Signaling Quality via Queues*

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Abstract

We consider an M/M/1 queueing system with impatient consumers who observe the length of the queue before deciding whether to buy the product. The product may have high or low quality, and consumers are heterogeneously informed. The firm chooses a slow or (at a cost) a fast service rate. In equilibrium, informed consumers join the queue if it is below a threshold. The threshold varies with the quality of the good, so an uninformed consumer updates her belief about quality on observing the length of the queue. The strategy of an uninformed consumer has a "hole": she joins the queue at lengths both below and above the hole, but not at the hole itself. We show that if the prior probability the product has high quality and the proportion of informed consumers are both low, a high-quality firm may select a slower service rate than a low-quality firm. The queue can therefore be a valuable signaling device for a high-quality firm. Strikingly, in some scenarios, the high-quality firm may choose the slow service rate even if the technological cost of speeding up is zero.

1 Introduction

Consumers frequently have to wait before they can consume a product or service. Lines outside nightclubs, rides at amusement parks and waiting lists for new products are part of everyone's experience. Toys and innovative products, whose value cannot easily be communicated, exhibit similar phenomena. For example, Cabbage Patch Kids in 1983 and Beanie Babies in the 1990s had significant waiting times, and queues formed in front of stores when these products came on the market. Both these goods are mass-produced, and the producers could easily have increased their capacity to reduce consumer waiting times. Since waiting is costly for consumers, it may appear that the firms were not maximizing profit. Rather, as we show in this paper, in some settings firms have a strategic incentive to manipulate impatient consumers' waiting times to generate greater demand. In other words, the producers benefited from the "buzz" that was created by the long queues.

We develop a model in which a firm sells a good that can be of either high or low quality. Impatient consumers arrive at the market according to a Poisson process. Purchasing the good entails joining a "first-come, first-served" queue. The firm knows the quality of its good and controls the distribution of the queue length by choosing its service rate. It can choose either a slow service rate or, at a cost, a fast one. Some consumers are informed, and know the quality of the good. Others are uninformed, and must infer the quality from the length of the queue. An arriving consumer sees only the queue. She does not know how many people arrived before her, and cannot directly observe the service rate.

As is standard in queuing models, informed consumers adhere to a "threshold" strategy, where the threshold depends on the quality of the good. That is, they join the queue if it is short enough, and balk at longer queues. In contrast, uninformed consumers, who have to infer the quality of the firm from the queue, play a non-threshold strategy. Their Bayesian updating process leads to a "hole" in their joining decision. That is, there is exactly one queue length at which they balk, which lies between the thresholds at which informed consumers balk when faced with low- and high-quality firms. In a pure strategy equilibrium, they join the queue at both smaller and larger queue lengths. In a mixed strategy equilibrium, they randomize at some queue lengths below the hole, and join with probability 1 at queue lengths above the hole up to the threshold at which the informed consumers join the high-quality firm's queue. Since no uninformed consumer joins the queue at a hole, longer queues can only arise if an informed consumer who knows the firm has high quality were to join at that length. Hence, an uninformed consumer arriving when the queue is above the hole infers that the firm has high quality. The hole thus serves as a natural filter of information about quality for uninformed consumers.[1]

The hole in the uninformed consumer's strategy affects the incentives of each type of firm to speed up (i.e., choose the fast service rate) in different ways. We assume each type of firm has the same profit margin and maximizes the revenue rate per unit time. The queue at a low-quality firm never rises above the hole, since informed consumers balk at a weakly lower threshold. Thus, the low-quality firm prefers to keep the queue at lengths below the hole, and naturally has an incentive to choose the fast service rate. The high-quality firm, on the other hand, loses uninformed consumers at the hole, but wins them back at higher queue lengths. If the proportion of informed consumers is high, the high-quality also has an incentive to select a fast service process. However, if there is a large proportion of uninformed consumers, the high-quality firm wishes to avoid having the queue stall at the hole. Enabling the queue to cross the hole allows it to capture the uninformed demand that exists at longer queue lengths. The high-quality firm can do this by selecting the slow service process, even if increasing the service rate comes at a low (or no) cost.

In the economics literature, consumers (investors) have imperfect information about the value of an asset and observe the actions of all previously arriving consumers. Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992) show that herd behavior may result; that is, consumers may ignore their own information and adopt the actions of previous agents. Instead, if each agent observes only her predecessor's action, beliefs and actions can cycle forever. There are long periods of uniform behavior punctuated by rare switches (Celen and Kariv, 2004). Smith and Sorensen (1997) consider a model in which agents receive a random sample of the history. They find that the true quality of the good will never be learned, even on the long run. Our model may be interpreted as a model in which each consumer observes a random history that is determined by the queueing process. Chamley (2004) provides a comprehensive review of the herding literature.

In the queueing literature, equilibrium joining strategies are examined by Naor (1969) and the subsequent literature on economic aspects of queueing (see Hassin and Haviv, 2003, for an excellent overview). When there are positive waiting costs, agents play a threshold strategy: they join the queue as long as it is not too long. Beyond some threshold, there is a congestion effect: the waiting costs imply that joining the queue is not worthwhile. The service quality is typically assumed to be common knowledge. Hassin and Haviv (1997) consider a queue with positive externalities in which each agent wishes to "follow

[1] Debo, Parlour and Rajan (2008) consider a model in which all consumers have noisy signals and demonstrate that there can be multiple holes in the consumer joining strategy. In the current paper, consumers either have perfect information or are completely uninformed, which leads to at most one hole.

the crowd," and show that threshold behavior continues to obtain. Veeraraghavan and Debo (2008, 2011) show that a queue may communicate information about quality. In both papers, consumers obtain an imperfect signal about which of two servers provides the highest value. With no waiting costs consumers join the longest queue (Veeraraghavan and Debo, 2008). With waiting costs, the equilibrium queue joining strategy is a complex function of both queue lengths, and no structural properties are obtained (Veeraraghavan and Debo, 2011). Debo and Veeraraghavan (2011) consider a model in which quality and service rate are correlated, but, both unknown to all consumers that only observe the queue length upon arrival before deciding whether to join or balk. They show that non-threshold queue joining equilibria may arise. In all these papers, the service rates are exogenous, so the firm is not strategic.

The literature on service rate decisions with observable queues is sparse (see Hassin and Haviv, Chapter 8). Hassin (1986) considers a firm choosing the profit-maximizing number of single-server facilities. If it is profitable to have the queue length observable, it is also socially optimal.

2 Model

We consider the market for an experience good, which may be a physical product or a service. The sequence of events is as follows. At stage 0, nature chooses the type of the firm, which corresponds with the quality of the good. With probability p, the firm has type h and with probability $1-p$ it has type ℓ. At stage 1, the firm, which privately observes its own type, chooses an exponential service rate $\mu \in \{\underline{\mu}, \bar{\mu}\}$. The mean service time per consumer is then $\frac{1}{\mu}$. Without loss of generality, we normalize the cost of choosing rate $\underline{\mu}$ to zero; let $\kappa \geq 0$ be the technological cost of choosing rate $\bar{\mu}$. We assume that κ does not depend on the quality of the firm. Further, the firm cannot communicate its service rate to the consumers. κ thus represents a non-verifiable expenditure of resources. Once chosen, the service rate is held constant for the rest of the game. Throughout this paper, we restrict attention to the case in which each firm type plays a pure strategy; for $\theta \in \{h, \ell\}$, we let μ_θ denote the service rate that a firm of type θ chooses.

At stage 2, after the firm has chosen its service rate, risk-neutral consumers arrive at the market according to a Poisson process with parameter Λ. A proportion q of consumers is informed, and knows the quality of the good. The remaining proportion $1-q$ is uninformed. Uninformed consumers have a prior belief that the good is high quality with probability p. The utility a consumer obtains from purchasing and consuming a good of quality θ is v_θ, with $v_h > v_\ell$. This utility is net of the good's price, which is not explicitly modeled.

If consumers arrive faster than they are serviced, they form a queue. The queue is served on a first-come first-served basis. A consumer suffers a disutility $c > 0$ per unit of time that she has to wait to obtain the good, starting from her initial arrival to the market. Upon arrival, a consumer (whether informed or uninformed) does not directly observe the service rate of the firm, but does observe the number of people waiting to be served by the firm.

Each consumer takes an action $a \in \{\text{join,balk}\}$, where $a = \text{join}$ is the decision to acquire the good, or to join the queue. Once she joins the queue, she may not renege; i.e., she cannot leave until she has been served. Joining the queue is therefore synonymous with consuming the good. If she chooses $a = \text{balk}$ (i.e., to not acquire the good), she obtains a reservation utility of zero. Thus, each consumer will join the queue only if the expected utility from joining exceeds zero.

We assume that an informed consumer obtains a positive utility from consuming the low-quality good whenever she finds no one else ahead of her, even if the firm has chosen the slow service rate. This ensures that the low-quality firm can earn a strictly positive profit. Otherwise, the low-quality firm may exit the market, in which case consumers would know any good offered was of high quality.

Assumption 1 $v_\ell > c/\underline{\mu}$.

In what follows, we characterize Markov-perfect Bayesian equilibria of the game. In the consumer continuation game (i.e., after the firms have chosen their service rates), we consider equilibria in which the strategy of a consumer depends only on whether she is informed and on the length of the queue when she arrives at the market, but not on the exact time at which she arrives or on the number of consumers that have preceded her. Thus, the equilibria are both symmetric across consumers and time-invariant.

2.1 Consumers' Objective Functions

Consider an informed consumer who knows the firm has type θ and finds that there are already n consumers in the queue (including the consumer currently being served) when she arrives at the market. Suppose she joins the queue. Although she does not directly observe the service rate, in equilibrium her beliefs over the service rate must be consistent with the firm's actual choice. The expected service time for each consumer is $\frac{1}{\mu_\theta}$. Therefore, the total expected time to service the n consumers already in the queue and the new arrival is $\frac{n+1}{\mu_\theta}$. Hence, her expected utility from joining the queue is $w_i(n, \theta, \mu_\theta) = v_\theta - (n+1)\frac{c}{\mu_\theta}$.

Suppose an uninformed consumer who finds n consumers waiting in line ascribes a posterior probability $\gamma(n)$ to the firm being of high quality. Then, her expected utility if she joins the queue when there are already n consumers waiting in line is

$$w_u(n, \gamma, \mu_h, \mu_\ell) = (1 - \gamma(n))w_i(n, \ell, \mu_\ell) + \gamma(n)w_i(n, h, \mu_h). \quad (1)$$

Define $\bar{N} = \lfloor v_h \frac{\bar{\mu}}{c} \rfloor$, where $\lfloor x \rfloor$ is the largest integer less than or equal to x. Then, $\bar{N}$ is the largest queue length that can be observed in any equilibrium of the game, because no informed consumer will join a queue of $\bar{N}$ or longer, so that uninformed consumers will also balk at these queue lengths. Let $\mathcal{N} = \{0, 1, \cdots, \bar{N}\}$.

In general, a consumer may play a mixed strategy, and join the queue with some probability between 0 and 1. Thus, a mixed strategy for an informed consumer is a mapping $\sigma_i : \{h, \ell\} \times \mathcal{N} \rightarrow [0, 1]$, and a mixed strategy for an uninformed consumer is a mapping $\sigma_u : \mathcal{N} \rightarrow [0, 1]$. The overall consumer strategy profile may then be represented as $\sigma = (\sigma_i, \sigma_u)$.

2.2 Firm's Objective Function

Each type of firm chooses its own service rate μ_θ to maximize its expected payoff, given σ. Recall that the price of the good is not a choice variable for the firm; rather the price is implicit in the values v_h, v_ℓ. We therefore assume that the firm earns a revenue of r per consumer that it serves, irrespective of its type. Then, each type of firm maximizes its throughput per unit time in the steady state, given the equilibrium strategy of the consumers. The throughput per unit time in the long-run depends on the stationary distribution over queue length induced by σ and μ. Let $\pi_\theta(n, \mu, \sigma)$ denote the stationary probability of observing n consumers in the queue when the firm has quality θ.

