Equilibria in a Hotelling model: First-mover advantage?

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Abstract

We consider the presence of first-mover advantage or disadvantage in a duopoly model of product positioning in which firms are symmetric, except for order of entry. We study a generalization of the Hotelling model, in which a consumer's utility from a product depends on the location of product and consumer in product attribute space, a random utility term that captures idiosyncratic preferences, and the price of the product. Since the model is analytically intractable, we computationally study location equilibria, with prices decided simultaneously after locations have been chosen.

We analyze the equilibrium outcomes based on the sensitivity of a firm's profit to prices and locations. A first-mover advantage obtains when profits are highly sensitive to location but only moderately sensitive to price. In contrast, when the location sensitivity is low, a moderate price sensitivity results in a first-mover disadvantage. High price sensitivity leads to maximum differentiation in product attributes. Finally, low sensitivity to both price and location has both firms locating at the market center.

1 Introduction

Is there a first-mover advantage in a market for a new product? Golder and Tellis (1993) report that two famous examples (among others) of this pioneer advantage are Campbell Soup and Crisco Shortening. However, they also point out that many markets have been characterized by a first-mover disadvantage, rather than an advantage, such as laundry detergent (Tide, a later entrant, is the market leader over Dreft) and chocolate (Hershey over Whitman's). Boulding and Christen (2003) report that, over the long term, there is typically a first-mover disadvantage in terms of profit.

Previous explanations for a first-mover disadvantage have typically relied on dynamic frictions that violate a notion of symmetry between a pioneer and a later entrant. For example, a follower may use a leader's experience to learn about the consumer distribution (Gal-Or 1987, Fershtman et al. 1990), or gain a technological advantage by developing a lower cost product (Tyagi 2000). There are countervailing forces as well: For example, Schmalensee (1982) recognizes that consumer loyalty is a factor that typically benefits the first mover in a dynamic setting. Chen and Xie (2007) show that, if a firm sells two products, asymmetry in customer loyalty can be a source of first-mover advantage or disadvantage. The existence or lack of a first-mover advantage can affect a pioneer's incentives to enter a market (Narasimhan and Zhang 2000).

In this paper, we consider the existence of first-mover advantage in a model in which firms are symmetric, except for entry order. That is, the entrant or follower firm does not have a cost advantage, does not learn about demand from the pioneer firm, and more generally, does not have any asymmetric advantage or disadvantage compared to the pioneer or leader firm. Instead, we build upon the canonical Hotelling model, with both firms symmetric in terms of how they are affected by their relative locations with respect to consumer distribution and their prices. In such a situation, it is widely believed that the first mover has an advantage, since it can locate at the market center.

Products have many attributes, and modeling consumer preferences along a single dimension represents a stylized simplification that models only the most important attribute. For example, if the product is a soft drink such as Coke or Pepsi, taste is arguably the most important attribute. Nevertheless, a consumer's preferences over other attributes, such as brand image or celebrity endorsements, may also influence her decision. One way to represent preferences over other unmodeled or unobserved attributes is via a random utility for the product. The random utility term makes a consumer's choice across the two products probabilistic, rather than deterministic. Of course, the price of the product is also an important input into the consumer's decision.

The relative importance of prices and product features in a consumer's decision problem in turn affects the sensitivity of a firm's profit to its price and its location in the product attribute space. We investigate the effect of these sensitivities on equilibrium outcomes. Specifically, we generalize the Hotelling model along two dimensions. Changing the variance of the random utility term reduces price sensitivity, and mitigates price competition among firms. We also vary a parameter that affects the distance cost (i.e., the cost to a consumer who does not obtain her most preferred product). Together, these two parameters affect the price and location sensitivity of a firm's profit.

As mentioned by Anderson et al. (1992), it is common to model choice probabilities as coming from a multinomial logit distribution. The multinomial logit model does not admit closed-form solutions for prices or locations. In fact, even the deterministic choice version of the Hotelling model with prices may be intractable rendering a numerical analysis necessary (Eaton and Lipsey 1976, Prescott and Visscher 1977). Nevertheless, we show analytically that, in equilibrium, the firm that is closer to the market center has a higher price, greater expected demand, and a higher profit than its competitor. To analyze equilibrium locations, we then turn to a computational analysis of the model.

Our primary focus is on sequential location choice: the leader firm picks a location, the follower firm observes this choice and picks a location, and then both firms simultaneously choose prices. Numerically, we show that the interplay between price sensitivity and location sensitivity of profit critically affects the degree of price competition between firms, and hence the equilibrium outcomes. Our findings are as follows.

- When price sensitivity is moderate in magnitude, consumers respond to changes in a firm's price, but not strongly. In such a situation, if location sensitivity is large in magnitude, both firms have a desire to move away from the center. A follower's desire to avoid price competition creates a distinct advantage for the first firm to enter the market. However, the leader also prefers to mitigate price competition, and typically does not locate at the market center—instead, it locates relatively close to the market center to extract a first-mover advantage, but allows enough space to differentiate and avoid a disastrous price war.

- When price sensitivity is moderate in magnitude but location sensitivity is low, a parameter region with first-mover disadvantage occurs. While firms still have an incentive to differentiate, the follower's best response is closer to the leader than the leader would prefer. In such a situation, the leader yields a more central location to the follower. That is, the follower is closer to the market center than the leader, and has a higher profit.

- With a small magnitude of price sensitivity and a small location sensitivity, price competition is relatively weak, and both firms locate at the market center.

- Finally, when the price sensitivity is sufficiently large, the market is characterized by maximal differentiation. The intense price competition can only be mitigated by firms locating far apart from each other. As a result, firms locate at opposite ends of the unit interval.

In Section 6, we discuss these results in greater detail and provide examples of markets in each of the above categories.

Our paper builds on the work of Rhee (2006), who incorporates a random utility term into a standard Hotelling model with a quadratic distance cost for consumers. He demonstrates numerically that, depending on the variance of the random utility term, either a first-mover advantage or disadvantage may result. By varying the relative importance of the distance cost, we find that the interaction of price sensitivity and location sensitivity of profit plays a critical role in determining equilibrium outcomes. When the price sensitivity is sufficiently large, the result is maximal differentiation.

Finally, for completeness, we consider simultaneous location choice by the two firms, followed by simultaneous determination of prices. Again, a large price sensitivity results in an equilibrium with maximum differentiation. When price sensitivity is low, depending on the location sensitivity, either both firms are at the market center, or firms are symmetrically located on either side of the market center.

In earlier work, Rhee et al. (1992) consider a simultaneous game with a linear distance cost. They show that when idiosyncratic preferences are unimportant, firms differentiate, whereas when they are important, firms locate at the center. We find that the level of location sensitivity of consumers adds an additional dimension to the determination of equilibria.

In the Hotelling model without prices, of course, both firms locate at the market center. d'Aspremont et al. (1979) show that, when prices are introduced and transportation costs are quadratic in distance, maximal differentiation in product position results. Maximal differentiation also occurs under sequential entry if firms are restricted to locating in the unit interval, as shown by Tabuchi and Thisse (1995).

There nevertheless appears to be a popular belief that symmetric location models display a first-mover advantage, and that the leader will inevitably occupy the market center. For example, Golder and Tellis (1993) suggest that firms which enter early and position near the center of the market can receive higher profits. Similar suggestions about the benefit (in terms of profit, market share, or both) of entering first and locating near the center appear in Lieberman and Montgomery (1988), Lilien et al. (1992), and Bohlmann et al. (2002). Theoretically, Tabuchi and Thisse (1995) show that if product positions are unrestricted relative to the demand distribution (i.e., firms can locate anywhere on the real line), a large first-mover advantage exists, with the leader occupying the market center. On the practical side, Song et al. (1999) find that managers to a significant extent believe in a first-mover advantage.

Some recent work, however, has found instances of a first-mover disadvantage, even in games with symmetric payoffs. In pure location games without pricing, the first mover may experience a disadvantage when the product attribute space has two or more dimensions (Chawla et al. (2006) and Wagener (2006)).

The rest of this paper is organized as follows. We present the model in detail in Section 2. In Section 3, we provide some analytic results on pricing. The computational analysis of location equilibria in the sequential game and the impact of price and location sensitivity of profit on these equilibria is provided in Section 4. We analyze the simultaneous location game in Section 5. The implications of our results are discussed in Section 6.

