On multiple-principal multiple -agent models of moral hazard*

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Abstract

We provide two examples in a pure moral hazard setting with two principals and two agents. Example 1 shows that a strongly robust equilibrium in simple (direct) mechanisms can no longer be sustained as an equilibrium when a principal can deviate to an indirect communication scheme. Conversely, an equilibrium with one principal offering an indirect mechanism cannot be replicated as an equilibrium in simple mechanisms. Example 2 shows more directly that a payoff profile that can be achieved in equilibrium when one principal offers an indirect mechanism cannot be achieved as an equilibrium profile in simple mechanisms.

Key words: Moral hazard, multiple principals, multiple agents, simple mechanisms.
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1 Introduction

Consider an environment with two principals, two agents, and pure moral hazard. That is, there is complete information over agents' types and effort is non-contractible. In this environment, we define a simple mechanism as one in which a principal privately recommends a specific effort to each agent. A simple mechanism then corresponds to what Myerson (1982) terms a direct mechanism.

We provide two examples in such a setting. Example 1 shows that an equilibrium outcome in simple mechanisms that is strongly robust (as defined by Peters (2001)) is no longer sustained as an equilibrium when principals can deviate to richer communication schemes. When effort is directly contractible, Han (2007) shows that a strongly robust pure strategy equilibrium of a game in which only simple mechanisms are allowed continues to be a strongly robust equilibrium when principals can offer indirect mechanisms. Our example thus demonstrates that his result does not extend to the case of non-contractible effort. That is, in a multi-principal multi-agent model with moral hazard, payoff profiles supported via an equilibrium in simple mechanisms may not survive the introduction of a more complex communication scheme by a principal.

Example 1 also suggests a failure of the revelation principle in our setting: the probability distribution over principals' decisions in an equilibrium in which principals offer indirect communication mechanisms cannot be replicated by restricting principals to the use of simple mechanisms. This failure differs from those identified in multi-principal games with a single agent (see Martimort and Stole (2002) and Peters (2001)), since we explicitly allow principals to make recommendations on agents' effort choices.

However, in our first example, there is an equilibrium in simple mechanisms which replicates the payoffs that both principals and agents obtain in the equilibrium of the indirect mechanism game. We therefore construct a second example which demonstrates a more conclusive failure of the revelation principle: a payoff profile is achievable in equilibrium when one principal can offer an indirect mechanism, but not when all principals are restricted to simple mechanisms.

2 Model

There are 2 principals (denoted as P1 and P2) and 2 agents. There is complete information about agents' types. Each principal j chooses an allocation $y_j \in Y_j$. Each agent i chooses an unobservable effort $e^i \in E^i$. The profile of allocations and efforts, (y_1, y_2, e^1, e^2) results in final payoffs to each principal and agent.

We use the communication structure for principal-agent models introduced by Myerson (1982). At stage 1, principals offer mechanisms. A mechanism offered by principal j is defined as $\gamma_j = \left(M_j, R_j, \pi_j\right)$. Here, $M_j = M_j^1 \times M_j^2$ is a message space, from which agents can choose messages to send to principal j, and $R_j = R_j^1 \times R_j^2$ represents a recommendation space, from which private recommendations to agents will be made. The mechanism of principal j also includes a stochastic choice rule $\pi_j : M_j \to \Delta(Y_j \times R_j)$. At stage 2, each agent i sends a message $m_i^j \in M_i^j$ to each principal j. At stage 3, each principal j chooses one element from the distribution $\pi_j(m_j^1, m_j^2)$, and privately communicates the recommendation r_j^i to each agent i. Each agent i chooses her effort e^i at stage 4. Finally, at stage 5, payoffs are realized.

Note that principal j is allowed to choose a lottery over allocations and recommendations for any message array m_j. However, it is standard to consider the mechanism γ_j as a pure strategy for principal j, with a mixed strategy consisting of a randomization over such mechanisms (see, for example, Peters (2001)).

Standard models of moral hazard allow a principal to observe an outcome that is correlated with agents' effort, and to condition compensation (or allocations in our language) on the outcome. Our model is easily generalized to this framework. In our examples, the outcome space may be thought of as a singleton, which allows us to directly consider allocations rather than mappings from outcomes to allocations.