In general, the distribution defined by $\pi_h(\cdot)$ will be different than the distribution defined by $\pi_\ell(\cdot)$ for two reasons. First, even if both types of firm choose the same service rate, informed consumers follow a strategy that varies with firm type. Second, the two types of firm may adopt different service rate strategies. Since the distributions over queue lengths induced by the two firms are different, uninformed consumers update their prior beliefs about firm quality on observing the length of the queue when they arrive.

As is standard, the number of consumers in the queue at any given point of time can be represented as a birth-death process, from which the stationary distribution over queue length may be derived. Suppose a firm has type θ. Let $s_\theta(n, \sigma)$ be the probability that a newly-arrived consumer playing strategy σ joins a queue that already has n consumers.

Then,

$$s_\theta(n,\sigma) \quad = \quad (1-q)\sigma_u(n) + q\sigma_i(\theta,n) \text{ for } \theta \in \{h,\ell\}. \tag{2}$$

If there are n consumers already in the queue or in service, the rate at which a new consumer joins is $\Lambda s_\theta(n,\sigma)$. The rate at which consumers leave the queue is equal to the service rate μ. The stationary probability for each queue length n, $\pi_\theta(n,\mu,\sigma)$, is then derived in a standard manner from the resulting flow balance equations (see Lemma 3 in Appendix A). Let $R_\theta(\mu,\sigma)$ be the firm θ's revenue per unit of time when the consumer strategy profile is σ and the firm chooses service rate μ. Under these conditions, the firm is busy with stationary probability $1-\pi_\theta(0,\mu,\sigma)$. Since the expected time to service a consumer is $\frac{1}{\mu}$, the revenue per unit of time is $r\mu$. Hence, the expected revenue of firm θ when it chooses service rate μ is $R_\theta(\mu,\sigma) = r\mu(1-\pi_\theta(0,\mu,\sigma))$.

2.3 Definition of Equilibrium

We consider Markov-perfect Bayesian equilibria of the game: both types of firm and both types of consumer maximize their respective expected payoffs, and, where possible, the beliefs of uninformed consumers obey Bayes' rule. By the PASTA (Poisson Arrivals See Time Averages) property of queueing systems (see Wolff, 1982), in the limit a newly-arriving consumer will be faced with a distribution over queue length equal to the stationary distribution.

Definition 1 A Markov-perfect Bayesian equilibrium in which each type of firm plays a pure strategy is defined by a 4-tuple $(\mu_h, \mu_\ell, \sigma, \gamma)$ that satisfies the following properties.

(i) Each type of firm maximizes its expected payoff:

$$\mu_\theta \in \arg\max_{\mu \in \{\bar{\mu},\underline{\mu}\}} R_\theta(\mu,\sigma) - \kappa 1_{\{\mu_\theta=\bar{\mu}\}} \text{ for } \theta \in \{h,\ell\}, \tag{3}$$

where $1_{\{\mu=\bar{\mu}\}}$ is an indicator variable equal to one if $\mu=\bar{\mu}$ and zero otherwise.

(ii) Each type of consumer maximizes her expected utility:

$$\sigma_i(\theta,n) \in \arg\max_{\tilde{\sigma}\in[0,1]} \tilde{\sigma} w_i(n,\theta,\mu_\theta), \text{ for } \theta \in \{h,\ell\} \text{ and for all } n \in \mathcal{N}, \tag{4}$$

$$\sigma_u(n) \in \arg\max_{\tilde{\sigma}\in[0,1]} \tilde{\sigma} w_u(n,\gamma,\mu_h,\mu_\ell), \text{ for all } n \in \mathcal{N}. \tag{5}$$

(iii) Where possible, the beliefs γ are derived from the strategies using Bayes' rule. That is, whenever the denominator is strictly positive,

$$\gamma(n) \quad = \quad \frac{p\pi_h(n,\mu_h,\sigma)}{p\pi_h(n,\mu_h,\sigma) + (1-p)\pi_\ell(n,\mu_\ell,\sigma)}. \tag{6}$$

Suppose firms have chosen their service rates at stage 1. At stage 2, consumer arrive; we refer to this stage as the "consumer game." In this game, a subset of consumers does not know the type of the firm, and so is playing a game with incomplete information. As is standard, we refer to the consumer game as a "continuation game." Each consumer has a finite set of actions and the set of feasible states (i.e., queue lengths observed by a newly-arrived consumer) is also finite. As equilibria exist in finite games, it is immediate that an equilibrium (possibly in mixed strategies) exists in the consumer game, regardless of the service rate chosen by each type of firm. Further, each type of firm has only two service rates to choose from, so it is immediate to conclude that an equilibrium (again, possibly in mixed strategies) exists in the firms' game. Hence, existence of an equilibrium in the overall game follows. That is, for any values of the parameters $(\Lambda, \bar{\mu}, \underline{\mu}, p, q, c, v_h, v_\ell)$, there is at least one Markov-perfect Bayesian equilibrium in the overall game. As is standard, all these parameters are assumed to be common knowledge across all players.

When we consider the firms' choice of service rates, we focus on pure strategy equilibria. There may also exist equilibria in which firms mix over service rates. If firm θ is mixing over $\bar{\mu}$ and $\underline{\mu}$, informed consumers have beliefs over the service rate, and uninformed consumers over (firm type, service rate) pairs. Each kind of consumer, informed and uninformed, updates its prior beliefs on observing the queue length. For expositional simplicity, we consider only equilibria in which firms choose pure service rate strategies. Our focus is on equilibria in which firm h slows down and firm ℓ chooses the fast service rate. If any equilibrium in which firm h mixes over service rate, it slows down with positive probability. In particular, mixed strategy equilibria in which firm h plays $\underline{\mu}$ with greater probability than firm ℓ are consistent with the spirit of our analysis.

3 Equilibria in the Consumer Game

We first consider equilibria in the consumer game that arises after each firm type θ has chosen its service rate μ_θ; that is, in this section, we take μ_h and μ_ℓ to be fixed. The optimal strategy of an informed consumer is straightforwardly characterized as a threshold strategy, which is standard in the queueing literature (see, for example, Naor, 1969, and Hassin and Haviv, 2003). An informed consumer knows the quality of the good, and hence joins the queue whenever it is short enough. That is, she joins as long as the queue length n satisfies $n < n_\theta = \lfloor v_\theta \frac{\mu_\theta}{c} \rfloor$. Further, if $n_\theta \frac{c}{\mu_\theta} < v_\theta$, it is a unique best response for an informed consumer to balk at n_θ. For the rest of the paper, we assume that this condition holds (Part (i) of Assumption 2). For analytic convenience, we further assume that $n_\ell < n_h$. That is, regardless of the service rate strategies chosen by the firms, the threshold at which informed consumers balk from the high-quality firm strictly exceeds the corresponding threshold for the low-quality firm. Part (ii) of Assumption 2 essentially implies that the range in consumer valuations $(v_h - v_\ell)$ has a greater impact on the consumer's choice than the range in service rates $(\underline{\mu} - \bar{\mu})$. If the assumption is satisfied, then, for any pair of strategies (μ_h, μ_ℓ) chosen by the different types of the firm, it will be the case that $n_h > n_\ell$.

Assumption 2 *(i)* $n_\theta \frac{c}{\mu_\theta} < v_\theta$*, for each* $\theta = h, \ell$*. (ii)* $\lfloor v_\ell(\bar{\mu}/c) \rfloor < \lfloor v_h(\underline{\mu}/c) \rfloor$*.*

We show that the equilibrium strategy of an uninformed consumer is typically not a threshold strategy. Rather, it is characterized by a "hole." That is, there exists exactly one queue length $\hat{n}$ between n_ℓ and n_h at which the uninformed consumer does not join the queue. At every other queue length between 0 and n_h, she follows the strategy of an informed consumer who knows the firm has high quality; that is, she joins the queue.

Proposition 1 *In any pure strategy equilibrium of the consumer game, there exists a* $\hat{n} \in \{n_\ell, n_\ell + 1, \cdots, n_h\}$ *such that* $\sigma_u(\hat{n}) = 0$ *and* $\sigma_u(n) = \sigma_i(h, n)$ *for all* $n \in \mathcal{N} \backslash \hat{n}$*.*

To understand the existence of a hole in the uninformed consumer's strategy, consider her Bayesian updating problem. If the queue length she observes is below n_ℓ, her beliefs are irrelevant to her optimal action: even a fully informed consumer would join such a queue for a low-quality firm, so it is optimal for the uninformed consumer to also join. However, at n_ℓ, informed consumers no longer join the queue for the low-quality firm. Thus, any queue length strictly above n_ℓ the beliefs of the uninformed consumer over the quality of the firm are critical in determining her action. As long as she believes it is sufficiently likely the firm has high quality, she will join. Suppose there is a queue length $\hat{n} \geq n_\ell$ at which uninformed consumers do not join the queue. Then, any queue length strictly above $\hat{n}$ can only be reached if an informed consumer joins the queue at $\hat{n}$. But informed consumers have perfect information, so that observing a queue of length strictly greater than $\hat{n}$, an uninformed consumer must believe the firm has high quality with probability 1. Thus, the uninformed consumer again joins the queue at lengths between $\hat{n} + 1$ and $n_h - 1$, leading to a hole at $\hat{n}$.

Suppose uninformed consumers play a pure strategy with a hole at n. Define the resulting likelihood ratio at queue length 0 (i.e., the ratio $\frac{\pi_\ell(0,\mu_\ell,\sigma)}{\pi_h(0,\mu_h,\sigma)}$) as

$$\Phi(n) \quad = \quad \frac{\sum_{m=0}^{n} \left(\frac{\Lambda}{\mu_h}\right)^m + q \sum_{m=n+1}^{n_h} \left(\frac{\Lambda}{\mu_h}\right)^m}{\sum_{m=0}^{n_\ell} \left(\frac{\Lambda}{\mu_\ell}\right)^m + \sum_{m=n_\ell+1}^{n} (1-q)^{m-n_\ell} \left(\frac{\Lambda}{\mu_\ell}\right)^m}.$$

The number of pure strategy equilibria in the game depends on whether μ_ℓ is greater or less than $(1-q)\mu_h$. At the length n_ℓ, only uninformed consumers join the queue for the low-quality firm. Thus, the arrival rate for firm ℓ falls discretely to $(1-q)\Lambda$, whereas the arrival rate for the high-quality firm remains Λ. Consumers leave firm θ at the rate μ_θ. From the expressions for the long-run probability of the Birth-Death process (see Lemma 3 in Appendix B), it follows that the probability of observing $n \in \{n_\ell, \ldots, \hat{n}\}$ is proportional to $((1-q)\Lambda/\mu_\ell)^{n-n_\ell}$ for the low-quality firm and to $(\Lambda/\mu_h)^{n-n_\ell}$ for the high-quality firm. Hence, beyond the queue length n_ℓ, if $(1-q)\mu_h > \mu_\ell$, a higher queue length implies it is less likely that the firm has high quality. Conversely, if $(1-q)\mu_h < \mu_\ell$, the likelihood a firm has high quality increases in queue length.

Proposition 2 *(i) Suppose $\mu_\ell < (1-q)\mu_h$ and $\left(\frac{\mu_\ell}{\mu_h}\right)^{n_\ell+1} < \Phi(n_\ell)$. Then, the consumer game has at least one pure strategy equilibrium.*
(ii) Conversely, if $\mu_\ell > (1-q)\mu_h$ and $\left(\frac{\mu_\ell}{\mu_h}\right)^{n_\ell+1} > \Phi(n_\ell)$, the consumer game has at most one pure strategy equilibrium.

In case (ii) of the proposition, the consumer game may not have any pure strategy equilibrium. As argued in Section 2.3, an equilibrium always exists, so in this case there must be a mixed strategy equilibrium.

Observe that the conditions in Proposition 2 do not depend on the parameter p, the prior probability the firm has high quality. By imposing conditions on p, we can be more precise about the nature of the pure strategy equilibria in the game. In particular, for each queue length n we identify upper and lower thresholds for p, the prior probability the firm has high quality. These thresholds $\bar{p}(n)$ and $\underline{p}(n)$ depend on all other parameters in the game, $(\Lambda, \mu_h, \mu_\ell, q, c, v_h, v_\ell)$. For convenience, we suppress this dependence in the notation. If $p \in (\underline{p}(n), \bar{p}(n))$, there exists a a pure strategy equilibrium in the consumer game with a hole exactly at the queue length n if either an added condition on the parameters is satisfied (condition (i) (a) in Proposition 3 below) or n is close to either n_ℓ or n_h. However, if condition (i) (a) below is violated and n is sufficiently far from both n_ℓ and n_h, a pure strategy equilibrium with a hole at n cannot be supported, regardless of the value of p.