2 Model

We consider a generalization of the Hotelling location model. There are two firms, a pioneer firm that is referred to as Leader (L) and an entrant called the Follower (F), competing in a market. There is a continuum of consumers distributed uniformly over $[0, 1]$. A point in $[0, 1]$ denotes a level of a product attribute, and the location of a consumer reflects her ideal level. At stage 1 of the game, the two firms each position their product in the attribute space, by choosing a location in $[0, 1]$. We consider both simultaneous and sequential location in the first stage. In the sequential move game, firm L chooses its location first, and firm F chooses its location after observing

L's location choice. At stage 2, after observing each other's locations, the firms simultaneously choose prices.[1] At stage 3, each consumer buys one unit of the product from either L or F.

Both firms are assumed to have zero marginal costs, allowing us to focus on the mechanics of the location and pricing game. In addition, we assume there are no fixed costs. These assumptions simplify the analysis, but do not affect our results in any qualitative manner. As described below, given the prices and locations of both firms, each consumer's decision is probabilistic. Both firms maximize expected profit, which is the product of their price and overall expected demand.

Suppose a consumer located at y purchases the good at price p_j from firm j located at x_j. The consumer suffers a disutility from purchasing a product that does not possess her ideal attributes. This disutility is modeled as a distance cost $|y-x_j|^\alpha$, where $|y-x_j|$ is the distance between x_j and y and $\alpha \geq 1$ is a parameter that affects the relative importance of distance compared to price. For example, if $\alpha = 1$, the distance cost is linear in distance, whereas it is quadratic if $\alpha = 2$. The restriction of α to be at least 1 ensures convexity of the distance cost. Since $|y-x_j|$ will be strictly less than 1 for almost all consumers, an increase in α corresponds to a reduction in disutility from not obtaining the consumer's preferred product. Hence, an increase in α corresponds to a weakening of consumer preferences for the product attribute being modeled on the unit interval.

The consumption value of the good, assumed to be the same across the two firms, is denoted by $v_0 > 0$. Then, the consumer's deterministic utility in this scenario is

$$u(y, x_j, p_j) = v_0 - |y - x_j|^\alpha - p_j. \tag{1}$$

In addition, the utility of the consumer from good j is affected by unmodeled factors. These unmodeled factors could include other attributes of the good, marketing considerations such as branding, and purely idiosyncratic reactions of the consumer

[1] Some of the previous literature allows firms to locate outside the consumer space $[0, 1]$. See, for example, d'Aspremont et al. (1979) and the references in Anderson et al. (1992), Chapter 8.

to the good. Formally, the consumer at y obtains a random utility ϵ_{yj} from the good of each firm j, which captures the consumer's idiosyncratic preference. Thus, overall utility of the consumer located at y, if he buys good j at price p_j, is

$$\tilde{u}(y, x_j, p_j) = v_0 - |y - x_j|^\alpha - p_j + \epsilon_{yj}. \tag{2}$$

Each consumer purchases one unit of the good from the firm that provides him the highest overall utility. We assume the consumer does not have an outside option, and must purchase one of the two goods.[2] Since v_0 is common across the two goods, it does not affect the relative choice between the goods. Hence, for the rest of the paper, we assume $v_0 = 0$.

Following Anderson et al. (1992) (see Chapter 2 in particular), we consider a multinomial logit model for the idiosyncratic utility term. That is, we assume that for each consumer location $y \in [0, 1]$ and each firm j, the random utility term ϵ_{yj} is independent and identically distributed with distribution $F(x) = e^{e^{-x/\mu+\gamma}}$, where $\mu > 0$ is a parameter of the distribution and $\gamma \approx 0.5772$ is Euler's constant. The distribution has mean zero and variance $\mu^2\pi^2/6$, so that μ is proportional to the standard deviation of the random utility terms.

For each consumer y and each firm j, let $u_j(y) = u(y, x_j, p_j)$ denote the deterministic utility to the consumer from purchasing and consuming the good of firm j. Then, the consumer buys the good from L if $u_L(y) + \epsilon_L > u_F(y) + \epsilon_F$, or $\epsilon_L - \epsilon_F > u_F(y) - u_L(y)$. Thus, the probability that the consumer at y buys from L is given by $q_L(y, x_L, x_F, p_L, p_F) = \text{Prob}(\epsilon_L - \epsilon_F > u_F(y) - u_L(y))$, and the probability he buys from F is $q_F(y, x_L, x_F, p_L, p_F) = 1 - q_L(y, x_L, x_F, p_L, p_F)$. Going forward, for brevity we write these probabilities as $q_L(y)$ and $q_F(y)$, suppressing the locations and prices.

As shown in Proposition 2.2 from Anderson et al. (1992) (on page 39; attributed to

[2]It is straightforward to extend both the model and our numerical results to the case of an outside option, which may be thought of as a third good that provides the consumer with a deterministic reservation utility.

an unpublished document by Holman and Marley) the expected demands at location y for L and F are obtainable in closed form as follows:

$$q_L(y) = \frac{e^{u_L(y)/\mu}}{e^{u_L(y)/\mu} + e^{u_F(y)/\mu}} \tag{3}$$

$$q_F(y) = \frac{e^{u_F(y)/\mu}}{e^{u_L(y)/\mu} + e^{u_F(y)/\mu}} \tag{4}$$

The overall expected demand of each firm j is then given by $\int_0^1 q_j(y)dy$, since the consumer density is uniform over $[0, 1]$. Since the total demands across the two firms sum to one, firm j's demand may also be interpreted as its the overall market share. The expected profit of firm j is $\Pi_j = p_j \int_0^1 q_j(y)dy$.

Thus, the two key parameters in the model are α and μ. An increase in α implies a weakening of product attribute preferences. As a result, the price of the product becomes a more important factor in the consumer's decision. This has two effects on firms' location choices. First, keeping firm locations fixed, the degree of substitutability between the products increases. The resultant increase in price competition creates an incentive for firms to move further apart from each other. Second, since distance costs are mitigated, the degree to which a firm can alleviate price competition by moving away from its rival is also decreased. If price competition cannot be avoided by moving away, a firm may instead come closer to the market center as α increases.

These effects are in turn affected by changes in μ. An increase in μ implies a strengthening of idiosyncratic preferences. As a result, price competition is reduced, and firms are more willing to come toward the market center.

Equilibrium locations are determined by the interaction of all these effects. As we show below, the sensitivity of a firm's profit to both locations and prices varies with changes in the parameters, and ultimately determines the locations that obtain in equilibrium.

2.1 Solving the Game

We solve the game by backward induction. Consumer demands at each location at stage 3 are given by equations (3) and (4). Consider stage 2, where firms choose their prices, having chosen locations at stage 1. Given locations x_L, x_F and prices p_L, p_F, the expected profit of firm j, for $j = L, F$, is

$$\Pi_j(x_L, x_F, p_L, p_F) = p_j \int_0^1 q_j(y)dy, \tag{5}$$

where $q_j(y)$ is a function of x_L, x_F, p_L, p_F, which are suppressed for brevity.

The prices are chosen in a Nash equilibrium of the game at this stage. Each firm chooses its price to maximize its expected profit, holding fixed the price of the other firm. The best response function of firm j is given by the first-order condition $\frac{\partial \Pi_j}{\partial p_j} = 0$, or

$$\int_0^1 q_j(y)\Big[1 - \frac{p_j}{\mu}\,\Big(1 - q_j(y)\Big)\Big]dy = 0. \tag{6}$$

For now, we assume that the second-order condition $\frac{\partial^2 \Pi_j}{\partial p_j^2} < 0$ is satisfied. We show this analytically when locations are symmetric about the market center in Proposition 2 below, and confirm it computationally in our numeric analysis.

Noting that $q_F(y) + q_L(y) = 1$, the best response conditions can be equivalently written as:

$$\int_0^1 q_j(y)dy - \frac{p_j}{\mu}\int_0^1 q_L(y)q_F(y)dy \quad = \quad 0, \qquad \text{for } j = L, F. \tag{7}$$

The Nash equilibrium prices at stage 2, p_L^*, p_F^*, simultaneously satisfy equations (7). Considering the expressions for $q_L(y)$ and $q_F(y)$ in equations (3)–(4), and noting that $u_L(y) = -|y - x_L|^\alpha - p_L$ and $u_F(y) = -|y - x_F|^\alpha - p_F$, it is immediate to see that, in general, the best response conditions that determine prices at stage 2 do not admit a closed-form solution.