In our framework, a simple mechanism for principal j is defined as follows. For each agent, the principal sets a singleton message space and directly suggests the action the agent should take.[1] That is, $|M_j^i| = 1$ and $R_j^i = E^i$ for every i. Thus, a simple mechanism corresponds to a "direct mechanism" as defined by Myerson (1982).[2] We refer to any mechanism in which, for any agent i, either $|M_j^i| > 1$ or $R_j^i \neq E^i$, or both, as an indirect mechanism.

The set of feasible message spaces for principal j is denoted as $\mathcal{M}_j$, with $\mathcal{R}_j$ the set of feasible recommendation spaces. In a simple mechanism game, principals are restricted to using simple mechanisms, so for each principal j and agent i, $|M_j^i| = 1$ for every $(M_j^1, M_j^2) \in \mathcal{M}_j$, and $\mathcal{R}_j = E^1 \times E^2$. In an indirect mechanism game, these sets are rich enough to allow for simple or indirect mechanisms to be chosen.

In this context, one way to state the revelation principle is as follows. Fix the sets $\mathcal{M}_j$

[1] Setting a singleton message space is equivalent to not asking for a message, and allows us to preserve the interpretation of π as a mapping that depends on messages.

[2] A different route to define direct mechanisms is suggested in Epstein and Peters (1999), who include the communication about other principals' mechanisms in the set of messages available to each single agent.

and $\mathcal{R}_j$ so as to allow each principal j the choice of an indirect mechanism. Consider any equilibrium of the resultant game. Then, there is an equilibrium of the simple mechanism game that reproduces the same probability distribution over allocations and efforts.[3] Note that in a multi-principal environment with moral hazard, the notion of incentive compatibility is not well-defined, since agents may receive conflicting recommendations from the two principals. Thus, rather than impose incentive compatibility, we only require that agents play a continuation equilibrium of the game once the mechanisms are chosen.

A weaker statement of the revelation principle is as follows. Again, fix the sets $\mathcal{M}_j$ and $\mathcal{R}_j$ so as to allow each principal j the choice of an indirect mechanism and consider any equilibrium of the resultant game. Then, there is an equilibrium of the simple mechanism game which delivers the same expected payoffs to each principal and agent. However, the distribution over allocations and efforts may be quite different in the two equilibria.

Following Peters (2001), we define strong robustness as follows. In the competing mechanism game, fix spaces $\mathcal{M}_j$ and $\mathcal{R}_j$ for each principal j. Since the allocation space Y_j is also fixed, this in turn fixes the set of feasible choice rules and hence of feasible mechanisms, Γ_j. Suppose that for any mechanisms $(\gamma_1, \gamma_2) \in \Gamma_1 \times \Gamma_2$ chosen by principals, agents play some continuation equilibrium (in which they choose messages at stage 2 and efforts at stage 4). An equilibrium of the simple (indirect) mechanism game is strongly robust if no principal j can improve his own payoff by offering any simple (indirect) mechanism $\gamma_{\tilde{j}} \in \Gamma_j$, even if agents coordinate on the continuation equilibrium that maximizes the payoff of principal j.

3 Robustness

Example 1.

Let the allocation spaces be $Y_1 = \{y_1\}$ for P1 and $Y_2 = \{y_{21}, y_{22}\}$ for P2. The effort spaces are $E^1 = \{a_1, a_2\}$ for agent 1 and $E^2 = \{b_1, b_2\}$ for agent 2. The payoffs of the game are given in Table 1. The first payoff is that of P1, who chooses the row in the table, the second payoff is that of P2, who chooses the column, and the last two payoffs are those of agents 1 and 2, respectively.

We develop our argument via a series of claims.

Claim 1: Suppose both principals are restricted to offering simple mechanisms. Then, the

[3] This way of defining the revelation principle may be seen as a natural extension of the notions suggested in the literature on competing mechanisms in the presence of a single agent; see Martimort and Stole (2002) and Peters (2001).

	y_{21}			y_{22}		
		b_1	b_2		b_1	b_2
y_1	a_1	$(12,1,5,5)$	$(0,1,2,7)$	a_1	$(12,1,6,6)$	$(0,1,2,7)$
	a_2	$(0,1,7,2)$	$(-12,1,0,0)$	a_2	$(0,1,7,2)$	$(-48,1,0,0)$

Table 1: Payoffs in Example 1

payoff profile $(4,1,\frac{14}{3},\frac{14}{3})$ is supported in a strongly robust equilibrium in which principals adopt pure strategies.