Proposition 3 *There exist threshold queue lengths $\underline{n} \geq n_\ell$ and $\bar{n} \leq n_h$ and threshold probabilities $\bar{p}(n), \underline{p}(n)$ for each $n \in \{n_\ell, \cdots, n_h\}$ such that $0 \leq \underline{p}(n) < \bar{p}(n) \leq 1$, $\underline{p}(n_\ell) = 0$, $\bar{p}(n_h) = 1$, and:*
(i) If $p \in [\underline{p}(n), \bar{p}(n)]$ and either (a) $\frac{\mu_\ell}{(1-q)\mu_h} < \frac{v_h\mu_h - v_\ell\mu_\ell + 2c}{v_h\mu_h - v_\ell\mu_\ell - 2c}$ or (b) $n \in \{n_\ell, \cdots, \underline{n}\} \cup$

$\{\bar{n}, \cdots, n_h\}$, *there is a pure strategy equilibrium in the consumer game with a hole at* n.
(ii) If $\frac{\mu_\ell}{(1-q)\mu_h} < \frac{v_h\mu_h - v_\ell\mu_\ell + 2c}{v_h\mu_h - v_\ell\mu_\ell - 2c}$ *and* $n \in \{\underline{n}+1, \cdots, \bar{n}-1\}$, *there does not exist a pure strategy equilibrium in the consumer game with a hole at* n *for any* $p \in [0, 1]$.

The thresholds $\underline{p}$ and $\bar{p}$ are increasing in n. However, they may overlap (i.e., it is possible that $\underline{p}(n+1) < \bar{p}(n)$). Therefore, for some parameter values, there may be multiple pure strategy equilibria in the consumer game. In Appendix A, we provide algorithms to determine all pure and mixed strategy equilibria in the consumer game. For generic parameter values, characterizing the equilibrium of the consumer game is non-trivial. We exhibit an example in which there are two pure strategy and three mixed strategy equilibria.

Example 1 Let $p = 0.5$, $q = 0.45$, $v_h = 1.70$, $v_\ell = 0.75$, $c = 0.35$, $\Lambda = 1$, $\mu_h = 1.40$, and $\mu_\ell = 0.75$. Then, $n_h = 6$ and $n_\ell = 1$. That is, in every equilibrium, the informed consumer joins the queue at all queue lengths less than 6 if the firm has high quality and only at the queue length 0 if the firm has low quality. An equilibrium can then be characterized by the strategy of an uninformed consumer.

Consider the condition in Proposition 3 (i) (a): we have $\frac{v_h\mu_h - v_\ell\mu_\ell + 2c}{v_h\mu_h - v_\ell\mu_\ell - 2c} = 2.2528 > \frac{\mu_\ell}{(1-q)\mu_h} = 0.974$. Therefore, for every $n \in \{1, 2, \cdots, 6\}$, there exists a range of p such that there is a pure strategy equilibrium in which the uninformed consumer's strategy has a hole at n. Using the expressions shown in the proof of Proposition 3, we compute the probability thresholds $\underline{p}(n)$ and $\bar{p}(n)$ for all $n \in \{1, 2, \cdots, 6\}$. With these thresholds, we find that for $p = 0.5$ there are two pure strategy equilibria:
(P1) The hole in the uninformed consumer's strategy is at $n = 2$, so the uninformed consumer joins at queue lengths $n = 0, 1, 3, 4, 5$.
(P2) The hole in the uninformed consumer's strategy is at $n = 3$, so the uninformed consumer joins at queue lengths $n = 0, 1, 2, 4, 5$.

Next, applying the algorithm in Appendix A, we find three mixed strategy equilibria. In each of these, the uninformed consumer joins the queue with probability 1 at $n = 0$, 1 and 3, and with probability 0.0924 at $n = 2$, and balks at any queue length $n \geq 6$. The uninformed consumer's strategy in each equilibrium is further characterized as follows:
(M1) Join with probability 0.0866 at $n = 4$ and probability 1 at $n = 5$.
(M2) Join with probability 0.4934 at $n = 4$ and balk at $n = 5$ or higher.
(M3) Join the queue with probability 0.1843 and $n = 4$ and probability 0.7588 at $n = 5$. ■

4 Equilibrium Choice of Service Rates

Now we turn to the optimal choice of service rates by the firms. In making their decision, the firms take the consumer strategy as given. For any pure consumer strategy $\sigma = (\sigma_i, \sigma_u)$, let $\hat{n}(\sigma) = \min\{n \mid \sigma_u(n) = 0\}$ denote the queue length at which σ_u has a hole. If the firm has low-quality, its queue never extends beyond $\hat{n}(\sigma)$. At queue lengths below n_ℓ, both informed and uninformed consumers join the queue, and at queue lengths between n_ℓ and $\hat{n}(\sigma) - 1$, only uninformed consumers join. In contrast, the high quality firm has all consumers joining the queue at all queue lengths below n_h, except for the queue length $\hat{n}(\sigma)$, at which only informed consumers join.

For each $\theta = h, \ell$, let $\tilde{n}_\theta(\sigma) = \min\{n \mid \sigma_i(\theta, n) = 0\}$ be the first queue length at which informed consumers balk. We say a pure consumer strategy σ satisfies the necessary conditions of equilibrium in the consumer game if $\tilde{n}_h(\sigma) \geq \tilde{n}_\ell(\sigma)$ and $\hat{n}(\sigma) \in \{\tilde{n}_\ell(\sigma), \tilde{n}_\ell(\sigma)+1, \cdots, \tilde{n}_h(\sigma)\}$. As the next Lemma shows, conditional on consumers playing a pure strategy, both types of firm would prefer to have the hole in the uninformed consumer's strategy at as high a queue length as possible.

Lemma 1 *Fix a pure strategy for consumers, σ, that satisfies the necessary conditions for equilibrium in the consumer game. Suppose a firm of type $\theta \in \{h, \ell\}$ chooses service rate μ, and consumers play σ. Then,*
(i) The expected revenue per unit of time for each type of firm is

$$
\begin{aligned}
R_h(\mu, \sigma) &= r\mu \left(1 - \frac{1}{\sum_{j=0}^{\hat{n}(\sigma)} (\Lambda/\mu)^j + q \sum_{j=\hat{n}(\sigma)+1}^{\tilde{n}_h(\sigma)} (\Lambda/\mu)^j}\right), \\
R_\ell(\mu, \sigma) &= r\mu \left(1 - \frac{1}{\sum_{j=0}^{\tilde{n}_\ell(\sigma)} (\Lambda/\mu)^j + \sum_{j=\tilde{n}_\ell(\sigma)+1}^{\hat{n}(\sigma)} (1-q)^{j-\tilde{n}_\ell(\sigma)} (\Lambda/\mu)^j}\right).
\end{aligned}
$$

(ii) Fixing $\tilde{n}_h(\sigma)$ and $\tilde{n}_\ell(\sigma)$, for each $\theta \in \{h, \ell\}$, the expected revenue R_θ increases in $\hat{n}(\sigma)$.

Therefore, all else equal, each type of firm will prefer that the hole in an uninformed consumer's strategy, $\hat{n}$, occur at a high rather than low queue length. The intuition for our results in this section is that, if the hole occurs at a low queue length (specifically, n_ℓ), the high-quality firm has an incentive to slow down (i.e., choose the slow service rate $\underline{\mu}$) to ensure that a relatively large proportion of time is spent at queue lengths above the hole.

One complication that arises is that speeding up (i.e., choosing the fast service rate $\bar{\mu}$) increases n_h, the threshold at which informed consumers balk, thereby potentially increasing the revenue of the firm. We begin with an extended example of equilibrium in the overall game that fully addresses this complication. We then turn to analytic results in Section 4.2, where we consider the more restricted case in which service rates are sufficiently close to each other to leave the thresholds n_h, n_ℓ unaffected by the choices of each type of firm. In Section 4.3, we report on an extensive numerical study that demonstrates the robustness of our analytic results.

4.1 Service Rate Choices: Example

We first provide an example to illustrate the tradeoffs faced by the two types of firm in choosing their respective service rates. Let $v_h = 20, v_\ell = 1, c = 0.4005, \Lambda = 1, \bar{\mu} = 1.15$ and $\underline{\mu} = 0.85$. We consider two cases for q, the proportion of informed consumers: $q = 0.05$ (Case I) and $q = 0.9$ (Case II). We analyze how the equilibrium varies with p and κ.

We begin by analyzing the consumer game at stage 2. Consider condition (i)(a) in Proposition 3, $\frac{\mu_\ell}{(1-q)\mu_h} < \frac{v_h\mu_h+v_\ell\mu_\ell+2c}{v_h\mu_h+v_\ell\mu_\ell-2c}$. It is straightforward to check that for Case I ($q = 0.05$) this condition is satisfied given the service rate choices in the first three rows of the table, but not when firm h is slow and firm ℓ is fast. Therefore, in Case I, whenever $\mu_h = 1.15$ or $\mu_h = \mu_\ell = 0.85$, for every $n \in \{n_\ell, n_\ell + 1, \cdots, n_h\}$ there exists a range of p such that there is a pure strategy equilibrium in which the uninformed consumer's strategy has a hole at n. However, if $\mu_h = 0.85$ and $\mu_\ell = 1.15$, the condition is violated. Here, in Case I, $n_\ell = 2$, $\underline{n} = 5$ and $\bar{n} = n_h = 42$. Therefore, a pure strategy equilibrium with a hole at n can be supported only if $n \in \{2, 3, 4, 5, 42\}$.

In Case II (with $q = 0.9$), the condition i(a) is violated for any pair of service rate choices. As a result, a hole in the uninformed consumer's strategy can be supported in equilibrium only if the hole is either at queue length n_ℓ or at n_h, but no queue lengths in-between those two. That is, here $\underline{n} = n_\ell$ and $\bar{n} = n_h$. Note that $n_h = 57$ if firm h chooses $\bar{\mu}$ and 42 if firm h chooses $\underline{\mu}$, and $n_\ell = 2$ regardless of whether firm ℓ chooses $\bar{\mu}$ or $\underline{\mu}$.

There are four possible pairs of service rate outcomes in the model, since each type of firm could choose either a fast or a slow service rate. We focus on values of p that, regardless of firms' service rate choices, support a pure strategy equilibrium in which the uninformed consumer's strategy has a hole at n_ℓ; that is, we consider $p \leq \bar{p}(n_\ell)$. We then select the consumer equilibrium in which the uninformed consumer's strategy has a hole at n_ℓ.

Next, we compute the revenue of each type of firm for each of the four pairs of service rate choices. The difference in revenues for firm θ between the high and low service rates, $R_\theta(\bar{\mu}, \sigma) - R_\theta(\underline{\mu}, \sigma)$, determines the maximum the firm is willing to pay to increase its service rate from $\underline{\mu}$ to $\bar{\mu}$. In any pure strategy equilibrium of the overall game, if firm θ chooses the fast service rate $\bar{\mu}$, it must be that $R_\theta(\bar{\mu}, \sigma) - R_\theta(\underline{\mu}, \sigma) \leq \kappa$, with the inequality being reversed if it chooses the slow rate $\underline{\mu}$.

Table 1 provides the ranges of p (the prior probability the firm has high quality) and κ (the cost of speeding up) for which each type of equilibrium is sustained. That is, in each row, for all values of p and κ in the specified range, it is an equilibrium for firms to choose the service rates mentioned in column 1 of the same row and for uninformed consumers to play a strategy that has a hole at n_ℓ. For each type of equilibrium, $n_\ell = 2$, $\hat{n} = 2$, and $n_h = 57$ when $\mu_h = 1.15$ and 42 when $\mu_h = 0.85$.