Given equilibrium prices p_L^*, p_F^* at stage 2, we work back to stage 1. Let $\hat{\Pi}_L(x_L, x_F) = \Pi_L(x_L, x_F,\ p_L^*(x_L, x_F), p_F^*(x_L, x_F))$. That is, $\hat{\Pi}_L(x_L, x_F)$ is the profit accruing to L

if the two firms locate at x_L and x_F respectively, and choose equilibrium prices as described above. Let $\hat{\Pi}_F(x_L, x_F)$ be similarly defined.

2.1.1 Sequential move game

In the sequential move game, given a location x_L selected by L, firm F chooses a location x_F which maximizes its profit once both firms set their equilibrium prices:

$$x_F^*(x_L) = \arg \max_{x_F \in [0,1]} \hat{\Pi}_F(x_L, x_F) \tag{8}$$

The corresponding first-order condition for F's maximization problem is $\frac{\partial \hat{\Pi}_F(x_L, x_F)}{\partial x_F} = 0$.

Continuing backward, L's problem is to select a location x_L^* that maximizes profit Π_L assuming that F will respond with the optimal location choice $x_F^* = x_F^*(x_L^*)$, with both firms choosing equilibrium prices, $p_L^* = p_L^*(x_L^*, x_F^*)$ and $p_F^* = p_F^*(x_L^*, x_F^*)$, at stage 2. Let $\bar{\Pi}(x_L)$ be the profit of firm L if it chooses a location x_L, firm F then chooses an optimal location $x_F^*(x_L)$, and both firms choose prices optimally in the pricing subgame. Formally, $\bar{\Pi}_L(x_L) = \Pi_L(x_L, x_F^*(x_L), p_L^*(x_L, x_F^*(x_L)), p_F^*(x_L, x_F^*(x_L)))$.

Since the consumer distribution is uniform, and hence symmetric about the market center at 0.5, we can without loss of generality restrict L to locating in the sub-interval $[0, 0.5]$. Then, L's optimal location x_L^* maximizes $\bar{\Pi}(x_L)$ over the sub-interval $[0, 0.5]$. If its optimal location is in the interior of this sub-interval, it satisfies the first-order condition $\frac{\partial \bar{\Pi}_L(x_L)}{\partial x_L} = 0$.

2.1.2 Simultaneous move game

In the simultaneous move game, both firms simultaneously choose locations to maximize their own profit. Again, the optimum for each player is described by a first-order condition. In a pure strategy equilibrium, if the equilibrium locations x_L^* and x_F^* are in the interior of $[0, 1]$, they must simultaneously satisfy the conditions

$$\frac{\partial}{\partial x_L} \hat{\Pi}_L(x_L^*, x_F^*) = 0; \qquad \frac{\partial}{\partial x_F} \hat{\Pi}_F(x_L^*, x_F^*) = 0 \tag{9}$$

For the simultaneous move game, we restrict attention to pure-strategy equilibria that are symmetric (that is, $x_L^* + x_F^* = 1$). Based on our numeric findings, we conjecture that the only pure-strategy equilibria in the simultaneous move game are indeed symmetric.

Since we do not have a closed-form expression for equilibrium prices at stage 2, we cannot analytically solve for equilibrium locations in either the simultaneous or the sequential move game. Instead, we numerically solve for equilibria. The results are presented in Section 4. First, to obtain some insight into the trade-offs faced by the firms, we consider the pricing subgame at stage 2.

3 Pricing Subgame

Consider the pricing subgame at stage 2. Suppose each firm $j = L, F$ has chosen its location, x_j. Assume that, for any pair of locations (x_L, x_F) chosen at stage 1, there exists a pure-strategy pricing equilibrium at stage 2. This assumption is borne out in our numeric calculations. Through much of this section, we also assume that the solution to the first-order conditions (7) does indeed constitute a pair of optimal prices for F and L. In Proposition 2, we prove this when firms have symmetric locations. More broadly, we confirm in our numeric solution that prices are optimal for each set of parameter values and firm locations. That is, we verify numerically that prices found by solving equations (7) do indeed maximize profits.

Location is critical to relative outcomes in the game. First, we consider asymmetric locations, with one firm (say firm i) being closer to the market center than its rival (firm j). We show that the firm that locates closest to the center will, in equilibrium, have a higher price, market share, and profit than the other firm. The proof of this proposition, and all other analytic results, is in the Appendix, Section A.

Recall that the overall expected demand for firm i is $d_i = \int_0^1 q_i(y)dy$, which is also the market share of firm i in our model.

Proposition 1 *In an equilibrium of the pricing subgame at stage 2, the firm closer to the center has a higher price, a higher overall expected demand, and a higher profit than its rival. That is, if* $|x_i - 0.5| < |x_j - 0.5|$*, then* $p_i^* > p_j^*$*,* $d_i^* > d_j^*$*, and* $\Pi_i^* > \Pi_j^*$*.*

In the next section, we show numerically that for a large region of parameter values, equilibria in the simultaneous-move game are characterized by one firm being closer to the market center than its rival. Given Proposition 1, in such equilibria, a first-mover advantage will exist if and only if firm L is closer to the market center (i.e., the point 0.5) than firm F. Equilibria with F closer to the market center will necessarily involve a first-mover disadvantage.

Next, consider the special case in which firms have located symmetrically around the market center (the point 0.5), i.e., when $x_L + x_F = 1$. We first show that in this case, the equilibrium prices and profits of the two firms are equal. Further, it is reasonable to conjecture that, as the distance between the firms increases in the location stage, the equilibrium prices will increase in the pricing subgame. The absence of an outside option then implies that overall profits will also go up. While this is hard to prove for the general case (i.e., keeping one player's location fixed while moving the other player further away), we are able to prove this for the symmetric locations case. Finally, we show that, when locations are symmetric about the market center, the solution to the best response equations (7) indeed determines optimal prices; that is, a second-order condition for profit maximization is satisfied.

Proposition 2 *Suppose firms' locations are symmetric about the market center (i.e.,* $x_i + x_j = 1$*). Then, in the pricing subgame at stage 2,*
(i) equilibrium prices, market shares, and profits of the firms are equal. That is, $p_i^* = p_j^*$*,* $d_i^* = d_j^*$*, and* $\Pi_i^* = \Pi_j^*$*,*
(ii) equilibrium prices increase as firms' locations become more distant from the center, and
(iii) the solution to the first-order conditions (7) maximizes the profit of each firm j*,*

keeping fixed the price of the other firm, and therefore characterizes an equilibrium of the pricing subgame.

4 Location Equilibria in the Sequential-move Game

Determining the optimal location essentially requires determining the equilibrium prices for each pair of feasible locations. However, as mentioned above, the equilibrium prices, which satisfy equations (7) do not admit a closed-form solution in general. Hence, we numerically determine equilibrium locations and prices for a range of (α, μ) parameters. Specifically, we consider α varying from 1 to 3, and μ varying from 0.2 to 2. Details of our numerical procedure are provided in Appendix B.

Since our focus is on the existence of a first-mover advantage or disadvantage, we begin by analyzing the sequential game. The equilibria that result for different parameter values are exhibited in Figure 1.

