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THREE-DIMENSIONAL BLOCKMODELS

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In the classic blockmodel formulation, a social network among members of a population with n actors and k relations (types of tie) is arrayed as k n X n matrices. Though this is a three-dimensional data structure, it is typically reduced to a two-way analysis. In this paper, a three-way procedure for analyzing multigraph data is developed. Specifically, in addition to applying the rule of structural equivalence to collapse actors, it is also applied to the relations (the third dimension), and structurally equivalent sets of relations are collapsed. The result is a three-dimensional blockmodel (image) of social structure that is a more parsimonious representation of social structure than the classic two-dimensional blockmodel images. The three-dimensional approach is illustrated by application to three case studies: Homan's Bank Wiring Room, Sampson's monastery, and a local economy of hospital services. The structural equivalence approach to relations is further explored by applying it to the individual-level Liking and Antagonism relations and their compounds (of length four or less) in the Bank Wiring Room. This application demonstrates that the structural equivalence approach can be used to identify equality equations for primitive and compound relations.

THREE-DIMENSIONAL BLOCKMODELS*

In the classic blockmodel formulation, a social network among members of a population with n actors and k relations (types of tie) is

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arrayed as k n X n matrices. Within each matrix, a unique row and a corresponding column are assigned to each actor. The rule of structural equivalence is applied to the *columns* of the stacked matrices, and the partitions derived from the columns are applied to both the columns and the rows of the original data matrices. This approach identifies positions in a social structure, as well as patterns among positions (e.g., White *et al.*, 1976; Breiger *et al.*, 1975).

In the formulation outlined above, the social network is analyzed essentially as a two-dimensional problem. Though the analysis of social structure is based on the simultaneous blocking of multigraph data (i.e., a three-dimensional data structure) the classic approach analyzes the social network with a two-way procedure. This two-way approach is exemplified by the typical use of CONCOR, the main clustering technique used in blockmodeling. As Breiger *et al.* (1975:338) summarize in their classification of CONCOR in taxonomies of data and data analysis:

> In the useful terminology of Carroll and Chang (1970), the application of CONCOR to multiple types of relation constitutes two-way scaling, since the result of forming M_1 on the stacked raw matrices is to study *subjects by subjects*. We began with a three-way data structure (the k distinct relations constituting the third level), but by stacking we reduced the problem to a two-way analysis.

While the classic two-way approach has become a powerful tool for analyzing social structure, reducing a three-dimensional data structure to a two-way structure fails to fully exploit the model-building potential of multigraph data. The approach presented here is one way to analyze multigraph data that produces a three-way model of social structure. Thus, the paper builds upon the classic blockmodeling approach and extends its usefulness by explictly recognizing and analyzing social structure along the third dimension.

One of the two main goals of blockmodeling is to derive a comprehensible model of social structure from complex social networks of multiple types of tie among (often) large numbers of actors. This model (the blockmodel image) is a simpler, reduced representation of the underlying data, one that preserves the intrinsic structure of the data. Operationally, such a model is generated by collapsing nodes into distinct sets, using the rule of structural equivalence (Lorrain and White 1971), and treating each set as internally homogeneous and homogeneous in its relations to every other set. This process of model building, I suggest, should be extended to apply to the *relations* *themselves*(the third dimension). Collapsing structurally equivalent relations into distinct sets fully exploits the inherent dimensionality of the data, and produces a more parsimonious model of social structure.

Because the three-dimensional blockmodel approach includes a focus on the equivalence of relations, it is related to the second main goal of blockmodeling — the analysis of role structures (Lorrain and White 1971; Boorman and White 1976; Bonacich 1979; Bonacich and McConaghy 1980). Overall, the three-dimensional blockmodel approach can be viewed as occupying a position midway between and connecting the two levels of analysis in blockmodeling. At the first level, blockmodeling focuses on the pattern of relationships among actors and positions in a network, over different types of relations. The three-dimensional approach is likewise concerned with the pattern of relationships among actors, but with greater explicit emphasis on the pattern of relations that underlie the actors and positions in a social structure. At the second level, blockmodeling focuses on the pattern of relations among relations, using the algebra of semigroups. Some of the algebraic concepts of blockmodeling are also used in the three-dimensional approach, with the intent of identifying the relations among (primitive) relations in a social structure. Though I focus on the relations among primitive relations, it is possible to apply the structural equivalence approach to primitive and compound relations. As demonstrated in the Appendix, the structural equivalence approach yields results that are very similar to those of other approaches.

The paper is organized in four sections, plus an appendix. In Section 1, I outline the various substantive forms that three-dimensionality can take. In Section 2, I present definitions of the structural equivalence of relations, and present operational methods for analyzing multigraph social network data with a three-way procedure. In Section 3, three case studies are reported to illustrate the application of the three-dimensional blockmodel approach. Two of these cases studies — Homan's bank wiring room and Sampson's monastery — are used because they were analyzed in the seminal works on blockmodeling (Breiger *et al.*, 1975; White *et al.*, 1976) and social network analysts are very familiar with them, facilitating comparisons of the original analyses and the analysis presented here. The third case study — a local economy of hospital services (Baker 1984c) — was selected because it illustrates the applicability of the three-dimensional approach to structures in which the relations represent social institutions (in this case, markets) instead of the usual network case in which relations represent types of affect or sentiment. In fact, the origin of the three-dimensional approach is found in the context of markets; it was first proposed and developed to model the pattern of relations among markets (Baker 1982, 1983, 1984c, 1985).

