Central Bank Intervention and the Intraday Process of Price Formation in the Currency Markets

Paolo Pasquariello[1]

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Abstract

We study the impact of sterilized spot interventions on the microstructure of currency markets. We analyze their major channels of effectiveness, imperfect substitutability and signaling, in a model of sequential trading in which a stylized Central Bank is rational, but not necessarily profit-maximizing. In this setting, and consistent with available empirical evidence, we find that interventions have endogenous long-lived effects on quotes when informative about policy objectives and fundamentals, or when the threat of future actions by the Central Bank is significant and credible, for these circumstances lead uninformed investors or dealers to permanently revise their beliefs. Portfolio balance effects of interventions are instead short-lived because of trading occurring sequentially. We also find that a Central Bank attempting to lean against the wind or chase the trend of the domestic currency is generally more successful under competitive dealership. Intuitively, competition induces the dealers to pass all gains or losses they expect from trading with the Central Bank onto the population of investors. This is accomplished by greater, and generally asymmetric, revisions of their bid and ask quotes. The resulting equilibrium process of intraday price formation is shown to depend crucially on dealers' market power, on the sign and magnitude of the intervention, on the transparency of the order flow induced by it, and on the perceived likelihood of future interventions.

1　Introduction and Motivation

The foreign exchange (forex) market is probably the most active financial market in the world in terms of volume, frequency, and intensity of trading.[1] However, progress in computer and communications technology has only recently made intraday pricing data available, stimulating greater interest in the microstructure of currency markets. Yet in most cases, with the notable exceptions of Lyons (1997, 2001), Naranjo and Nimalendran (2000), and Evans and Lyons (2001), the empirical analysis of these rich datasets has not been preceded by theoretical investigations of how institutional features specific to the forex market may affect its functioning. The main objective of this paper is to contribute to closing this gap. We focus on some important aspects of the currency markets that are not shared by any centralized equity market, in particular the presence of a rational, but not necessarily profit-maximizing player like the Central Bank (CB), and derive from their inclusion several empirically testable implications for quotes and bid-ask spreads.

Many macroeconomics textbooks describe the exchange rate as an *intermediate* target of monetary policy: CBs choose levels (or bands of fluctuation) for the domestic currency compatibly with the "ultimate" trade-off of monetary policy, between sustainable economic growth (the "output gap") and moderate inflation. Each monetary authority weighs growth and price stability differently, but all participate in the currency markets in pursuing their economic policies. CBs may also serve less stringent agendas, under pressure from political power, interest lobbies, etc. In both cases, their actions may not be motivated by pure profit. There is in fact much anecdotal and empirical evidence that policy objectives and wealth maximization often collide.[2] In the microstructure literature on asymmetric information, insiders always profit at the expense of market-makers and uninformed (noise) traders.[3] Forex dealers may instead reasonably expect not only to suffer potential losses but also to earn potential gains from trading against a better informed CB, if its actions, albeit rational, are inconsistent with profit maximization. CBs may also take positions in the currency markets for purely speculative motives, as in the frequently cited example of Bank Negara, the Malaysian CB, in the early 1990s.[4] In such circumstances,

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[1]The Bank for International Settlements (BIS, 2002) triennial survey of global currency market activity for 2001 reports a daily volume of \$1.2 trillion.

[2]For example, Taylor (1982) uses the profit criterion of Friedman (1953) to show that during the 1970s CBs were only partially successful in resisting currency fluctuations, but lost billions of dollars in the process.

[3]O'Hara (1995) reviews the market microstructure literature on information economics.

[4]Bank Negara is deemed responsible for many speculative transactions from 1989 to 1992. According to Brown (2000), Bank Negara was "using its inside information as a member of the club of central bankers to speculate in currencies, sometimes to an amount in excess of \$1 billion a day." At that time, Dr. Mahathir, Malaysia's Prime

currency dealers may incur losses against those potentially better-informed agents. The dealers' market power would then determine how much of these potential gains or losses might be passed to investors via quotes and spreads.

Motivated by these considerations, we explore the short- and long-term impact of official spot interventions on different dimensions of the intraday process of price formation in the forex market: quotes, quotes' revisions, absolute and proportional bid-ask spreads, price volatility, and investors' order flow.[5] At the core of our work is Madhavan's (2000) recognition of "the potential value from combining microstructure and macro variables within a single model" to enhance the understanding of the global currency markets. To that end, we first develop a model of sequential exchange rate trading in the spirit of Garman (1976), Brock and Kleidon (1992), and Saar (2000a, b). In this model there are two assets, a domestic and a foreign currency-denominated riskless bond. Price-taking CARA investors reach the market according to an exogenous arrival process and trade just once. Investors are of two different types, depending on their initial endowment of the domestic asset. Liquidity is provided, and prices are set by a monopolist or competitive risk-neutral market-makers maximizing expected instantaneous profits from trading under a market-clearing constraint. We then extend this basic setup by introducing a stylized CB, whose motives and actions are consistent with the literature on official intervention, and information asymmetry about them. We concentrate on the most common form of intervention, *sterilized* transactions which, by leaving the domestic monetary base unchanged, do not affect monetary policy and economic fundamentals. The effectiveness of these trades is still controversial and, as such, at the center of the current theoretical and empirical debate. The resulting model, albeit parsimonious, allows us to consider the two major channels of effectiveness of sterilized intervention, imperfect substitutability (or portfolio balance) and signaling, without sacrificing its analytical tractability.[6]

The inclusion of a rational, but not necessarily profit-maximizing CB in a purely microstructure setting in which orders do not clear simultaneously and dealers hold different degrees of market power, and the joint analysis of its effectiveness in managing the exchange rate represent

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Minister, was reported claiming that the CB's practices were simply "active reserve management."

[5]Lyons (2001) defines the spot market "the essence of the forex market." CBs can (and sometimes do) use alternative financial instruments for their interventions, like interest rates, forward contracts, or derivatives. There is, however, some agreement (e.g., Eaton and Turnovsky, 1984; Miller, 1998; Sarno and Taylor, 2001) that spot trades are the most effective, since they have a direct cost for the CB.

[6]In the first case (e.g., Branson, 1983, 1984), CB trades alter the composition of risk-averse agents' portfolios. The exchange rate must then shift to induce the market to adjust to the new currency supply. In the second case (e.g., Mussa, 1981; Bhattacharya and Weller, 1997), intervention affects exchange rates by providing investors and dealers with supposedly new and relevant information.

an original contribution to the economic and financial literature. Indeed, our model generates both macro and microstructure implications. We show that endogenous sterilized intervention, or merely its likelihood, has long-lived effects on the exchange rate when informative about policy objectives or fundamentals, or when the threat of future intervention is significant and credible, since these circumstances lead uninformed investors or dealers to permanently revise their beliefs. Portfolio balance effects of CB trades are instead short-lived because trades occur sequentially. We also find that interventions are more successful when dealers compete with each other for the incoming trade. Intuitively, competition induces dealers to transfer any additional cash flow they expect from trading with the CB onto the population of investors. This is accomplished by greater, and generally asymmetric, revisions of their bid and ask quotes. The resulting impact of intervention on intraday quotes and spreads is not unidirectional, and depends not only on dealers' market power but also on sign and magnitude of CB trades, on the perceived likelihood of future interventions, and on the transparency of the order flow induced by the intervention, consistently with empirical evidence in Payne and Vitale (2001), Pasquariello (2002), and Dominguez (2003), among others.

Closely related to our study are two papers: Naranjo and Nimalendran (2000) and Evans and Lyons (2001). Naranjo and Nimalendran explain the estimated increase in daily bid-ask spreads around unexpected interventions in the Deutschemark/U.S. Dollar market by adverse selection. However, they assume that CBs, acting as large insiders, attempt to disguise their presence among uninformed investors and that their trades are exogenously effective. Evans and Lyons instead concentrate exclusively on the portfolio balance effects of order flow uncertainty induced by exogenous interventions on inter-dealer trading by extending the "hot potato" model of Lyons (1997). Information asymmetry and signaling, although of greater empirical significance (e.g., Edison, 1993; Pasquariello, 2002), are ruled out by construction. Furthermore, their model is in the spirit of simultaneous-move games, and does not study the intraday dynamics of bid-ask spreads.

The paper is organized as follows. Section 2 describes the basic model. In Section 3 we derive our first set of results for quotes and spreads, under full information and different degrees of dealers' market power. Section 4 extends the model to a unifying example of information asymmetry between the CB and investors and dealers. Section 5 explores the effects of this additional uncertainty on the equilibrium process of price formation using sequential stages of a stylized trading day in the forex market. Section 6 concludes. All proofs are in the Appendix unless otherwise noted.

2 The Basic Model

Currency markets share with most financial markets the attribute that investors do not necessarily hit dealers's quotes with orders at the same time. Hence, uncertainty regarding the composition of the investors' population affects the way dealers set prices. The financial literature has developed several models for the sequential arrival of orders to a market.[7] The basic setup of our model is similar in spirit to Garman (1976), Block and Kleidon (1992), and Saar (2000a, b). In the following subsections we describe the traded assets, define how three categories of players, investors, market-makers (MMs), and a CB interact, and solve for equilibrium quotes and spreads under monopoly and perfect competition and no CB intervention. Finally, we allow the CB to trade, and derive our first set of results assuming full information. We will introduce information asymmetry in Section 4.

2.1 Assets

There are two assets in the economy, a riskless bond paying $R > 1$ units of the domestic currency (the numeraire, e.g., U.S. dollars, USD) and a riskless bond paying $R_F > 1$ units of the foreign currency (e.g., British pounds, GBP) at time T'. The dollar payoff at time T' of the GBP-denominated bond, F, is uncertain because so is $S_{T'}$, the amount of USD necessary to buy one GBP at time T' (USDGBP), i.e., $F = R_F S_{T'}$. W.l.o.g., we let $R_F = 1$. The long-term exchange rate F is normally distributed with (random) mean f and variance $\sigma_F^2 > 0$. The forex market opens at time zero, and trading occurs until time $T < T'$. The interval $[0, T]$ should be interpreted as a short period of time, for example one trading day. Intraday interest rates are assumed to be zero. Arrivals of orders are exogenous and driven by an orderly point process $G(t)$ (as in Saar, 2000a, b), so that only one arrival is allowed at any point in time t. At each $t \in [0, T]$, we assume there is a probability $l \in [0, 1]$ of a CB order arriving to the market.

2.2 Investors

Investors are risk-averse and of types 1 and 2, depending on their initial endowment of risky GBP-denominated T-bills, $\overline{X}_1$ and $\overline{X}_2$, and of riskless USD-denominated T-bills, $\overline{B}_1$ and $\overline{B}_2$. A fraction q (known to all agents) of investors is of type 1; hence q can be interpreted as the

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[7]Models of non-simultaneous trading have been used to study the problem of a dealer facing incoming orders that move him away from his desired inventory position (as in Amihud and Mendelson, 1980 and Ho and Stoll, 1981), or of a market-maker facing incoming orders from potentially better-informed traders (e.g., Glosten and Milgrom, 1985 or Easley and O'Hara, 1987, 1991, 1992).

probability that an incoming investor's order is from a type 1 trader. We assume that $\overline{X}_1 > \overline{X}_2$, i.e., that type 1 investors are potentially net sellers, and type 2 investors potentially net buyers, of GBP. All investors maximize the expected CARA utility of their final wealth, $W_{T'}$, with the same degree of risk-aversion α. All investors take market prices as given and trade only once.[8] Thus, the optimal demands for GBP ($Q_{i,t}$) and USD ($B_{i,t}$) of an investor of type $i = \{1, 2\}$, with information set I_t, solve the following problem:

$$\max_{Q_{i,t}, B_{i,t}} E\left[-e^{-\alpha W_{i,T'}} | I_t\right]$$

$$s.t. \quad B_{i,t} + S_t Q_{i,t} = \overline{B}_i + S_t \overline{X}_i \tag{1}$$

$$RB_{i,t} + FQ_{i,t} = W_{i,T'},$$

where the USD (GBP) price of the USD (GBP)-denominated riskless bond is one. The resulting optimal net demand for GBP by an investor of type i, $Q_{i,t} - \overline{X}_i$, is given by

$$X_{i,t} = \frac{1}{\pi}\left(\frac{E\left[f|I_t\right]}{R} - S_t\right) - \overline{X}_i, \tag{2}$$

where $\pi = \frac{\alpha \sigma_F^2}{R}$ is a parameter characterizing the elasticity of investors' demand for risky assets, i.e., the less than perfect substitutability of domestic and foreign currency.[9]

2.3 Dealers

According to Lyons (1995), most currency dealers carefully monitor their inventory during each working day. Lyons suggests that, because forex trading might continue during the evening, the prospect of carrying an open position through the night is not appealing to them. Anecdotal evidence further indicates that, in the fast-paced currency markets, dealers concentrate on extracting rents from the order flow, rather than maximizing the terminal value of their positions, as investors do instead.[10] It then seems reasonable that forex dealers, in setting their quotes, would impose no expected drift to their inventory at any point in time, but profit from incoming orders by charging a bid-ask spread as compensation for providing liquidity. It seems equally reasonable that dealers' market power would have an effect on their ability to extract rents from

[8]This assumption is reasonable if we interpret the interval $[0, T]$ as one trading day.

[9]See Grossman and Stiglitz (1980). We use negative exponential utility and normally distributed payoffs for analytical tractability, despite their known shortcomings (e.g., unlimited liability and possibly negative prices). In what follows, we assume the parameters of the economy are such that a draw $F < 0$ is extremely unlikely and exchange rates are non-negative. Bhattacharya and Weller (1997) use a similar description of investors' activity in the currency markets and label them forex speculators.

[10]Mayer (1988) makes a similar observation about NYSE specialists.

the order flow. For most currency pairs of G7 countries, competition among dealers is intense and may exert a downward pressure on such compensation. However, for some other exchange rates, especially in emerging economies, there are often very few dealers providing that service on a regular basis and for significant trade sizes.

Consistently with this view of the activity of MMs in the forex markets, we consider the extreme cases of monopolist and competitive dealership. To isolate the impact of interventions on quotes and spreads, we assume our stylized currency market is frictionless. Then, the expected instantaneous profit Π_t per unit of time earned by the MMs can be expressed as

$$E\left[\Pi_t | M_t\right] = (1 - l)\, q X_{1,t} S_{1,t} + (1 - l)\,(1 - q)\, X_{2,t} S_{2,t} + l X_t^{CB} S_t^{CB}, \tag{3}$$

where M_t is the dealers' information set at time t, before the next incoming order arrives, $S_{i,t}$ and S_t^{CB} are the prices quoted to investors of type i and to the CB, respectively, and X_t^{CB} is a CB order. In the full information scenario, M_t includes all parameters controlling F, l, and all past transactions. In the monopoly case, at each t the dealer sets quotes that maximize Eq. (3).[11] Competitive dealers instead set identical quotes such that, before an order arrives, $E\left[\Pi_t | M_t\right] = 0$ for each MM, by virtue of Bertrand competition.[12]

In both cases, dealers are subject to the constraint that their inventory has no expected drift at each point in time t. Hence, before an order arrives, quotes are set so the market is always cleared by balancing the expected flow of currency bought and sold. This implies that

$$E\left[Z_t | M_t\right] = (1 - l)\, q X_{1,t} + (1 - l)\,(1 - q)\, X_{2,t} + l X_t^{CB} = 0, \tag{4}$$

where $(-Z_t)$ is the instantaneous inventory position of the dealers. This assumption captures our earlier observations about the nature of MMs' activity in the currency markets.

At each time t, dealers set *potential* (or *reservation*) prices, i.e., quotes at which they are willing to buy, if a type 2 investor arrives, to sell, if a type 1 investor arrives, and to trade with a CB, if the monetary authority intervenes. When an investor's order hits the dealers' screens, the only equilibrium price is going to be the one resulting from the most recent transaction. We define the expected transaction price with investors as $S_t^* = q S_{1,t} + (1 - q) S_{2,t}$. Nonetheless, it is that declaration of intents by the dealers that constitutes the bid and offer quotes. Trading is not anonymous with respect to whether the incoming order is from investors or the CB, as

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[11] Amihud and Mendelson (1980) use a similar specification, but allow the MM to set prices so that his expected inventory position is inside some pre-specified bounds.

[12] This is also the case in many other sequential trading models (e.g., Glosten and Milgrom, 1985 and Madhavan, 1992, just to name a few) under the assumptions of risk-neutrality and no capital constraint for the MMs.

appears to be the case in most forex markets. However, MMs can distinguish the type of an arriving investor just from the sign and size of the submitted order.

We start by assuming full information ($F \tilde{~} N\left(f, \sigma_F^2 > 0\right)$, $E\left[f|I_t\right] = f$, and l is known to all market participants) and $l = 0$. We then solve the problem of both the monopolist and the competitive MMs in Proposition 1, using the convention that each incoming order is split evenly among all dealers quoting the same price, as in Saar (2000a). The resulting equilibrium prices serve as a benchmark to evaluate the impact of an active CB in the next subsection.