Consider the following simple mechanisms. That is, the message spaces are singletons, and $R^i_j = E^j$ for each i and j. The choice rules of the principals are as follows:

$$\pi_1 = \begin{cases} (y_1, a_1, b_1) & \text{with probability } \frac{1}{3} \\ (y_1, a_2, b_1) & \text{with probability } \frac{1}{3} \\ (y_1, a_1, b_2) & \text{with probability } \frac{1}{3} \end{cases}$$

$$\pi_2 = \begin{cases} (y_{21}, a_1, b_1) & \text{with probability } \frac{1}{2} \\ (y_{22}, a_2, b_2) & \text{with probability } \frac{1}{2} \end{cases}$$

On receiving the recommendation from P2, each agent knows which column of the large matrix in Table 1 has been chosen. Indeed, the payoffs in the agents' continuation game are common knowledge among the agents. Thus, in any equilibrium, for each of the two cells in the large matrix, the distribution over efforts E in the agents' continuation game must be a correlated equilibrium (as in Aumann (1987)).

Observe that the recommendations of P1 do induce a correlated equilibrium of the agents' continuation game in each column of the large matrix. Hence, it is a best response for each agent to to obey the recommendation of P1. As noted, the mechanisms offered by the principals represent pure strategies. The resultant payoff profile is $(4,1,\frac{14}{3},\frac{14}{3})$.

Since P2 is indifferent across all outcomes, the mechanism of P2 remains a best response regardless of the strategy of P1. Fix the mechanism of P2 as π_2. Then, to show that the offered mechanisms represent a strongly robust equilibrium, we need to show that, following any simple mechanism P1 may offer, there is no continuation equilibrium in the agents' game that yields P1 a payoff strictly greater than 4.

First, consider continuation equilibria in which agents obey the recommendation of P1.

In a simple mechanism, P1 cannot offer recommendations that are contingent on the recommendations of P2. Thus, P1 is limited to inducing the same correlated equilibrium in the agents' game, regardless of whether P2 offers y_{21} or y_{22}. If both agents obey the recommendations of P1, the maximal payoff he can attain is then 4, via the correlated equilibrium which places probability $\frac{1}{3}$ on each of (a_1,b_1), (a_2,b_1) and (a_1,b_2).

Next, consider continuation equilibria in which agents ignore the recommendation of P1 in one column of the large matrix in Table 1. It is straightforward to show that when P2 offers y_{21}, the continuation equilibrium that maximizes the payoff of P1 is the correlated equilibrium that places probability $\frac{1}{3}$ on each of (a_1,b_1), (a_2,b_1), and (a_1,b_2), and yields P1 an payoff of 4. Similarly, when P2 offers y_{22}, the payoff of P1 is maximized by the correlated equilibrium that places probability $\frac{1}{2}$ on (a_1,b_1), and probability $\frac{1}{4}$ each on (a_1,b_2) and (a_2,b_1), which yields a payoff of 6 to P1.

Suppose P1 deviates to a simple mechanism which induces a correlated equilibrium in only one column of the large matrix in Table 1. Then, in the other column, agents will instead play a Nash equilibrium of their continuation game. The agents' continuation game in each cell of the large matrix has exactly three Nash equilibria (two in pure and one in mixed strategies), each of which offer a payoff of zero to P1. Thus, given the choice rule π_2 offered by P2, any simple mechanism offered by P1 that induces a correlated equilibrium in only one column of the large matrix yields a payoff of at most 3 to P1.

Thus, the choice rules (π_1,π_2), with agents obeying the recommendations of P1, constitute a strongly robust equilibrium in simple mechanisms.

Claim 2: Fix the mechanism of P2 (i.e., fix the recommendation space $R_2 = E$ and the choice rule π_2). If P1 can deviate to an indirect mechanism, he can attain a payoff of 5.

Consider the following indirect mechanism offered by P1. The sets of recommendations for agents 1 and 2 are, respectively $R = \{r_1,r_2,r_3,r_4\}$ and $S = \{s_1,s_2,s_3,s_4\}$. Let $\tilde{\pi}_1$ denote P1's choice rule. The allocation offered by P1 is, of course, y_1. The probabilities over recommendations are as shown in Table 2.