Conclusions are presented in Section 4. The Appendix demonstrates the utility of the structural equivalence approach to the analysis of role structures.

1. SOCIAL STRUCTURE AS THREE-DIMENSIONAL SOCIAL NETWORK

Single Population/Multiple Relations

The typical blockmodel problem involves relationships among members of a single population across multiple relations. For example, in Breiger's (1976) study of career attributes and network structure in a biomedical research specialty, awareness relationships in a population of 107 scientists were arrayed across three types of relations — Mutual Contact, Both Unaware, and One Unaware (the last two relations, representing reciprocated and unreciprocated unawareness, were derived from a single unaware relation). Each scientist was assigned a unique row and corresponding column, resulting in a 107 by 107 matrix for each relation.

The scientists' awareness networks are represented as a three-way data structure: 3 107 by 107 ($k\ n \times n$) matrices. This structure may be conceptualized as a "box" formed by placing the three matrices one behind the other, taking care to preserve the same ordering of rows and columns in each matrix. This box, representing three relations among a single population, is the simplest three-dimensional data array.*

* There are similarities and differences between the "box" developed here and Winship's "relation box" (Winship 1974; Mandel 1978, 1983; Mandel and Winship 1979; Winship and Mandel 1983) in their construction and purpose. As data arrays, they are quite similar. As shown in Winship and Mandel (1983:326), the relation box is arrayed as a $N \times N \times T$ array, where N is the size of the population and T is the number of different types of relations. Each entry in the relation box represents the presence or absence of a direct or indirect relationship between two individuals. In the simplest form of the box discussed in this paper (e.g., the scientists' awareness networks), the data are arrayed in exactly the same way, except that the compound relations are not considered (although they could be — see Appendix). Differences occur when the other forms the box can take are considered. For example, the box can represent the relationships between members of two different populations, such as film producers and freelance music composers (Faulkner 1983), or even three distinct populations (film producers, directors, and composers). Two (or more) populations could be included in the relation box (see fns. 18 and 20 in Winship

A variant of the single population/ multiple relations three-dimensional data structure is the case in which the third dimension represents the formal groups of which the actors may be members. The resulting person by person by group box is one way to operationalize the "duality of persons and groups" (Breiger 1974).

Single Population/Single Relation Over Time

The third dimension can also represent time. In this situation, each n × n matrix in the box represents relationships of a single type among members of one population at a specific point in time. Given k time-points (observations), the box would contain k n × n matrices placed in temporal sequence, one behind the other. (Note that if multiple relations are included, the network becomes a four-dimensional problem.)

Networks over time are not treated in the present paper (but see White *et al.* (1976); Doreian (1979/80) and Galaskiewicz and Wasserman (1981) for analyses of networks over time). The three-dimensional blockmodels developed in this paper pertain to the analysis of social structure at one interval of time.

Dual Populations/Multiple Relations

Though single populations are typically analyzed in network studies, networks between members of different populations are also studied. In Faulkner's (1983) study of the social structure of the Hollywood film

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and Mandel 1983:325–328) by treating them as if they were in the same network, but without any ties between them; the first two dimensions of the relation box would be N = N1 + N2 (where N1 and N2 are the number of actors in the two populations). In contrast, the box developed here is intended to focus on the pattern of connections *between* two populations, often where ties within a single population are undefined or rare (e.g., film producers do not compose music for their films). Another difference between the box developed here and the relation box is that the former considers situations in which relations represent institutions (e.g., markets). Though it may be possible to modify the relation box to include some of these features of the three-dimensional blockmodeling box, they are not as yet part of the state of the art.

Probably the main difference is the purpose of the relation box and the box developed here. The purpose of the relation box (along with associated concepts and methods of the role-set approach) is to identify *roles* (as theoretically distinct from positions) in one or across more than one population (see also White and Reitz 1982, 1983). This is not the purpose of the three-dimensional blockmodel box; its purpose is to study positions and patterns among positions in a way that makes explicit the similarities and differences among the relations underlying them.

industry, relationships between two distinct populations — film producers (buyers) and film music composers (sellers) — were arrayed as a n × m rectangular matrix (in this case, however, only one relation was analyzed).

When relationships between members of two populations are measured over multiple types of relations, social structure may be represented by a three-dimensional array (box). Assuming k relations, the box would contain k n × m matrices, one placed behind the other. Market economies are prime examples of social structures composed of dual populations and multiple relations. A market economy may be conceptualized as a "network of interlocking markets" represented by a "market box," a three-dimensional array of buyers by sellers by markets (e.g., Baker 1984c). In the case of a hospital economy, for example, the two populations are communities (buyers) and hospitals (sellers) and the relations are the various health services provided and obtained in the economy (e.g., pediatrics, medicine, surgery, psychiatry). Such a hospital economy is analyzed in this paper.