Proposition 1 *With full information and $l = 0$, the competitive (^{C}) MMs' reservation bid and ask quotes at time t, before an investor's order arrives, are*

$$S_{1,t} = S_{2,t} = {}^C S_t^* = \frac{f}{R} - \pi X^*, \tag{5}$$

where $X^* = q\overline{X}_1 + (1-q)\overline{X}_2$. Therefore, $S_{2,t} - S_{1,t} = 0$. The monopolist (^{M}) MM's reservation bid and ask quotes are instead given by

$$S_{1,t} = \left(\frac{f}{R} - \pi X^*\right) - \frac{\pi(1-q)}{2}\left(\overline{X}_1 - \overline{X}_2\right), \tag{6}$$

$$S_{2,t} = \left(\frac{f}{R} - \pi X^*\right) + \frac{\pi q}{2}\left(\overline{X}_1 - \overline{X}_2\right), \tag{7}$$

which in turn imply that

$$^M S_t^* = \frac{f}{R} - \pi X^* = {}^C S_t^* \tag{8}$$

and

$$S_{2,t} - S_{1,t} = \frac{\pi}{2}\left(\overline{X}_1 - \overline{X}_2\right). \tag{9}$$

We define the bid-ask spread as the difference between the prices dealers quote to type 2 and type 1 investors, $S_{2,t} - S_{1,t}$. In the remainder of the paper, we specify conditions on the model's parameters such that type 1 investors are always net sellers hitting the dealers' bid quotes $S_{1,t}$ and type 2 investors are always net buyers hitting the dealers' ask quotes $S_{2,t}$. For example, in Proposition 1 we have that $X_{2,t} > 0$ and $X_{1,t} < 0$ in both the monopoly and competitive scenarios if and only if $\overline{X}_1 > \overline{X}_2$, as we previously assumed. Under these conditions, we can interpret our spread as a wedge between the price at which the dealers are willing to sell GBP to type 2 investors and the price at which the dealers are willing to buy GBP from type 1 investors.

The expected competitive transaction price $^C S_t^*$ corresponds to the price we would observe if all investors arrived at the same time in a competitive market, i.e., to the market-clearing

price. Consequently, the no-expected drift condition of Eq. (4) is equivalent to the market-clearing condition in a competitive equilibrium. The price of GBP-denominated bonds is equal to their discounted future payoff minus a risk adjustment factor to induce risk-averse, USD-based investors to hold GBP assets. No spread emerges in this case, because of the assumption of full information in this otherwise frictionless forex market. A monopolist dealer uses his market power to extract positive rents from investors by charging a higher ask to net buyers and by paying a lower bid to net sellers. A positive spread ensues. The expected transaction price $^M S_t^*$ is nevertheless equal to $^C S_t^*$. These results do not depend on the relative magnitude of the investors' endowments of GBP, but only on whether there is trading in this economy $(\overline{X}_1 \neq \overline{X}_2)$.[13]

2.4   Central Bank

Monetary authorities frequently intervene in the forex market, to manage otherwise free-floating rates, to comply with international currency agreements, to serve macroeconomic agendas, or as a result of domestic political pressure.[14] In some circumstances, CBs have also acted in pursuit of purely speculative motives, as in the case of Bank Negara in the early 1990s. Lewis (1995) identified several common features in the interventions conducted by the Federal Reserve (Fed), Bank of Japan (BoJ), and Bundesbank (BuBa) between 1985 and 1987. First, most of those actions were aimed at preventing their domestic currencies from moving away from some target levels. Second, in a few significant cases the interventions went in the opposite direction with respect to the exchange rate. Third, although the interventions may have been announced to or observed by the market, the magnitude of the incoming CB orders was usually not, as also suggested by Goodhart and Hesse (1991).

Policy and *wealth-preservation* (or *speculative*) motives may be conflicting. Suppose in fact that a CB believes, based on superior information, the USD is fundamentally overvalued $(S_t < f)$. A sudden devaluation, however, could create excessive inflationary pressures. To attenuate those pressures, the CB could set an intermediate target level for the exchange rate, $\overline{S}$, between S_t and f, and sell some amounts of GBP to prevent the USDGBP from breaking its current trend too rapidly toward its long-term value. Given its knowledge of f, the CB action, if effective on S_t, is not profit-maximizing (as buying GBP would instead be) and leads to a reduction of its expected future wealth.

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[13]It is easy to show that, by plugging the equilibrium monopoly or competitive quotes in the investors' optimal demands (Eq. (2)), both $X_{1,t}$ and $X_{2,t}$ are equal to zero for $\overline{X}_1 = \overline{X}_2$. Therefore, when otherwise identical investors have the same initial endowments, no risk-sharing (hence no trading) occurs.

[14]Taylor (1995) offers an overview of the economics of official interventions.

In this paper, we model this potential trade-off in a parsimonious way by assuming that a price-taking CB chooses the net amount of foreign (domestic) currency to buy or sell, X_t^{CB} (B_t^{CB}), that, given the true probability of that trade to occur (l), minimizes the loss function

$$L\left(\overline{S}, \lambda\right) = \left[E\left(S_t^*|F_t\right) - \overline{S}\right]^2 - \lambda E\left(W_{T'}^{CB}|F_t\right), \tag{10}$$

in which $W_{T'}^{CB} = R\left(B_t^{CB} + \overline{B}_t\right) + F\left(X_t^{CB} + \overline{RES}_t\right)$, subject to the budget constraint

$$B_t^{CB} = -S_t^{CB} X_t^{CB}, \tag{11}$$

where F_t is the CB's information set at time t, $E\left(S_t^*|F_t\right) = qS_{1,t} + (1-q)S_{2,t}$ is the expected transaction price from trading between MMs and investors, $\tau \in (0, t)$ is when the most recent past intervention occurred, and $\overline{RES}_t = \overline{RES}_\tau + X_\tau^{CB}$ and $\overline{B}_t = \overline{B}_\tau + B_\tau^{CB}$ are the endowments of GBP and USD held by the CB at time t before intervening.[15]

The specification of Eq. (10) is similar in spirit to Stein (1989), Bhattacharya and Weller (1997), and Vitale (1999). The first component measures policy motives by the squared distance between $E\left(S_t^*|F_t\right)$ and $\overline{S}$.[16] $E\left(S_t^*|F_t\right)$ is the expected market-clearing exchange rate. At each time t the CB is myopic, i.e., like the investors, it does not expect to return to the market by T. Therefore, the CB's trading activity is controlled exogenously by the parameter l.[17] The second component of $L\left(\overline{S}, \lambda\right)$ incorporates wealth-preservation motives: Interventions are costly when CB actions are unprofitable from a speculative perspective. The parameter λ controls for the relevance of the ensuing potential trade-off between policy and speculation in $L\left(\overline{S}, \lambda\right)$. The budget constraint in Eq. (11) implies that each GBP trade is accompanied by an open-market trade in the opposite direction. In addition, as mentioned in Section 2.1, both f and $S_{T'}$ are independent of any CB action. From these assumptions it follows that, in our setting, interventions are always sterilized.

Nonetheless, our model does not exogenously impose a relationship between CB trades and the exchange rate, as, for example, in Naranjo and Nimalendran (2000). The market friction allowing interventions to be potentially effective in this economy is instead derived from the

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[15] Because we interpret the interval $[0, T]$ as a short period of time, we abstract from the issue of a CB being unable to trade because of lack of reserves by assuming the initial endowments $\overline{RES}_0$ and $\overline{B}_0$ are big enough so that $\overline{RES}_t > 0$ and $\overline{B}_t > 0$ before and after an intervention.

[16] The CB's loss function can be easily generalized to the case of a target band of fluctuation $(\overline{S}_L, \overline{S}_H)$ for the currency by specifying the policy component in Eq. (10) as $\left[E\left(S_t^*|F_t\right) - \frac{1}{2}\left(\overline{S}_L + \overline{S}_H\right)\right]^2$.

[17] This assumption allows us to abstract from the complex issue of analyzing the CB's endogenous intertemporal strategic behavior (explored, for example, by Cadenillas and Zapatero, 1999, 2000) and to make the problem of the monetary authority more tractable, albeit at the cost of less realism.

market-clearing condition (Eq. (4)) and the investors' optimal net demand for GBP (Eq. (2)). The CB in fact conjectures that the expected transaction price $qS_{1,t} + (1-q)S_{2,t}$ is equal to

$$E\left(S_t^*|F_t\right) = \frac{E\left[f|I_t\right]}{R} - \pi X^* + \pi L X_t^{CB}, \tag{12}$$

where $L = \frac{l}{1-l}$.[18] Hence, for a given $l > 0$, the optimal intervention schedule X_t^{CB} for each price level S_t^{CB} minimizes $L\left(\overline{S}, \lambda\right)$ subject to Eq. (12) and the wealth constraint of Eq. (11). We have the following result.

Proposition 2 *With full information and $l > 0$, the demand function of the CB is given by*

$$X_t^{CB} = \gamma \left[\overline{S} - \left(\frac{f}{R} - \pi X^*\right) + \frac{\lambda}{2\pi L}\left(f - RS_t^{CB}\right)\right], \tag{13}$$

where $\gamma = \frac{1}{\pi L}$.[19]

It is clear from Eq. (13) that, ceteris paribus for S_t^{CB}, the optimal intervention size declines for higher l. Intuitively, a lower l makes the threat of intervention less significant for the dealers, and makes a bigger X_t^{CB} necessary to move $E\left(S_t^*|F_t\right)$ toward $\overline{S}$. This property is consistent with Naranjo and Nimalendran (2000), where it is reported that BuBa's interventions are more frequent than the Fed's, but smaller in absolute dollar size. The CB's optimal demand function depends in an intuitive fashion on both the policy and wealth-preservation motives. The CB needs to buy (sell) GBP to push the expected transaction rate closer to the target level, if the difference between $\overline{S}$ and the competitive market-clearing price when $l = 0$ ($\frac{f}{R} - \pi X^*$), is positive (negative). If $\overline{S} > \frac{f}{R} - \pi X^*$, the CB is *chasing the trend*, attempting to induce a faster depreciation of S_t^* toward its long-term fundamental value, hence it buys GBP.[20] If instead $\overline{S} < \frac{f}{R} - \pi X^*$, the CB is *leaning against the wind*, attempting to resist S_t^*'s long-term trend, hence it sells GBP.[21] Furthermore, the CB buys more GBP if the expected net future value

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[18]Eq. (12) follows immediately from the market-clearing condition $E\left[Z_t|M_t\right] = 0$, the definition of $E\left(S_t^*|F_t\right)$, and the optimal demands for GBP by type i investors.

[19]The proof is straightforward from the F.O.C. of the constrained minimization of $L\left(\overline{S}, \lambda\right)$ w.r.t. X_t^{CB}. The S.O.C., $2\pi^2 L^2 > 0$, is always satisfied.

[20]If $\overline{S} > \frac{f}{R}$, the CB is *riding the wave*, i.e., is aggressively pursuing a depreciation of the dollar beyond the long-term risk-neutral rate $\frac{f}{R}$. These competitive devaluations, conducted with the purpose of remedying balance of payments problems, are explicitly prohibited by the IMF Article 4, Section 1.

[21]In Section 2.1 we assumed that $R > 1$ and $R_F = 1$ (or, more generally, $\frac{R_F}{R} < 1$). If we had assumed that $R < 1$ (or $\frac{R_F}{R} > 1$) insomuch that, when $l = 0$, the dollar was weaker than its long-term expected value ($S_t^* > E\left[S_{T'}\right] = f$), then the CB would be chasing the trend if $\overline{S} < \frac{f}{R} - \pi X^*$, and leaning against the wind if $\overline{S} > \frac{f}{R} - \pi X^*$. For simplicity, we ignore this possibility and concentrate only on the scenario in which $R > 1$.

(NFV) in dollars of that investment, $f - RS_t^{CB}$, is higher. The amount of GBP bought (or sold) by the CB also depends on the trade-off between policy and speculative motives. This trade-off is, not surprisingly, highest when the CB is trying to lean against the wind, in doing so reducing its expected future wealth.

The elasticity of investors' demand for GBP (via π) affects the magnitude of CB orders, consistently with the intuition of portfolio balance theories of CB intervention. Indeed, ceteris paribus for S_t^{CB}, if investors are more risk-averse (higher α), if there is more uncertainty surrounding the long-term exchange rate F (higher σ_F^2), or if investors have a higher expected endowment of GBP (higher X^*), then their demand for GBP is less elastic. Hence, a bigger increase (decrease) in the equilibrium prices is needed to induce them to hold more (less) GBP and push S_t^* toward a high (low) $\overline{S}$. Therefore, a bigger positive (negative) intervention is necessary.

3 The Full Information Case

In this section, we allow the CB to intervene $(l > 0)$. We then solve for equilibrium quotes for monopoly and dealership competition under the assumption of full information. Full information restricts the effectiveness of CB's actions to the portfolio balance channel where investors have to be compensated for having to hold more (or less) of the foreign currency than they would if $l = 0$.

We construct the equilibrium in three steps. We first assume that, in equilibrium, dealers can always distinguish whether the incoming order is from an investor or the CB, they can conjecture the investor's type from the size and sign of his order, and they use this knowledge to formulate their reservation prices for each potential arrival.[22] We further assume the CB conjectures an expression for the expected transaction price in its loss function $L\left(\overline{S}, \lambda\right)$. Second, we compute type 1 and type 2 investors' optimal demands for GBP and the optimal intervention at $S_{1,t}$, $S_{2,t}$, and S_t^{CB}. Finally, we show that the resulting investors' orders, given those prices, are indeed different, and that $E\left[S_t^*|F_t\right]$ is indeed equal to $qS_{1,t} + (1 - q) S_{2,t}$, confirming the MMs' and CB's initial guesses, as in classic fixed-point problems.

Full information also implies that all agents observe the trade executed by the CB. Available empirical evidence (in particular Goodhart and Hesse, 1991; Lewis, 1995; Peiers, 1997) seems to suggest otherwise. Nevertheless, we use this simplified scenario to explain the basic intuition for

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[22]This assumption is reasonable, because it captures two typical aspects of OTC currency markets: lack of anonymous trading and, consequently, price discrimination. For more on the trading relationship between the CB and MMs for the domestic currency, see Peiers (1997).

how dealers adjust their quotes in response to CB actions, and to understand why the adjustment does (or does not) induce a change in the bid-ask spread with respect to the benchmark of Eqs. (5) to (9). In Section 4 we introduce information asymmetry regarding some of the model's parameters and study the impact of interventions on the process of price formation when the signaling channel of effectiveness is active as well.

3.1 The monopolist dealer

Proposition 1 showed that a monopolist dealer widens an otherwise zero spread (in our frictionless market) to extract rents from the order flow. When we allow the CB to trade, the problem of the monopolist dealer is now

$$
\begin{aligned}
\max_{S_{1,t}, S_{2,t}, S_t^{CB}} & \quad (1-l)\, q X_{1,t} S_{1,t} + (1-l)\,(1-q)\, X_{2,t} S_{2,t} + l X_t^{CB} S_t^{CB} \\
s.t. \quad & E\left[Z_t | M_t\right] = (1-l)\, q X_{1,t} + (1-l)\,(1-q)\, X_{2,t} + l X_t^{CB} = 0 \\
& X_t^{CB} = \gamma \left[\overline{S} - \left(\tfrac{f}{R} - \pi X^*\right) + \tfrac{\lambda}{2\pi L}\left(f - R S_t^{CB}\right)\right] \\
& X_{i,t} = \tfrac{1}{\pi}\left(\tfrac{f}{R} - S_{i,t}\right) - \overline{X}_i \qquad i = \{1,2\}.
\end{aligned}
\tag{14}
$$

Equilibrium construction in this setting generates the following result.