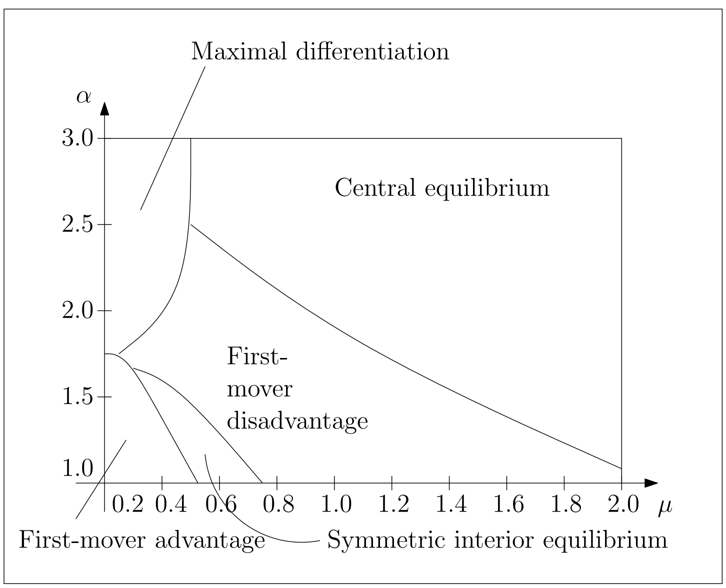

Figure 1: Regions of different equilibria in sequential move game

We find that there are five classes of equilibria. Three of these are symmetric, with firms having equal prices and profits. The other two exhibit either a first-mover advantage or a first-mover disadvantage.

1. Central equilibrium: Both firms are at the market center, with $x_L^* = x_F^* = 0.5$. The equilibria obtain in the north-east segment of the parameter region.

2. First-mover advantage: Firm L is closer to the center than the follower, with $0.5 - x_L^* < x_F^* - 0.5$. Proposition 1 implies that L has a profit advantage in these equilibria, which is confirmed in our numerical results. The parameter region in which these equilibria obtain is roughly the triangle bounded by $(\mu = 0.2, \alpha = 1.8)$ and $(\mu = 0.5, \alpha = 1)$.

   The maximum Leader advantage we observe is when $\alpha = 1.5$ and $\mu = 0.2$. At these values, we find $x_L^* = 0.2525$, $x_F^* = 0.9775$, $p_L^* = 0.9382$ and $p_F^* = 0.8098$. Thus, L is approximately halfway between one end of the market and the market center, while F is almost at the other end of the market. This locational advantage of L is reflected in the higher profit observed: $\Pi_L^* = 0.5036$, with $\Pi_F^* = 0.3751$, so that the leader has a profit advantage of 34.24%.

3. First-mover disadvantage: Firm F is closer to the center $(0.5 - x_L^* > x_F^* - 0.5)$, and so, by Proposition 1, has a profit advantage. As shown in Figure 1, the parameter region in which these equilibria obtain consists of intermediate values of μ, for α ranging from 1 to approximately 2.5.

   The maximum follower advantage is when $\alpha = 2.5$ and $\mu = 0.6$. At these values, L locates at $x_L^* = 0$ and F locates at $x_F^* = 0.7425$, with prices $p_L^* = 1.2648$ and $p_F^* = 1.3846$. The resultant profits are $\Pi_L^* = 0.6038$ and $\Pi_F^* = 0.7236$ respectively, translating to a 16.56% profit disadvantage for L.

4. Symmetric interior equilibrium: Firms are strictly in the interior of the location space, with $x_L^* \in (0, 0.5)$, $x_F^* \in (0.5, 1)$, and $x_F^* = 1 - x_L^*$. These equilibria occur

in a small region of the parameter space, when α is low and μ is in a low to intermediate range.

5. Maximal differentiation: The firms are at the extremes of the location space, with $x_L^* = 0$ and $x_F^* = 1$. These equilibria are obtained when α is high and μ is low.

4.1 Sensitivity of Profit to Price and Location

To explain the equilibrium outcomes in the sequential game, we consider the sensitivity of profit with respect to price and location as the parameters α and μ change. These sensitivities will depend on the location of both firms and on the prices chosen by them. To standardize the computation, we fix symmetric firm locations at $x_L = 0.25$ and $x_F = 0.75$. Given these locations, we determine equilibrium prices p_L^* and p_F^* for each (α, μ) pair.

Let $\bar{\Pi}$ denote the profit of firm L when prices are optimal chosen given the above locations. Define a discrete change in price for firm L of Δ_p. Suppose firm L charges $p_L^* + \Delta_p$, firm F continues to charge p_F^*, and firm locations remain fixed at $x_L = 0.25$ and $x_F = 0.75$. Let $\hat{\Pi}_L$ be the new profit of firm L. Since the new price of firm L is no longer a best response to firm F, it follows that $\hat{\Pi}_L < \bar{\Pi}_L$.

We define the sensitivity of L's profit with respect to its price as

$$\eta_p = \frac{\bar{\Pi}_L - \hat{\Pi}_L}{\bar{\Pi}_L}.$$

Note that as $\Delta_p \to 0$, η_p will also approach zero, since we are starting with optimal prices. In our computations, we take $\Delta_p = 0.1$. This corresponds to approximately a 5% change in the average optimal price over the range of α and μ that we consider.

The interpretation of price sensitivity is as follows. As defined, η_P is positive, since $\bar{\Pi}_L$ is the profit earned by firm L when it prices opimtally. Thus, larger values of η_P indicate that a given change in price has a greater effect on a firm's profit.

Hence, there is stronger price competition between the firms. In such situations, we expect that firms will have an incentive to move apart from each other.

Next, consider the sensitivity of firm L's profit with respect to its own location. As before, we take $x_L = 0.25$ and $x_F = 0.75$. Suppose firm L moves to a new location $\tilde{x}_L = x_L + \Delta_x$. Hold firm F fixed at $x_F = 0.75$. Given the new locations, firms choose optimal prices, which in turn determine firm profits. Let $\tilde{\Pi}_L$ be firm L's profit in this scenario. We define the sensitivity of firm L's profit with respect to its location as

$$\eta_x = \frac{\tilde{\Pi}_L - \bar{\Pi}_L}{\bar{\Pi}_L}.$$

In our computations, we take $\Delta_x = 0.05$.

Unlike price sensitivity, location sensitivity may be positive or negative. If it is negative, firm L would benefit from moving away from the center. In contrast, if it is positive, firm L prefers to come toward the market center.

We first consider the effect on equilibrium as one of these sensitivities changes, keeping the other one fixed.

4.1.1 Effect of price sensitivity of profit

For each value of α between 1 and 3, we determine a value of μ so that the location sensitivity η_x as computed above is approximately zero. We then determine both the equilibrium outcome and the price sensitivity for these (α, μ) pairs. Figure 2 displays the equilibrium locations on the X-axis and the price sensitivity of profit on the Y-axis.

As seen from the figure, when the price sensitivity of profit is high (above 0.01 in the figure), price competition is intense enough to drive firms toward the corners, and maximal differentiation results. For lower values of price sensitivity, given the location sensitivity chosen, there is a first-mover disadvantage: the leader is further away from the center than the follower. Although there may be gains to differentiation, the small price sensitivity implies that the follower will in general be closer to the leader than the

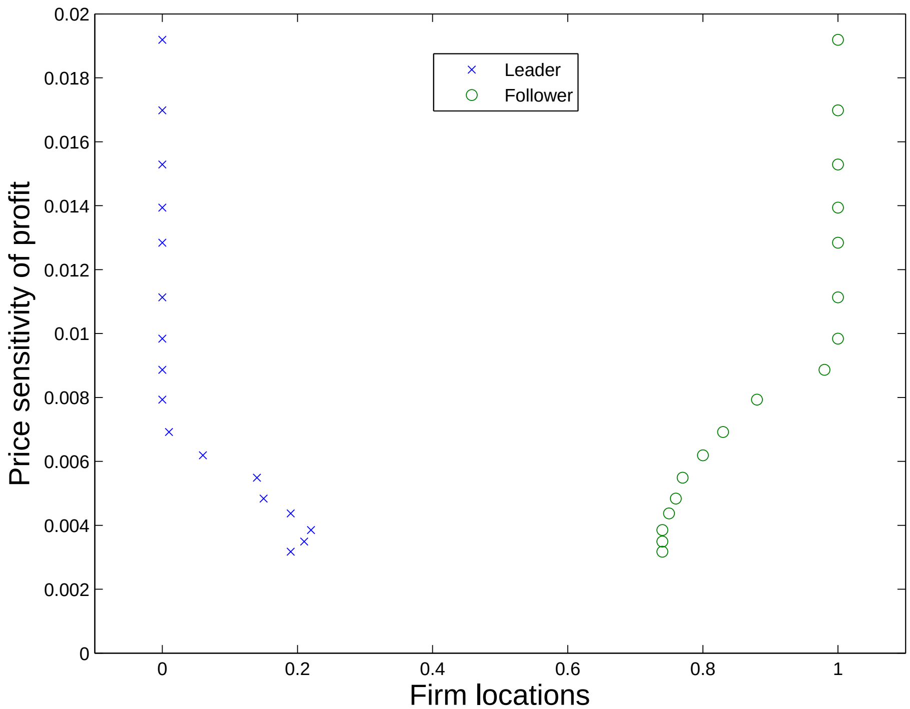

Figure 2: Equilibrium outcomes for different values of responsiveness to changes in price, keeping location sensitivity fixed

leader would like. Hence, to extract the gains from differentiation, the leader must take the initiative by yielding the center. Resultantly, a first-mover disadvantage occurs.