While in many network examples relations represent types of affect or sentiment (e.g., like, dislike, antagonism, esteem), the market box — which is a special case of three-dimensional data structures — exemplifies some of the other types of tie that occur in network data. In the market box representing a hospital economy, relations represent different health services. In the film music market, the main type of tie is the "work transaction," the link between a freelance composer and producer formed when they work together on a feature film (Faulkner 1983). In the stock options market, the main type of tie is the "trade," represented by the number of options exchanged between a buyer and a seller (e.g., Baker 1981, 1984a, 1984b). And in the world system, each type of tie is a specific commodity type, such as raw materials or finished goods (e.g., Breiger 1981; Snyder and Kick 1979).

2. DEFINITIONS AND METHODS FOR THREE-DIMENSIONAL BLOCKMODELING

Analyzing multigraph data as a three-dimensional blockmodel problem is a straightforward extension of the classic two-way approach. In addition to applying the rule of structural equivalence to the columns (or columns *and* rows, in the case of two populations), the rule is also applied to the relations themselves. After definitions are presented, this technique is illustrated by analyzing a small hypothetical three-dimensional network consisting of two distinct populations of actors and four relations.

Definitions

The structural equivalence of actors in three-dimensional blockmodeling is defined precisely the same way as in the classic two-way approach:

> *Definition 1.* Two actors are structurally equivalent if they have the same relationships to other actors. That is, a and b are structurally equivalent if and only if:
>
> $$aRc \Leftrightarrow bRc$$
> $$cRa \Leftrightarrow cRb$$
>
> for any c and any relation (e.g., Breiger *et al.* 1975:330). It is well known that this strict definition of structural equivalence is rarely met in real social structures; in practice, a and b are considered structurally equivalent if they more or less meet the strict definition.

Though Definiton 1 is applied to actors in social networks, it is not usually applied to the relations themselves. The usual definition of the equivalence of relations is:

> *Definition 2.* Two relations are defined to be equal if they connect precisely the same people. Let A and B be two different relations. A = B if and only if:
>
> $$iAj \Leftrightarrow iBj$$
>
> for every i, j. (Definition 2 here is the same as Definition 3 in Bonacich and McConaghy (1980:494)).

Though Definition 2 is as strict as Definition 1, it is the definition of the equivalence of relations most commonly used in practice.

When applied to real social structures, the strictness of Definition 2 limits its usefulness and causes analytic problems, especially in the algebra of blockmodeling. Empirical network data may be noisy due to measurement error, or due to the presence of what Schwartz and Sprinzen (1984) call "structurally weak" idiosyncratic relational ties. In either situation, unless such ties are excluded, the algebra can become overly complicated and cumbersome. One rule in forming compound relations, for example, is to continue forming new compounds until no new ones are found. When forming compounds from the raw relations, the presence of just a few idiosyncratic ties can force the creation of an excessive number of new compounds. Another problem is the inability of programs such as BLOCKER and GENTAB to operate properly on matrices containing impure zeroblocks (Schwartz and Sprinzen 1984).*

The most common way to circumvent this problem is to use the blockmodel images instead of the raw matrices. Although the algebra of blockmodeling is as appropriate for analyzing role structure from individual-level data as it is from blocks of individuals (as Bonacich and McConaghy (1980:491) emphasize), it is often impractical to work with the raw matrices directly. The individual-level matrices rarely satisfy Definition 2. In contrast, image matrices never suffer from the problem of impure zeroblocks. However, while image matrices are convenient to work with, because they simplify social structure, equality equations based on them are not always correct (i.e., different equalities are evident in the data themselves). The comparison of relations in Sampson's monastery is a case in point (presented in Section 3).

A structural equivalence definition for relations is one way to avoid the restrictiveness of Definition 2. As with the structural equivalence of actors, a strict definition is developed first:

Definition 3. Let A, B, and C be relations on a network of actors. A and B are structurally equivalent if and only if:

$$(iAj \Leftrightarrow iCj) \Leftrightarrow (iBj \Leftrightarrow iCj)$$

OR

$$(iAj \Leftrightarrow iC'j) \Leftrightarrow (iBj \Leftrightarrow iC'j)$$

where $C'ij = 1 - Cij$ (for binary data).

for any i, j and any relation C.

If relations A and B are structurally equivalent under Definition 3 in its strict form, then $A = B$ (as in Definition 2). The difference is that the equivalence of A and B is based on their similarity to C, not to each other.

Definition 3 must be weakened to apply to real social structures. Thus, A and B will be structurally equivalent if both are relatively similar to (or different from) all C.

* Schwartz and Sprinzen (1984) present an ingenious procedure for circumventing the noisy data problem and working directly with the individual-level matrices. By identifying and *temporarily* removing structurally weak ties, they only include ties that are supported by the underlying structure.

Illustration with Hypothetical Data

Consider the box below which represents the relationships among members of two populations, (A, B, C, D) and (V, W, X, Y, Z), across four relations (R1, R2, R3, R4). To display the box on the printed page, it has been unfolded along the k-dimension (i.e., the relations) resulting in a cross-sectional view of the box.