Proposition 3 *With full information and $l > 0$, the monopolist MM's reservation bid and ask quotes at time t, before an investor's order arrives, are*

$$
S_{1,t} = \left(\frac{f}{R} - \pi X^*\right) - \frac{\pi(1-q)}{2}\left(\overline{X}_1 - \overline{X}_2\right) + \pi L X_t^{CB}
\tag{15}
$$

$$
S_{2,t} = \left(\frac{f}{R} - \pi X^*\right) + \frac{\pi q}{2}\left(\overline{X}_1 - \overline{X}_2\right) + \pi L X_t^{CB},
\tag{16}
$$

while the reservation exchange rate if a CB intervenes at t is given by

$$
S_t^{CB} = \omega_1 \frac{f}{r} + \omega_2 \overline{S} + \omega_3 \pi X^*,
\tag{17}
$$

where ω_1, ω_2, and ω_3 are defined in the Appendix. The resulting absolute spread is unchanged with respect to the benchmark of Eq. (9). The proportional spread does instead change, and is now equal to

$$
PS_t = \frac{S_{2,t} - S_{1,t}}{{}^M S_t^*} = \frac{\pi\left(\overline{X}_1 - \overline{X}_2\right)}{2\left(\frac{f}{R} - \pi X^* + \pi L X_t^{CB}\right)}.
\tag{18}
$$

Proposition 3 states that, in equilibrium, the monopolist dealer revises upward (downward) his quotes if there is a positive probability that the CB will intervene at time t buying (selling)

GBP. Although the absolute spread is unchanged, the proportional spread declines (increases). The quote revision is symmetric and given by

$$\Delta S_{i,t} = \Delta S_t^* = \pi L X_t^{CB} = \frac{\pi L}{2\pi L + \lambda R}\left(\overline{S} - \frac{f}{R}\right) + \frac{\pi}{2}X^*. \tag{19}$$

To explain the intuition for these results, assume that it becomes known to the dealer (or the CB announces) that from time t onward, with probability $l > 0$, an order $X_t^{CB} > 0$ might arrive. At the benchmark ($l = 0$) prices, this potential buy order creates an imbalance in the expected dealer's inventory by adding to the originally flat position a negative drift component. In order for the no-inventory condition (Eq. (4)) to be satisfied, the MM increases $S_{1,t}$ (to reduce the size of the expected incoming sales) and $S_{2,t}$ (to increase the size of the expected incoming purchases). Because at the revised quotes net buyers buy less and net sellers sell more, investors' net demand is now expected to be a sale. This allows the dealer to clear the market when the CB is expected to be a net buyer of GBP. Consequently, $\Delta S_{i,t} > 0$, the expected transaction price $^M S_t^*$ increases, and the proportional spread declines. Vice versa, if $X_t^{CB} < 0$, then $\Delta S_{i,t} < 0$, the expected transaction price decreases, and the proportional spread increases. Thus, in our setting, even a fully anticipated CB intervention does have an impact on quotes and PS_t, in the direction of its expected sign.[23]

As suggested by Eq. (19), CB trades are effective in moving S_t^* by an amount $\Delta S_{i,t}$ toward $\overline{S}$. In Figure 1 we plot the behavior of $^M S_t^*$ at different levels of l, for a CB chasing the trend.[24] In this example, $\Delta S_{i,t}$ induces an undershooting (overshooting) of $^M S_t^*$ with respect to $\overline{S}$ if the likelihood of intervention is relatively high (low). This occurs because, for higher (lower) l, the CB is expected to hit the dealer with a smaller (bigger) order. Therefore, the MM exploits his market power by charging a higher (lower) S_t^{CB}, thus making the intervention more (less) expensive, and reducing its endogenous magnitude. The ensuing needed adjustment in the bid and offer rates to clear the market is smaller (bigger). Furthermore, Eq. (19) implies that the price impact ΔS_t^* is higher, and the intervention more effective, when the coefficient of risk-aversion (α) and the volatility of the long-term value of GBP(σ_F^2) are higher or government action is less likely.[25] Indeed, in all those circumstances, the monopolist dealer needs a bigger quote revision to clear the market.

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[23] PS_t is computed using $^M S_t^*$, and not the conventional mid-quote, because the probability that an investor's order is of type 1 (q) is public information.

[24] In all the simulations that follow, the model's parameters were chosen to ensure the ratio between the expected absolute size of investors' and CB orders approximates available empirical estimates (roughly 12%).

[25] It is easy to show that, for any $\overline{S} < \frac{f}{R}$, $\frac{\partial \Delta S_t^*}{\partial \alpha} > 0$, $\frac{\partial \Delta S_t^*}{\partial \sigma_F^2} > 0$, and $\frac{\partial \Delta S_t^*}{\partial l} < 0$.

Eventually, when the intervention occurs, there is no additional impact on quotes and spreads. This is because, in full information, the market is strong-form efficient and CB actions do not affect investors, and dealers' beliefs about F. However, in our setting, trading is sequential and the market does not clear just once. Hence, S_t^* returns immediately to pre-intervention levels as soon as $l = 0$ again, because no new investor needs to revise his optimal GBP holdings. Therefore, the impact of CB intervention remains on quotes and transaction prices just as long as the threat of its arrival is present.

3.2 The competitive dealers

We now consider the case of dealers with no market power. Proposition 1 shows that, in this case, no spread arises in our frictionless currency market when $l = 0$. Does that conclusion still hold if $l > 0$? Along the lines of Sections 2.1 and 3.1, the equilibrium rates $S_{1,t}$, $S_{2,t}$, and S_t^{CB} are those which, given investors' and CB's optimal demands, satisfy both $E[\Pi_t | M_t] = 0$ and $E[Z_t | M_t] = 0$. These two restrictions are not sufficient to identify three reservation prices, hence we express the bid and offer quotes as functions of a free variable, S_t^{CB}. We then have the following proposition.[26]

Proposition 4 *With full information and $l > 0$, the competitive MMs' reservation bid and ask quotes as a function of S_t^{CB} at time t, before an investor's order arrives, are*

$$S_{1,t} = \left(\frac{f}{R} - \pi X^*\right) + \pi L X_t^{CB} - \left[\frac{\pi}{2}(1-q)\left(\overline{X}_1 - \overline{X}_2\right) - \frac{\pi}{2}(\Gamma)\right] \tag{20}$$

$$S_{2,t} = \left(\frac{f}{R} - \pi X^*\right) + \pi L X_t^{CB} + \left[\frac{\pi q}{2}\left(\overline{X}_1 - \overline{X}_2\right) - \frac{\pi q}{2(1-q)}(\Gamma)\right], \tag{21}$$

where $\Gamma > 0$ is given in the Appendix. The spread between the offer and the bid quotes is then

$$S_{2,t} - S_{1,t} = \frac{\pi}{2}\left(\overline{X}_1 - \overline{X}_2\right) - \frac{\pi}{2(1-q)}(\Gamma). \tag{22}$$

There exists a price $S_t^{CB} = S_t^{CB}(*)$ such that, for $X_t^{CB} \neq 0$, the above spread is equal to zero:

$$S_t^{CB}(*) = S_{1,t} = S_{2,t} = \left(\frac{2\pi L}{2\pi L + \lambda R}\right)\overline{S} + \left(\frac{\lambda}{2\pi L + \lambda R}\right)f. \tag{23}$$

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[26]For tractability, we again impose that each incoming order, hence X_t^{CB} as well, is split evenly among MMs. Empirical evidence on intraday official intervention activity is scarce. This assumption is nevertheless consistent with data on the intraday transactions by the Swiss National Bank on the Swiss Franc (one of the most actively traded currencies), described in Fisher and Zurlinden (1999), Payne and Vitale (2001), and Pasquariello (2002).

The equilibrium bid and ask quotes differ from those reported in Proposition 1. Both $S_{1,t}$ and $S_{2,t}$ depend on three components. The first is $\frac{f}{R} - \pi X^*$, the competitive benchmark exchange rate for $l = 0$ (Eq. (5)). The remaining two are revisions induced by the positive likelihood of the CB arriving, but only one of them, $\pi L X_t^{CB}$, affects symmetrically both bid and offer quotes. This is the adjustment needed to clear the market when $X_t^{CB} \neq 0$ and $l > 0$, as in the monopoly scenario. If, for example, $X_t^{CB} < 0$, MMs symmetrically decrease bid and ask prices to achieve two related objectives. First, the resulting lower $^C S_t^*$ is closer to $\overline{S}$ and induces a smaller intervention. Second, investors buy more GBP than they would if $l = 0$, thus facilitating the dealers' efforts to have a driftless expected inventory.

Most interestingly, $l > 0$ induces a wedge between bid and offer quotes. Indeed, the bid-ask spread (Eq. (22)) is generally nonzero, unless either $S_t^{CB} = S_t^{CB} (*)$ or $l = 0$. To interpret this result, recall that in our frictionless market the benchmark spread is zero when $l = 0$ (Proposition 1). Therefore, we can think of Eq. (22) as the change in an otherwise positive spread resulting from other market frictions, such as inventory control, order processing costs, etc. When $l > 0$, a wedge arises because competitive pressure among dealers obliges them to pass all extra revenues (costs) from the potential arrival of a positive (negative) CB order onto investors. The elasticity of their demand for GBP then determines whether this effort eventually makes that wedge positive or negative.[27]

Not surprisingly, as $\Gamma > 0$, this wedge is always lower than the absolute spread set by a monopolistic dealer, $\frac{\pi}{2} \left(\overline{X}_1 - \overline{X}_2 \right)$, because competition erodes the MMs' ability to be compensated for providing liquidity. Therefore, the more binding is the no-profit condition, $E [\Pi_t | M_t] = 0$, the smaller is the competitive spread with respect to $\frac{\pi}{2} \left(\overline{X}_1 - \overline{X}_2 \right)$. Consequently, for an exogenous X_t^{CB}, the terms $\frac{\pi}{2} (\Gamma)$ and $-\frac{\pi q}{2(1-q)} (\Gamma)$ in $S_{1,t}$ and $S_{2,t}$, respectively, can be interpreted as the effect of the binding no-profit condition on the monopolistic quotes of Section 3.1. However, X_t^{CB} is **not** exogenous in our model. Hence, the wedge of Eq. (22) also depends on the parameters controlling for the CB's optimal intervention schedule.

To clarify the interpretation of the multiple equilibria of Proposition 4, we parametrize the model for different target levels $\overline{S}$ and plot the resulting spread and X_t^{CB} as functions of l in Figure 2. We start by setting the free parameter $S_t^{CB} = \overline{S}$. Later, we consider the robustness of the analysis to this assumption. Here we comment only on the case of a CB chasing the trend (Figure 2a). The same reasoning however applies for a CB leaning against the wind (Figure 2b). The bid-ask spread is positive, but declining for increasing l as is the magnitude of X_t^{CB}. As

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[27]The apparent arbitrage opportunity offered by a negative wedge cannot be exploited by the investors, since they can trade with the dealers only once.

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we suggested above, both bid and ask quotes go up in response to $X_t^{CB} > 0$. The dealers need to receive bigger sell orders and smaller buy orders from the investors to clear the market. This is achieved by revising the ask more than the bid, and a positive spread arises. The revision of quotes is asymmetric because the selected S_t^{CB} (fixed by assumption at $\overline{S}$) cannot be reduced and, at the given endowment ratio ($\frac{\overline{X}_1}{\overline{X}_2} = 3$), type 1 investors are bigger "net sellers" than otherwise identical type 2 investors are "net buyers." The intensity of these effects is reduced for increasing l, as it implies smaller CB orders.

Figure 3a displays the spread and MMs' net revenues from type 1 and type 2 investors and the CB for increasing values of $\overline{X}_1$, hence of the endowment ratio. For higher $\overline{X}_1$, type 1 investors are more sensitive to changes in $S_{2,t}$. Thus, the spread increases, as a smaller $\Delta S_{1,t}$ (with respect to $\Delta S_{2,t}$) is now needed to induce them to sell and to ensure that both $E\left[\Pi_t | M_t\right] = 0$ and $E\left[Z_t | M_t\right] = 0$. It is worth observing that, in this example, $[q X_{1,t} S_{1,t} + (1-q) X_{2,t} S_{2,t}] < 0$: The competitive dealers are incurring an expected net loss versus the investors to compensate for the gains they expect to earn versus the CB. In other words, the CB is transferring rents to investors to induce them to hold more GBP, but competitive dealers are unable to retain even part of these rents.

Finally, we relax the assumption that $S_t^{CB} = \overline{S}$. We plot in Figure 3b the wedge of Eq. (22) for different values of S_t^{CB} centered around $S_t^{CB}(*)$. The wedge is negative for any $S_t^{CB} > S_t^{CB}(*)$. Why is this occurring? The model's parameters imply that wealth-preservation is weighted less than policy in $L\left(\overline{S}, \lambda\right)$: X_t^{CB} is in fact insensitive to higher S_t^{CB}, although the NFV of buying GBP $(f - R S_t^{CB})$ is declining. However, for higher S_t^{CB}, potential revenues from the CB are higher as well. Competitive MMs then need to make the investors more "net sellers" to pass these increasing revenues to them, while still clearing the market. To do so, they further increase the bid quote, as type 1 investors are the needed net sellers. Hence, $\Delta S_{1,t} > \Delta S_{2,t}$, and a negative spread is observed. Along the same lines, $S_{2,t} - S_{1,t} < 0$ for any $S_t^{CB} < S_t^{CB}(*)$ if the CB were leaning against the wind.

3.3   A first look at the effectiveness of CB intervention

The previous subsections showed that CB intervention has a significant impact on the process of intraday price formation in the forex markets, even assuming full information, when investors' orders do not arrive simultaneously to the dealers' screens. Indeed, one of the novelties of this study is extending the classic portfolio balance setting (with imperfect substitutability of domestic and foreign currency-denominated assets), previously restricted to stylized markets clearing only once, to sequential trading.

CB interventions have both transient and persistent effects on the exchange rate depending on the likelihood of the incoming order being from the CB. Prices are initially revised because, when dealers learn that $l > 0$, instantaneous market clearing forces them to effectively pass the incoming CB order to risk-averse investors at a premium. This adjustment is temporary if l immediately reverts to zero, since compensation is no longer needed for new investors. The effect is instead persistent if the threat of future CB trades is significant and credible (i.e., for as long as $l > 0$), since those investors have to be compensated for possibly having to rebalance their optimal portfolios. The effects of CB actions on equilibrium quotes also depend on the degree of market power held by the currency dealers. Propositions 3 and 4 imply that the expected transaction price induced by $l > 0$ is given by

$$^{M}S_t^* = \left(\frac{f}{R} - \pi X^*\right) + \frac{\pi L}{2\pi L + \lambda R}\left(\overline{S} - \frac{f}{R}\right) + \frac{\pi}{2}X^* \tag{24}$$

in the case of a monopolist MM, and by

$$^{C}S_t^* = \overline{S} + \frac{\lambda}{2\pi L}\left(f - RS_t^{CB}\right) \tag{25}$$

in the case of dealership competition. We measure the relative effectiveness of interventions under the two regimes by computing $EM_t = \left(^{M}S_t^* - \overline{S}\right)^2 - \left(^{C}S_t^* - \overline{S}\right)^2$. Positive values for EM_t indicate that the threat of the arrival of CB trades pushes S_t^* closer to $\overline{S}$ in the competitive scenario of Section 3.2. We then have the following proposition.

Proposition 5 *With full information and $l > 0$, CB interventions are maximally effective when $S_t^{CB} = \frac{f}{R}$. CB interventions are always more effective for competitive MMs if $\lambda = 0$. When instead $\lambda > 0$, the same is true only if the absolute NFV of currency trading for the CB is "small" and/or if λ is "small."*

Proof. See the Appendix, where we also show that, if λ is "small" or if there is no trade-off between wealth-preservation and policy motives, the CB is always better off when dealers compete for the incoming trade, i.e., $L^C\left(\overline{S}, \lambda\right) < L^M\left(\overline{S}, \lambda\right)$. ∎

That the effectiveness of intervention is generally hindered by GBP trading being a positive NFV decision should not be surprising. Indeed, if wealth-preservation is conflicting with achieving $\overline{S}$, that trade-off shifts downward the CB's optimal intervention schedule for each S_t^{CB}. If instead wealth-preservation and policy motives reinforce each other, the resulting S_t^* may overshoot $\overline{S}$. In addition, interventions tend to be more effective when dealers' market power is minimal. When market power is significant, the MMs' quest for profit maximization prevents interventions from

being fully effective. The monopolist dealer does not adjust his quotes completely, to extract some rents from the CB or to pass most of the costs of its intervention onto investors. Consequently, absolute X_t^{CB} is smaller and the intervention is less effective. Competition instead induces the dealers to transfer expected costs and revenues from CB actions fully to investors. Hence, quotes' revisions are more substantial.

Speculative motives also affect the impact of interventions on the exchange rate. Proposition 5 states that, when $\lambda = 0$, intervention is unequivocally more effective in the competitive scenario. In that case, the CB is a pure price-manipulator interested in managing the currency at any cost, thus offering the monopolist MM more opportunities to profit from its activity by not fully adjusting S_t^* toward $\overline{S}$. When instead $\lambda > 0$, there are some circumstances in which the reverse might be true (and $EM_t < 0$). Proposition 5 tells us this could occur when wealth-preservation is "very important," because it is "very profitable" to trade currencies and/or CB's loss function is "very sensitive" to its final wealth $W_{T'}^{CB}$.

Market power may have an economically significant influence on the effectiveness of CB intervention. In Figure 4 we plot S_t^* and X_t^{CB} as functions of l for a CB chasing the trend under monopoly and competition. In the latter case, we assume that $S_t^{CB} = S_t^{CB}(*)$. For higher l, the expected transaction price $^C S_t^*$ converges to the target level $\overline{S}$, although with some initial overshooting. The intervention is much less effective in a market with a monopolist dealer: $^M S_t^*$ undershoots $\overline{S}$ and never approaches it. At the same time, the absolute magnitude of X_t^{CB} is greater in the competitive scenario, but the difference shrinks when the arrival of such order is more likely. This occurs because $S_t^{CB}(*) < \omega_1 \frac{f}{r} + \omega_2 \overline{S} + \omega_3 \pi X^*$, hence the opportunity cost of not trading GBP is higher. In other words, when dealers compete for the incoming trade, and l and S_t^{CB} are low, wealth-preservation reinforces the CB's resolve to weaken the USD. Overshooting of $^C S_t^*$ with respect to $\overline{S}$ may ensue. When instead dealers' market power is significant, $f - R S_t^{CB}$ is smaller and may even become negative, thus inducing a trade-off between speculation and the pursuit of $\overline{S}$. Consequently, $^M S_t^*$ undershoots $\overline{S}$. Empirical evidence on the relationship between the effectiveness of intervention and dealers' market power is scarce and mostly anecdotal. There is nonetheless a consensus that smaller, less frequent, and less successful interventions are observed for less intermediated exchange rates of emerging economies.[28]

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[28]See, for example, Brown (2000) and Chancellor (2000) for anecdotal evidence on the actions of Asian CBs during the turmoil of 1997 and 1998, and Moloney (2000) on interventions by G-7 CBs over the past 25 years.