Service Rate in Equilibrium Firm h	Firm ℓ	Supporting Range of	Case I $q = 0.05$	Case II $q = 0.9$
Fast	Fast	p	$\emptyset$	$p \leq 0.006$
		κ	$\emptyset$	$\kappa \leq 0.101$
Slow	Slow	p	$p \leq 0.660$	$p \leq 0.972$
		κ	$\kappa \geq 0.101$	$\kappa \geq 0.139$
Fast	Slow	p	$\emptyset$	$p \leq 0.074$
		κ	$\emptyset$	$\kappa \in [0.101, 0.139]$
Slow	Fast	p	$p \leq 0.135$	$\emptyset$
		κ	$\kappa \leq 0.101$	$\emptyset$

This table shows the range of κ and p that supports different kinds of service rate equilibria for two sets of parameters, when the equilibrium in the continuation game has $\hat{n} = n_\ell = 2$.

Table 1: **Equilibrium service rate choices**

Consider Case I first, in which the proportion of informed consumers is low, with $q = 0.05$. Here, we find that $R_h(\bar{\mu}, \sigma) - R_\ell(\underline{\mu}, \sigma) < 0$. That is, even if it is costless to improve its technology, firm h prefers the slow service rate. Therefore, there is no equilibrium in which firm h chooses the fast service rate $\bar{\mu}$. The intuition follows from the hole in the uninformed consumer's strategy being at a low queue length ($\hat{n} = 2$). Since there are many uninformed consumers, it is valuable for firm h to signal its quality by ensuring that the queue generally remains greater than 2 in length.

For the low-quality firm in Case I, we find that $R_\ell(\bar{\mu}, \sigma) - R_\ell(\underline{\mu}, \sigma) = 0.101$ in all cases. Therefore, if $\kappa \leq 0.101$, the firm chooses the fast service rate, and if $\kappa \geq 0.101$, it chooses to slow down. Thus, in equilibrium firm h is slow and firm ℓ is either fast or slow, depending on the cost of speeding up. For each of these two sets of equilibria, the maximal value of p under which the equilibrium can be sustained is given by $\bar{p}(n_\ell)$.

In Case II, with a high proportion of informed consumers, it is no longer important to the high-quality firm that uninformed consumers do not join at queue length 2. Regardless of service rate choices, the revenue gain from speeding up is greater for the high-quality firm than the low-quality firm. Therefore, it is not feasible to sustain an equilibrium in which the high-quality firm is slow and the low-quality firm is fast, although the converse case is now feasible. Now, when κ is large both firms are slow, and when κ is sufficiently small, over a narrow range of p, both firms are fast. It may be noted that there are values of p and κ at which multiple equilibria exist in the service rate game. For example, when $p = 0.05$ and $\kappa = 0.139$, there is an equilibrium in which both firms are slow and an equilibrium in which the high-quality firm is fast and the low-quality firm is slow. In Section 4.3, in our numerical study, whenever there are multiple equilibria we choose the equilibrium that maximizes the ex ante profit of the firm.

It is important to appreciate that the service rate equilibria in Table 1 depend on the equilibrium in the consumer game. There exist other ranges of p and κ that support different kinds of equilibria. Suppose, for example, $p \geq 0.907$ and $\kappa \leq 0.101$. Then, in both cases (i.e., whether q is high or low), there exists an equilibrium in which the both firms choose the fast service rate, and the hole in the uninformed consumer's strategy is at $n_h = 57$.

4.2 Analytic Results

The overall intuition from the previous example is that if q, the proportion of informed consumers, is low and the hole in the uninformed consumer's strategy is at a low queue length (which relies in turn on a sufficiently low prior probability the firm has high quality, p), the high-quality firm has a greater incentive than the low-quality firm to choose the slow service rate. We now formalize this intuition. To obtain the same results analytically, we have to make some additional assumptions.

Recall that $n_\theta = \lfloor v_\theta \frac{\mu_\theta}{c} \rfloor$ is the threshold at which informed consumers balk when the firm has type θ. We assume that n_h and n_ℓ are invariant to the service rate chosen by each type of firm. This assumption takes away one incentive the high-quality firm may have to speed up, which is increasing the maximal length of the queue.

Assumption 3 $\lfloor v_\ell(\bar{\mu}/c)\rfloor = \lfloor v_\ell(\underline{\mu}/c)\rfloor$ *and* $\lfloor v_h(\bar{\mu}/c)\rfloor = \lfloor v_h(\underline{\mu}/c)\rfloor$.

Assumption 3 effectively implies that the fast and slow service rates ($\bar{\mu}$ and $\underline{\mu}$) are close to each other. In other words, if a firm speeds up the rate at which it services consumers, it does so through an incremental operational improvement. Let $\mu_0 > 0$ be the mean of the fast and slow service rates. Then, for any $\epsilon \in (0, \mu_0)$, define $\bar{\mu} = \mu_0 + \epsilon$ and $\underline{\mu} = \mu_0 - \epsilon$.

We focus on equilibria in the consumer game in which uninformed consumers play a pure strategy with a hole at exactly n_ℓ. Let $\hat{\sigma}$ denote the consumer strategy in which informed consumers join the queue of a firm of type θ at all queue lengths up to and including $n_\theta - 1$, and uninformed consumers join the queue at all queue lengths up to $n_h - 1$ except at the length n_ℓ. Observe that for a given service rate choice μ_h, μ_ℓ, if $p \leq \bar{p}(n_\ell \mid \mu_h, \mu_\ell)$ (where $\bar{p}(\cdot)$ is strictly positive and identified precisely in the proof of Proposition 3), such an equilibrium exists. Define

$$p_0 \quad = \quad \min_{\mu_h, \mu_\ell \in \{\underline{\mu}, \bar{\mu}\}} \bar{p}(n_\ell \mid \mu_h, \mu_\ell). \tag{7}$$

If all consumers follow the strategy $\hat{\sigma}$, p_0 is the lowest value of the prior at which, for some service rate strategy, an uninformed consumer is exactly indifferent between joining and balking the queue when there are already n_ℓ consumers waiting. If $p \leq p_0$, regardless of the service rate strategy chosen by each type of firm, there is an equilibrium of the consumer continuation game in which the hole in an uninformed consumer's strategy is exactly at n_ℓ. In this subsection, we select this equilibrium in the consumer game whenever it exists, ignoring any other equilibria. It can be shown that if p is sufficiently small, the consumer equilibrium in which uninformed consumers play a strategy with a hole at n_ℓ is welfare-maximizing for uninformed consumers. In the next subsection, we conduct a numerical study in which we compute the welfare of uninformed consumers in every equilibrium and explicitly choose the welfare-maximizing one. As we show, in the cases of interest, the chosen equilibrium is often one in which uninformed consumers play a strategy with a hole at n_ℓ.

Now, consider a firm of type θ. Let Δ_θ denote the marginal revenue the firm gains from choosing the fast service rate $\bar{\mu} = \mu_0 + \epsilon$ rather than the slow service rate $\underline{\mu} = \mu_0 - \epsilon$, given that consumers are playing the strategy $\hat{\sigma}$. Then,

$$\Delta_\theta(\mu_0, \epsilon) \quad = \quad R_\theta(\mu_0 + \epsilon, \hat{\sigma}) - R_\theta(\mu_0 - \epsilon, \hat{\sigma}). \tag{8}$$

Observe that, from Lemma 1, for each θ the revenue rate R_θ depends on q, the proportion of informed consumers. Hence, Δ_θ also depends on q. For any μ_0, the firm will strictly prefer the fast service rate $\mu_0 + \epsilon$ over the slow service rate $\mu_0 - \epsilon$ if and only if $\Delta_\theta(\mu_0, \epsilon) > \kappa$, the marginal cost of installing added capacity (i.e., of choosing the fast service rate).

We set $\mu_0 = \Lambda$, and consider the relative incentive each type of firm has to choose the fast service rate rate $\bar{\mu} = \Lambda + \epsilon$ over the slow service rate $\underline{\mu} = \Lambda - \epsilon$. We obtain clean analytic expressions for the derivative of revenue of type θ with respect to μ, $\frac{\partial R_\theta}{\partial \mu}(\mu, \hat{\sigma})$, in the limit as ϵ goes to zero (i.e., $\underline{\mu}$ and $\bar{\mu}$ each approach Λ). Using these limiting analytic expressions as our starting point, we are then able to provide comparisons in a neighborhood of service rates around Λ.

Lemma 2 *Suppose $\mu = \Lambda$ and $p \leq p_0$. Then,*

$$\begin{aligned} \frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) &= \frac{r\left[2(n_h - n_\ell)^2 q^2 - (n_h - 3n_\ell - 1)(n_h - n_\ell)q + n_\ell(n_\ell + 1)\right]}{2[n_\ell + 1 + q(n_h - n_\ell)]^2} \\ \frac{\partial R_\ell}{\partial \mu}(\Lambda, \hat{\sigma}) &= \frac{r n_\ell}{2(n_\ell + 1)}. \end{aligned}$$

Consider the rate of change of the revenue of the high-quality firm in the limit, when $\mu = \Lambda$. First, suppose that $q = 1$; i.e., all consumers are informed. Then, $\frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) = \frac{r n_h}{2(n_h+1)}$, which exceeds $\frac{\partial R_\ell}{\partial \mu}(\Lambda, \hat{\sigma}) = \frac{r n_\ell}{2(n_\ell+1)}$ since $n_h > n_\ell$. That is, when all consumers are informed, the high-quality firm has a greater incentive to speed up than the low-quality firm. Next, suppose that $q = 0$, so that all consumers are uninformed. Then, the two derivatives in Lemma 2 are each exactly equal to $\frac{r n_\ell}{2(n_\ell+1)}$. This is intuitive: if there are no informed consumers, the two types of firm have exactly the same stationary distribution over queue length. Hence, each type of firm has the same incentive to speed up.

Define a threshold proportion of informed consumers, $\hat{q}$ as follows:

$$\hat{q} = \left(1 - \frac{1}{n_h - n_\ell}\right) \frac{n_\ell + 1}{n_\ell + 2}. \tag{9}$$

It is immediate that the threshold $\hat{q}$ is increasing in $n_h - n_\ell$.

Our main result is described as follows. Consider any $q > \hat{q}$. Then, if $\underline{\mu}$ and $\bar{\mu}$ are sufficiently close to Λ, the marginal revenue of the high-quality firm exceeds the marginal revenue of the low-quality firm; that is, $\Delta_h(\Lambda, \epsilon) > \Delta_\ell(\Lambda, \epsilon)$. The high-quality firm therefore has a greater incentive to speed up than the low-quality firm. Thus, there is a range of costs at which the high-quality firm chooses the fast service rate $\bar{\mu}$ and the low-quality firm chooses the slow service rate $\underline{\mu}$. Similarly, if $q < \hat{q}$ and $\underline{\mu}$ and $\bar{\mu}$ are sufficiently close to Λ, the low-quality firm has a greater incentive to speed up than the high-quality firm; that is, $\Delta_h(\Lambda, \epsilon) < \Delta_\ell(\Lambda, \epsilon)$. Thus, there is another range of costs at which the high-quality firm chooses the slow service rate $\underline{\mu}$ and the low-quality firm chooses the fast service rate $\bar{\mu}$.

Proposition 4 *Suppose $\bar{\mu} = \Lambda + \epsilon$ and $\underline{\mu} = \Lambda - \epsilon$. For every $q \in (0, 1)$, there exists an $\hat{\epsilon}(q) > 0$ such that if $\epsilon \in (0, \hat{\epsilon}(q)]$ and $p \leq p_0$:*

1. *If $q < \hat{q}$, then $\Delta_h(\Lambda, \epsilon) < \Delta_\ell(\Lambda, \epsilon)$. Hence, there exists a range for κ such that in equilibrium the high-quality firm chooses the slow service rate $\Lambda - \epsilon$ and the low-quality firm chooses the fast service rate $\Lambda + \epsilon$. However, the converse service rate strategies cannot be sustained in equilibrium.*

2. *If $q > \hat{q}$, then $\Delta_h(\Lambda, \epsilon) > \Delta_\ell(\Lambda, \epsilon)$. Hence, there exists a range for κ such that in equilibrium the high-quality firm chooses the fast service rate $\Lambda + \epsilon$ and the low-quality firm chooses the slow service rate $\Lambda - \epsilon$. However, the converse service rate strategies cannot be sustained in equilibrium.*

Consider the decision faced by the high-quality firm. Suppose the queue length is exactly n_ℓ when an informed consumer arrives. Recall that $n_\ell < n_h$, hence, the informed consumer will join, and the queue length will become $n_\ell + 1$. Therefore, if the proportion of informed consumers is relatively high, the hole is irrelevant, since it will be crossed whenever an informed consumer arrives. However, if the proportion of informed consumers is low, the hole is unlikely to be crossed simply by an arriving consumer. In this case, choosing the slow service rate is potentially valuable, since a greater amount of time is then spent at queue lengths above the hole.