	A	B	C	D	
V	1	1	0	0	
W	1	0	1	1	
X	0	0	1	1	R1
Y	1	1	0	0	
Z	1	1	0	0	
V	0	0	1	1	
W	0	0	1	1	
X	1	1	0	0	R2
Y	1	1	0	0	
Z	0	0	1	1	
V	1	1	0	0	
W	0	0	1	1	
X	0	0	1	1	R3
Y	1	1	0	0	
Z	1	1	0	0	
V	1	0	1	1	
W	0	0	1	1	
X	1	1	0	0	R4
Y	1	1	0	0	
Z	0	0	1	1	

For simplification, binary data are used; note that no such restriction exists for this approach (e.g., case studies 2 and 3 in Section 3).

Because this box contains two populations, the rule of structural equivalence must be applied three times, once along each dimension. (For simplification, bipartite divisions are used along each dimension.) The steps are outlined below:

Step 1: To determine the subsets of structurally equivalent actors in the first population (A, B, C, D), stack the original four matrices to form a single matrix with 4 columns and 20 rows (5 × 4). Apply the rule of structural equivalence to the columns of the stacked matrix to identify the members of each subset.

Step 2: To determine the subsets of structurally equivalent actors in the second population, transpose each individual matrix, and stack the transposed matrices to form a single matrix with 5 columns (the former rows) and 16 rows (4 × 4). Apply the rule of structural equivalence to the columns of the stacked matrix to identify the members of each subset.

Step 3: To determine the subsets of structurally equivalent relations, "unravel" each individual matrix (relation) into a single vector of length 20 (5 × 4), taking care to unravel each matrix in the same order. Concatenate the four vectors into a single matrix with 4 columns and 20 rows. Apply the rule of structural equivalence to the columns of this matrix to identify the members of each subset.

Now that all sets of structurally actors and relations have been determined, two- and three-dimensional images are formed as follows:

Step 4: Reorder the original four matrices according to the permutations derived in Steps 1-3. Calculate the submatrix densities for each individual matrix. Select a density cutoff, and map the corresponding image (at the two-dimensional level). The resulting matrix of submatrix densities and the corresponding image are:

	SUBMATRIX DENSITIES		IMAGE	
R1	.8333	.3333	1	0
	.5000	.5000	0	0
R3	.6667	.3333	1	0
	.5000	.5000	0	0
R2	.0000	1.0000	0	1
	1.0000	.0000	1	0
R4	.1667	1.0000	0	1
	1.0000	.0000	1	0

Step 5: Calculate the "cross-relational" submatrix densities by calculating the density of each submatrix formed when (R1, R3) are joined, and when (R2, R4) are joined.* Using the same cutoff as in the two-dimensional case, produce the three-dimensional block-

* Instead of averaging structurally equivalent relations, one may collapse relations by forming their union. For example, in their comparison of relations in Sampson's monastery, Breiger *et al.* (1975:350–6) formed the Boolean union of the Affect, Esteem, Influence, and Sanctioning relations, and the Boolean union of the Dislike, Disesteem, Negative Influence, and Negative Sanction relations. That is, they formed a "Negative Affect" and "Positive Affect" group.

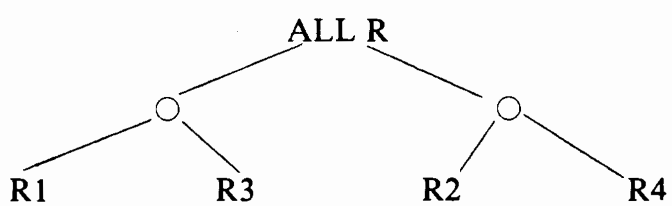
model image. In the case of these hypothetical data, the three-dimensional matrix of submatrix densities and the corresponding image are:

	CROSS-RELATION SUBMATRIX DENSITIES		IMAGE	
(R1, R3)	.7500	.3333	1	0
	.5000	.5000	0	0
(R2, R4)	.0833	1.0000	0	1
	1.0000	.0000	1	0

The construction of blockmodel images is now complete. It is useful to form two-dimensional blockmodels in the conventional manner (Step 4) and develop a typical interpretation. The last step is to form the three-dimensional blockmodel images. It is obvious that the number of potential three-dimensional images is greater than the 16 unique 2 × 2 binary images (in the case of a single population). With explicit consideration of the third dimension, there are 256 possible 2 × 2 × 2 binary images (assuming a single population). Substantive interpretations of some of these images are provided in the three examples analyzed in Section 3.

Trees, Equations, and Equivalence Classes

As the successive division of nodes into structurally equivalent sets is naturally represented by a tree diagram, so too may the division of relations be represented as a tree diagram. Below is the tree diagram that represents the hypothetical relations modeled above:



The three diagram implies certain equality equations and equivalence classes of relations. At the bottom of the tree, for example, two equations are indicated,

1. R1 = R3
2. R2 = R4

which form two equivalence classes,

R1, R3/R2, R4.

Because the structural equivalence approach does not rely on the customary definition of relational equality (Definition 2) and does not require that two relations be identical to be equivalent, the two equations implied by the tree diagram should be restated as R1 ~ R3 and R2 ~ R4.