4 Information Asymmetry

We have proceeded so far in a full information setting. This assumption prevented CB intervention from having any information content. In this section we close this gap by introducing uncertainty about $\overline{S}$, the CB's objective, f, the mean of the long-term exchange rate (F), and l, the likelihood of intervention.[29] In particular, we examine a stylized case in which all those forms of uncertainty interact, and study their combined impact on quotes and spreads with respect to the benchmark cases of no intervention (Proposition 1) and intervention with full information (Section 3).

To that purpose, we redefine M_t, I_t, and F_t, the information sets available to dealers, investors, and the CB, respectively. We assume that I_t and F_t contain all past transaction prices $S_{i,t-j}$ and orders $X_{i,t-j}$, while M_t also includes CB trades before time t.[30] Because the MMs do not maximize expected utility of future wealth, they do not need to formulate beliefs about f. However, being in the business of making and clearing the market, they need to learn investors' beliefs. For simplicity, we assume that dealers know exactly those beliefs, and so does the CB, but still formulate their own to estimate the sign and magnitude of X_t^{CB}.[31] As a result of information asymmetry about $\overline{S}$, f, and/or l, CB actions can surprise the market in their direction and size (thus being *unexpected*, along the lines of Naranjo and Nimalendran, 2000) and/or for their timing (thus being *unannounced*). This allows us to study the effectiveness of CB interventions on the exchange rate in terms of its speed of adjustment toward, proximity to, and persistence around the target level.

4.1 The extended model

We start by assuming that at $t = 0$ nature first chooses f (the mean of $F \tilde{} N(f, \sigma_F^2 > 0)$), between f_H and f_L with probability p_f and $(1 - p_f)$, respectively, then picks $\overline{S}$, between $\overline{S}_H$ and $\overline{S}_L$, and eventually l. If f_H (f_L) occurs, then $\overline{S} = \overline{S}_H$ ($\overline{S} = \overline{S}_L$) with probability ψ known to all market participants. If $\psi > \frac{1}{2}$, then f and $\overline{S}$ are positively correlated: the pairs $(f_H, \overline{S}_H)$ and $(f_L, \overline{S}_L)$

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[29]CBs may not declare explicitly the degree of resolution of their trade-off between policy and wealth-preservation (λ) as well. We explored this issue in a previous version of this paper, and found its implications for quotes and spreads similar to those analyzed in Sections 4 and 5.

[30]This assumption makes the myopia of investors and the CB less restrictive, since it implies that, at each point in time t, the new incoming investor will have different, more precise beliefs than those of the investors who preceded him. In turn, his demand and the MMs' ensuing quotes may provide the CB with new incentives to intervene, regardless of its past actions.

[31]Therefore, Proposition 2 implies that CB's optimal intervention schedule is now given by $X_t^{CB} = \gamma \left[\overline{S} - \left(\frac{E[f|I_t]}{R} - \pi X^* \right) + \frac{\lambda}{2\pi L} \left(f - R S_t^{CB} \right) \right]$.

are more likely than $(f_H, \overline{S}_L)$ and $(f_L, \overline{S}_H)$. If $\psi < \frac{1}{2}$, then f and $\overline{S}$ are negatively correlated. Finally, if $\psi = \frac{1}{2}$, f and $\overline{S}$ are uncorrelated: for a given f, $\overline{S}_H$ and $\overline{S}_L$ are equally likely. Hence, the parameter ψ can be interpreted as a measure of the consistency of the CB's intervention policy with the long-term behavior of the exchange rate or, alternatively, of the uncertainty surrounding its activity.

The CB, although aware of $\overline{S}$ and l, is informed about f just with probability v. This implies that not in all circumstances CB trades may be informative about f: With probability $(1 - v)$, X_t^{CB} depends on $E\left[f|F_t^U\right]$, where F_t^U is the information set of the *uninformed* CB.[32] Investors and MMs do not observe any of the variables chosen by nature, but form beliefs about p_f, $p_{\overline{S}}$, and l. Dealers formulate beliefs about f to forecast X_t^{CB}. Consistently with available (albeit scarce) empirical evidence (e.g., Fischer and Zurlinden, 1999), in our setting all dealers observe the sign and size of the most recent CB transaction. Vice versa, as in most currency markets (e.g., Goodhart and Hesse, 1991; Lewis, 1995), investors only know if an intervention occurred, but do not observe that trade. This lack of transparency slows down their learning about f. We accommodate this circumstance by assuming that investors observe the most recent transaction after an intervention at time t $(X_{i,t+1})$ and update their beliefs about f from the sign of the observed quote revision $(S_{i,t+1} - S_{i,t-j})$. This form of order flow uncertainty is between dealers and investors (and not among dealers, as in Evans and Lyons, 2001) and has not been previously explored by the currency microstructure literature.[33]

For sake of simplicity, we assume that λ is "small enough" so that wealth-preservation is not the CB's dominating concern, and that $\overline{S}$ and f control the sign and magnitude of X_t^{CB}, respectively, regardless of l. In particular, at the given λ, the realization f_H (f_L) is such that the informed CB's expected NFV of investing in GBP is positive (negative). This assumption *de facto* restricts that CB to four distinct transactions. When both $\overline{S}$ and f are high (low), there is no trade-off between policy and wealth-preservation in $L\left(\overline{S}, \lambda\right)$, since they both push the CB to buy (sell) GBP. The resulting intervention is then endogenously big and positive (negative), $_BX_t^{CB} > 0$ $(_BX_t^{CB} > 0)$. For low (high) values of $\overline{S}$ and high (low) values of f, the small λ ensures that the resulting trade-off between policy and speculation is resolved in favor of the former, and that eventually the CB sells (buys) small amounts of GBP, $_SX_t^{CB}$.[34] Hence, when

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[32] The information set of the *informed* CB is then given by $F_t^I = \left\{F_t^U, f\right\}$. Furthermore, we impose the logic restriction that the uninformed CB cannot use its knowledge of $\overline{S}$ to infer f, i.e., that $E\left[f|F_t^U\right]$ is given.

[33] One exception is the empirical study of Peiers (1997), who applies Granger causality tests to DEMUSD quotes and shows that Bundesbank trades are revealed first to dealers and then to the general public.

[34] If, for example, $\overline{S} = \overline{S}_L$ and $f = f_H$, the CB leans against the wind (as $\overline{S}_L < \frac{f_H}{R} - \pi X^*$) by selling GBP $(X_t^{CB} < 0)$; however, when $f = f_H$, the NFV of investing in GBP is positive, inducing the CB to sell less of it.

ψ is high (low), f is positively (negatively) correlated with $\overline{S}$, the trade-off in $L\left(\overline{S}, \lambda\right)$ is more (less) significant and the informed CB more likely to intervene with a big (small) order.

What is the optimal intervention strategy of the uninformed CB? We focus on a specific *pooling* equilibrium in which the uninformed CB, aware of $\overline{S}$ but not f, always buys (sells) GBP by the amount $_SX_t^{CB} > 0$ ($_SX_t^{CB} < 0$) when $\overline{S} = \overline{S}_H$ ($\overline{S} = \overline{S}_L$). This mimicking behavior is interesting to us because it prevents interventions from being fully revealing of CB's information about f. Proposition 6 summarizes the resulting optimal actions by the CB.

Proposition 6 *The following intervention schedules of the informed ($^IX_t^{CB}$) and uninformed ($^UX_t^{CB}$) CB,*

$$^IX_t^{CB} = \begin{cases} _BX_t^{CB} > 0 & if \ \overline{S} = \overline{S}_H \ \& \ f = f_H \\ _SX_t^{CB} > 0 & if \ \overline{S} = \overline{S}_H \ \& \ f = f_L \\ _SX_t^{CB} < 0 & if \ \overline{S} = \overline{S}_L \ \& \ f = f_H \\ _BX_t^{CB} < 0 & if \ \overline{S} = \overline{S}_L \ \& \ f = f_L \end{cases} \tag{26}$$

and, for a given $E\left[f|\mathcal{F}_t^U\right]$,

$$^UX_t^{CB} = \begin{cases} _SX_t^{CB} > 0 & if \ \overline{S} = \overline{S}_H \ \forall \ f \\ _SX_t^{CB} < 0 & if \ \overline{S} = \overline{S}_L \ \forall \ f, \end{cases} \tag{27}$$

respectively, are optimal when MMs use the pricing schedule

$$S_t^{CB} = \begin{cases} 0 & if \ X_t^{CB} < 0 \ \& \ X_t^{CB} \neq \ _SX_t^{CB} \\ \infty & if \ X_t^{CB} > 0 \ \& \ X_t^{CB} \neq \ _SX_t^{CB} \\ (0, \infty) & otherwise \end{cases} \tag{28}$$

and a set of restrictions on the model's parameters, reported in the Appendix, apply.

Intuitively, those restrictions ensure that policy motives are significant enough to determine the direction of CB intervention. The extreme pricing hypothesis of Eq. (28) guarantees that the uninformed CB finds optimal to pool with the informed one by choosing $_SX_t^{CB}$. In fact, S_t^{CB} in Eq. (28) is "too low" ("too high") when $^UX_t^{CB}$ is negative (positive).

4.2   Beliefs and beliefs' revisions

Dealers choose potential prices and investors formulate their demands for GBP based on their beliefs about f, $\overline{S}$, and l. We now describe how these beliefs are updated, starting with the

dealers. At each point in time t, before trading, MMs' beliefs about f and $\overline{S}$ are given by $E[f|M_t] = q_{f,t}f_H + (1 - q_{f,t})f_L$ and $E[\overline{S}|M_t] = q_{\overline{S},t}\overline{S}_H + (1 - q_{\overline{S},t})\overline{S}_L$, respectively, where $q_{f,t} = \Pr\{f_H|M_t\}$ and $q_{\overline{S},t} = \Pr\{\overline{S}_H|M_t\} = q_{f,t}\psi + (1 - q_{f,t})(1 - \psi)$; MMs' prior about l at time t is

$$q_{l,t} = E[l|M_t] = \int_0^1 l f_{M_t}(l)\, dl, \tag{29}$$

where $f_{M_t}(l)$ is their prior distribution of l given M_t. We assume that all priors are updated according to Bayes' Rule every time a new order arrives to the market. The ensuing Proposition 7 summarizes how dealers' beliefs $q_{l,t}$, $q_{f,t}$, and $q_{\overline{S},t}$ evolve into their posteriors $p_{l,t}$, $p_{f,t}$, and $p_{\overline{S},t}$.

Proposition 7 *The arrival of investors' (CB's) orders at time t induces a downward (upward) revision of the MMs' prior $q_{l,t}$ into $p_{l,t}$, where*

$$p_{l,t} = \begin{cases} E[l|M_t] - \frac{Var[l|M_t]}{1 - E[l|M_t]} < E[l|M_t] & \text{if } X_{i,t} \text{ arrives} \\ \begin{cases} E[l|M_t] + \frac{Var[l|M_t]}{E[l|M_t]} > E[l|M_t] & \text{if } _SX_t^{CB} \text{ arrives} \\ l & \text{if } _BX_t^{CB} \text{ arrives.} \end{cases} \end{cases} \tag{30}$$

MMs use $p_{l,t}$ in their reservation quotes at time t and as their best prior at time $t+1$ ($q_{l,t+1} = p_{l,t}$). The sign of an intervention always fully reveals $\overline{S}$. The arrival of $_BX_t^{CB} > 0$ or $_BX_t^{CB} < 0$ fully reveals f (and l) to the MMs. If $_SX_t^{CB} > 0$ arrives, then MMs update their beliefs about f according to:

$$q_{f,t+1} = p_{f,t} = \frac{q_{f,t}(1 - v)}{q_{f,t}(1 - v) + (1 - q_{f,t})v}, \tag{31}$$

while, if $_SX_t^{CB} < 0$ arrives,

$$q_{f,t+1} = p_{f,t} = \frac{q_{f,t}v}{q_{f,t}v + (1 - q_{f,t})(1 - v)}. \tag{32}$$

According to Proposition 7, only the CB trades, because potentially informative, may induce a permanent revision of MMs' expectations about f. This is the information asymmetry channel of effectiveness of intervention. Proposition 7 also states that the arrival of $X_{i,t}$ ($_SX_t^{CB}$) reduces (increases) the perceived likelihood of a future CB action. The effect of the potential arrival of an investor or the CB on MMs' beliefs is crucial in our model. Dealers set reservation prices conditional on the arrival of an investor of type 1, an investor of type 2, or the CB. The likelihood of such arrivals is, by assumption, independent from those quotes. If an investor arrives, then the MMs are induced to believe that an intervention is less likely, and $p_{l,t} = E[l|M_t, X_{i,t}] < q_{l,t}$.

If instead $_SX_t^{CB}$ arrives, $p_{l,t} = E\left[l|M_t, _SX_t^{CB}\right] > q_{l,t}$. Therefore, the prices quoted by the MMs already discount that information, i.e., are computed based on $p_{l,t}$ and not $q_{l,t}$. These updates depend on the degree of dispersion in the dealers' priors about l, $Var\left[l|M_t\right]$. We can interpret $Var\left[l|M_t\right]$ as a measure of the CB's credibility. Widely dispersed beliefs around l, for a given $E\left[l|M_t\right]$, induce a bigger impact of the order flow on beliefs' revisions. The arrival of $_BX_t^{CB}$, by fully revealing f and $\overline{S}$, induces $p_{l,t} = l$, for the magnitude of the intervention depends on its likelihood as well. However, because of the mimicking behavior by the uninformed CB (Proposition 6), observing $_SX_t^{CB}$ does not fully reveal l. The dealers then use Eq. (30) to generate a new posterior for it.[35]

We now turn to how investors revise their beliefs about f when the CB intervenes. In our economy, investors observe every past transaction between investors and MMs; furthermore, they are aware that an intervention may have actually occurred, but not of its sign and magnitude. Therefore, only transaction prices can convey (albeit noisy) information about f to those otherwise less informed agents. If we define $^Iq_{f,t} = \Pr\{f_H|I_t\}$ as the investors' priors for p_f, and $^Ip_{f,t}$ as their posteriors, the following proposition applies.

Proposition 8 *When an intervention occurs at time t, investors observe the positive (negative) change in the first transaction price $S_{i,t+1}$ after the intervention and revise their beliefs about f at $t+2$ according to*

$$
^Iq_{f,t+2} = \ ^Ip_{f,t+1} = \begin{cases} \dfrac{^Iq_{f,t}[\psi+(1-\psi)v]}{^Iq_{f,t}[\psi+(1-\psi)v]+\left(1-\ ^Iq_{f,t}\right)[(1-\psi)(1-v)]} & if\ \Delta S_{i,t+1} > 0 \\[4mm] \dfrac{^Iq_{f,t}[(1-\psi)(1-v)]}{^Iq_{f,t}[(1-\psi)(1-v)]+\left(1-\ ^Iq_{f,t}\right)[\psi+(1-\psi)v]} & if\ \Delta S_{i,t+1} < 0, \end{cases} \tag{33}
$$

where $\Delta S_{i,t+1} = S_{i,t+1} - S_{i,t-j}$ is the revision in the bid ($i=1$) or ask ($i=2$) price, and $t-j$ is when the latest $X_{i,t-j}$ arrived.

Investors revise upward (downward) their expectations about f using Bayes' Rule if there is a positive (negative) drift in the most recent transaction price at the bid or at the offer($S_{i,t+1}$), after the intervention occurred. This update structure is less refined than the one MMs rely upon after observing X_t^{CB}. Consequently, dealers and investors may disagree on f ($p_{f,t+1} \neq\ ^Ip_{f,t+1}$) even after this trade occurred.

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[35]In both cases, we further assume that the resulting $p_{l,t}$ becomes MMs' best prior for the likelihood of the next order coming from the CB, to reflect the fact (reported, for example, in Fischer and Zurlinden, 1999) that, following a CB trade, the likelihood of future interventions tends usually to decline.

5 A Particular Day in the Currency Market

Armed with these results, we finally investigate how dealers with different market power revise their quotes when investors' orders and CB interventions arrive at our stylized forex market with information asymmetry, and examine the equilibrium behavior of S_t^* during the interval $[0, T]$. To do so, we specify the following sequence of events (stages) over the trading day:

0. At $t = 0$, MMs set potential bid and ask quotes using their initial beliefs for f, $\overline{S}$, and l. If $q_{l,0} > 0$, these quotes are revised with respect to the benchmark of Proposition 1.

1. Order flow from investors $(X_{i,t})$ arrives. In response to it, MMs revise their priors about l according to Proposition 7, thus quotes and spreads with respect to stage 0.

2. When the CB eventually intervenes, both MMs and investors are informed of its arrival. However, MMs observe the sign and magnitude of X_t^{CB}, and update their beliefs about $\overline{S}$, f, and l (as described in Section 4.2) only if the actual intervention is unexpected ($\overline{S} \neq E\left[\overline{S}|M_t\right]$ and/or $f \neq E\left[f|M_t\right]$), or unannounced ($l \neq E\left[l|M_t\right]$); next, they execute it at a price S_t^{CB} incorporating all the resulting revealed information. This information is then discounted in their reservation bid and ask quotes for any future incoming order.

3. The first investor's order coming to the market after the intervention is processed at the new bid or ask price. After this trade occurs, investors observe the corresponding transaction price change $\Delta S_{i,t+1}$ and revise $E\left[f|I_t\right]$ according to Proposition 8.