Intuitively, the benefit of slowing down for the high-quality firm will depend on the number of queue lengths above the hole; i.e., on $n_h - n_\ell$. If n_h is substantially greater than n_ℓ, there is a greater incentive to slow down and reach the higher queue lengths. Conversely, if n_h is close to n_ℓ, the incentive to slow down is weaker. Indeed, it may be observed that the threshold proportion of informed consumers in equation (9), $\hat{q}$, is strictly increasing in $(n_h - n_\ell)$. Thus, the range of q for which the high-quality firm has a stronger incentive to slow down, compared to the low-quality firm, increases in $(n_h - n_\ell)$.

Further, note that when $n_h = n_\ell + 1$, $\hat{q} = 0$. Thus, at the minimal distance between n_h and n_ℓ, the high-quality firm always has a stronger incentive to speed up, compared to the low-quality firm. It is only when the distance between n_h and n_ℓ increases to two or more that the high-quality firm may have a weaker incentive to speed up.

Next, as a special case, consider v_h becoming large, keeping v_ℓ fixed. Observe that, as v_h (and hence n_h) becomes large, the threshold $\hat{q}$ (as defined in equation (9)) goes to $\frac{n_\ell+1}{n_\ell+2}$. Thus, it follows from Proposition 4 that for large values of v_h, if $q > \frac{n_\ell+1}{n_\ell+2}$, there is a range of κ for which the high-quality firm speeds up and the low-quality firm slows down. Conversely, if $q < \frac{n_\ell+1}{n_\ell+2}$, there is a range of κ for which the high-quality firm slows down and the low-quality firm speeds up. In fact, as v_h becomes large, we obtain even stronger results for the high-quality firm. Strikingly in this case, if the proportion of informed consumers is sufficiently low (in particular, $q < \frac{1}{2}$), the high-quality firm actually obtains a lower revenue when it speeds up to the fast service rate, compared to staying at the slow service rate. Hence, even if there is no cost to increasing the service rate, the high-quality firm will prefer to provide slow service.

Proposition 5 *There exist an $\bar{\epsilon}(q)$ and a threshold value for the high-quality good $\bar{v}(q)$ with the following properties: Suppose $v_h \geq \bar{v}(q)$, $\epsilon \in (0, \bar{\epsilon}(q)]$, $\bar{\mu} = \Lambda + \epsilon$, $\underline{\mu} = \Lambda - \epsilon$, and $p \leq p_0$. Then, $\Delta_h(\Lambda, \epsilon) < 0$ if $q < \frac{1}{2}$ and $\Delta_h(\Lambda, \epsilon) > 0$ if $q > \frac{1}{2}$. Hence, if $q < \frac{1}{2}$, there is no value of κ (including $\kappa = 0$) at which the high-quality firm chooses the fast service rate in equilibrium.*

The intuition behind our results as follows. Speeding up service has two potential benefits for the high-quality firm. First, it may increase the threshold n_h at which informed consumers balk. Assumption 3 rules out this effect since n_h is fixed at the same level at both service rates $\bar{\mu}$ and $\underline{\mu}$. Second, it shifts the stationary distribution of consumers toward the lower parts of the queue, in particular to queue lengths below the length at which the uninformed consumers have a hole in their strategy. When the proportion of informed consumers is low, speeding up service is less valuable, since the queue is less likely to cross the length at which uninformed consumers have a hole.

When the proportion of informed consumers is relatively high, however, even at that queue length, the likelihood is that the next consumer is an informed consumer. Hence, queue lengths above the hole are more likely to be reached, making speeding up more valuable. This value is enhanced when v_h is large, so that there are many queue lengths above the length at which the uninformed consumers' strategy has a hole.

Recall that we assume $\mu_0 = \Lambda$. In Debo, Parlour and Rajan (2010), we consider the case in which the service rates $\bar{\mu}$ and $\underline{\mu}$ are close to each other and are both strictly greater than the arrival rate Λ. We show that if the mean of feasible service rates, μ_0, is sufficiently close to Λ and v_h is sufficiently high the high quality firm will never choose the fast service rate, even if there is no cost to speeding up. If p and q are low, the result is similar to that in Proposition 5 above. However, if q is high, the hole in the uninformed consumers' strategy is at n_h rather than n_ℓ. If v_h is high, n_h is also high so few arrivals balk when the firm has high quality. With a low v_ℓ and n_ℓ, the low-quality firm suffers larger losses from balking and hence has a greater incentive to speed up. Therefore, although the phenomenon of the high-quality firm slowing down exists, the queue does not communicate information about quality.

4.3 Robustness: Numerical Study

To confirm the robustness of our analytic results in Section 4.2, we conduct an extensive set of numerical computations. For the computations, we fix $v_\ell = 1.51$, $c = 1$, $\Lambda = 1$ and $\mu_0 = 1$. The values of the other parameters are shown in Table 2. Importantly, for every set of parameter values, Assumption 3 is violated: the threshold at which informed consumers balk when the firm has high quality, n_h, depends on the service rate chosen by the firm. By combining different values of the parameters, we generate a total of 1,176 examples.

Parameter	Values considered	Mean Value
p	0.01, 0.05, 0.25, 0.5, 0.75, 0.95, 0.99	0.5
q	0.01, 0.05, 0.25, 0.5, 0.75, 0.95, 0.99	0.5
v_h	4.01, 6.01, 8.01, 10.01, 12.01, 14.01	9.01
ϵ	0.01, 0.025, 0.125, 0.25	0.1025

Table 2: **Parameter values for numerical computations** ($v_\ell = 1.51, c = \Lambda = \mu_0 = 1$)

For each set of feasible parameter values, we consider the four possible service rate strategies of the firms; i.e., (h fast, ℓ fast), (h fast, ℓ slow), (h slow, ℓ fast), and (h slow, ℓ slow). For each service rate strategy, we first determine all equilibria in the consumer game. If there are multiple equilibria, we select the equilibrium that maximizes the welfare of the uninformed consumers. We find that there are an average of 1.43 equilibria in the consumer game across our parameter values, with the average number of consumer equilibria increasing with v_h. Once we have chosen equilibria in the consumer game, the revenue to each type of firm from each of the four pairs of service rate choices is easily computed.

We then pose the following question: For each set of feasible parameter values, does there exist a range of κ such that, if κ lies in this range, firm h will choose the slow service rate $\underline{\mu}$ and firm ℓ the high service rate $\bar{\mu}$? For some ranges of κ, we find there are multiple equilibria in the firms' game. In this case, we select the equilibrium that maximizes the ex ante profit of the firm, that is, maximizes $p[R_h(\mu,\sigma)-\kappa 1_{\{\mu_h=\bar{\mu}\}}]+(1-p)[R_\ell(\mu,\sigma)-\kappa 1_{\{\mu_\ell=\bar{\mu}\}}]$. In Table 3, we report for each value of p the proportion of cases with that p for which we find a range of κ such that the service rate strategy (h slow, ℓ fast) is played in the firms' game.[2] We also report the average values of q, v_h, and ϵ in these cases, and the mean value of κ and range of κ that support the (h slow, ℓ fast) equilibrium. As a benchmark, both the mean and range of κ are scaled by 2ϵ, which is the highest possible increase in throughput from speeding up. Finally, for the cases in which the (h slow, ℓ fast) equilibrium exists, we also report the proportion of cases in which the resulting equilibrium in the consumer game has the uninformed consumers playing a pure strategy with the hole at n_ℓ.

p	Percent	Mean	Mean	Mean	κ values		Percent
	of cases	q	v_h	ϵ	Mean	Range	hole at n_ℓ
0.010	50.000	0.160	9.296	0.095	0.203	0.100	95.238
0.050	46.429	0.164	9.625	0.084	0.233	0.085	63.636
0.250	16.071	0.062	9.491	0.037	0.350	0.014	0.000
0.500, 0.750, 0.950, 0.990	0.000	—	—	—	—	—	—
Overall	16.071	0.148	9.460	0.082	0.236	0.082	68.617

Table 3: **Average parameter values at which (h slow, ℓ fast) equilibria exist, for different values of p**

As can be seen from the Table, the (h slow, ℓ fast) equilibrium exists only at low values of p ($p \leq 0.25$ in the set of values chosen for our computations) and q (the average q value at which this equilibrium exists is 0.148, compared to a mean of 0.5 in the computations). As p increases from 0.01 to 0.05 and 0.25, the frequency of this equilibrium and the values of ϵ and the range of κ that support it all fall. Also, the proportion of cases in which the uninformed consumers play a strategy with a hole at n_ℓ falls: as p increases, the hole

[2] An alternative equilibrium selection rule, for example, would be to choose the equilibrium that maximizes the profit of firm ℓ if $p \leq 0.5$ and the equilibrium that maximizes the profit of firm h if $p > 0.5$. Such a rule makes no difference to our results.

advances beyond n_ℓ. Overall, in 99.5% of the cases in which the (h fast, ℓ slow) equilibrium exists, a pure strategy equilibrium is selected in the consumer game and in 68.6% of the cases the resulting strategy of uninformed consumers has a hole at n_ℓ.

Overall, our numerical study confirms the analytical results from Section 4.2. When the prior probability the firm has high quality and the proportion of informed consumers are both low, there often exists a range of κ such that in equilibrium the high-quality firm chooses the slow service rate and the low-quality firm chooses the fast service rate. That is, the high-quality firm finds it valuable to use a long queue to signal its type to uninformed consumers.

For approximately 30% of the parameter values, we find there is a range of κ at which there is no pure strategy equilibrium in the firms' game. Since the firms' game has a finite strategy space, we know a mixed strategy equilibrium must exist in these cases. The range of κ at which there is no pure strategy equilibrium is typically small, and averages 0.023 (or approximately 70% smaller than the average κ range of 0.082 that supports the (h slow, ℓ fast) equilibrium).

5 Conclusion and Discussion

Our results on service rate selection translate quite naturally into predictions on how different types of firms strategically manipulate queues. In a pure strategy equilibrium in the consumer game, the equilibrium strategy of an uninformed consumer has a hole. Queues below the hole are short enough such that low waiting costs make joining rational. At the hole, uninformed consumers balk. Above the hole, they correctly infer that the good has high quality.

Both firm types lose the uninformed consumers at the hole. The high-quality firm knows it will win back these consumers at queue lengths above the hole. However the hole will only be filled (or crossed) if an informed consumer arrives at the market. Therefore, a high-quality firm is worst off facing a market with few informed consumers and in which the uninformed ascribe a low prior probability to it being of high quality. In this case, the hole is at low queue lengths, with little likelihood it will be filled by an informed consumer. To keep the queue above the hole, the firm chooses a slow service rate. Intuitively, the high-quality firm can compensate for the relative shortage of informed consumers by slowing down the service rate, ensuring a higher probability that the queue is above the hole.

In our model, the queue serves to communicate information to uninformed consumers about the strength of demand, and hence about their own valuation for the product. More broadly, our (infinite horizon) queuing model can be considered as an approximation of the initial phase of a product life-cycle. The informational effect of excess demand will be strongest at this point as there are likely to be few informed consumers in the market. Later phases of the product life-cycle can also be approximated with our model, with a larger proportion of informed consumers. Our analysis suggests that it may be optimal for such firms to gradually increase the service rate over time as the fraction of informed consumers in the market increases. In case such firms cover the demand rate too early in the product life cycle, when the fraction of informed consumers is low, further service rate expansion would stall until the fraction of informed consumers is high enough. Indeed, if excess demand provides an informational externality that generates even more demand, it is no longer true that supply creates its own demand. Rather, the lack of supply may lead to an increase in demand.