If the structural equivalence approach has identified the correct equations, then there should be fewer disagreements between the relations equated in R1, R3/R2, R4 than any other partition of the four elements. To make this comparison, I have adapted a strategy used to select preferred homomorphisms (Bonacich and McConaghy 1980). First, the dissimilarity of two relations is calculated as the number of unequal entries between their matrices. Second, all possible equivalence classes are enumerated. Third, the maximum dissimilarity of each equivalence class — the largest number of unequal entries in any pair of matrices equated in a class — is calculated. The largest dissimilarity of any class in an equivalence partition defines the maximum diameter (Umax) of a particular partition. All equivalence partitions are then compared with the equivalence partitions suggested by the structural equivalence approach.

Before the rule of structural equivalence is applied, all relations are equated, and Umax should indicate a great deal of dissimilarity. As one moves down the tree diagram, Umax should become smaller. In general, Umax at successive levels should follow the order:

$$\text{Umax}_L \geq \text{Umax}_{L+1} \geq \text{Umax}_{L+2} \geq \ldots \text{Umax}_{L+n}$$

where L = level 1, L + 1 = level 2, . . . L + n = final level, at which each element occupies its own group.

In the hypothetical data, the similarity of each relation with every other relation is:

	R1	R2	R3	R4
R1	0			
R2	13	0		
R3	1	12	0	
R4	12	1	11	0

As shown, the most similar relations are (R1, R3) and (R2, R4); the most dissimilar relations are (R1, R2).

All possible equivalence partitions and Umax for each are shown in Table I. Each partition that corresponds to classes indicated by structural equivalence is so labeled. As shown, maximum dissimilarity occurs whenever R1 and R2 are in the same class (partitions 1–5), which

TABLE I
Equivalence partitions of four elements in hypothetical data and their diameters

Equivalence classes	Umax	Tree level
1. R1, R2, R3, R4	13	Level 1
2. R1, R2/R3/R4	13	—
3. R1, R2/R3, R4	13	—
4. R3/R1, R2, R4	13	—
5. R4/R1, R2, R3	13	—
6. R1/R2, R3, R4	12	—
7. R1, R4/R2/R3	12	—
8. R2, R3/R1/R4	12	—
9. R1, R4/R2, R3	12	—
10. R2/R1, R3, R4	12	—
11. R3, R4/R1/R2	11	—
12. R1, R3/R2, R4	1	Level 2
13. R2, R4/R1/R3	1	Level 3A
14. R1, R3/R2/R4	1	Level 3B
15. R1/R2/R3/R4	0	Level 3

includes Level 1 (all four elements in the same class). The least amount of dissimilarity occurs at partitions 12, 13, and 14 (excluding the partition at 15). Parsimony suggests that the equivalence partition R1, R3/R2, R4 is the best fit: it contains the least dissimilarity and the fewest number of equivalence classes. This partition is identified by the rule of structural equivalence, indicating that this rule can be used to locate and equate the proper relations.

Though this paper focuses on three-dimensional blockmodels in which the structural equivalence of *primitive* relations is considered, the structural equivalence approach can also be used to assess the equivalence of primitive and compound relations in a way that yields results comparable with those of algebraic and other approaches (see Appendix).

Interpretations of Equated Relations

In three-dimensional blockmodels, structurally equivalent relations are collapsed into a single relation. Structurally equivalent relations suggest one or more interpretations:

1) The empirical relations measure the same underlying (but unobserved) relation.

2) The empirical relations are socially distinct, yet they are mutually

reinforcing. "Playing sports" and "after-hours socializing" may be two distinct types of tie, but each type supports the other (e.g., if people tend to socialize with team-mates, and if people tend to play sports with those they socialize with).

3) The structural equivalence of two or more relations is spurious.

The specific interpretation of structurally equivalent relations depends on the substantive context of the social structure.

3. THREE CASE STUDIES

The three-dimensional blockmodeling approach is illustrated by application to three different social networks. The first case study, Homan's (1950) Bank Wiring Room, is the simplest case: relationships among members of a single population across multiple relations. The second case study, Sampson's (1969) monastery, is similar to the Bank Wiring Room — relationships among members of a single population over multiple relations — but the data are not binary as in Homan's case. The third case study, a hospital economy analyzed in Baker (1984c), is the most complex: two populations (hospitals as sellers; communities as buyers) across multiple relations (various hospital services) with continuous data (number of discharges).

Homan's Bank Wiring Room

Data — Social networks among the 14 members of Homan's (1950) Bank Wiring Room at a Western Electric plant (originally studied by Roethlisberger and Dicksen 1939) has been modeled by a number of network analysts (e.g., Breiger *et al.* 1975; White *et al.* 1976; Winship and Mandel 1983; Schwartz and Sprinzen 1984). Ties were inferred for two inspectors (I1, I3), nine wiremen (W1 - W9), and three soldermen (S1, S2, S4) across five types of tie: Like, Antagonism, Games (for a form of affectionate horseplay), Help (with production tasks), Windows (concerning quarrels over opening windows) (White *et al.*, 1976:755–6). (These relations are abbreviated as L, A, G, H, and W, respectively).