4. Incoming investors use their new beliefs to buy/sell GBP. Dealers use the investors' new $E\left[f|I_{t,\Delta S_{i,t+1}}\right]$ to update their potential bid and ask quotes with respect to stage 3.

5. All future orders $X_{i,t}$ induce additional revision only of MMs' priors for l.

In the remainder of this section, we solve for (and simulate) the process of intraday price formation implied by this sequence of events for a monopolist and competitive MMs.[36]

5.1 The monopolist dealer

Under information asymmetry, the monopolist MM's problem at each point in time $t_n \in [0, T]$ over the sequence of stages described above is similar to the one reported in Eq. (14), with

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[36] As in Section 3, under both circumstances we impose similar sets of restrictions to the model's parameters (and agents' beliefs) to interpret the wedge $S_{2,t} - S_{1,t}$ as a bid-ask spread.

$E\left[f|M_{t_n}\right]$, $E\left[\overline{S}|M_{t_n}\right]$, and $E\left[l|M_{t_n}\right]$ replacing f, $\overline{S}$, and l, the investors' X_{i,t_n} depending on $E\left[f|I_{t_n}\right]$, and the informed CB's $X_{t_n}^{CB}$ equal to $\gamma\left[\overline{S}-\left(\frac{E[f|I_{t_n}]}{R}-\pi X^*\right)+\frac{\lambda}{2\pi L}\left(f-RS_{t_n}^{CB}\right)\right]$. We start by assuming that at $t_0=0$ the CB announces it may intervene in the future, but not its objectives.[37] In response, the MM formulates priors for l and f ($q_{l,0}$ and $q_{f,0}$) and computes $q_{\overline{S},0}$ using ψ: $q_{\overline{S},0}=q_{f,0}\psi+(1-q_{f,0})(1-\psi)$. Investors formulate their own beliefs about p_f ($^I q_{f,0}$). As in Section 3, we focus on the quote revision induced at each stage with respect to the benchmark of Proposition 1 ($l=0$).

At stage 0, before any order arrives, the dealer sets the following bid and ask quotes:

$$S_{1,0} = S_{1,0}^{l=0}+\Delta^M S_0^* = S_0^{*,l=0}-\frac{\pi(1-q)}{2}\left(\overline{X}_1-\overline{X}_2\right)+\Delta^M S_0^* \tag{34}$$

$$S_{2,0} = S_{2,0}^{l=0}+\Delta^M S_0^* = S_0^{*,l=0}+\frac{\pi q}{2}\left(\overline{X}_1-\overline{X}_2\right)+\Delta^M S_0^*, \tag{35}$$

where $S_0^{*,l=0}=\frac{E[f|I_0]}{R}-\pi X^*$, $S_{i,0}^{l=0}=S_{i,t}$ of Proposition 1, and $\Delta^M S_0^*=\pi E\left[L|M_0\right]E\left[X_0^{CB}|M_0\right]$ is given by

$$\Delta^M S_0^* = \frac{\pi E\left[L|M_0\right]E\left[\overline{S}|M_0\right]}{2\pi E\left[L|M_0\right]+\lambda R}-\frac{1}{2}\left\{S_0^{*,l=0}\right\}+\frac{\lambda R}{4\pi E\left[L|M_0\right]+2\lambda R}\left\{\frac{E\left[f|M_0\right]}{R}\right\}. \tag{36}$$

The monopolist MM uses both $E\left[f|I_0\right]$ and $E\left[f|M_0\right]$ to compute $E\left[X_0^{CB}|M_0\right]$. Therefore, he increases (decreases) his benchmark quotes if there is a positive probability ($E\left[L|M_0\right]>0$) that the CB will buy (sell) GBP. Relaxing the full information assumption does not affect the absolute spread ($S_{2,0}-S_{1,0}=\frac{\pi}{2}\left(\overline{X}_1-\overline{X}_2\right)$) because the MM, aware of the identity of his counterparty, does not experience any adverse selection, in contrast to Naranjo and Nimalendran (2000). However, the proportional spread PS_0 does decrease (increase) if $E\left[X_0^{CB}|M_0\right]$ is positive (negative): The MM can clear the market just by passing to investors part of the expected revenues (costs) from a positive (negative) intervention. Therefore, he revises PS_0 downward (upward).

In stage 1 the first investors' order arrives at time t_1. The reservation bid (S_{1,t_1}) and ask (S_{2,t_1}) prices already account for the MM's ensuing new belief that the CB is less likely to intervene in the future, p_{l,t_1}, where

$$p_{l,t_1} = E\left[l|M_0,X_{i,t_1}\right]=E\left[l|M_0\right]-\frac{Var\left[l|M_0\right]}{1-E\left[l|M_0\right]}<q_{l,t_1}. \tag{37}$$

The difference between quote revisions at stage 0 and stage 1 depends on the dispersion of the dealer's beliefs about l, i.e., on the CB's credibility. If this credibility is lower ($Var\left[l|M_0\right]$ is

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[37]See the survey article by Sarno and Taylor (2001) for an analysis of the issue of secrecy usually permeating government activity in currency markets.

higher for a given $q_{l,t_1} = p_{l,0} = E[l|M_0]$), then $E[L|M_0, X_{i,t_1}]$ is lower, $E\left[X_{t_1}^{CB}|M_0, X_{i,t_1}\right]$ is higher (as argued in Section 2.4), and so is $\Delta^M S_{t_1}^*$ (for $\frac{\partial \Delta^M S_0^*}{\partial L} < 0$). Consequently, investors' order flow induces more volatility in the transaction price, via the process of belief updating for q_{l,t_n}, if the CB is less credible.

In stage 2, when the CB eventually intervenes at time t_2, it is easy to show that the price at which that transaction is executed is given by

$$S_{t_2}^{CB} = \frac{1}{R} \left\{ E\left[\omega_{1A}|M_{t_1}, X_{t_2}^{CB}\right] E\left[f|M_0, X_{t_2}^{CB}\right] \right\} +$$
$$+ E\left[\omega_{1B}|M_{t_1}, X_{t_2}^{CB}\right] \frac{E[f|I_0]}{R} + E\left[\omega_2|M_{t_1}, X_{t_2}^{CB}\right] \overline{S} + E\left[\omega_3|M_{t_1}, X_{t_2}^{CB}\right] \pi X^*, \quad (38)$$

where $E\left[\omega_{1A}|M_{t_1}, X_{t_2}^{CB}\right] = \frac{\lambda R + \pi E\left[L|M_{t_1}, X_{t_2}^{CB}\right]}{2\pi E\left[L|M_{t_1}, X_{t_2}^{CB}\right] + \lambda R} > 0$, $E\left[\omega_{1B}|M_{t_1}, X_{t_2}^{CB}\right] = -E\left[\omega_3|M_{t_1}, X_{t_2}^{CB}\right] = -\frac{\pi E\left[L|M_{t_1}, X_{t_2}^{CB}\right]}{\lambda R} < 0$, and $E\left[\omega_2|M_{t_1}, X_{t_2}^{CB}\right] = \frac{2\pi E\left[L|M_{t_1}, X_{t_2}^{CB}\right]\left(\lambda R + \pi E\left[L|M_{t_1}, X_{t_2}^{CB}\right]\right)}{\lambda R\left(2\pi E\left[L|M_{t_1}, X_{t_2}^{CB}\right] + \lambda R\right)} > 0$. As in Proposition 3, $S_{t_2}^{CB}$ is a weighted average of the MM's resulting new posteriors (and investors' expectations) about f, the revealed target $\overline{S}$, and the risk-premium πX^*.[38] MM's beliefs are revised with respect to stage 1 according to Proposition 7, as long as $X_{t_2}^{CB}$ is unexpected and/or unannounced. $E[f|I_0]$ has a negative weight in Eq. (38) because, if investors are more pessimistic about the USD, ceteris paribus, the CB is expected to trade a smaller $X_{t_2}^{CB} > 0$ to chase the trend or a bigger $X_{t_2}^{CB} < 0$ to lean against the wind. Vice versa, $E[f|M_{t_2}]$ has a positive weight in $S_{t_2}^{CB}$ because, at a greater expected NFV of buying GBP, $X_{t_2}^{CB} > 0$ is expected to be bigger or $X_{t_2}^{CB} < 0$ is expected to be smaller. In both cases, the MM's optimal response is to bid/offer less for GBP to clear the market. $S_{t_2}^{CB}$ increases for higher $E\left[f|M_0, X_{t_2}^{CB}\right]$ because, in that case, the speculative component of the CB's demand for GBP is expected to be bigger.

At this point, the investors only know that an intervention occurred, but do not observe $X_{t_2}^{CB}$. Therefore, they cannot revise their beliefs about f until a new transaction with an investor is executed. When this happens in stage 3 at time t_3, that order is settled either at the new bid $S_{1,t_3} = S_{1,0}^{l=0} + \Delta^M S_{t_3}^*$ or at the new ask $S_{2,t_3} = S_{2,0}^{l=0} + \Delta^M S_{t_3}^*$, where $\Delta^M S_{t_3}^*$ is given by

$$\Delta^M S_{t_3}^* = \frac{\pi E\left[L|M_{t_2}, X_{i,t_3}\right] \overline{S}}{2\pi E\left[L|M_{t_2}, X_{i,t_3}\right] + \lambda R} - \frac{1}{2}\left\{S_0^{*,l=0}\right\} +$$
$$+ \frac{\lambda R}{4\pi E\left[L|M_{t_2}, X_{i,t_3}\right] + 2\lambda R}\left\{\frac{E[f|M_{t_2}]}{R}\right\}. \quad (39)$$

The quotes' revision at t_3 $(\Delta^M S_{t_3}^*)$ is due to both the imperfect substitutability $(p_{l,t_3} > 0)$

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[38] $S_{t_2}^{CB}$ also accounts for p_{l,t_2}, therefore is regret-free in the sense of Glosten and Milgrom (1985), since it depends on $X_{t_2}^{CB}$. Because we assume that investors' optimal demands X_{i,t_n} are fully anticipated by the MMs, the prices at which they are cleared, S_{i,t_n}, are regret-free as well.

and the signaling $(E[f|M_{t_2}] \neq E[f|M_0])$ effects of the intervention. Then, the investors adjust their beliefs about p_f using the signed difference $S_{i,t_3} - S_{i,t_1}$, along the lines of Proposition 8:

$E[f|I_0, S_{i,t_3} - S_{i,t_1}] = {}^I p_{f,t_3} f_H + (1 - {}^I p_{f,t_3}) f_L.$

This new expectation enters investors' demands and is discounted into the MM's new quotes.[39] Any incoming order, at time $t_n > t_3$, is then executed at rates reflecting not only the MM's and investors' new beliefs about f and $\overline{S}$ but also the MM's new prior about l (q_{l,t_n}) induced by those arrivals, so that ${}^M S_{t_n}^* = {}^M S_{t_3}^* + \pi E[L|M_{t_n}] E[X_{t_n}^{CB}|M_{t_n}] = S_0^{*,l=0} + \Delta^M S_{t_n}^*$, where

$$\Delta^M S_{t_n}^* = \frac{\pi E[L|M_{t_n}, X_{i,t_n}] \overline{S}}{2\pi E[L|M_{t_n}, X_{i,t_n}] + \lambda R} + \frac{1}{2} \left\{ \frac{E[f|I_{t_3}]}{R} - \pi X^* \right\} +$$
$$+ \frac{\lambda R}{4\pi E[L|M_{t_n}, X_{i,t_n}] + 2\lambda R} \left\{ \frac{E[f|M_{t_2}]}{R} \right\} - \left\{ \frac{E[f|I_{t_0}]}{R} - \pi X^* \right\} \qquad (40)$$

and $E[L|M_{t_n}, X_{i,t_n}] = E[L|M_{t_2}, X_{i,t_3}, ..., X_{i,t_n}]$. If q_{l,t_n} drops to zero, both S_{i,t_n} and ${}^M S_{t_n}^*$ revert toward pre-intervention levels, for the portfolio balance effect is smaller on the incoming risk-averse traders. However, the reversion is less than complete, since in the limit, if $t_N \leq T$ such that $q_{l,t_N} = 0$, ${}^M S_{t_n}^*$ converges to:

$$^M S_{t_N}^* = \frac{E[f|I_{t_3}]}{R} - \pi X^* \neq S_0^{*,l=0}. \qquad (41)$$

The long-lived difference between ${}^M S_{t_N}^*$ and $S_0^{*,l=0}$ depends only on the difference between $E[f|I_{t_3}]$ and $E[f|I_0]$, i.e., on a permanent revision of the investors' beliefs. Indeed, unless the CB keeps the threat of intervention alive or investors' expectations about f have been altered permanently by the previous intervention (as in Eq. (41)), the exchange rate eventually reverts to the pre-announcement levels.

To provide further intuition for Eqs. (34) to (41), we simulate the above sequence of events for a specific parametrization of the economy. We assume that 30 trades arrive during the interval $[0, T]$ and that (for simplicity) the same time elapses between each trade. W.l.o.g., we also assume that (consistent with evidence in Fischer and Zurlinden, 1999 and Payne and Vitale, 2001) the CB order arrives in the morning, at time t_6. However, this information is unknown to the dealer. We choose $l = 0.105$ and let $f_{M_0}(l)$ be a $Beta(a, b)$ with $a = 266$ and $b = 1215$, implying $q_{l,0} = \frac{a}{a+b} = 0.18$ and $Var[l|M_0] = \frac{ab}{(a+b+1)(a+b)^2} = 0.0001$.[40] We finally impose that $\psi = 0.30$, so that the true correlation between f and $\overline{S}$ is negative, as it is generally the case for G-7 currencies (e.g., Sarno and Taylor, 2001).

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[39] Therefore, the delay in the revision of investors' beliefs induces positive serial correlation in the transaction prices immediately following a CB trade, a testable implication of our model, regardless of dealers' market power.

[40] Any prior distribution for l with support on $[0, 1]$ is suitable for this simulation. We choose the Beta distribution for computational ease, as its posterior given by Bayes' Rule is a Beta as well.

We start analyzing the case in which $f = f_L = \$1.45$, $\overline{S} = \overline{S}_L = \$1.31$, and the informed CB attempts to lean against the wind with $_BX_{t_6}^{CB} < 0$ in Figure 5a. The initial benchmark exchange rate is $S_0^{*,l=0} = \$1.4345$. Following the CB announcement at $t_0 = 0$, the model's parameters imply that $E\left[X_0^{CB}|M_0\right]$ is small and negative, and S_0^* declines (according to Eq. (34)). Therefore, when it arrives, $_BX_{t_6}^{CB}$ is unexpectedly big. The proportional spread PS_0 instead increases, as the MM needs to pass some of the expected cash outflows from a potential CB trade onto the investors to avoid a drift on his inventory while simultaneously profiting from trading. Because $Var\left[l|M_0\right] > 0$, our CB is not fully credible. Hence, the next few investors' orders, from t_1 to t_5, lower p_{l,t_n}, inducing $^MS_{t_n}^*$ to slightly increase (and PS_{t_n} to slightly decrease). When the CB arrives at t_6, the MM learns from $_BX_{t_6}^{CB} < 0$ the true f and $\overline{S}$ (as shown in Proposition 7) and embeds this knowledge in his reservation quotes for the next incoming trades. Hence, $^MS_{t_7}^*$ drops to $\$1.3807$. Afterwards, investors observe $\Delta S_{i,t_7} < 0$, infer that the CB was leaning against the wind (Proposition 8), and incorporate their resulting more optimistic beliefs about the USD $(E\left[f|I_{t_7}\right] < E\left[f|I_0\right])$ in their demands. So does the MM in pricing all future incoming orders. An additional downward (upward) adjustment of S_{1,t_8} and S_{2,t_8} (PS_{t_8}) results.

In stage 4 there are no more informative trades arriving. Nonetheless, investors' order flow from t_9 to t_{30} drives down q_{l,t_n} and its conditional variance $Var\left[l|M_{t_n}\right] = E\left[l^2|M_{t_n}\right] - q_{l,t_n}^2$, and drives up the absolute size of $E\left[X_{t_n}^{CB}|M_{t_n}\right]$. Thus, the threat of a future intervention becomes less credible, the impact of imperfect substitutability on the quotes is weaker, $^MS_{t_n}^*$ starts rising and PS_{t_n} falls. If we assume that eventually both q_{l,t_n} and $Var\left[l|M_{t_n}\right]$ drop to zero by time $t_{30} = T$, so does the portfolio balance effect on the reservation prices. Thus, $^MS_{t_n}^*$ moves toward $S_0^{*,l=0}$, but does not return to its pre-announcement level. Indeed, $^MS_T^* = \$1.3743 < S_0^{*,l=0}$, i.e., closer to $\overline{S}$ than it was at $t_0 = 0$, and $PS_T > PS_0$ (when $l = 0$). Both $\Delta^MS_T^* < 0$ and $\Delta PS_T > 0$ stem from the signaling effect of $X_{t_6}^{CB}$: at time T, investors are more willing to sell GBP than they were at stage 0 $(E\left[f|I_T\right] < E\left[f|I_0\right])$; therefore, the MM reduces his bid and ask prices to clear the market, but not the absolute spread to maximize his expected profits.