It is natural to ask whether a firm may adjust its price to signal the quality of its product. If a firm can choose the price of the product, there are two possibilities. First, both types of firm may pool on price (i.e., choose the same price). Our model corresponds to this case. Second, the high and low quality firms may separate on prices (i.e., choose different prices). In this case, all consumers are effectively informed about quality, since the price reveals quality to uninformed consumers.

There is typically no direct cost to a low-quality firm of simply setting a price equal to that of the high-quality firm. There is an indirect cost: If the price is sufficiently high, the low-quality firm may lose all informed consumers. However, uninformed consumers are still uncertain about the quality of the firm, and will have to learn about quality from the queue. If the proportion of informed consumers is low, the cost to the low-quality firm of imitating the high-quality firm is correspondingly low. Therefore, in this case we expect the low-quality firm to pool on price with the high-quality firm. As a result, our finding that the high-quality firm has an incentive to slow down when there are few informed consumers is robust to firms using the price of the product to signal quality.

Similarly, other signals about quality, such as directly reporting the service rate, may also be infeasible. For example, if a firm reports its service rate, the low-quality firm has an incentive to lie and make the same report as the high-quality firm, so the problem is substantially the same as it is in this paper. However, if the service rate is verifiable, a separating equilibrium may exist in which the quality is known to all consumers. Our model applies to the case in which the service rate is not verifiable. A formal study of the signaling value of prices and/or service rates when uninformed consumers can also learn about the quality of the firm via queues is an interesting avenue for future research.

Appendix

A Algorithms to Identify Equilibria in the Consumer Game

Pure Strategy Equilibria:

Recall that, in the consumer game, we treat μ_h and μ_ℓ as fixed. The best response of an informed consumer is to join a queue for firm θ as long as $n < n_\theta$. Let σ_i denote this strategy. An uninformed consumer joins if $n < n_\ell$. For $n \geq n_\ell$, it is a best response to join if $w_u(n, \gamma, \mu_h, \mu_\ell) \geq 0$. With a slight abuse of notation, let $\pi_\theta(n, \mu_\theta, \sigma)$ denote the stationary probability of queue length n at firm type θ when the overall consumer strategy is σ.

Let $\phi(n) = \frac{\pi_\ell(n,\mu_\ell,\sigma)}{\pi_h(n,\mu_\ell,\sigma)}$ be the likelihood the firm has low quality, given only that the queue length is n. Then, from equation (6), the posterior probability the firm has high quality is $\gamma(n) = \frac{p}{p+(1-p)\phi(n)}$. Hence, for $n \in \{n_\ell, \cdots, n_h\}$, the condition $w_u(n, \gamma, \mu_h, \mu_\ell) \geq 0$ is equivalent to $\phi(n) \leq \frac{p}{1-p} V(n)$, where (suppressing the dependence of $V(\cdot)$ on other parameters)

$$V(n) = \frac{v_h - (n+1)c/\mu_h}{(n+1)c/\mu_\ell - v_\ell}. \tag{10}$$

From equation (13), it follows that $\phi(n) = \phi(0) \left(\frac{\mu_h}{\mu_\ell}\right)^n \prod_{j=0}^{n-1} \frac{s_\ell(j,\sigma)}{s_h(j,\sigma)}$. Therefore, given $\phi(0)$, the best response of an uninformed consumer at queue length n depends only on consumer actions up to queue length $n-1$. We exploit this structure to transform the characterization of equilibrium from an $|\bar{N}|$-dimensional problem to a single dimensional one. Let φ represent a conjecture for $\phi(0)$, and define

$$\hat{n}(\varphi) = \min\{n_\ell \leq n \leq n_h \mid \varphi \left(\frac{1}{1-q}\right)^{n_\ell} \left(\frac{(1-q)\mu_h}{\mu_\ell}\right)^n > \frac{p}{1-p} V(n)\}. \tag{11}$$

Then, $\hat{n}(\varphi)$ represents the first queue length at which an uninformed consumer will not join, given that all consumers at shorter queue lengths have joined. That is, it denotes the hole in the uninformed consumer's best response strategy.

Consider $\Phi(n)$ from equation (7), and the mapping $\varphi \mapsto \Phi(\hat{n}(\varphi))$. Then, a pure strategy equilibrium in the consumer continuation game is a fixed point of this mapping. For each $n \in \{n_\ell, \cdots, n_h\}$, compute $\Phi(n)$, set $\varphi(n) = \Phi(n)$, and determine whether $\Phi(\hat{n}(\varphi(n))) = \varphi(n)$. If the last equation is satisfied for any n, there is a pure strategy equilibrium in the consumer game with a hole at n.

Mixed Strategy Equilibria:

We engage in an exhaustive search to identify mixed strategy equilibria. Let $\mathcal{Z} = 2^{\{n_\ell,\cdots,n_h\}}$ be the set of all combinations of queue lengths at which the uninformed consumer may mix between joining and balking. For each $Z \in \mathcal{Z}$, we evaluate whether it can support a mixed strategy equilibrium as follows.

Step 1. Sort the elements in Z from lowest to greatest. In increasing order of queue length, denote them as $z_1, z_2, \cdots, z_m$. Observe that to support an equilibrium in which the uninformed consumer mixes at all lengths in Z, she must join with probability 1 at all lengths in the set $\{n_\ell, \cdots, z_m - 1\}\backslash Z$.

Step 2. For the consumer to mix at z_1, it must be that $w_u(z_1, \gamma, \mu_h, \mu_\ell) = 0$. Let $\hat{\varphi}$ be the corresponding likelihood the firm has low quality at $n = 0$. Following the same calculations as in finding pure strategy equilibria, that implies the likelihood ratio at $n = 0$ must satisfy

$$\hat{\varphi}(z_1) \quad = \quad \frac{p}{1-p} V(z_1)\,(1-q)^{n_\ell} \left(\frac{\mu_\ell}{(1-q)\mu_h}\right)^{z_1}. \tag{12}$$

Set $\varphi = \hat{\varphi}(z_1)$.

Step 3. For each $n = 0, 1, \cdots, z_1 - 1$, set $\sigma_u(n \mid Z) = 1$ for $n = 0, 1, \cdots z_1 - 1$. Set $\sigma = (\sigma_i, \sigma_u)$. Compute $\phi(n) = \varphi \left(\frac{\mu_h}{\mu_\ell}\right)^n \prod_{j=0}^{n} \frac{s_\ell(j,\sigma)}{s_h(j,\sigma)}$. Verify that, given $\phi(n)$, $w_u(n, \gamma, \mu_h, \mu_\ell) \geq 0$. If the last inequality is violated for any n, there cannot be a mixed strategy equilibrium with mixing at all queue lengths in Z. In this case, skip all remaining steps and move on to the next $Z \in \mathcal{Z}$.

Set $t = 1$.

Step 4. If $t = m$, move on to Step 7.

If $t \neq m$: Denote by a_t the probability that the uninformed consumer joins the queue at length z_t. Set $\sigma_u(n \mid Z) = 1$ for all $n \in \{z_t + 1, \cdots, z_{t+1} - 1\}$. Choose a_t so that $\phi(z_{t+1}) = \frac{p}{1-p} V(z_{t+1})$, where $\phi(z_{t+1}) = \varphi \left(\frac{\mu_h}{\mu_\ell}\right)^{z_{t+1}} \prod_{j=0}^{z_{t+1}-1} \frac{s_\ell(j,\sigma)}{s_h(j,\sigma)}$.

Step 5. For all $n \in \{z_t + 1, \cdots, z_{t+1} - 1\}$, compute $\phi(n) = \varphi \left(\frac{\mu_h}{\mu_\ell}\right)^n \prod_{j=0}^{n} \frac{s_\ell(j,\sigma)}{s_h(j,\sigma)}$. Verify that, given $\phi(n)$, $w_u(n, \gamma, \mu_h, \mu_\ell) \geq 0$. If the last inequality is violated for any n, there cannot be a mixed strategy equilibrium with mixing at all queue lengths in Z. In this case, skip all remaining steps and move on to the next $Z \in \mathcal{Z}$.

Step 6. Set $t = t + 1$, and repeat from Step 4 onwards.

Step 7. If $t = m$: There is at most one hole in the consumer strategy over queue lengths $z_m + 1, \cdots, n_h - 1$. For each $j \in \{z_m + 1, \cdots, n_h - 1\}$, do the following. First, set $\sigma_u(j \mid Z) = 0$. Then, set $\sigma_u(i \mid Z) = 1$ for $i \in \{z_m + 1, \cdots, n_h - 1\} \backslash \{n\}$. Observe that once a_t is fixed, σ_u is completely determined and the stationary probabilities $\pi_\theta(n, \mu_\theta, \sigma)$ can be computed for each queue length n. Next, compute a_t so that $\frac{\pi_\ell(0,\mu_\ell,\sigma)}{\pi_h(0,\mu_h,\sigma)} = \varphi$. Finally, verify that $a_t \in (0, 1)$ and verify that $w_u(i, \gamma, \mu_h, \mu_\ell) \leq 0$ and $w_u(j, \gamma, \mu_h, \mu_\ell) \geq 0$ for each $j \in \{z_m + 1, \cdots, n_h - 1\} \backslash \{n\}$. If both these conditions are met, σ_u is an equilibrium consumer strategy. If not, σ_u is not an equilibrium strategy. In either case, move on to the next $j \in \{z_m + 1, \cdots, n_h - 1\}$.

B Proofs

Lemma 3 *Suppose the firm has type $\theta \in \{h, \ell\}$ and chooses service rate μ, and all consumers follow the strategy profile σ. Then, the stationary probabilities over different queue lengths are given by:*

$$\pi_\theta(0, \mu, \sigma) = \frac{1}{1 + \sum_{n=1}^{\bar{N}} \left(\frac{\Lambda}{\mu}\right)^n \prod_{j=0}^{n-1} s_\theta(j, \sigma)}, \quad \pi_\theta(n, \mu, \sigma) = \pi_\theta(0, \mu, \sigma) \left(\frac{\Lambda}{\mu}\right)^n \prod_{j=0}^{n-1} s_\theta(j, \sigma).$$

Proof of Lemma 3

Straightforward; hence omitted here. Please see Debo et al. (2010) for a complete proof. ■

Proof of Proposition 1

Suppose there is a pure strategy equilibrium σ in the consumer game. Consider a newly-arrived uninformed consumer. Suppose first that $n < n_\ell$. Since an informed consumer joins the queue of the low-quality firm for all $n < n_\ell$, it must be that $v_\ell > (n + 1)\frac{c}{\mu_\ell}$. Now, by Assumption 2, it follows that $v_h > (n + 1)\frac{c}{\mu_h}$. Hence, for any value of $\gamma(n) \in [0, 1]$, $w_u(n, \gamma, \mu_h, \mu_\ell) > 0$. Thus, the uninformed consumer should join the queue whenever $n < n_\ell$, so that in equilibrium $\sigma_u(n) = 1$ for n in this range.

Now, suppose $n \geq n_\ell$. In a pure strategy equilibrium, $\sigma_u(n) = 1$ or $\sigma_u(n) = 0$ for all n. Suppose first that $\sigma_u(n) = 1$ for all $n < n_h$. By definition of n_h, $v_h - (n_h + 1)\frac{c}{\mu_h} < 0$. Hence, regardless of her belief $\gamma(n_h)$, the uninformed consumer does not join at n_h, so that $\sigma_u(n_h) = 0$.

Next, suppose there exists an $n \in \{n_\ell, n_\ell + 1, \cdots, n_h - 2\}$ at which $\sigma_u(n) = 0$. Then, $s_\ell(n, \sigma) = 0$, since an informed consumer does not join the queue of a low-quality firm when

$n > n_\ell$. However $s_h(n,\sigma) = q > 0$. Now, consider queue length $n+1$. Since $n+1 < n_h$, it follows that $\sigma_i(h, n+1) = 1$. Then, from Lemma 3, it follows that $\pi_\ell(n+1,\mu,\sigma) = 0$ and $\pi_h(n+1,\mu,\sigma) > 0$. Therefore, from Bayes' rule, it must be that $\gamma(n+1) = 1$; that is, the uninformed consumer believes the firm has high quality with probability 1. Since the best response of an informed consumer is strict for every n, it must be that $\sigma_u(n+1) = \sigma_i(n+1)$. The same reasoning applies to any queue length $\tilde{n} > n+1$; at any such queue length, $\gamma(\tilde{n}) = 1$, so that $\sigma_u(\tilde{n}) = \sigma_i(h, \tilde{n})$.