Note also that the equilibrium locations are non-monotonic in the price sensitivity of profit. Although we have fixed the location sensitivity at zero, as firms change their location, the location sensitivity itself varies. The non-linear interaction between location and price sensitivities determines the actual equilibrium locations.

4.1.2 Effect of location sensitivity of profit

We perform the same exercise as earlier, this time keeping price sensitivity fixed at 0.013. Again, for each value of α, we find the value of μ that results in a price sensitivity of approximately 0.013. We then determine the equilibrium outcomes and the location sensitivity for each (α, μ) pair. The equilibrium locations are shown on the X-axis in Figure 3, with the location sensitivity on the Y-axis.

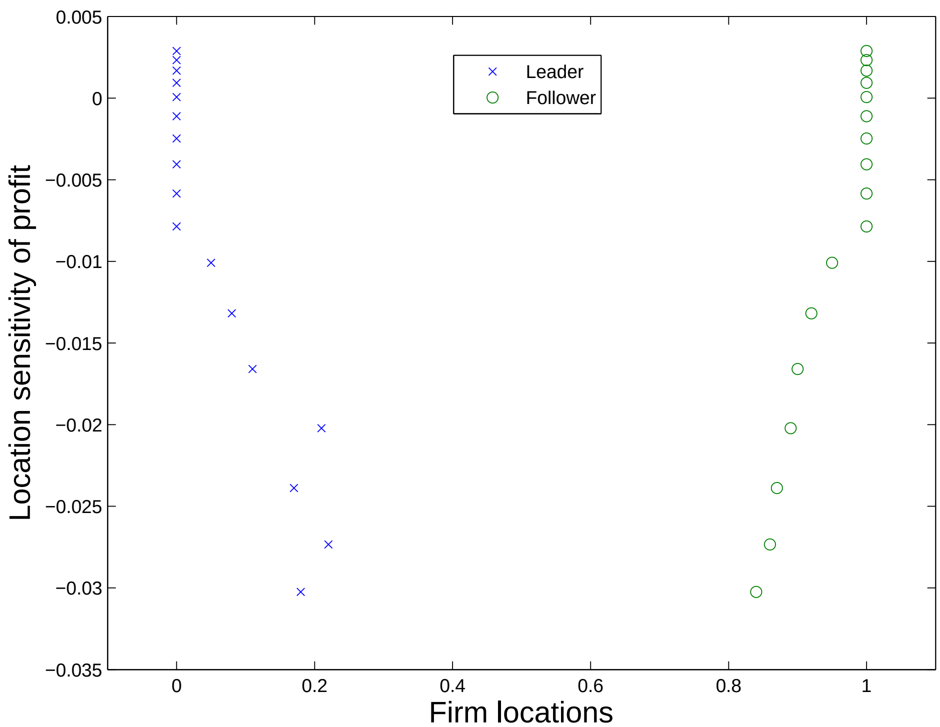

Figure 3: Equilibrium outcomes for different values of responsiveness to changes in location, keeping price sensitivity fixed

When the location sensitivity of profit is close to zero, we find that maximal differentiation results. A zero location sensitivity implies that a small change in location has a negligible impact on a firm's profit. However, the price sensitivity is relatively high (0.013), implying that there are potential benefits to mitigating price competition. These benefits are only realized if firms move sufficiently far apart from each other, resulting in maximal differentiation.

When the location sensitivity of profit is negative and large in magnitude, there is a first-mover advantage: firm L is closer to the center than firm F. For any symmetric pair of locations, a negative location sensitivity implies that firm F would like to be even further away from the center. Hence, a first-mover advantage results.

4.1.3 Analysis of equilibria in terms of sensitivities

We now analyze the various equilibria in the sequential-move game using the notions of price and location sensitivity of profit. Figure 4 shows a scatter plot of the sensitivities η_p and η_x for the same values of μ and α used to compute the equilibria. That is, α varies from 1 to 3, and μ from 0.2 to 2. In the figure, the points are marked according to the equilibrium outcome generated in the sequential game. Each contiguous segment of points corresponds to a fixed value of μ, with μ increasing as we move from the upper left corner to the lower right corner. Within each contiguous segment of points, α increases from left to right.

Consider the equilibria we attain in the sequential game in terms of price and location sensitivity of profit.

1. **Central Equilibrium**: As expected, the central equilibrium only occurs when location sensitivity is positive (i.e., a firm's profit increases as it comes toward the center). However, it is important to note that even in the region of positive location sensitivity, the central equilibrium only occurs if price sensitivity is small in magnitude. Otherwise, staying sufficiently far away from the center, and increasing prices as a result, can result in an overall benefit to the firm.

2. **First-mover advantage**: This occurs in a region diametrically opposite to the central equilibrium region. Here, location sensitivity is negative and large in magnitude, so firms have an incentive to differentiate in location. Further, price sensitivity is large in magnitude, indicating that consumers respond strongly to changes in price. On both counts, firms wish to locate far apart from each

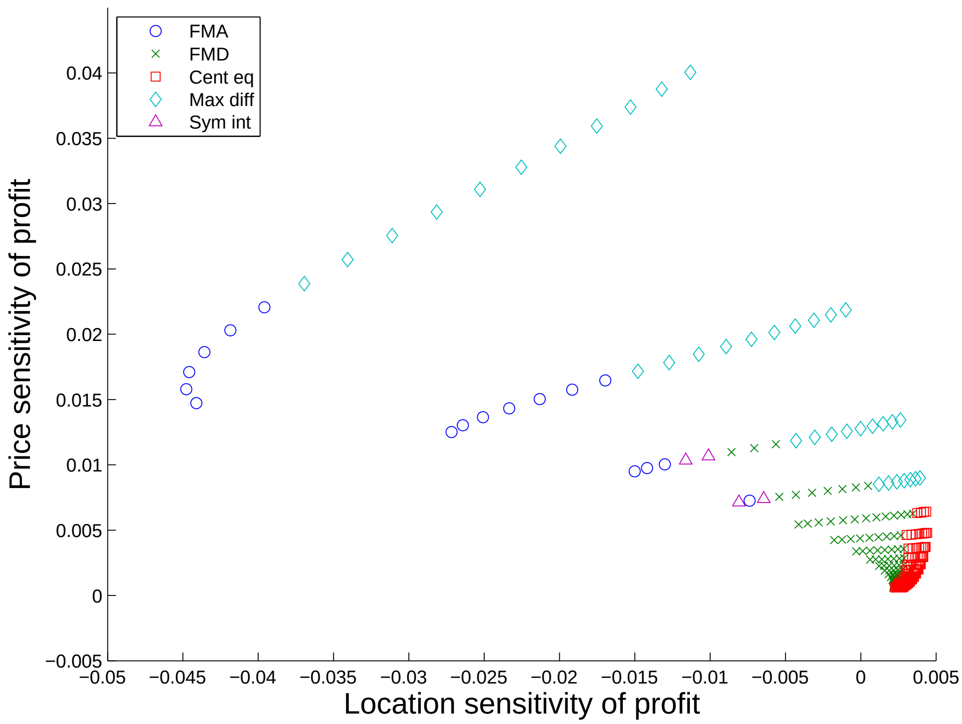

Figure 4: Equilibrium outcomes for different values of responsiveness to changes in location and price

other. The strong price sensitivity means that the leader can count on the follower to do the differentiation. Thus, the leader stakes out a location closer to the market center, resulting in a first-mover advantage.