Unexpectedly small interventions may instead generate undesired short- and long-lived effects on quotes and spreads. For example, this is the case when both $q_{f,0}$ and $q_{\overline{S},0}$ are low, but a trade-off between policy and speculation $(f = f_H$ and $\overline{S} = \overline{S}_L)$ implies that $_SX_{t_6}^{CB} < 0$ is optimal. Because there is a positive probability $(1 - v = 0.2)$ that $_SX_{t_6}^{CB} < 0$ comes from the uninformed CB (i.e., that $f = f_L$), beliefs' updates following its arrival are less than complete $(q_{f,0} < p_{f,t_6} < 1)$, along the lines of Proposition 7. The resulting price update $\Delta^MS_{t_7}^*$ at stage 3, albeit negative, may then be smaller than $\Delta^MS_{t_5}^*$. Hence, $S_{i,t_7} = S_{i,0}^{l=0} + \Delta^MS_{t_7}^*$ may increase (and PS_{t_7} decrease) with respect to pre-intervention levels, although the CB actually sold GBP

to achieve $\overline{S}_L$. At stage 4, the investors, after observing a $S_{i,t_7} - S_{i,t_{7-j}} > 0$, revise upward their beliefs about p_f, according to Proposition 8, and the ensuing $E\left[f|I_0, S_{i,t_7} - S_{i,t_{7-j}}\right] > E\left[f|I_0\right]$. Thus, ${}^M S_{t_n}^*$ rises again and remains higher than $S_0^{*,l=0}$, even if q_{l,t_n} goes to zero.

The existence of these perverse effects of CB interventions on exchange rates has been documented in the empirical literature. For example, Dominguez and Frankel (1993) reported that Fed purchases of USD during the period following the Louvre Accord in 1987 were consistently accompanied by its depreciation in a market environment in which investors were becoming more pessimistic about the long-term perspectives of the dollar. Additionally, we showed that CB sales of GBP, when unexpectedly small, may induce the proportional spread to fall, and not to increase (as in the case of unexpectedly big sales). These results suggest that both sign and magnitude of the intervention play a crucial role in explaining its impact on the process of price formation in a currency market dominated by a monopolist dealer.

5.2 The competitive dealers

We now relax the full information assumption on equilibrium quotes and spreads under competitive dealership. More specifically, we use the previously described sequence of intraday events to derive, at each point in time t_n, explicit solutions for the problem of competitive MMs: the prices S_{1,t_n} and S_{2,t_n} such that $E\left[Z_{t_n}|M_{t_n}\right] = 0$ and $E\left[\Pi_{t_n}|M_{t_n}\right] = 0$ when MMs' expectations $E\left[f|M_{t_n}\right]$, $E\left[\overline{S}|M_{t_n}\right]$, and $E\left[l|M_{t_n}\right]$ replace f, $\overline{S}$, and l and investors' optimal demands (Eq. (2)) depend on $E\left[f|I_{t_n}\right]$. To do so, we further assume that $S_{t_n}^{CB} = S_t^{CB}(*)$ of Eq. (23), i.e., we impose that the full information spread of Proposition 4 (when $l > 0$) is equal to zero. At each stage of the trading day, following a CB announcement at stage 0, the MMs update $E\left[f|M_{t_n}\right]$, $E\left[\overline{S}|M_{t_n}\right]$, and $E\left[l|M_{t_n}\right]$ as in Section 5.1, and consequently revise their potential prices with respect to the benchmark of Proposition 1 ($l = 0$), so that

$$S_{1,t_n} = S_0^{*,l=0} + \pi E\left[L|M_{t_n}\right] E\left[X_{t_n}^{CB}|M_{t_n}\right] - \left\{(1-q)\frac{\pi}{2}\left(\overline{X}_1 - \overline{X}_2\right) - \frac{\pi}{2}E\left[\Gamma|M_{t_n}\right]\right\} \quad (42)$$

$$S_{2,t_n} = S_0^{*,l=0} + \pi E\left[L|M_{t_n}\right] E\left[X_{t_n}^{CB}|M_{t_n}\right] + \left\{\frac{\pi q}{2}\left(\overline{X}_1 - \overline{X}_2\right) - \frac{\pi q}{2(1-q)}E\left[\Gamma|M_{t_n}\right]\right\}, \quad (43)$$

where $E\left[\Gamma|M_{t_n}\right]$ is obtained by replacing A, C, and $X_{t_n}^{CB}$ in Γ (Proposition 4) with their expectations conditional on the information set M_{t_n}: $E\left[A|M_{t_n}\right] = S_0^{*,l=0} + \pi E\left[L|M_{t_n}\right] E\left[X_{t_n}^{CB}|M_{t_n}\right]$, $E\left[C|M_{t_n}\right] = E\left[A|M_{t_n}\right]\left(\frac{E[A|M_{t_n}]}{\pi q} + \overline{X}_1 - \frac{R}{\pi}E\left[f|I_{t_n}\right]\right) - E\left[L|M_{t_n}\right] E\left[X_{t_n}^{CB}|M_{t_n}\right] S_t^{CB}(*)$, and

$$E\left[X_{t_n}^{CB}|M_{t_n}\right] = \frac{1}{\pi E\left[L|M_{t_n}\right]}\left\{E\left[\overline{S}|M_{t_n}\right] - \left(\frac{E\left[f|I_{t_n}\right]}{R} - \pi X^*\right)\right\} +$$

$$+ \frac{\lambda}{2\left(\pi E\left[L|M_{t_n}\right]\right)^2}\left\{E\left[f|M_{t_n}\right] - RS_t^{CB}(*)\right\}. \quad (44)$$

Therefore, information asymmetry induces a wedge between equilibrium ask and bid quotes:

$$S_{2,t_n} - S_{1,t_n} = \frac{\pi}{2}\left(\overline{X}_1 - \overline{X}_2\right) - \frac{\pi}{2\left(1-q\right)}E\left[\Gamma|M_{t_n}\right].\tag{45}$$

This wedge is generally different from zero and from the monopoly spread in Section 5.1, $\frac{\pi}{2}\left(\overline{X}_1 - \overline{X}_2\right)$. When $E\left[l|M_{t_n}\right] > 0$, competitive MMs must dissipate all the additional expected cash flows from CB trades onto the investors while clearing the market. Because of imperfect substitutability, this can be accomplished only by an asymmetric revision of reservation prices, as in Proposition 4. The monopolist MM is instead able to retain part of those expected revenues, or to pass part of those expected costs to the risk-averse investors. When there is information asymmetry between competitive MMs and the informed CB ($E\left[\Gamma|M_{t_n}\right] \neq \Gamma$), this task is made more difficult by the uncertainty surrounding l ($Var\left[l|M_{t_n}\right] > 0$) and sign and magnitude of the expected intervention ($E\left[X_{t_n}^{CB}|M_{t_n}\right] \neq X_{t_n}^{CB}$). Therefore, an equilibrium nonzero wedge arises in Eq. (45). This uncertainty, however, does not affect the spread set by the monopolist MM, for he still finds optimal to adjust his quotes symmetrically to maximize his expected profits from trading (as in the full information scenario).

To clarify the intuition for these results, we simulate the same sequence of events described in Section 5.1 and display the equilibrium dynamics for $^{C}S_{t_n}^{*}$ and PS_{t_n} in Figure 5b. At the beginning of the day, the CB announcement of a small GBP sale induces a small downward revision in quotes, while both $S_{2,0} - S_{1,0}$ and PS_0 decline with respect to Proposition 1 (i.e., become negative, for $S_{1,0} = S_{2,0}$ when $l = 0$). Indeed, to clear the market, MMs need to attract more purchases and less sales of GBP. To do so, they mark down bid and offer prices. Additionally, because $E\left[X_0^{CB}|M_0\right] < 0$, MMs also need to generate positive expected net revenues from trading with the investors to satisfy the binding condition that $E\left[\Pi_0|M_0\right] = 0$. For the given set of the model's parameters, this is achieved by reducing absolute and proportional spreads.

Spreads instead increase for the next few uninformative trades with investors, as q_{l,t_n} declines along the lines of Proposition 7, as does $Var\left[l|M_{t_n}\right]$. When eventually the CB intervenes (at time t_6) with a big negative order, that trade is fully revealing of $\overline{S}_L$ and f_L. Quotes are then again revised downward, but both absolute and proportional spreads now increase (as in Figure 5a) because at the new posteriors the MMs expect bigger future $X_{t_n}^{CB} < 0$. The spread at t_7, in stage 3, is very close to zero by construction, as $E\left[X_{t_7}^{CB}|M_{t_7}\right] = X_{t_7}^{CB}$ and $S_{t_7}^{CB} = S_t^{CB}(*)$, but not exactly so because $E\left[f|I_{t_7}\right] > f_L$. Only afterwards (at t_8) do investors learn from the observed transaction prices that a negative intervention must have occurred, become more optimistic about the USD, and demand less GBP, thus inducing smaller expected future interventions and a bigger increase in the proportional spread. For the rest of the trading day, only uninformative investors' orders arrive, and the MMs update only q_{l,t_n}. Arrival of X_{i,t_n} implies more uncertainty

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about whether the CB will trade again (higher $Var\left[l|M_{t_n}\right]$) and a bigger $E\left[X_{t_n}^{CB}|M_{t_n}\right] < 0$, hence higher $^{C}S_{t_n}^{*}$ and smaller absolute and proportional spreads. Consequently, as evident from Figure 5b, the time series of observed (S_{i,t_n}) and expected $(^{C}S_{t_n}^{*})$ transaction prices are more volatile.

Intuitively, at the beginning of the day, price volatility is high because the CB announcement increases the dispersion of beliefs among market participants. Indeed, at $t_0 = 0$ the MMs revise downward their quotes, although investors forecast a weaker dollar ($^{I}q_{f,0} = 0.75$). When the intervention actually occurs, and the information learned from it is conveyed to all market participants, then exchange rate volatility subsides. However, lower q_{l,t_n} from investors' order flow and increasing uncertainty around it make more difficult for the MMs to clear the market at each point in time, for the composition of the incoming demand for GBP is becoming more difficult to predict. In addition, investors, being now more optimistic about the USD, are less willing to buy GBP. The resulting bigger (but less likely) $E\left[X_{t_n}^{CB}|M_{t_n}\right]$ induces absolute and proportional spreads to decrease, and transaction prices and price volatility to increase. When q_{l,t_n} eventually converges toward zero, $Var\left[l|M_{t_n}\right]$ declines sharply as well: A less significant and credible threat of future CB trades affects less the process by which dealers formulate quotes. Therefore, intraday transaction price volatility subsides, the absolute spread rises toward pre-announcement levels, and so does PS_{t_n} toward $PS_T = 0$.

Nonetheless, even without portfolio balance effects, if $q_{l,T} = 0$ then $^{C}S_T^{*} = {}^{M}S_T^{*} = \$1.3743 < S_0^{*,l=0}$. More generally, Eq. (41) applies to competitive dealership as well, for each possible $X_{t_n}^{CB}$. This result has two interesting normative implications for an active monetary authority. First, the CB trade has the same long-lived effect on quotes (but not on spreads), regardless of dealers' market power, if the day-end threat of future intervention is not significant ($E\left[X_T^{CB}|M_T\right] = 0$) or not credible ($q_{l,T} = 0$). That effect is due exclusively to the signaling channel, consistent with the empirical findings of Pasquariello (2002). Portfolio balance effects are instead short-lived. Indeed, if $E\left[f|I_T\right] = E\left[f|I_0\right]$ then $^{C}S_T^{*} = S_0^{*,l=0}$ unless $q_{l,T} > 0$. In our model this stems from the structure of currency trading (sequential trading), and not from increasing substitutability of domestic and foreign assets (e.g., Edison (1993); Sarno and Taylor, 2001). To our knowledge, this observation has not been made in the literature.

Second, with information asymmetry the intervention is more effective (in the short- and long-term) than in the monopoly case if $q_{l,t_n} > 0$. In our example, $^{C}S_{t_8}^{*} = \$1.3270$ is lower than $^{M}S_{t_8}^{*} = \$1.3506$ (thus closer to $\overline{S}_L = \$1.31$), and remains so as long as $q_{l,t_n} > 0$: portfolio balance effects of $E\left[X_{t_n}^{CB}|M_{t_n}\right] \neq 0$ on S_{i,t_n} are in fact more significant when the MMs are unable to extract rents from the trading process, as suggested in Section 3.3. However, if the perceived

threat of a future intervention becomes increasingly less credible (if q_{l,t_n} declines rapidly toward zero), then $S_{t_n}^*$ converges to $\frac{E[f|I_T]}{R} - \pi X^*$ of Eq. (41) regardless of dealers' market power. The unsuccessful attempts by the CBs of several Asian countries during 1997 and 1998 to lean against the wind and rescue their ailing exchange rates seem to offer some anecdotal support to these findings, for their domestic currency markets were characterized by dealers holding some market power.

6   Conclusions

This study developed a theory of the impact of sterilized CB intervention on the microstructure of currency markets. To that end, we devised a model of sequential trading in which prices are set by a monopolist or by competitive risk-neutral dealers and the demand schedule of the monetary authority results endogenously from the optimal resolution of a trade-off between policy and wealth-preservation. This model generates a rich set of implications of the presence of an active CB for intraday price revisions, spreads, exchange rate returns, and return volatility.

Under full information, our stylized forex market is strong-form efficient, restricting the effectiveness of intervention to portfolio balance considerations. Under the more realistic assumption of information asymmetry, the market is instead semi-strong efficient and CB actions may also signal information about policy motives and fundamentals. In both settings, official intervention has a significant impact on the resulting process of intraday price formation. Dealers' reservation quotes are revised upward (downward) as soon as the CB is expected to buy (sell) the foreign currency. The effect of CB trades on absolute and proportional spreads depends crucially on dealers' market power. In the extreme scenario of a monopolist market-maker, the absolute spread is unchanged by the intervention. However, the proportional spread increases (decreases) when the monetary authority is leaning against the wind (chasing the trend). This occurs because the dealer is attempting to retain (pass) at least some of the ensuing expected revenues (costs) to maximize his expected profits while clearing the market. When dealers compete for the incoming trade, all potential cash flows from the intervention have to be transferred onto the population of investors. Because of imperfect substitutability of domestic and foreign assets, this can be achieved only by creating a wedge between bid and offer prices. This intuition is critical as well to understanding why interventions are generally more successful when dealers hold less market power. Indeed, a monopolist market-maker, in his quest for profit-maximization, does not adjust his quotes completely in response to a CB trade. Under competitive dealership, costs and revenues from CB transactions are instead transmitted fully to the risk-averse investors,

thus quotes' revisions are more substantial. We also demonstrated that interventions have a per-manent effect on the exchange rate only when deemed informative or when the threat of future CB actions is significant and credible, while portfolio balance effects are short-lived because of sequential trading.

In our setting, changes in bid and offer quotes, absolute and proportional spreads, and trans-action prices are found to be related to sign and magnitude of the intervention, consistently with recent empirical evidence (e.g., Payne and Vitale, 2001; Pasquariello, 2002). Small CB trades are more easily accommodated by forex dealers and more easily absorbed by risk-averse investors. Small orders are also not fully revealing of the CB's information advantage. Hence, they may in-duce smaller or, if unexpected, even undesired revisions in beliefs, demands, quotes, and spreads. The speed of adjustment in transaction prices is shown to depend on the transparency of the CB's order flow for uninformed investors. Finally, simulations of the equilibrium process of intraday price formation revealed that exchange rate volatility tends to be high before an intervention occurs, subsides when new information is conveyed by its arrival to dealers and investors, then increases following greater uncertainty surrounding future CB actions, and eventually declines when a new intervention is perceived to be more unlikely.