Finally, suppose that $\sigma_u(n_h - 1) = 0$. The queue length n_h is never observed in equilibrium, so an uninformed consumer's beliefs are arbitrary at that queue length. However, since $\sigma_i(h, n_h) = 0$, regardless of beliefs, $w_u(n_h, \gamma, \mu_h, \mu_\ell) < 0$, so that in equilibrium it must be that $\sigma_u(n_h) = 0 = \sigma_i(h, n_h)$. ∎

Proof of Proposition 2

Consider the algorithm for determining pure strategy equilibria in Section A. From the definition of $\hat{n}$ in equation (11), it is easy to see that $\hat{n}(\varphi)$ is decreasing in φ.

Now, consider the mapping $\Phi(n)$ introduced in equation (11). We proceed to find conditions under which $\Phi(n)$ is increasing in n and decreasing in n.

Let $a_n = 1+\sum_{m=1}^{n}\left(\frac{\Lambda}{\mu_h}\right)^m + q\sum_{m=n+1}^{n_h}\left(\frac{\Lambda}{\mu_h}\right)^m$ and $b_n = 1+\sum_{m=1}^{n_\ell}\left(\frac{\Lambda}{\mu_\ell}\right)^m + \sum_{m=n_\ell+1}^{n}(1-q)^{m-n_\ell}\left(\frac{\Lambda}{\mu_\ell}\right)^m$. Then, $\Phi(n) = \frac{a_n}{b_n}$, and we can write $\Phi(n+1) = \frac{a_n+c_{n+1}}{b_n+d_{n+1}}$, where $c_{n+1} = (1-q)\left(\frac{\Lambda}{\mu_h}\right)^{n+1}$ and $d_{n+1} = (1-q)^{n+1-n_\ell}\left(\frac{\Lambda}{\mu_\ell}\right)^{n+1}$.

It is immediate that if $c_{n+1} > d_{n+1}$ above, then $\frac{a_n+c_{n+1}}{b_n+d_{n+1}} > \frac{a_n}{b_n}$. Evaluating these terms at $n = n_\ell$, suppose

$$\left(\frac{\mu_\ell}{\mu_h}\right)^{n_\ell+1} > \frac{1+\sum_{m=1}^{n_\ell}\left(\frac{\Lambda}{\mu_h}\right)^m + q\sum_{m=n_\ell+1}^{n_h}\left(\frac{\Lambda}{\mu_h}\right)^m}{1+\sum_{m=1}^{n_\ell}\left(\frac{\Lambda}{\mu_\ell}\right)^m}. \tag{13}$$

Then, $\Phi(n_\ell+1) > \Phi(n_\ell)$.

Now, $\Phi(n+2)$ can be written generically as $\frac{a_n+c_{n+1}+c_{n+2}}{b_n+d_{n+1}+d_{n+2}}$, where $a_n, b_n, c_{n+1}, d_{n+1}$ are as defined above, $c_{n+2} = c_{n+1}\frac{\Lambda}{\mu_h}$ and $d_{n+2} = d_{n+1}(1-q)\frac{\Lambda}{\mu_\ell}$. In particular, $\frac{c_{n+2}}{d_{n+2}} > \frac{c_{n+1}}{d_{n+1}}$ if and only if $\mu_\ell > (1-q)\mu_h$, or $q > 1 - \frac{\mu_\ell}{\mu_h}$. Whenever $\Phi(n+1) > \Phi(n)$, it follows that $\frac{c_{n+1}}{d_{n+1}} > \frac{a_n}{b_n}$, and hence $\frac{c_{n+2}}{d_{n+1}} > \frac{c_{n+1}}{d_{n+1}}$ implies that $\frac{c_{n+2}}{d_{n+2}} > \frac{a_n}{b_n}$. That is, under these conditions, it follows that $\Phi(n+2) > \Phi(n+1)$. Further, if $\mu_\ell > (1-q)\mu_h$, it must be that $\frac{c_{n+2}}{d_{n+2}} > \frac{c_{n+1}}{d_{n+1}}$ for any n, so that $\Phi(n)$ is monotonically increasing in n.

Finally, observe that if inequality (13) is reversed and $\mu_\ell < (1-q)\mu_h$, the argument above is entirely reversed, and $\Phi(n)$ is monotonically decreasing in n.

(i) Now, consider part (i) of the proposition. Observe that under the assumptions in the statement, $\Phi(n)$ is monotonically decreasing in n. Further, $\hat{n}(\varphi)$ is weakly decreasing in φ. Therefore, the mapping $\varphi \mapsto \Phi(\hat{n}(\varphi))$ is weakly increasing in φ. Now, as $\varphi \to 0$, $\Phi(\hat{n}(\varphi))$ remains strictly positive. Similarly, as $\varphi \to \infty$, $\Phi(\hat{n}(\varphi))$ remains finite. As a consequence, the correspondence $\Phi(\hat{n}(\varphi))$ has at least one intersection with the 45° line. Suppose the intersection occurs at a vertical segment. As the correspondence is increasing, there must exist a smaller value φ', at which $\Phi(\hat{n}(\varphi')) = \varphi'$. That is, there is at least one pure strategy equilibrium in the consumer game.

(ii) Under the assumptions in the statement of part (ii) of the proposition, $\Phi(n)$ is monotonically increasing in n. Since $\hat{n}(\varphi)$ is weakly decreasing in φ, it follows that, the mapping $\varphi \mapsto \Phi(\hat{n}(\varphi))$ is weakly decreasing in φ. As a result, there can be at most one point φ^* at which $\Phi(\hat{n}(\varphi^*)) = \varphi^*$. That is, there is at most one pure strategy equilibrium in the consumer game. ■

Proof of Proposition 3

(i) Consider the algorithm for determining pure strategy equilibria in Section A. From equations (11) and (10), we can equivalently write $\hat{n}$ as

$$\hat{n}(\varphi) \quad = \quad \min \left\{ n_\ell \leq n \leq n_h \mid (1-q)^{n_\ell} \left(\frac{\mu_\ell}{(1-q)\mu_h} \right)^n V(n) < \frac{1-p}{p} \varphi \right\}. \tag{14}$$

We begin by exploring sufficient conditions to ensure that $\left(\frac{\mu_\ell}{(1-q)\mu_h} \right)^n V(n)$ is strictly decreasing in n. Observe that

$$\left(\frac{\mu_\ell}{(1-q)\mu_h} \right)^n V(n) > \left(\frac{\mu_\ell}{(1-q)\mu_h} \right)^{n+1} V(n+1) \quad \iff \quad \frac{\mu_\ell}{(1-q)\mu_h} < \frac{V(n)}{V(n+1)}. \tag{15}$$

Consider the right-hand side of inequality (15), $\frac{V(n)}{V(n+1)}$. We establish the following claim:

Claim: Suppose $n \leq n_h - 3$, so that $V(n+2) > 0$. Then, $\frac{V(n)}{V(n+1)} \lesseqgtr \frac{V(n+1)}{V(n+2)} \iff n \lesseqgtr \frac{v_h\mu_h + v_\ell\mu_\ell}{2c} - 2$.

Proof of Claim: Observe that $\frac{V(n)}{V(n+1)} > \frac{V(n+1)}{V(n+2)} \iff V(n)V(n+2) > V(n+1)^2$. The latter inequality is true if and only if

$$\frac{\{v_h\mu_h - (n+1)c\}\,\{v_h\mu_h - (n+3)c\}}{\{(n+1)c - v_\ell\mu_\ell\}\,\{(n+3)c - v_\ell\mu_\ell\}} \quad > \quad \frac{\{v_h\mu_h - (n+2)c\}^2}{\{(n+2)c - v_\ell\mu_\ell\}^2}. \tag{16}$$

Write $z = v_h\mu_h$, $y = v_\ell\mu_\ell$, $f = (n+1)c$, $g = (n+2)c$ and $h = (n+3)c$. Then, we can write (16) as $\frac{(z-f)(z-h)}{(f-y)(h-y)} > \frac{(z-g)^2}{(g-y)^2}$, or $(z-f)(z-h)(g-y)^2 > (f-h)(h-y)(z-g)^2$. Multiply out fully, and eliminate common terms on both sides (using the relationship $2g = f + h$).

Rearranging the remaining terms, the inequality reduces to $(z^2 - y^2)(g^2 - fh) > (z - y)g[(f+h)g - 2fh]$. Now, $z - y = v_h\mu_h - v_\ell\mu_\ell > 0$ by assumption (2). Further, since $f + h = 2g$, we can write the right-hand side as $2(z-y)g(g^2 - fh)$. Here, $g^2 - fh = (n+2)^2 - (n+1)(n+3) = 1 > 0$. Divide throughout by $(z-y)(g^2 - fh)$, we obtain $z + y > 2g$, or $v_h\mu_h + v_\ell\mu_\ell > 2(n+2)c$, which reduces to $n < \frac{v_h\mu_h}{v_\ell\mu_\ell}2c - 2$.

Therefore, $\frac{V(n)}{V(n+1)} > \frac{V(n+1)}{V(n+2)}$ if and only if $n < \frac{v_h\mu_h}{v_\ell\mu_\ell}2c - 2$. Following through the same steps starting with $\frac{V(n)}{V(n+1)} < \frac{V(n+1)}{V(n+2)}$, it is easy to see that the latter inequality holds if and only if $n > \frac{v_h\mu_h + v_\ell\mu_\ell}{2c} - 2$. Finally, it also follows that $\frac{V(n)}{V(n+1)} = \frac{V(n+1)}{V(n+2)}$ if and only if $n = \frac{v_h\mu_h + v_\ell\mu_\ell}{2c} - 2$. ■

Now, we return to the proof of part (i) of the proposition. It follows from the claim that if the quantity $\frac{v_h\mu_h + v_\ell\mu_\ell}{2c} - 2$ is an integer, the expression $\frac{V(n)}{V(n+1)}$ attains a global minimum at this queue length. A lower bound for the value of $\frac{V(n)}{V(n+1)}$ is thus $\frac{v_h\mu_h - v_\ell\mu_\ell + 2c}{v_h\mu_h - v_\ell\mu_\ell - 2c}$. Therefore, if $\frac{\mu_\ell}{(1-q)\mu_h} < \frac{v_h\mu_h - v_\ell\mu_\ell + 2c}{v_h\mu_h - v_\ell\mu_\ell - 2c}$, the expression $\left(\frac{\mu_\ell}{(1-q)\mu_h}\right)^n V(n)$ is strictly decreasing in n.

Now, let $\delta(n) = (1-q)^{n_\ell}\left(\frac{\mu_\ell}{(1-q)\mu_h}\right)^n V(n)$, and for $n \in \{n_\ell, \cdots, n_h\}$, define $\bar{p}(n) = \min\{\frac{\Phi(n)}{\Phi(n)+\delta(n)}, 1\}$, with $\underline{p}(n) = \max\{\frac{\Phi(n)}{\Phi(n)+\delta(n-1)}, 0\}$. Since $\delta(n) < \delta(n-1)$, it follows that $\bar{p}(n) > \underline{p}(n)$.

Now, for a given $n \in \{n_\ell + 1, \cdots, n_h\}$, suppose $p \in [\underline{p}(n), \bar{p}(n)]$. Then, $\frac{p}{1-p}\delta(n) \le \Phi(n) \le \frac{p}{1-p}\delta(n-1)$. It follows that there exists a pure strategy equilibrium in the consumer game in which the uninformed consumer's strategy has a hole at exactly n. Recognizing that $\underline{p}(n) = 0$, the same argument extends to an equilibrium in which the uninformed consumer's strategy has a hole at n_ℓ.

(ii) Next, suppose $\frac{\mu_\ell}{(1-q)\mu_h} > \frac{v_h\mu_h - v_\ell\mu_\ell + 2c}{v_h\mu_h - v_\ell\mu_\ell - 2c}$. Then, the expression $\left(\frac{\mu_\ell}{(1-q)\mu_h}\right)^n V(n)$ is not strictly decreasing in n. Therefore, it may be for some n that $\bar{p}(n) < \underline{p}(n)$, so that there cannot be a pure strategy equilibrium in which the uninformed consumer's strategy has a hole at n.

