing nonsingular covariance matrices Σ_v, Σ_δ, and Σ_c, and the matrix $\Sigma_\delta^{-1} \Sigma_c \Sigma_\delta^{-1}$ are all SPD for any $N \times F$ matrix β in the economy of Eq. (1) by properties of matrices (e.g., Brookes, 2017). Because we can write Eq. (A16') as

$$\frac{K}{4} \left(\Sigma_z^{1/2} \Gamma \Sigma_z^{1/2} \right) = \left(\Sigma_z^{1/2} A_1 \Sigma_z^{1/2} \right) \left(\Sigma_z^{1/2} A_1 \Sigma_z^{1/2} \right) \tag{A17'}$$

(where $\Sigma_z^{1/2}$ is the unique SPD square root of Σ_z), and because the left-hand-side of Eq. (A17') is itself SPD, the matrix $\Sigma_z^{1/2} A_1 \Sigma_z^{1/2}$ represents its unique SPD square root (e.g., Bellman, 1970, pp. 93-94).

It then ensues that

$$A_1 = \frac{\sqrt{K}}{2} \left(\Sigma_z^{-1/2} \Psi^{1/2} \Sigma_z^{-1/2} \right) \equiv \frac{\sqrt{K}}{2} \Lambda, \tag{A18'}$$

where $\Psi \equiv \Sigma_z^{1/2} \Gamma \Sigma_z^{1/2}$, is clearly the unique SPD matrix that solves Eq. (A17'). Since $A_0 = \overline{v} - A_1 \overline{z}$, Eq. (A18') implies that $A_0 = P_0 - \frac{\sqrt{K}}{2} \Lambda \overline{z}$. The matrix B_1 is derived by plugging the above expression for A_1 into Eq. (A14'), and is equal to $\frac{2}{\sqrt{K}} \Lambda^{-1} H$ (i.e., C in Eq. (10)). Because the matrix Λ in Eq. (A18') is SPD, it is simple to verify that B_1 is invertible, consistently with our initial assumptions, using Theorem A1 and the definition of H in Proposition 1.

Lastly, it remains to prove that, given any linear pricing rule, the symmetric linear strategies

X_k in Eq. (10), for $k = 1, \ldots, K$, represent the unique Bayesian Nash equilibrium of the Bayesian game among insiders. This is shown by extending to our setting the "backward reaction mapping" introduced by Novshek (1984) to find n-firm Cournot equilibria. Proposition 1 is in fact equivalent to a symmetric Cournot equilibrium with K speculators. The key to Novshek's (1984) argument is to look for the actions of each informed trader that are consistent with utility maximization and the aggregate demand ω_1, instead of specifying the actions for each speculator which are consistent with the choices of the others. Uniqueness then follows from observing that, because the optimal demands X_k depend only on individual attributes δ_k, there is only one ω_1 that can be decomposed into the sum of those vectors X_k and liquidity trading, but also that such aggregate order flow ω_1 can be decomposed only in one way into those vectors X_k, given z.

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