Given $\tilde{\pi}_1(.)$ and $\pi_2(.)$, consider the following strategies for the agents. Agent 1 plays the following strategy, based on the recommendation received from P1. If she receives

r_1: play a_1 regardless of the recommendation offered by P2.

r_2: play a_1 if P2 sends recommendation a_1 and a_2 if P2 sends recommendation a_2

r_3: play a_2 if P2 sends recommendation a_1 and a_1 if P2 sends recommendation a_2

r_4: play a_2 regardless of the recommendation offered by P2.

	s_1	s_2	s_3	s_4
r_1	$1/6$	$1/12$	$1/6$	$1/12$
r_2	$1/12$	0	$1/12$	0
r_3	$1/6$	$1/12$	0	0
r_4	$1/12$	0	0	0

Table 2: P1's choice rule in the indirect mechanism

Agent 2 plays an exactly similar strategy, substituting s_i for r_i and b_j for a_j above. It is straightforward to check that the strategies constitute a continuation equilibrium in the agents' effort game. Thus, P1 earns an expected payoff of 5 from the indirect mechanism, higher than the 4 she can earn from a simple mechanism.

Thus, if P2 offers a mechanism with a recommendation space $R_2 = E$, P1 earns a higher payoff from a mechanism with a recommendation space $E \times R_2$ than with a simple mechanism. From the viewpoint of P1, the recommendation offered by P2 is equivalent to an unknown type for each agent. Although P1 does not explicitly ask the agents to report the recommendation offered by P2, his own recommendations may be interpreted as contingent recommendations. That is, the recommendations of P1 may be interpreted as asking agents to take an action that varies with the recommendations of P2.

In the example, P2 is indifferent across all outcomes. It is easy to see that, if P2 had non-trivial preferences over outcomes, she may, in turn, wish to make recommendations contingent on the (contingent) recommendations of P1, and so on, leading to the usual infinite regress problem (see, for example, Epstein and Peters (1999)).

In a setting with complete information and contractible actions, Han (2007) identifies a set of equilibria in simple mechanisms which survive the introduction of indirect communication mechanisms. In particular, his main theorem shows that any pure strategy strongly robust equilibrium in a game in which principals are restricted to simple mechanisms remains strongly robust in the game where principals can offer indirect mechanisms. A main implication of Example 1 is that the theorem of Han (2007) cannot be extended to moral hazard environments. In our example, the pure strategy played by P2 introduces a form of asymmetric information between P1 and agents. When the game is played, agents will observe their private recommendations and will know the allocation chosen by P2. However, P1 only knows the probability distribution selected by P2. It follows that P1 has an incentive to use a more sophisticated communication mechanism with agents to extract their private information. With contractible actions, if each principal plays a pure strategy, P1 cannot gain by learning the recommendation offered by P2.

Claims 1 and 2 also suggests a failure of the revelation principle in multiple-principal games. Fixing the message and recommendation spaces as in Claim 2, it is immediate that $\tilde{\pi}_1$ and π_2 constitute equilibrium choice rules in the principals' game. The resultant payoff profile from is $(5, 1, \frac{119}{24}, \frac{119}{24})$. Since 4 is the maximal payoff P1 can attain via simple mechanisms when P2 plays π_2, it follows that the distribution over outcomes induced by the equilibrium in indirect mechanisms cannot be replicated in an equilibrium with simple mechanisms. To the extent that the revelation principle is formulated in terms of probability distributions over players' actions, Claims 1 and 2 demonstrate a failure. Similar failures have been demonstrated in common agency (multi-principal, single agent) environments by Peters (2001) and Martimort and Stole (2002), using mechanisms without recommendations. Our example thus shows that, when there is moral hazard, the failure persists despite the use of recommendations by principals.

It turns out there is another equilibrium in simple mechanisms in which the payoff profile $(5, 1, \frac{119}{24}, \frac{119}{24})$ is obtained in equilibrium.

Claim 3: Suppose principals are restricted to simple mechanisms. There is an equilibrium in pure strategies with choice rules $\hat{\pi}_1, \hat{\pi}_2$ which results in a payoff profile $(5, 1, \frac{119}{24}, \frac{119}{24})$.