Three-Dimensional Blockmodel — In the present analysis, I use the Breiger *et al.* (1975:346) 4-block division of the 14 actors into the following sets:

(W1, W3, S1, W4, I1) (W2, W5, I3) (W6, S2) (W7, W8, W9, S4)

The 5 relations were partitioned by applying CONCOR to the unraveled relations, after removing the major diagonal from each matrix (because reflexive ties were not permitted). The resulting tree diagram and diameter matrix are presented in Figure 1.

I have selected a three-class division of the relations into (L, G) (H) (A, W); the relations below are collapsed in accordance with this partition. The matrix of submatrix densities and the corresponding image for the three-dimensional blockmodel are:

	SUBMATRIX DENSITIES				3-D IMAGE
L, G	.8	.3	0	.025	1 0 0 0
	.3	0	0	.04	0 0 0 0
	0	0	0	.185	0 0 0 0
	.025	.04	.184	.915	0 0 0 1
H	.2	.07	.1	.1	0 0 0 0
	.27	0	0	0	0 0 0 0
	.1	0	.5	.37	0 0 1 0
	.05	0	.12	.42	0 0 0 1
A, W	0	.2	.1	.125	0 0 0 0
	.2	.165	.335	.415	0 0 0 1
	.1	.335	0	.31	0 0 0 0
	.125	.415	.31	.415	0 1 0 1

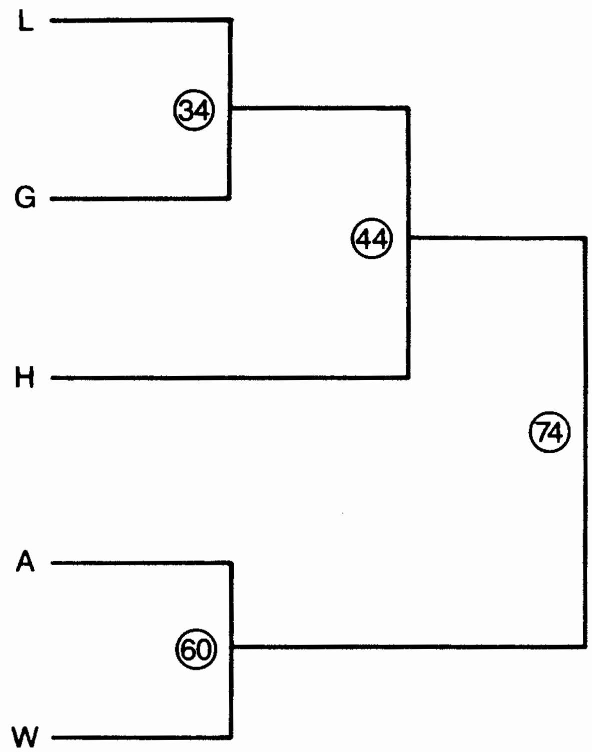
Interpretation — As noted in Breiger *et al.* (1975:345), blocks 1 and 4 distinguish the two cliques in Homan's analysis (cliques A and B, respectively); and the blocked Liking and Games relations each separately indicate the positive sentiment within each clique. As shown in Figure 1b, the dissimilarity (U) of Liking and Games is only 34, or 18.7% (34/182).*

* As an adjunct to U, it is useful to examine the inclusions of structurally equivalent relations. For example, there are 26 Liking ties and 56 Games ties in the Bank Wiring Room. All but two Liking ties are included in Games ties; 92.3% of all Liking ties are repeated in Games ties. In other words, people who Like each other almost always play Games with each other. The matrix below shows the inclusions among the five primitive relations in the Bank Wiring Room. Each percentage is calculated by counting the number of times iAj = iBj = 1 for two relations A and B, and dividing this sum by the number of ties in the sparser relation A or B.

	L	G	H	A	W
L	100%				
G	92%	100%			
H	38%	75%	100%		
A	0	26%	4%	100%	
W	31%	53%	38%	21%	100%

FIGURE 1 Bank wiring room — tree diagram and diameter matrix.

a. Tree Diagram of 5 Relations



b. Diameter Matrix

	L	G	A	H	W
L	0				
G	34	0			
A	64	74	0		
H	32	44	60	0	
W	48	54	60	44	0

KEY

○ = Umax (maximum diameter of two relations in subset)

The application of CONCOR to the relations indicates that these two relations are structurally equivalent. Therefore, they are collapsed into a single positive relation, as shown above, which preserves the structure indicated in each individual relation. The theoretical importance of collapsing these two relations is that they individually and jointly indicate the same type of positive relation. Collapsing these relations produces a simpler model of social structure.

The Helping relation, while a positive relation (and hence structurally equivalent with Liking and Games at the bipartite level) is "less equivalent" with Liking or Games, compared with the equivalence of Liking and Games. Since Helping is not collapsed with any other relation in this three-dimensional model, it is interpreted in the same way as in Breiger *et al.* (1975:347). To summarize, the hangers-on groups tend to help their respective cliques, and there is some intra-clique helping (Block 4). However, the incidence of ties and densities are too low to draw reliable conclusions.