7   Appendix

Proof of Proposition 1.   We construct the equilibrium for both monopoly and dealership competition in three steps, similarly to Saar (2000a, Section 1.2). This procedure is described in greater detail in Section 3. We start by assuming that MMs conjecture the investors' type from sign and magnitude of their orders and set prices accordingly. The monopolistic MM solves:

$$\max_{S_{1,t},S_{2,t}} E\left[\Pi_t | M_t\right] = qX_{1,t}S_{1,t} + (1-q)X_{2,t}S_{2,t}$$

$$s.t. \quad E\left[Z_t | M_t\right] = qX_{1,t} + (1-q)X_{2,t} = 0 \tag{A-1}$$

$$X_{i,t} = \frac{1}{\pi}\left(\frac{f}{R} - S_{i,t}\right) - \overline{X}_i \qquad i = \{1,2\}.$$

Plugging Eq. (2) into the no-inventory constraint allows us to rewrite Eq. (A-1) in terms of $S_{2,t}$. It is then easy to show that the ensuing F.O.C. generates the optimal prices $S_{1,t}$ and $S_{2,t}$ of Proposition 1 (Eqs. (6) and (7)). We then plug these prices into Eq. (2) to find $X_{1,t} = -\frac{1}{2}(1-q)\left(\overline{X}_1 - \overline{X}_2\right)$ and $X_{2,t} = \frac{q}{2}\left(\overline{X}_1 - \overline{X}_2\right)$. Finally, we verify that the MM's conjecture was correct. Our initial assumption that $\overline{X}_1 < \overline{X}_2$ in fact implies not only that $X_{1,t} \neq X_{2,t}$ but also that $X_{1,t} < 0$ and $X_{2,t} > 0$. Bertrand competition among MMs moves prices away from the levels of Eqs. (6) and (7) until $E\left[\Pi_t | M_t\right] = 0$ and $S_{1,t} = S_{2,t}$. Indeed, using Eq. (4) for $l = 0$ to rewrite

$X_{1,t}$ in terms of $X_{2,t}$ and plugging it into $E[\Pi_t|M_t] = 0$ leads to $(1-q) X_{2,t} (S_{2,t} - S_{1,t}) = 0$, which, with trading $(X_{2,t} \neq 0$ and $X_{1,t} \neq 0)$, can be satisfied only by $S_{1,t} = S_{2,t} = S_t^*$. Eq. (5) then follows from plugging Eq. (2) into Eq. (4) and solving for S_t^*. Investors' optimal orders are then given by $X_{1,t} = -(1-q)(\overline{X}_1 - \overline{X}_2)$ and $X_{2,t} = q(\overline{X}_1 - \overline{X}_2)$, consistent with the MMs' initial conjecture and our interpretation of $S_{1,t}$ and $S_{2,t}$ for any $\overline{X}_1 < \overline{X}_2$. ∎

Proof of Proposition 3. The optimal S_t^{CB} of Eq. (17), with $\omega_1 = \frac{\lambda^2 R^2 - 2\pi^2 L^2}{\lambda R(2\pi L + \lambda R)}$, $\omega_2 = \frac{2\pi L(\pi L + \lambda R)}{\lambda R(2\pi L + \lambda R)} > 0$, and $\omega_2 = \frac{\pi L}{\lambda R} > 0$, follows straightforwardly from the F.O.C. of the MM's problem (Eq. (14)). We assume that model's parameters are such that $S_t^{CB} > 0$. Plugging S_t^{CB} in X_t^{CB} of Eq. (13), and then the resulting expression into Eqs. (15) and (16) gives

$$S_{i,t} = \overline{S}\left(\frac{\pi L}{2\pi L + \lambda R}\right) + \frac{f}{R}\left(\frac{\pi L + \lambda R}{2\pi L + \lambda R}\right) - \frac{\pi}{2}\overline{X}_i, \tag{A-2}$$

for $i = \{1, 2\}$. Eqs. (2) and (A-2) imply that $X_{2,t} - X_{1,t} = \frac{1}{2}(\overline{X}_1 - \overline{X}_2)$. Hence, as in Saar (2000a), $X_{1,t} \neq X_{2,t}$ (consistently with the MM's initial conjecture) unless $\overline{X}_1 = \overline{X}_2$, i.e., unless there is no trading (and risk-sharing). The CB's initial conjecture $(E[S_t^*|F_t] = qS_{1,t} + (1-q)S_{2,t})$ is also correct in equilibrium. In fact, it follows from Eq. (A-2) that:

$$qS_{1,t} + (1-q)S_{2,t} = \left(\frac{f}{R} - \pi X^*\right) + \left[\left(\overline{S} - \frac{f}{R}\right)\left(\frac{\pi L}{2\pi L + \lambda R}\right) + \frac{\pi}{2}X^*\right], \tag{A-3}$$

while plugging Eqs. (17) and (A-3) into Eq. (13) gives

$$X_t^{CB} = \frac{1}{\pi L}\left[\left(\overline{S} - \frac{f}{R}\right)\left(\frac{\pi L}{2\pi L + \lambda R}\right) + \frac{\pi}{2}X^*\right]. \tag{A-4}$$

It is then clear that $\left(\frac{f}{R} - \pi X^*\right) + \pi L X_t^{CB} = qS_{1,t} + (1-q)S_{2,t}$, as guessed by the CB. Finally, Eqs. (2) and (A-2) imply that $X_{2,t} > 0$ and $X_{1,t} < 0$ iff $\overline{X}_2 < 2\left[\frac{f}{R}\left(\frac{\pi L}{2\pi^2 L + \lambda R}\right) - \overline{S}\left(\frac{L}{2\pi L + \lambda R}\right)\right] < \overline{X}_1$, i.e., iff $\overline{X}_1$ is "sufficiently high" and $\overline{X}_2$ is "sufficiently low." ∎

Proof of Proposition 4. Construction of the equilibrium involves the same three steps as in the proof of Proposition 3. However, here we use $E[Z_t|M_t] = 0$ to express $S_{1,t}$ as a function of $S_{2,t}$ and S_t^{CB}. Then, plugging Eqs. (2) and (13) in $E[\Pi_t|M_t] = 0$ generates a quadratic equation with respect to $S_{2,t}$, whose solutions are

$$S_{2,t} = \frac{f}{R} - \pi X^* + \pi L X_t^{CB} + \frac{\pi q}{2}(\overline{X}_1 - \overline{X}_2) \pm \left[\frac{\pi q}{2(1-q)}(\Gamma)\right], \tag{A-5}$$

where $\Gamma = \left\{(1-q)^2\left[(\overline{X}_2 - \overline{X}_1) - \frac{2A}{\pi q}\right]^2 - 4\left(\frac{1-q}{\pi q}\right)C\right\}^{\frac{1}{2}} > 0$, $A = \overline{S} + \frac{\lambda}{2\pi L}(f - RS_t^{CB})$, and $C = A\left(\frac{A}{\pi q} + \overline{X}_1 - \frac{R}{\pi}f\right) - LX_t^{CB}S_t^{CB}$. We choose the expression for $S_{2,t}$ implied by the minus

sign in Eq. (A-5) (i.e., Eq. (21)) for it is the only solution reverting to Eq. (7) when $\overline{X}_1 > \overline{X}_2$. Eq. (20) then ensues. Because of the definition of Γ, we need to impose that

$$(1-q)^2 \left[(\overline{X}_2 - \overline{X}_1) - \frac{2A}{\pi q} \right]^2 - 4 \left(\frac{1-q}{\pi q} \right) C > 0. \tag{A-6}$$

This condition is satisfied when $\left| \overline{X}_1 - \overline{X}_2 \right|$ is not "too small." Finally, we verify that MMs' initial conjectures are confirmed in equilibrium. It easily follows from plugging Eqs. (20) and (21) into Eq. (2) that $X_{1,t} \neq X_{2,t}$ iff $(\overline{X}_1 - \overline{X}_2) \neq -\frac{1}{(1-q)}\Gamma$, which is indeed the case when Eq. (A-6) holds and $\overline{X}_1 > \overline{X}_2$. as assumed in Section 2.2. We leave to the reader to verify that the CB's conjecture that $E\left[S_t^*|F_t\right]$ is indeed correct in equilibrium. Now, we search for S_t^{CB} such that $S_{2,t} - S_{1,t} = 0$. It is clear from Proposition 1 that this is always the case if $l = 0$ and/or $X_t^{CB} = 0$. Hence, when $l > 0$, it is possible to find the (extremely high or extremely low) price such that $X_t^{CB} = 0$. Because in this study we focus on the impact of CB interventions on quotes and spreads, $S_t^{CB}(*)$ is instead the one price such that $S_{2,t} - S_{1,t} = 0$ but $X_t^{CB} \neq 0$. To find this price, we first observe that Eq. (4) implies that $qX_{1,t} + (1-q)X_{2,t} = -LX_t^{CB}$. Hence, for $E\left[\Pi_t|M_t\right] = 0$ to hold at a zero spread and $X_t^{CB} \neq 0$, it has to be true that $S_{1,t} = S_{2,t} = S_t^{CB}$. Lastly, plugging Eq. (13) in Eq. (12) and solving for $S_t^{CB}(*)$ gives Eq. (23). It is easy to verify that, when $S_t^{CB} = S_t^{CB}(*)$, investors' and CB's conjectures are correct in equilibrium. If $S_t^{CB} = S_t^{CB}(*)$, then $X_{i,t}$ are twice the amounts traded in monopoly, hence $X_{2,t} > 0$ and $X_{1,t} < 0$ under the same restriction reported in the proof of Proposition 3. If instead $S_t^{CB} \neq S_t^{CB}(*)$, it can be shown that $X_{2,t} > 0$ and $X_{1,t} < 0$ when the following restrictions hold:

$$\overline{X}_2 + \frac{q}{2}(\overline{X}_1 - \overline{X}_2) - \frac{q}{2(1-q)}(\Gamma) \quad < \quad \frac{1}{\pi}\left[\left(\frac{f}{R} - \overline{S} \right) - \frac{\lambda}{2\pi^2 L}\left(f - RS_t^{CB}\right) \right] \tag{A-7}$$

$$\overline{X}_1 - \frac{(1-q)}{2}(\overline{X}_1 - \overline{X}_2) + \frac{1}{2}(\Gamma) \quad > \quad \frac{1}{\pi}\left[\left(\frac{f}{R} - \overline{S} \right) - \frac{\lambda}{2\pi^2 L}\left(f - RS_t^{CB}\right) \right], \tag{A-8}$$

i.e., again when $\overline{X}_1$ is "sufficiently high" and $\overline{X}_2$ is "sufficiently low." For $l = 0$, Eqs. (A-7) and (A-8) reduce to $\overline{X}_1 > \overline{X}_2$. ∎

Proof of Proposition 5. The first statement of Proposition 5 ensues from $E\left[Z_t|M_t\right] = 0$ implying that $S_t^* = \overline{S} + \frac{\lambda}{2\pi L}\left(f - RS_t^{CB}\right)$. If $\lambda = 0$ in $L\left(\overline{S}, \lambda\right)$, then $^CS_t^* = \overline{S}$ while $^MS_t^* = \frac{1}{2}\overline{S} + \frac{1}{2}\left(\frac{f}{R} - \pi X^*\right)$. Clearly $^MS_t^* = \overline{S}$ just when $\frac{f}{R} - \pi X^* = \overline{S}$, i.e., when $X_t^{CB} = 0$. Finally, if we substitute Eqs. (24) and (25) into EM_t, it is easy to see that $EM_t < 0$ iff, under competitive dealership, $\left(f - R\, ^CS_t^{CB}\right)^2 > \frac{4\pi^2 L^2}{\lambda^2}\left[\left(\frac{f}{R} - \overline{S} \right)\left(\frac{\pi L + \lambda R}{2\pi L + \lambda R} \right) - \frac{\pi}{2}X^* \right]^2$. For reasonable parametrizations of the model, this occurs just for "very small" or "very high" values of $^CS_t^{CB}$ and/or for "very high" values of λ. In those circumstances, the CB resists the monopolist

MM's attempts to maximize profits at its expenses. Additionally, it is possible to show that if λ is "small" or if $\left(f - RS_t^{CB}\right)$ is positive when the CB is chasing the trend (but $^C S_t^{CB} < {}^M S_t^{CB}$) and negative when the CB is leaning against the wind (but $^C S_t^{CB} > {}^M S_t^{CB}$), Proposition 5 applies to $^M L\left(\overline{S}, \lambda\right)$ versus $^C L\left(\overline{S}, \lambda\right)$ as well. In fact, their difference is given by $EM_t + \lambda\left(^C W_{T'} - {}^M W_{T'}\right)$. Moreover, under those conditions, $^C X_t^{CB} > {}^M X_t^{CB}$. It then follows that $\lambda\left(^C W_{T'} - {}^M W_{T'}\right) = \lambda\left[\left(f - R^C S_t^{CB}\right)\; ^C X_t^{CB} - \left(f - R\;^M S_t^{CB}\right)\;^M X_t^{CB}\right] > 0$, and so is $^M L\left(\overline{S}, \lambda\right) - {}^C L\left(\overline{S}, \lambda\right)$. $\blacksquare$

Proof of Proposition 6. As suggested in Section 4.1, the model's parameters need to satisfy some rationality constraints to ensure the intervention schedule of Eqs. (26) and (27) is indeed optimal. We start with the informed CB. The following set of constraints, *Participation Constraints* (PC), guarantees that $^I X_t^{CB} \neq 0$ and its sign is optimal:

$$\left(\overline{S}_H - \frac{E\left[f|I_t\right]}{R}\right) + \pi X^* + \frac{\lambda}{2\pi L}\left(f_H - RS_t^{CB}\right) > 0 \qquad \text{PC-}\left(\overline{S}_H, f_H\right)$$

$$\left(\overline{S}_L - \frac{E\left[f|I_t\right]}{R}\right) + \pi X^* + \frac{\lambda}{2\pi L}\left(f_L - RS_t^{CB}\right) < 0 \qquad \text{PC-}\left(\overline{S}_L, f_L\right)$$

$$\left(\overline{S}_H - \frac{E\left[f|I_t\right]}{R}\right) + \pi X^* + \frac{\lambda}{2\pi L}\left(f_L - RS_t^{CB}\right) > 0 \qquad \text{PC-}\left(\overline{S}_H, f_L\right)$$

$$\left(\overline{S}_L - \frac{E\left[f|I_t\right]}{R}\right) + \pi X^* + \frac{\lambda}{2\pi L}\left(f_H - RS_t^{CB}\right) < 0. \qquad \text{PC-}\left(\overline{S}_L, f_H\right)$$

If we assume that the NFV of buying GBP assets is positive (negative) for f_H (f_L) and that λ is "small enough" (albeit not insignificant) for policy (speculation) to prevail in determining the sign (magnitude) of $^I X_t^{CB}$, those constraints are clearly satisfied when $\overline{S}_L << \frac{E[f|I_t]}{R} - \pi X^* << \overline{S}_H$. We also need to ensure that both $_B X_t^{CB} > {}_S X_t^{CB} > 0$ and $_B X_t^{CB} < {}_S X_t^{CB} < 0$, i.e., that $^I X_t^{CB}$ is compatible with the incentives from $L\left(\overline{S}, \lambda\right)$. It can be shown, however, that this is always the case for all draws for f and $\overline{S}$, given the above assumptions about λ, $\overline{S}$, and the sign of $\left(f - RS_t^{CB}\right)$. For example, it is clear that $L\left(\overline{S}_H, \lambda, f_H, _B X_t^{CB} > 0, S_t^{CB}\left(_B X_t^{CB}\right)\right) < L\left(\overline{S}_H, \lambda, f_H, _S X_t^{CB} > 0, S_t^{CB}\left(_S X_t^{CB}\right)\right)$ if policy is "sufficiently important" (e.g. for a "small" λ or an "ambitious" $\overline{S}_H$) and $\left(f_H - RS_t^{CB}\right) > 0$. Similarly, $L\left(\overline{S}_H, \lambda, f_L, _S X_t^{CB} > 0, S_t^{CB}\left(_S X_t^{CB}\right)\right) < L\left(\overline{S}_H, \lambda, f_L, _S X_t^{CB} < 0, S_t^{CB}\left(_S X_t^{CB}\right)\right)$. Indeed, the gain from selling GBP, weighted by the small λ, is, by assumption, not big enough to compensate the informed CB for the welfare loss from a

greater $\left[S_t^* - \overline{S}_H\right]^2$. We now turn to the uninformed CB. We need to impose some restrictions on its beliefs about f such that ${}^U X_t^{CB} > 0$ for $\overline{S}_H$ and ${}^U X_t^{CB} < 0$ for $\overline{S}_L$, i.e., such that

$$\left(\overline{S}_H - \frac{E\left[f|I_t\right]}{R}\right) + \pi X^* + \frac{\lambda}{2\pi L}\left(E\left[f|F_t^U\right] - RS_t^{CB}\right) > 0 \qquad \text{PC-}\left(\overline{S}_H, E\left[f|F_t^U\right]\right)$$

$$\left(\overline{S}_L - \frac{E\left[f|I_t\right]}{R}\right) + \pi X^* + \frac{\lambda}{2\pi L}\left(E\left[f|F_t^U\right] - RS_t^{CB}\right) < 0. \qquad \text{PC-}\left(\overline{S}_L, E\left[f|F_t^U\right]\right)$$

If $f_L \leq E\left[f|F_t^U\right] \leq f_H$, then a "small enough" λ again ensures that, for each $\overline{S}$, ${}^U X_t^{CB}$ of Eq. (27) is optimal. Finally, it is easy to verify that the pricing schedule of Eq. (28) is sufficient to ensure that mimicking is better than intervening with the amount implied by $L\left(\overline{S}, E\left[f|F_t^U\right], S_t^{CB} = 0 \text{ or } \infty\right)$. Intuitively, our previous assumptions about λ do not make wealth-preservation irrelevant in the CB's loss function. Thus, any ${}^U X_t^{CB} > 0$ $\left({}^U X_t^{CB} < 0\right)$ different from ${}_S X_t^{CB} > 0$ $\left({}_S X_t^{CB} < 0\right)$ is going to be transacted at a very high (low) price, thus reducing (increasing) the NFV of trading GBP with respect to the pooling scenario and making the uninformed CB worse off. ∎

Proof of Proposition 7. Under Section 4.1's assumption that $\left(f_H - RS_t^{CB}\right) > 0$ and $\left(f_L - RS_t^{CB}\right) < 0$, $\overline{S}$ controls the sign of the intervention, while f (or $E\left[f|F_t^U\right]$) explains its magnitude. Hence, the sign of X_t^{CB} fully reveals $\overline{S}$ to the MMs. It is also clear from Eqs. (26) and (27) that the arrival of ${}_B X_t^{CB} > 0$ or ${}_B X_t^{CB} < 0$ induces a full revelation of the true CB type to the MMs. When instead ${}_S X_t^{CB} > 0$ or ${}_S X_t^{CB} < 0$ arrive, the possibility that they are from an uninformed CB induces just a partial revision of the MMs' priors for f and $\overline{S}$. If $\xi_m = \Pr\left\{X_t^{CB}|f_m, \overline{S}\right\}$, for $m = \{H, L\}$, it follows from Section 4.1 that, by Bayes' Rule,

$$p_{f,t} = \frac{q_{f,t}\xi_m}{q_{f,t}\xi_m + \left(1 - q_{f,t}\right)\xi_m}. \tag{A-9}$$