3. **First-mover disadvantage**: This occurs in a region where location sensitivity and price sensitivity are both relatively small in magnitude. When the location sensitivity is small, minor changes in location have a very small effect on a firm's profit. To achieve a substantial improvement in profit, a firm must move far away from its rival. Since the price sensitivity is also small, the follower is willing to locate closer to the market center than the leader, causing a first-mover disadvantage.

4. **Symmetric interior equilibrium**: This occurs in a region of moderate price sensitivity and small but negative location sensitivity. Essentially, this is a case in between first-mover advantage and first-mover disadvantage. Firms wish to differentiate from each other, and both firms are willing to concede about as much as each other in this differentiation.

5. **Maximum differentiation**: This occurs in a region in which price sensitivity is large in magnitude, and location sensitivity is either negative or mildly positive. Consumers are very responsive to price, and a negative location sensitivity implies a strong desire for firms to differentiate. As a result, maximum differentiation results. A positive location sensitivity implies that firm L earns a higher profit by locating at 0.3 than at 0.25, given that firm F is at 0.75. However, in equilibrium, once firm L takes into account that firm F will play a best response, L earns an even higher profit by locating at 0.

Therefore, price and location sensitivity critically affect the nature of the equilibrium in the sequential game. It is useful to compare our findings to those of Rhee (2006), who analyzes the sequential game with quadratic transportation costs (i.e, $\alpha = 2$). He further allows firms to locate anywhere on the real line (in contrast to our model, where firm locations are restricted to the $[0, 1]$ interval). He finds that when idiosyncratic preferences are unimportant (i.e., μ is small), there is a first-mover advantage. However, for moderate values of μ, there is a first-mover disadvantage. As we show above, when μ is small, location sensitivity is negative. However, a first-mover advantage exists only when price sensitivity is also small. In contrast, when price sensitivity is large, the equilibrium exhibits maximal differentiation. When μ is small, price sensitivity is low when α is also small, but high when α is large.

5 Simultaneous-move Game

For completeness, we also analyze the simultaneous-move game, which has received much attention in the literature. The equilibria that emerge in the simultaneous-move game for different values of the parameters μ and α are shown in Figure 5. Observe that for several values of α and μ, multiple equilibria exist in the simultaneous move game (for example, consider the region with α approximately equal to 2 and μ between 0.5 and 0.8).

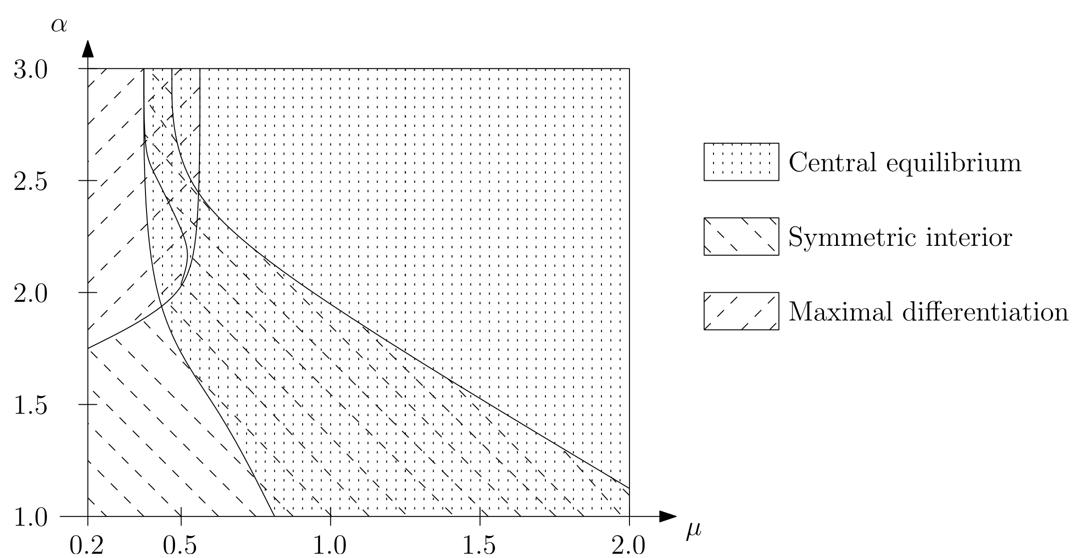

Figure 5: Regions of different equilibria in simultaneous move game

We identify three classes of equilibria in the simultaneous-move game. All the equilibria we found are symmetric in location (so $x_L^* + x_F^* = 1$). Thus, by Proposition 2, the two firms have equal prices and profits in each of these equilibria.

1. Maximal differentiation: The firms are at the extremes of the location space, with $x_L^* = 0$ and $x_F^* = 1$. These equilibria are observed for $\alpha \geq 1.7$, with small values of μ.

2. Central location: Both firms are at the market center, with $x_L^* = x_F^* = 0.5$. For each value of α, these equilibria are observed when μ is sufficiently high, with the threshold value of μ falling as α increases.

3. Symmetric interior location: Firms are strictly in the interior of the location space, with $x_L^* \in (0, 0.5)$, $x_F^* \in (0.5, 1)$, and $x_F^* = 1 - x_L^*$. These equilibria are observed for all values of α, with a range of μ that declines as α increases.

For α below 1.7, as seen from Figure 5, there are only two kinds of equilibria. For low values of μ, there is a symmetric interior equilibrium, in which firms move closer to the center as μ increases. For μ sufficiently high, both firms locate at the center.

When μ is low (below 0.8 or so) and α is high (above 2.0), price competition is intense, and firms locate at the corners of the unit interval, with L at 0 and F at 1. For μ above 0.4 and α above 1.8, there is also an equilibrium in which both firms are at the center. The simultaneous existence of equilibria with maximal differential and minimal differentiation emphasizes the two effects at work when α is high.

Finally, for some range of α above 2 and μ between 0.4 and 0.8, there is also a symmetric interior equilibrium. Each firm is highly sensitive to its rival's choice of location for parameters in this region; a small move by the rival can tip the best response either toward or away from the market center.

In comparing the equilibria of the simultaneous and sequential move games, a striking feature is the close overlap of the regions with central equilibrium and maximal differentiation. This is no coincidence: we prove below that if the sequential-move game leads to a symmetric equilibrium, then the same symmetric equilibrium must also be an outcome of the simultaneous move game.

Proposition 3 *Suppose that, for some (α, μ), the sequential location game has a symmetric equilibrium. Then, the same set of locations and prices constitute an equilibrium of the corresponding simultaneous move game.*

Note that the converse is not true; in fact, if it were, there would be no asymmetry in profits in the sequential move game.

In an analysis of the simultaneous game with $\alpha = 1$, Rhee et al. (1992) find that insufficient heterogeneity along the idiosyncratic attribute leads to some (but not maximal) differentiation among the firms, whereas sufficient heterogeneity leads to both firms at the center. Here too, we find that price and location sensitivity play a critical role in determining equilibrium outcomes. For low values of μ, we find that when price sensitivity is relatively low (corresponding to low values of α), a symmetric interior equilibrium results. However, when price sensitivity is large (corresponding to high values of α), maximal differentiation occurs. Further, as α increases, the central equilibrium is supported for smaller values of μ.

Interestingly, we further find that there are parameter values that support both maximal differentiation and minimal differentiation as an equilibrium. These parameter values correspond to a small magnitude of both location and price sensitivity. The slope of a firm's best response curve is highly sensitive to its rival's location under these circumstances. Suppose one firm occupies the market center. Since price sensitivity of profit is low, a change in prices has a small impact on profit. Thus, even if the other firm differentiates, it cannot raise its price high enough to compensate for the locational disadvantage. However, if its rival is at one corner of the location space, each firm maximizes profit by occupying the other corner.

The price sensitivity and location sensitivity of profit defined in Section 4.1 help explain the three kinds of equilibria we find in the simultaneous-move game. As in the sequential-move game, when the price sensitivity of profit is high, firms have a strong incentive to differentiate from each other. Hence, an equilibrium with maximum differentiation is supported. In contrast, when the price sensitivity is low and the location sensitivity is positive, each firm increases its profit by coming closer to the market center. This leads to the emergence of a central equilibrium. Finally, when price sensitivity is in a low-to-moderate range and location sensitivity is negative, the optimal balance between capturing the market center (which minimizes the average distance to a consumer) and differentiating from a rival firm (to mitigate price competition) results in a symmetric interior equilibrium.