As expected, the negative relations (A, W) were separated from the positive relations (L, G, H) at the bipartite level. But while Antagonism and Windows are grouped together, a U of 60 indicates a moderate amount of disagreement between these two. Though this might seem to indicate that it would be ill-advised to leave them together, collapsing them into a single negative relation admits of an interesting interpretation.

First, I will summarize Breiger *et al.*'s (1975:345–7) main findings in their separate discussions of the Antagonism and Window relations. The Antagonism relation shows: there is a lack of antagonism between the two central cliques and within each clique (zeroblocks in all four corners of the image); the central cliques are antagonistic only toward the marginal groups; and there is more antagonism between the two hangers-on groups than within either of these groups. The Window relation primarily shows that controversies about opening and closing windows in the work room occurred in Clique B (Block 4); this is attributed to the simple fact that this group was located closest to the windows.

While these two relations are clearly different types of tie, at a higher level of abstraction they can be viewed as types of *negative interaction*. Collapsing these two relations (as in the three-dimensional blockmodel above), creates an image that is very similar to the individual Antagonism relation with one important difference: there is a bond in the lower right hand submatrix, indicating the negative interaction about windows that occurred within Clique B (Block 4). The interpretation of this collapsed relation is the same as the Antagonism relation with the exception that a *type* of negative interaction also occurs within Clique B. Thus, collapsing these two relations does not obscure the antagonism relationships in this social structure, but instead suggests a type of negative interaction at a higher theoretical level. This arrangement has the additional advantage of generating a more parsimonious model of social structure.

Sampson's Monastery

Data — Sampson's (1969) study of relations in an American monastery, like Homan's study, is a classic that has become a favorite of network analysts (e.g., White *et al.* 1976; Breiger *et al.* 1975). Sampson defined four types of tie — Affect (Like), Esteem, Influence, and Sanction (L, E, I, and S, respectively). Each monk provided his first three positive choices (+1, +2, +3) and his first three negative choices (−1, −2, −3) for each type of tie. (The number highest in magnitude represents first choice; the number second highest in magnitude represents second choice; etc.) White *et al.* (1976) separated the positive ties from the negative ties, producing eight relations: the original four (with the negative ties removed) and four new ones containing the negative ties — Antagonism, Disesteem, Negative Influence, and Blame (A, D, N, and B, respectively). I have used all eight relations with valued data.

Three-Dimensional Blockmodel — I use White *et al.*'s (1975:750) three-block division of the 18 monks:

(10 5 9 6 4 11 8) (12 1 2 14 15 7 16) (13 3 17 18)

These three blocks correspond to Sampson's "Loyal Opposition," "Young Turks," and "Outcasts," respectively.

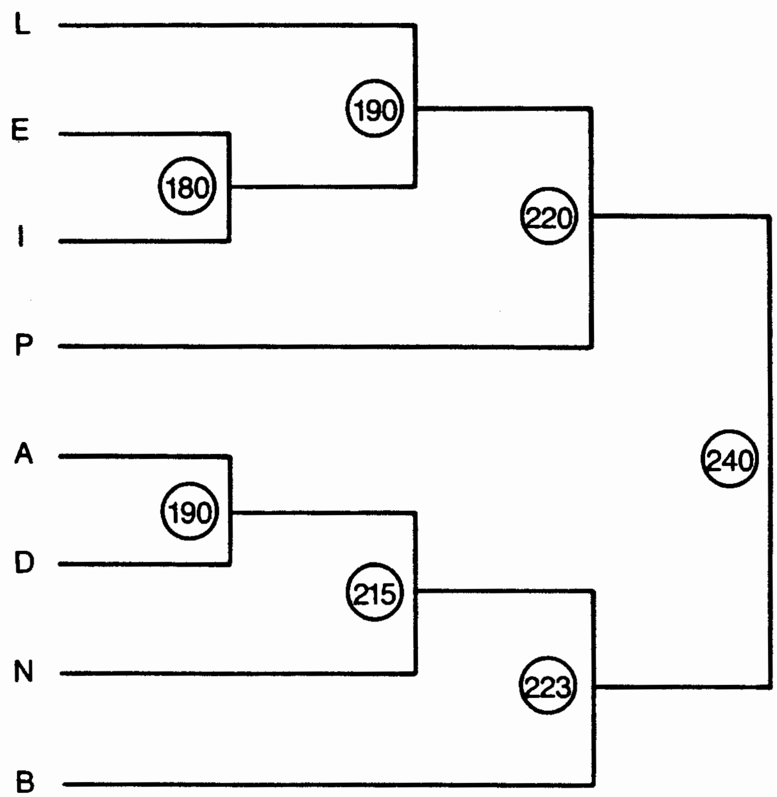
CONCOR was repeatedly applied to the unraveled eight relations (excluding the major diagonal in each matrix). The tree diagram and diameter matrix are presented in Figure 2.