The claim in part(i) establishes that $\frac{V(n)}{V(n+1)}$ is a U-shaped function, strictly decreasing for $n < \frac{v_h\mu_h + v_\ell\mu_\ell}{2c} - 2$, and strictly increasing for $n > \frac{v_h\mu_h + v_\ell\mu_\ell}{2c} - 2$. Therefore, the set of queue lengths at which $\frac{\mu_\ell}{(1-q)\mu_h} > \frac{V(n)}{V(n+1)}$ may be represented as $\{\underline{n}+1, \underline{n}+2, \cdots, \bar{n}-2, \bar{n}-1\}$, where $\underline{n} = \min_{n \in \{n_\ell, \cdots, n_h - 1\}}\{n \mid \delta(n) < \delta(n+1)\} - 1$ and $\bar{n} = \max_{n \in \{n_\ell, \cdots, n_h - 1\}}\{n \mid \delta(n) < \delta(n+1)\} - 1$. Then, following the same steps as in part (i) of the proposition, for each $n \in \{n_\ell, \cdots, \underline{n}\}$ and $\{\bar{n}, \cdots, n_h$, there exist values of p for which, in equilibrium, the uninformed consumer's strategy has a hole at n.

Finally, observe that $\bar{p}(n_\ell) > \underline{p}(n_\ell) = 0$, so that for $p \le \bar{p}(n_\ell)$, there is a pure strategy equilibrium with the uninformed consumer failing to join at queue length n_ℓ. Therefore, it must be that $\underline{n} \geq n_\ell$, Further, $\delta(n_h) < 0 < \delta(n)$ for any $n \in \{n_\ell, \cdots, n_h - 1\}$. Therefore, in equilibrium the queue length n_h can also be supported as a hole in the uninformed consumer's strategy; that is, $\bar{n} \leq n_h$. ■

Proof of Lemma 1

Suppose a firm with type θ chooses a service rate μ, and suppose consumers play σ, where σ is a pure strategy that satisfies the necessary conditions for equilibrium in the continuation game. Then, $s_h(j, \sigma) = 1$ for $j \leq \hat{n}(\sigma) - 1$ and $j \in \{\hat{n}(\sigma) + 1, \cdots, \tilde{n}_h(\sigma) - 1\}$, with $s_h(\hat{n}(\sigma)) = q$ (since only informed consumers join at $\hat{n}(\sigma)$). Substituting for $s_h(j, \sigma)$ in in the expression for $\pi_\theta(0, \mu, \sigma)$ in Lemma 3 yields

$$\pi_h(0, \mu, \sigma) \quad = \quad \frac{1}{\sum_{j=0}^{\hat{n}(\sigma)} (\Lambda/\mu)^k + q \sum_{j=\hat{n}(\sigma)+1}^{\tilde{n}_h(\sigma)} (\Lambda/\mu)^k}. \tag{17}$$

By inspection, we observe that regardless of whether Λ/μ is greater or less than 1, $\pi_h(0, \mu, \sigma)$ declines in $\hat{n}(\sigma)$ when $\tilde{n}_h(\sigma)$ is kept fixed.

Further, $s_\ell(j, \sigma) = 1$ for $j \leq \tilde{n}_\ell(\sigma) - 1$, with $s_\ell(j, \sigma) = 1 - q$ for $j \in \{\tilde{n}_\ell, \cdots, \hat{n}(\sigma) = 1$ (since only uninformed consumers join at these queue lengths) and $s_\ell(j, \sigma) = 0$ for $j \geq \hat{n}(\sigma)$. Substituting for $s_\ell(j, \sigma)$ in the expression for $\pi_\theta(0, \mu, \sigma)$ in Lemma 3 yields

$$\pi_\ell(0, \mu, \sigma) \quad = \quad \frac{1}{\sum_{j=0}^{\tilde{n}_\ell(\sigma)} (\Lambda/\mu)^k + \sum_{j=\tilde{n}_\ell(\sigma)+1}^{\hat{n}(\sigma)} (1-q)^{k-\tilde{n}_\ell(\sigma)} (\Lambda/\mu)^k}. \tag{18}$$

By inspection, we observe that $\pi_\ell(0, \mu, \sigma)$ declines in $\hat{n}(\sigma)$, keeping $\tilde{n}_\ell(\sigma)$ fixed.

The expected revenue of firm θ is $R_\theta(\mu, \sigma) = r\mu(1 - \pi_\theta(0, \mu, \sigma))$, from which the expressions in the statement of the Lemma follow. Since $\pi_\theta(0, \mu, \sigma)$ decreases as $\hat{n}(\sigma)$ increases (keeping fixed $\tilde{n}_h$ and $\tilde{n}_\ell$), it follows that for each θ, the expected revenue $R_\theta(\mu, \sigma)$ increases as $\hat{n}(\sigma)$ increases. ■

Proof of Lemma 2

Observe that under Assumption 3 (iii), regardless of firm's choices over service rate, there is a pure strategy equilibrium in which the strategy of the uninformed consumers has a hole at n_ℓ. Further, given Assumption 3 (ii), n_ℓ and n_h are invariant to firms' service rate choices.

Substitute $\hat{n} = n_\ell$ into the revenue rate expressions in Lemma 1. We obtain

$$R_h(\mu, \hat{\sigma}) = r\mu \left(1 - \frac{1}{\sum_{j=0}^{n_\ell}(\Lambda/\mu)^j + q\sum_{j=n_\ell+1}^{n_h}(\Lambda/\mu)^j}\right), \quad (19)$$

$$R_\ell(\mu, \hat{\sigma}) = r\mu \left(1 - \frac{1}{\sum_{j=0}^{n_\ell}(\Lambda/\mu)^j}\right). \quad (20)$$

Take the derivatives of these expressions with respect to μ, set $\mu = \Lambda$, and simplify. The expressions in the statement of the Lemma are obtained. ■

Proof of Proposition 4

Consider the expressions for $\frac{\partial R_\theta}{\partial \mu}$ when $\mu = \Lambda$ in Lemma 2. Write $n_h = n_\ell + m$, where $m \geq 1$ is an integer. Then,

$$\frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) = \frac{r\left[2m^2q^2 + m(2n_\ell + 1 - m)q + n_\ell(n_\ell + 1)\right]}{2(n_\ell + 1 + qm)^2}. \quad (21)$$

Let $\Gamma = \frac{1}{r}\left[\frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) - \frac{\partial R_\ell}{\partial \mu}(\Lambda, \hat{\sigma})\right] = \frac{2m^2q^2+m(2n_\ell+1-m)q+n_\ell(n_\ell+1)}{2(n_\ell+1+mq)^2} - \frac{n_\ell}{2(n_\ell+1)}$, so

$$2\Gamma = \frac{(n_\ell + 1)[2m^2q^2 + m(2n_\ell + 1 - m)q + n_\ell(n_\ell + 1)] - (n_\ell + 1 + mq)^2 n_\ell}{(n_\ell + 1 + mq)^2(n_\ell + 1)} \quad (22)$$

The sign of Γ equals the sign of numerator in equation (22), since the denominator is strictly positive. The numerator is equal to

$$\begin{aligned} & 2(n_\ell + 1)m^2q^2 + (2n_\ell + 1)(n_\ell + 1)mq - m^2(n_\ell + 1)q + n_\ell(n_\ell + 1)^2 \\ & -n_\ell[(n_\ell + 1)^2 + 2(n_\ell + 1)mq + m^2q^2] \\ = \; & (n_\ell + 2)m^2q^2 - (n_\ell + 1)mq(m - 1) = mq[(n_\ell + 2)mq - (n_\ell + 1)(m - 1)]. \end{aligned}$$

Hence, the sign of Γ is equal to the sign of $[(n_\ell + 2)mq - (n_\ell + 1)(m - 1)]$.

Now, recall that $\hat{q} = \left(1 - \frac{1}{n_h - n_\ell}\right)\frac{n_\ell+1}{n_\ell+2} = \left(\frac{m-1}{m}\right)\frac{n_\ell+1}{n_\ell+2}$. Hence, it follows that the numerator in equation (22) is positive if $q > \hat{q}$ and negative if $q < \hat{q}$. Therefore, if $q > \hat{q}$, $\frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) > \frac{\partial R_\ell}{\partial \mu}(\Lambda, \hat{\sigma})$, and if $q < \hat{q}$, $\frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) < \frac{\partial R_\ell}{\partial \mu}(\Lambda, \hat{\sigma})$.

We now turn to the two parts of the statement of the proposition.

(i) Suppose $q < \hat{q}$. Observe that for each $\theta = h, \ell$, $\lim_{\epsilon \to 0} \Delta_\theta(\Lambda, \epsilon) = \frac{\partial R_\theta}{\partial \mu}(\Lambda, \hat{\sigma})$. Since $\frac{\partial R_h}{\partial \mu} < \frac{\partial R_\ell}{\partial \mu}$, it follows that there exists an $\hat{\epsilon}(q) > 0$ such that, for all $\epsilon \in (0, \hat{\epsilon}]$, $\Delta_h(\Lambda, \epsilon) < \Delta_\ell(\Lambda, \epsilon)$. Now, for any $\kappa \in (\Delta_h(\Lambda, \epsilon), \Delta_\ell(\Lambda, \epsilon))$, the high-quality firm chooses the slow service rate $\Lambda - \epsilon$ and the low-quality firm chooses the fast service rate $\Lambda + \epsilon$.

Further, since $\Delta_h(\Lambda, \epsilon) < \Delta_\ell(\Lambda, \epsilon)$, there cannot exist a cost κ such that the high-quality firm chooses the fast service rate $\Lambda + \epsilon$ and the low-quality firm chooses the slow service rate $\Lambda - \epsilon$.

(ii) The argument when $q > \hat{q}$ is exactly similar to the argument in (i) above. ■

Proof of Proposition 5

(i) Consider the expression for $\frac{\partial R_h}{\partial \mu}$ when $\mu = \Lambda$, as shown in equation (21) in the proof of Proposition 4. Recall that $n_h = n_\ell + m$ in this expression. Consider the limit as $n_h \to \infty$, or, equivalently, $m \to \infty$.

In the limit as $n_h \to \infty$, we obtain

$$\lim_{n_h \to \infty} \frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) \quad = \quad \frac{r(2q^2 - q)}{2q^2} = r(1 - \frac{1}{2q}). \tag{23}$$

The last expression is strictly negative whenever $q < \frac{1}{2}$. Thus, for any $q < \frac{1}{2}$, there exists a threshold $\bar{n}(q)$ such that if $n_h \geq \bar{n}(q)$, $\frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) < 0$. Similarly, for any $q > \frac{1}{2}$, there exists a threshold $\bar{n}(q)$ such that if $n_h \geq \bar{n}(q)$, $\frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma}) > 0$. Define $\bar{v}(q) = \bar{n}(q) \frac{c}{\mu_0 - \epsilon}$. Then, the condition $n_h \geq \bar{n}(q)$ is equivalent to $v_h \geq \bar{v}(q)$.

Now, $\lim_{\epsilon \to 0} \Delta_h(\Lambda, \epsilon) = \frac{\partial R_h}{\partial \mu}(\Lambda, \hat{\sigma})$. Suppose $q < \frac{1}{2}$ and $v_h \geq \bar{v}(q)$. Then, if ϵ is sufficiently close to zero, if must be the case that $\Delta_h(\Lambda, \epsilon) < 0$. Formally, there exists some $\bar{\epsilon}(q)$ such that if $\epsilon \in (0, \bar{\epsilon}(q)]$, then $\Delta_h(\Lambda, \epsilon) < 0$. Hence, for parameters in this range, at any value of κ, including $\kappa = 0$, the high-quality firm will choose the slow service rate $\Lambda - \epsilon$.

Finally, suppose $q > \frac{1}{2}$ and $v_h \geq \bar{v}(q)$. Then, there exists some $\bar{\epsilon}(q)$ such that if $\epsilon \in (0, \tilde{\epsilon}(q)]$, then $\Delta_h(\Lambda, \epsilon) > 0$. ■

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