Consider the following simple mechanisms, represented by the choice rules

$$\hat{\pi}_1 = \begin{cases} (y_1, a_1, b_1) & \text{with probability } \frac{5}{12} \\ (y_1, a_2, b_1) & \text{with probability } \frac{7}{24} \\ (y_1, a_1, b_2) & \text{with probability } \frac{7}{24} \end{cases}$$

$$\hat{\pi}_2 = \begin{cases} (y_{21}, a_1, b_1) & \text{with probability } \frac{2}{5} \\ (y_{22}, a_1, b_1) & \text{with probability } \frac{3}{5} \end{cases}$$

In this case, on seeing the recommendation of P2, the agents do not know the allocation he has chosen. It is straightforward to check that a correlated equilibrium in the agents' effort game is induced if each agent obeys the recommendation of P1. Further, there is no correlated equilibrium in the agents' game that leads to a higher payoff for P1. The resultant payoffs are again $(5, 1, \frac{119}{24}, \frac{119}{24})$.

4 Revelation Principle

In Example 1, Claim 3 shows that the payoffs achieved by the equilibrium in indirect mechanisms can be replicated via an equilibrium in simple mechanisms, provided the mechanisms of both principals are changed. However, the probability distribution over equilibrium outcomes is clearly different across the two equilibria.

To emphasize the failure of the revelation principle in payoffs as well in our setting, we directly construct a second example showing that there exist payoff profiles supported at equilibrium by indirect mechanisms but not by simple ones.

Example 2.

The allocation spaces are $Y_1 = \{y_{11}, y_{12}\}$ for P1 and $Y_2 = \{y_{21}, y_{22}\}$ for P2. Effort spaces are again denoted as $E^1 = \{a_1, a_2\}$ and $E^2 = \{b_1, b_2\}$. As before, in the payoff matrix shown in Table 3, the first payoff is that of P1, the second is that of P2, and the last two are those of agent 1 and 2.

		y_{21}		y_{22}	
		b_1	b_2	b_1	b_2
y_{11}	a_1	$(12,2,5,5)$	$(0,2,2,7)$	$(12,1,-2,0)$	$(0,1,-1,-1)$
	a_2	$(0,2,7,2)$	$(-100,2,0,0)$	$(0,1,-1,-1)$	$(100,0,0,0)$
		b_1	b_2	b_1	b_2
y_{12}	a_1	$(-1,-1,-1,-1)$	$(-1,-1,-1,-1)$	$(-1,-1,-1,-1)$	$(-1,-1,-1,-1)$
	a_2	$(-1,-1,-1,-1)$	$(-1,-1,-1,-1)$	$(-1,-1,-1,-1)$	$(-1,-1,-1,-1)$

Table 3: Payoffs in Example 2

Claim: The payoff profile $(100,0,0,0)$ can be supported at equilibrium if and only if principals use indirect mechanisms.

Consider the following indirect mechanisms. Neither principal sends recommendations. P1 chooses message spaces $M_1^1 = M_1^2 = \{m_1, m_2\}$, with a choice rule

$$\pi_1 = \begin{cases} y_{11} & \text{if both agents send message } m_1 \\ y_{12} & \text{otherwise.} \end{cases}$$

P2 offers the allocation y_{22}. Given the offered mechanisms, it is a best reply for both agents to send the message m_1 to P1. Further, (a_2, b_2) is the unique Nash equilibrium in the agents' continuation game. Thus, the payoff profile $(100,0,0,0)$ is achieved.

Given the strategy of P2, P1 is achieving his maximal payoff, and hence has no incentive to deviate from his equilibrium strategy. What if P2 deviated to y_{11} (or more broadly, to another indirect mechanism with different message and recommendation spaces)? Following such a deviation by P2, we consider the equilibrium in the agents' continuation game in which both agents send message m_2 to P1. Observe that if one agent sends message m_2 to P1, it is a best response for the other agent to do the same. Thus, any deviation from the prescribed mechanism leads to a payoff of -1 for P2, compared to the payoff of 0 he attains in equilibrium. Therefore, P2 does not have a profitable deviation either.

Now, suppose principals are restricted to offering simple mechanisms. To achieve the outcome with payoffs $(100,0,0,0)$, the choice rule of P1 must assign a probability 1 to playing y_{11}. In that case, however, it is not optimal for P2 to choose a rule π_2 that places a strictly positive probability on y_{22}.

Therefore, the indirect mechanism selected by P1 at equilibrium cannot be reproduced through simple recommendations. It is worth noting that the communication mechanism π_1 cannot be reproduced by any mechanism, direct or indirect, that only involves recommendations from P1 to the agents. That is, the existence of messages from the agents to P1 is crucial. In the example, the messages sent from agents to P1 generate off-equilibrium threats for P2 that cannot be reproduced through private recommendations alone.

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