The above assumptions allow us to compute ξ_m for each CB order in Eqs. (26) and (27). For example, if ${}_S X_t^{CB} > 0$ arrives, $\overline{S}_H$ is revealed to the MMs and $\xi_H = \Pr\left({}_S X_t^{CB} > 0|f_H, \overline{S}_H\right) = (1 - v)$ because only the uninformed CB would intervene with that trade when $f = f_H$ and $\overline{S} = \overline{S}_H$. Similarly, $\xi_H = 1$ and $\xi_L = 0$ if ${}_B X_t^{CB} > 0$ arrives (and $\overline{S} = \overline{S}_H$), $\xi_H = (1 - v)$ and $\xi_L = v$ if ${}_S X_t^{CB} > 0$ arrives (and $\overline{S} = \overline{S}_H$), $\xi_H = v$ and $\xi_L = (1 - v)$ if ${}_S X_t^{CB} < 0$ arrives (and $\overline{S} = \overline{S}_L$), and $\xi_H = 0$ and $\xi_L = 1$ if ${}_B X_t^{CB} < 0$ arrives (and $\overline{S} = \overline{S}_L$). It then follows that, when $\overline{S} = \overline{S}_H$, $p_{f,t} = 1$ if $X_t^{CB} = {}_B X_t^{CB} > 0$, but $p_{f,t}$ is given by Eq. (31) if $X_t^{CB} = {}_S X_t^{CB} > 0$. However, when $\overline{S} = \overline{S}_L$, $p_{f,t} = 0$ if $X_t^{CB} = {}_B X_t^{CB} < 0$, but that $p_{f,t}$ is given by Eq. (32) if $X_t^{CB} = {}_S X_t^{CB} < 0$. Finally, we derive dealers' beliefs about l. If $v < 1$, small CB interventions

are not fully revealing about f, and MMs use Bayes' Rule to revise $q_{l,t}$. When instead a big intervention occurs, MMs know the true $\overline{S}$ and f, and use that information to learn the true l (and to set $p_{l,t} = l$) from the observed order sign, as $_BX_t^{CB}$ is function of l as well. However, the new prior $q_{l,t+1} = p_{l,t}$ is still subject to change due to the incoming order flow, because $Var\left[l|M_t\right]$, a proxy for CB's credibility, is still positive. When $X_{i,t}$ or $_SX_t^{CB}$ arrive, from Bayes' Rule the resulting MMs' conditional posterior distributions of l are given by $f_{M_t,X_{i,t}}(l) = \frac{(1-l)f_{M_t}(l)}{\int_0^1 (1-l)f_{M_t}(l)dl}$ or $f_{M_t,sX_t^{CB}}(l) = \frac{lf_{M_t}(l)}{\int_0^1 lf_{M_t}(l)dl}$. As in Saar (2000a), it is then easy to show that, if $X_{i,t}$ arrives,

$$
\begin{aligned}
p_{l,t} &= \int_0^1 lf_{M_t,X_{i,t}}(l)\,dl = \frac{\int_0^1 l(1-l)f_{M_t}(l)\,dl}{\int_0^1 (1-l)f_{M_t}(l)\,dl} = \\
&= \frac{E\left[l|M_t\right] - E\left[l^2|M_t\right]}{1 - E\left[l|M_t\right]} = E\left[l|M_t\right] - \frac{Var\left[l|M_t\right]}{1 - E\left[l|M_t\right]},
\end{aligned}
\tag{A-10}
$$

while, if $_SX_t^{CB}$ arrives,

$$
\begin{aligned}
p_{l,t} &= \int_0^1 lf_{M_t,sX_t^{CB}}(l)\,dl = \frac{\int_0^1 l^2 f_{M_t}(l)\,dl}{\int_0^1 lf_{M_t}(l)\,dl} = \\
&= \frac{E\left[l^2|M_t\right]}{E\left[l|M_t\right]} = E\left[l|M_t\right] + \frac{Var\left[l|M_t\right]}{E\left[l|M_t\right]}.
\end{aligned}
\tag{A-11}
$$

∎

Proof of Proposition 8. The assumptions made in Section 4.2 clearly imply that investors update their beliefs about f only after observing the sign of the first $\Delta S_{i,t+1}$ following the CB intervention. Investors are in fact assumed to be unaware of MMs' beliefs for $\overline{S}$, f, or l, hence cannot extrapolate them from the last transaction price. However, they know that CBs are uninformed with probability $1 - v$, that interventions produce an impact on quotes just if unexpected, and that, if it arrives, an intervention is big with probability ψ (as $\Pr\left\{\left(f_H, \overline{S}_H\right) \cup \left(f_L, \overline{S}_L\right)\right\} = p_f\psi + (1 - p_f)\psi = \psi$).It then follows from Eqs. (26) and (27) that $\Pr\left\{\Delta S_{i,t+1} > 0|f_H\right\} = \psi + (1 - \psi)v$, $\Pr\left\{\Delta S_{i,t+1} < 0|f_H\right\} = (1 - \psi)(1 - v)$, $\Pr\left\{\Delta S_{i,t+1} > 0|f_L\right\} = (1 - \psi)(1 - v)$, and $\Pr\left\{\Delta S_{i,t+1} < 0|f_L\right\} = \psi + (1 - \psi)v$. For example, given that an intervention actually occurred, a $\Delta S_{i,t+1} > 0$ can arise from an unexpected $_BX_t^{CB} > 0$, with probability ψ, or from an unexpected $_SX_t^{CB} < 0$, with probability $1 - \psi$. In the first case, $_BX_t^{CB} > 0$ may come only from an informed CB aware of $f = f_H$. In the second case, $_SX_t^{CB} < 0$ may come from a CB informed about f_H just with probability v, as an uninformed CB finds optimal to mimic that trade even if $f = f_L$. Finally, for simplicity, we impose that $Var\left[l|M_t\right]$ is "small enough" so that, following the arrival of $X_{i,t+1}$, $q_{l,t+2}$ does not decline "too much," i.e., $|q_{l,t+2} - q_{l,t+1}|$ is not "too big," and investors do not need to control for the fraction of

the quote's revision due to MMs' new beliefs about l when updating their prior $^I q_{f,0}$. Indeed, this assumption allows us to avoid the scenario in which CB's credibility is "so low" ($|q_{l,t+2} - q_{l,t+1}|$ is "so big") to more than compensate the effect of the MMs' new posteriors $p_{f,t}$, $p_{l,t}$, and $p_{\overline{S},t}$ on $\Delta S_{i,t+1}$, hence to induce the investors to false inference on f. Consequently, by Bayes' Rule, if for example $\Delta S_{i,t+1} > 0$ is observed, $^I p_{f,t+1}$ is given by

$$^I p_{f,t+1} = \frac{^I q_{f,t} \Pr\{\Delta S_{i,t+1} > 0 | f_H\}}{^I q_{f,t} \Pr\{\Delta S_{i,t+1} > 0 | f_H\} + (1 - ^I q_{f,t}) \Pr\{\Delta S_{i,t+1} > 0 | f_L\}}. \tag{A-18}$$

The expression for $^I p_{f,t+1}$ when $\Delta S_{i,t+1} < 0$ is similarly obtained. Eq. (33) then follows. ∎

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Figure 1. Intervention with a monopolist dealer and full information

Comparative statics: monopolist MM, full information, and CB chasing the trend. Exchange rate S_t on right axis, $\Delta S_{i,t}$ on left axis. Model's parametrization: $\alpha = 1$, $R = 1.05$, $\sigma_F^2 = 0.0045$, $\overline{X}_1 = 15$, $\overline{X}_2 = 5$, $q = 0.5$, $f = 1.7$, $\overline{S} = 1.58$, $\lambda = 0.00001$. Hence $^M S_t^* = 1.5761$ when $l = 0$.
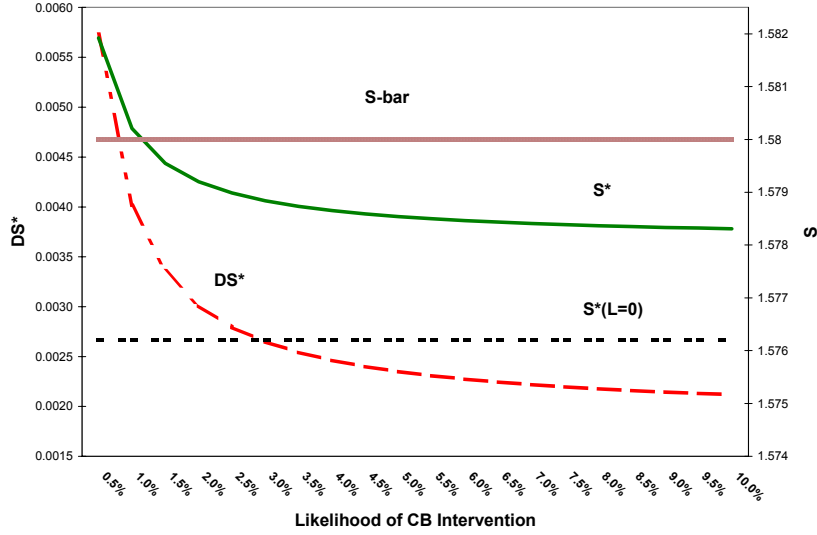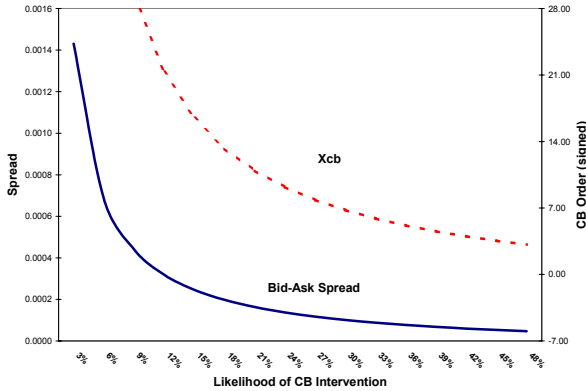

Figure 2. Intervention with competitive dealers and full information

Comparative statics: competitive MMs and full information. Units of foreign currency on right axis, spread on left axis. Model's parametrization: $\alpha = 1$, $R = 1.05$, $\sigma_F^2 = 0.025$, $\overline{X}_1 = 15$, $\overline{X}_2 = 5$, $q = 0.5$, $f = 1.7$, $\overline{S}_a = 1.45$, $\overline{S}_b = 1.35$, $\lambda = 0.00001$, $S_t^{CB} = \overline{S}$. Hence $^C S_t^* = 1.3810$ when $l = 0$.

a) CB chasing the trend

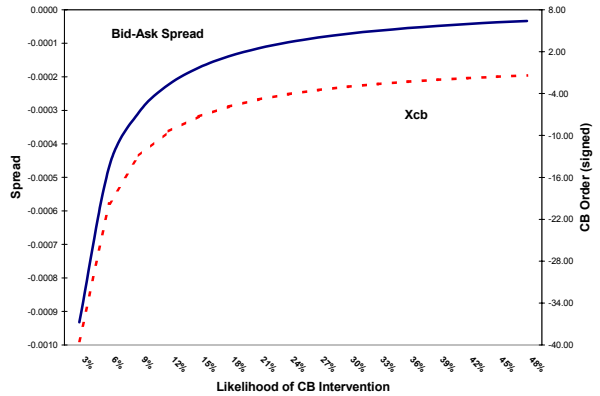### b) CB leaning against the wind

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Figure 3. Intervention with competitive dealers and full information: sensitivity analysis

Comparative statics: competitive MMs, full information, and CB chasing the trend. (a) Dollar revenues on right axis, spread on left axis. (b) Units of foreign currency on right axis, spread on left axis. Model's parametrization: $\alpha = 1, R = 1.05, \sigma_F^2 = 0.025, \overline{X}_1 = 15, \overline{X}_2 = 5, q = 0.5, f = 1.7, \overline{S}_a = 1.45, \lambda = 0.00001, l = 0.03$. Hence $^C S_t^* = 1.3810$ when $l = 0$.

a) Spread versus $\dfrac{\overline{X}_1}{\overline{X}_2}$ b) Spread and X_t^{CB} versus S_t^{CB}
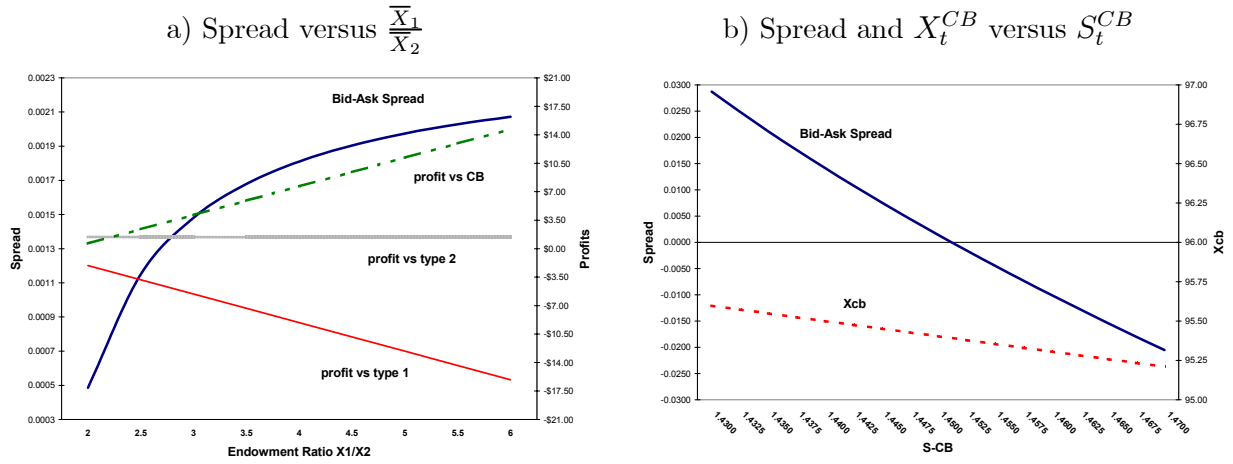

Figure 4. Effectiveness of intervention with full information

Comparative statics: full information and CB chasing the trend. Exchange rate S_t on right axis, GBP amounts on left axis. Model's parametrization: $\alpha = 1, R = 1.05, \sigma_F^2 = 0.025, \overline{X}_1 = 15, \overline{X}_2 = 5, q = 0.5, f = 1.7, \overline{S} = 1.45, \lambda = 0.00001$. Hence $^M S_t^* = 1.3810$ when $l = 0$.
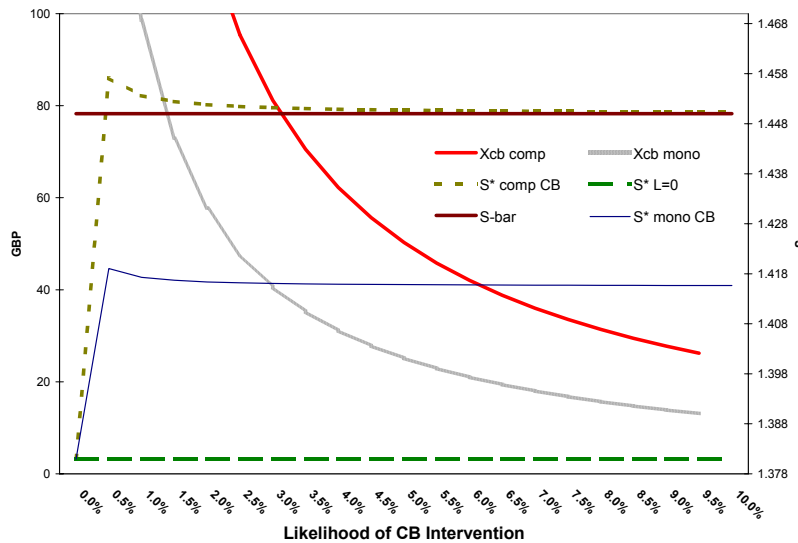

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Figure 5. Intervention with information asymmetry

Monopolist MM (a) and competitive MMs (b): CB leaning against the wind with $_B X_{t_6}^{CB} < 0$. Expected exchange rate $S_{t_n}^*$ on left axis (dark line), transaction prices S_{i,t_n} on left axis (light line), proportional spread PS_{t_n} on right axis (dotted line). Model's parametrization: $\alpha = 1$, $R = 1.05$, $\sigma_F^2 = 0.015$, $\overline{X}_1 = 12$, $\overline{X}_2 = 1$, $q = 0.25$, $f_H = 1.6$, $f_L = 1.45$, $\overline{S}_H = 1.5$, $\overline{S}_L = 1.31$, $\lambda = 0.001$. Market's beliefs: $^I q_{f,0} = 0.75$, $^U q_{f,0} = 0.5$, $q_{f,0} = 0.75$, $v = 0.8$, $\psi = 0.3$, $q_{\overline{S},0} = q_{f,0}\psi + (1 - q_{f,0})(1 - \psi) = 0.40$, $E\left[\overline{S}|M_0\right] = 1.3860$. The star symbol indicates when the intervention occurs in the simulated sequence of events.

a) Monopolist MM

b) Competitive MMs