6 Discussion

What are the implications of our model for a firm's decision on when to enter a new market? If the equilibrium of the sequential game is symmetric, the order of entry clearly does not matter. However, when asymmetric equilibria appear in the sequential-move game, the order of entry matters. If price sensitivity is large and location sensitivity is also large in magnitude, the first mover has an advantage, since the follower has a strong incentive to differentiate. There will therefore be a "rush to market" to realize the first-mover advantage. Conversely, if price sensitivity is relatively small and location sensitivity is also small in magnitude, we are in a region of first-mover disadvantage. Thus, both firms would prefer that the other firm move first.

In reality, of course, several other factors may also affect the timing of entry timing, including technology, entry barriers, evolving customer demand, and incomplete information. These, in conjunction with the impact of location and pricing, will then determine the optimal time to enter. Nevertheless, our model offers valuable insights for managers as they plan their entry strategies in new markets.

For example, consider the market for laundry detergent. As Golder and Tellis (1993) report, the pioneer in this market was Dreft, yet Tide became the market leader. This product can be characterized as having relatively low location sensitivity. Recall that the location sensitivity captures the effect on profit of a small change in location. Small changes in the characteristics of a detergent are likely to be undetectable for most consumers. Further, this product has moderate price sensitivity, since it is a low-priced product, yet is purchased several times a year. Our model predicts (see Figure 4) that such a market would exhibit a first-mover disadvantage. Indeed, today Dreft is a niche player with a main product away from the market center.

Next, consider the emergence of a first-mover advantage. In the market for canned soup, Campbell was among the early entrants and has long been a market leader (see Golder and Tellis (1993)). In the soup market, location sensitivity is relatively high: small changes in product features are quickly detectable by consumers. For example, adding more vegetables or a thicker broth changes consumer demand, and hence firm profit. Conversely, the price sensitivity is relatively low. From Figure 4, we expect such a market to exhibit a first-mover advantage.

The market for USB drives provides an example of a central equilibrium. Keeping capacity fixed, other changes in the product are unimportant and have little to no impact on consumer demand. Price sensitivity is also not too high, since it is a low-priced and infrequently-purchased good. Consistent with Figure 4, a central equilibrium is observed.

Finally, consider the market for the next generation of passenger aircraft. The two players, Boeing and Airbus, have chosen very different product features.[3] The Airbus A380 is the largest passenger aircraft in the market, whereas the Boeing 787 has a capacity less than half of the A380. The consumers in this market are passenger airlines. Airlines follow very different network strategies, so the product has a high location sensitivity. Of course, given the competition in the airline industry and the high cost of an aircraft, the price sensitivity is also high. As expected from our model, the market is characterized by maximum differentiation.

While we explicitly consider a Hotelling setting in this paper, we have confirmed that our insights into first-mover advantage do carry over more broadly to location models. Two other popular location models are those of Lane (1980), and the

[3]Indeed, an article in Time Magazine titled "Bigger vs. Faster" mentioned that Airbus and Boeing were "betting billions on very different visions of the future of flying." (May 7, 2001 issue).

Defender model of Hauser and Shugan (1983). In Lane's model, the leader again captures the market center and commands a profit which is more than twice that of the follower. We performed a numerical analysis similar to that in Section 4 in the Lane model (after adding the random utility term), and obtained qualitatively similar results about the existence of a first-mover advantage or disadvantage. We are not aware of a sequential location analysis of the Defender model.

A Proofs

Proof of Proposition 1

Suppose $|x_i - 0.5| < |x_j - 0.5|$. If $x_j < x_i < 0.5$, the result is immediate. Hence, assume that $x_i < 0.5 < x_j$. Then, we have $0.5 - x_i < x_j - 0.5$, or $x_i + x_j > 1$.

Let (p_i^*, p_j^*) denote the price equilibrium for these locations. Then, rewriting equations (7) slightly, for each firm l, f, the following best response condition for optimal price must be satisfied:

$$\int_0^1 q_j(y)dy = \frac{p_j}{\mu} \int_0^1 q_L(y)q_F(y)dy. \tag{10}$$

Suppose that $p_i^* \leq p_j^*$. Then, it follows from (10) that $\int_0^1 q_i^*(y)dy \leq \int_0^1 q_j^*(y)dy$. We will show instead that $\int_0^1 q_i^*(y)dy > \int_0^1 q_j^*(y)dy$, which provides a contradiction.

Let $y_0 = x_i + x_j - 1$, and $y_1 = \frac{x_i+x_j}{2} = \frac{y_0+1}{2}$. That is, the point y_1 is the midpoint of the interval $[x_i, x_j]$, and also the midpoint of the interval $[y_0, 1]$. Since $x_i + x_j > 1$, it follows that $y_0 > 0$.

Let z be any point in the interval $[y_0, 1]$. The symmetric image of z about the point y_1 is given by $\overline{z} = 1 + y_0 - z$. Therefore, we have $x_i - z = \overline{z} - x_j$, for any $z \in [y_0, 1]$. Since $p_i^* \leq p_j^*$ by assumption, it must be that $q_i(z) \geq q_j(\overline{z})$ for any $z \in [y_0, 1]$. Therefore, the expected demands of firm i over the interval $[y_0, 1]$ must be greater. That is,

$$\int_{y_0}^1 q_i^*(y)dy \geq \int_{y_0}^1 q_j^*(y)dy. \tag{11}$$

Now, consider the interval $[0, y_0]$. By construction, for any point $z \in [0, y_0]$, we have $|x_i - z| < |x_j - z|$. Further, we have assumed $p_i^* \leq p_j^*$. Therefore, it follows that $q_i^*(z) > q_j^*(z)$, so that

$$\int_0^{y_0} q_i^*(y)dy > \int_0^{y_0} q_j^*(y)dy. \tag{12}$$

Now, for each firm k, the overall expected demand is $\int_0^1 q_k^*(y)dy = \int_0^{y_0} q_k^*(y)dy + \int_{y_0}^1 q_k^*(y)dy$. Thus, it follows that $p_i^* \leq p_j^* \implies \int_0^1 q_i^*(y)dy > \int_0^1 q_j^*(y)dy$. However, by inspection of the best-response condition (10), it must be that $\int_0^1 q_i^*(y)dy > \int_0^1 q_j^*(y)dy \implies p_i^* > p_j^*$, which is a contradiction.

Therefore, it cannot be that $p_i^* \leq p_j^*$, so that we have $p_i^* > p_j^*$. Now, from equation (10), it follows immediately that $p_i^* > p_j^* \implies d_i^* = \int_0^1 q_i^*(y)dy > d_j^* = \int_0^1 q_j^*(y)dy$. Since firm i has a higher price and a higher market share than firm j, its profit is also higher. ∎

Proof of Proposition 2

(i) Suppose $x_i + x_j = 1$. Then, the point $z = \frac{1}{2}$ lies midway between x_i and x_j. Suppose that $p_i^* < p_j^*$. Now, for every $z \in [0, \frac{1}{2}]$, consider the point $\overline{z} = 1 - z \in [\frac{1}{2}, 1]$. Since $p_i^* < p_j^*$, it follows that $q_i^*(z) > q_j^*(\overline{z})$. Hence, $q_j^*(z) = 1 - q_i^*(z) < q_i^*(\overline{z}) = 1 - q_j^*(\overline{z})$. Therefore, it follows that

$$\int_0^1 q_i^*(y)dy > \int_0^1 q_j^*(y)dy, \tag{13}$$

or $d_i^* > d_j^*$. But, from the proof of Proposition 1, if $p_i^* < p_j^*$, it must be that $d_i^* < d_j^*$. Therefore, it cannot be that $p_i^* < p_j^*$.

By the same argument, we can rule out the case that $p_i^* > p_j^*$. Hence, it must be that $p_i^* = p_j^*$. Now, since locations are symmetric about the market center, and the prices are equal, it must be that $d_i^* = d_j^*$, and hence $\Pi_i^* = \Pi_j^*$.

(ii) Because symmetric locations imply equal prices (as shown in part (i)), the market share of each firm must be exactly 1/2. Using this, the best response condition for each firm j (10) reduces to:

$$\frac{p_j}{\mu} \int_0^1 q_L(y) q_F(y) dy \quad = \quad \frac{1}{2}. \tag{14}$$

Now consider two sets of locations, with (x_L, x_F) given by $(0.5 - h_1, 0.5 + h_1)$ and $(0.5 - h_2, 0.5 + h_2)$ respectively, where $h_2 > h_1 \geq 0$. Since the firms' locations are symmetric, the equilibrium prices must be equal. Let p_1 denote the equilibrium price of each firm when the firms locate at $(0.5 - h_1, 0.5 + h_1)$. Then, p_1 satisfies equation (14).

Consider what happens when firms locate at $(0.5 - h_2, 0.5 + h_2)$. Both firms must still price equally; let p_2 denote this price. Fix a consumer $y < 1/2$, and let $\delta(y, x_i^j)$

be defined as $|y - x_i^j|$, where $i \in \{L, F\}$, and $j \in \{1, 2\}$ represents the first and second set of locations respectively. Symmetric locations, equal prices, and $h_2 > h_1$ imply that for any $y < 1/2$, we have $\delta(y, x_L^2) - \delta(y, x_L^1) \leq \delta(y, x_F^2) - \delta(y, x_F^1)$. That is, any consumer at $y < 1/2$ sees their nominal distance $\delta(y, x)$ from L increase by an amount that is no more than the increase in distance from F. Now recall that $\alpha \geq 1$, which ensures that the deterministic consumer utility $-|y - x|^\alpha - p$ is convex in distance. Resultantly, we have $q_L(y, 0.5 - h_2, 0.5 + h_2, p_2, p_2) > q_L(y, 0.5 - h_1, 0.5 + h_1, p_1, p_1)$. The product $q_L(y)q_F(y)$ is therefore smaller for every $y \neq 1/2$ under the second set of locations compared to the first. For (14) to be satisfied, we therefore must have $p_2 > p_1$.

(iii) Consider the left-hand side of equation (7), which represents $\frac{\partial \Pi_j}{\partial p_j}$ for firm j. For convenience, let $j = l$ in what follows. Differentiating once again with respect to p_L and collecting terms, we have

$$\begin{aligned} \frac{\partial^2 \Pi_L}{\partial p_L^2} &= -\frac{1}{\mu}\Big[2\int_0^1 q_L(y)(1 - q_L(y))dy \\ &\quad -\frac{p_L}{\mu}\int_0^1 q_L(y)(1 - q_L(y))dy + \frac{2p_L}{\mu} q_L^2(y)(1 - q_L(y))dy\Big]. \end{aligned}$$

Now, using the fact that $q_F(y) = 1 - q_L(y)$, so that $q_L(y) + q_F(y) = 1$, we can write the last two terms in the parentheses as

$$\begin{aligned} &-\frac{p_L}{\mu}\int_0^1 q_L(y)q_F(y)[q_L(y) + q_F(y)]dy + 2\frac{p_L}{\mu}\int_0^1 q_L^2(y)q_F(y)dy \\ = &\quad \frac{p_L}{\mu}\int_0^1 q_L(y)q_F(y)(q_L(y) - q_F(y))dy. \end{aligned}$$

Define $\hat{q}(y) = q_L(y)q_F(y)$. Now, from part (i), symmetry implies that the solution to the best response conditions (7) satisfies $p_L^* = p_F^*$. Since locations are symmetric about the market center, and prices are equal, at any points $y \in [0, 1]$ and $z = 1 - y$, it follows that $\hat{q}(y)q_L(y) = \hat{q}(z)q_F(z)$. Therefore, $\int_0^1 \hat{q}(y)q_L(y) = \int_0^1 \hat{q}(y)q_F(y)$, so that

$$\frac{p_L}{\mu}\int_0^1 q_L(y)q_F(y)(q_L(y) - q_F(y))dy = 0.$$

Therefore, when locations are symmetric, evaluating the second partial derivative of profit with respect to price at the prices that satisfy the first-order conditions (7), we have

$$\frac{\partial^2 \Pi_L}{\partial p_L^2} \quad = \quad -\frac{1}{\mu}\left[2\int_0^1 q_L(y)(1-q_L(y))dy\right] \quad < \quad 0.$$

Hence, the second-order condition for profit maximization is satisfied for L. A similar analysis shows that the condition is satisfied for F. ■

Proof of Proposition 3

Since symmetric locations imply equal prices by Proposition 2 part (i), let $(x_L^*, 1-x_L^*, p^*, p^*)$ denote the equilibrium locations and prices in the sequential game. Suppose for contradiction that these locations and prices do not constitute an equilibrium in the simultaneous game. Then, given that firm L is located at x_L^*, the best response of F in the simultaneous move game must be given by some $x'_F \neq x_F^*$. That is, $\Pi_F(x_L^*, x'_F, p_L^*(x_L^*, x'_F), p_F^*(x_L^*, x'_F) > \Pi_F(x_L^*, x_F^*, p^*, p^*)$. But then, in the sequential game as well, when L locates at x_L^*, firm F should locate at $x_F x'$ rather than at x_L^*, contradicting the assumption that $(x_L^*, 1-x_L^*, p^*, p^*)$ constitutes an equilibrium outcome of the sequential game. ■

B Numerical Determination of Equilibria

To determine the equilibria of the overall game numerically, we trace the following steps:

1. Fix a finite location grid on $[0,1]$ for consumers. Let $\left[0, \frac{1}{m}, \frac{2}{m}, \cdots, 1\right]$ represent this grid. The mass at each point on this grid is $\frac{1}{m+1}$.

2. Fix finite location grids on $[0,1]$ for L and F. Let $\left[0, \frac{1}{2n}, \frac{2}{2n}, \cdots, \frac{1}{2}\right]$ represent the location grid for L, and let $\left[\frac{1}{2}, \frac{n+1}{2n}, \frac{n+2}{2n}, \cdots, 1\right]$ represent the grid for F.

Notice that we restrict L to locating in $[0, 0.5]$. As mentioned earlier, given the symmetry of the consumer distribution about the market center 0.5, this is without loss of generality. We also restrict F to locating in $[0.5, 1]$. While it is intuitive that F will want to locate on the other side of the market center as L, we first verified numerically that this property holds. That is, for each pair (α, μ) of parameter values, we allowed F to locate on a grid that spanned $[0, 1]$, and found that in each case F preferred to locate in the sub-interval $[0.5, 1]$.

3. For each point on the location grid of L, determine F's best response. This is done as follows:

   (a) For each point on F's location grid, given L's location, determine equilibrium prices by finding a solution to equations (7). Verify that the second-order condition for optimal prices holds.

   (b) Determine F's profit at that point.

   (c) F's best response to L's location is the point at which F attains maximum profit.[4]

4. Determine equilibrium in the simultaneous move game as follows. We search for all symmetric equilibria of this game (since the simultaneous-move game is symmetric by construction).

   In any symmetric equilibrium, L is as far from the market center (at 0.5) as F. Thus, $0.5 - x_L^* = x_F^* - 0.5$, or $x_L^* + x_F^* = 1$. Any point on F's best response function at which $x_F = 1 - x_L$ thus provides equilibrium locations in the simultaneous-move game. Notice that our numerical procedure finds only pure strategy equilibria.

5. Determine equilibrium in the sequential move game as follows. For each point

[4] In the numerical computations, the best response of F was unique for each set of parameters and fixed L location.

in L's location grid, determine the best response of F. Compute L's profit at this location. L's optimal action is given by the location that maximizes its profit. Again, we find pure strategy equilibria of the game.

In our numerical analysis, we set $m = 400$. That is, consumers are located at a set of 401 equidistant points in the interval $[0, 1]$, with a mass equal to $1/401$ at each point. Further, we set $n = 200$, so that the feasible location grid for L consists of 201 equidistant points in $[0, 0.5]$, and that for F consists of 201 equidistant points in $[0.5, 1]$.

Finally, we consider values of μ in the range 0.2 to 2.0, with steps of 0.1, and values of α in the range 1 to 3.0, with steps of 0.125. We explored larger values of μ and α, and found qualitatively similar equilibrium patterns. Our results extrapolate to (α, μ) values not explicitly displayed.

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