For reasons discussed below, I have collapsed the eight relations at the bipartite level. The resulting matrix of submatrix densities and the corresponding image for the three-dimensional blockmodel are:

SUBMATRIX DENSITIES	3-D IMAGE
.6276 .0179 .0714	1 0 0
.1339 .7344 .2500	0 1 1

FIGURE 2 Sampson's monastery — tree diagram and diameter matrix.

a. Tree Diagram for 8 Relations



b. Diameter Matrix

	L	E	I	P	A	D	N	B
L	0							
E	180	0						
I	190	177	0					
P	213	220	205	0				
A	212	216	219	177	0			
D	225	218	233	206	190	0		
N	236	235	240	227	215	212	0	
B	237	222	231	241	220	223	182	0

KEY
○ = Umax* (maximum diameter of two relations in subset, standardized)

.0357	.0357	.8214		0	0	1
.0051	.6429	.3010		0	1	1
.5625	.0938	.1696		1	0	0
.3980	.6786	.0663		1	1	0

(As in White *et al.* 1976:751, a cutoff of half the average density was used to form the blockmodel image.)

Interpretation — As should be expected, the first application of CONCOR to the relations split them exactly into the four positive and four negative relations. This indicates that these two sets are opposites at a global level. This split is obvious in the diameter matrix — small distances between elements within a subset, and large distances between elements from different subsets (Figure 2b).* Similarly, the zero-order correlation matrix of these 8 relations shows that all four positive (negative) relations are *positively* correlated with one another and *negatively* correlated with each of the negative (positive) relations.† Given these patterns in the original data, it seems advisable to remain at the bipartite level.

It is interesting to note that successive divisions of the two sets of relations parallel one another through the first and second levels (2-block and 4-block, respectively), but diverge at the third level (6-block). At the third level, Like, Esteem, and Influence are split into (L) and (E, I), but the relations derived from them, Antagonism, Disesteem, and Negative Influence, are split into (A, D) and (N). This indicates that in the positive set, Esteem and Influence are closely related, but in the negative set, their opposites are not; instead, Antagonism and Disesteem are closely related. These findings contrast with interpretations based on two-dimensional images. Comparing images, White *et al.* (1976:751) state that "[t]he Like image is identical with Esteem and Disesteem with Negative Influence." While this appears to

* When valued data are used, U should not be calculated in the same manner as in the case of binary data. One alternative is to use the city-block metric as a measure of dissimilarity, after standardizing the raw values. This technique was used on the monastery and hospital data to calculate dissimilarity (designated as U*). Of course, other measures of dissimilarity could be used.

† Note, however, that while each set is highly intracorrelated, there are relatively *weak* correlations between members of the two sets. This indicates that the relations within each set may be tapping the same underlying (but unobserved) relation, but the two underlying (unobserved) relations are not exactly opposites. Though every possible correlation between the sets is negative (and thus indicates some opposition), the strongest correlation is only −.19.

be so when comparing *images* (using Definition 2), the direct application of the rule of structural equivalence to the individual-level data indicates that the White *et al.* identities may not be true: as shown in the tree diagram, Esteem and Influence are structurally more alike than Like and Esteem, and Antagonism and Disesteem are structurally more alike than Disesteem and Negative Influence. These differences — comparing relations at the image level vs. the individual-data level — highlights one of the advantages of using the three-dimensional blockmodeling approach (see also Schwartz and Sprinzen 1984).

The interpretation of the three-dimensional blockmodel is similar to that provided by White *et al.* (1976), though a much simpler interpretation can be made. As shown in the positive image (collapsed L, E, I, and P), each group clearly holds positive feelings about itself (bonds along the diagonal). Only one group, the Young Turks, has positive feelings towards another group, the Outcasts (unreciprocated). The negative image (collapsed A, D, N, and B) is the exact opposite of the positive image. The Loyal Opposition receives negative feelings from both the Young Turks and Outcasts; these negative feeings are reciprocated by the Loyal Opposition in both cases; and the Outcasts also have negative feelings towards the Young Turks (unreciprocated). Overall, the three-dimensional blockmodel shows a clear within-group positive sentiment and a between-group negative feeling, with an unrequited "love-hate" relationship between the Young Turks and the Outcasts.

A Local Economy of Hospital Services

Data — The economy of hospital services used as a case study was reported in Baker (1984c). Only a brief description of the economy is given here. This economy is located in an area of about 170 square miles with a population of about 230,000 (1980 Census). A small city is located in the center of this area, surrounded by several townships, with an urban center to the northwest.

Each hospital is a provider of healthcare services in the economy; in other words, each hospital is a seller in the economy. Fifteen hospitals are included in this analysis (H1 to H15). They were chosen by selecting a central hospital (H1) and then including all other hospitals that were its main competitors.

Operationally, geographical boundaries and the central hospital's main competitors were identified by examining patient origin data arranged to show the sources of each hospital's patients and the hospitals utilized by residents of each town. First, the central hospital's primary service area was demarked by including all communities that combined comprised 95% of its patient base. In this manner, eight contiguous towns were selected. Second, the hospitals utilized by each town were ranked by percentage share. A cumulative percentage cut-off of 75–80% was used, and any hospital within this cut-off for any town was included in a master list of hospitals. This yielded fifteen hospitals that were the principal providers to this geographical area.

Patients are the users (buyers) of health services. In this analysis, patients are grouped by townships. A township delimits a geopolitical area containing a more or less demographically homogenous subpopulation. Various townships identify subpopulations that are relatively homogeneous within and heterogeneous between.

Unlike the relations in the Bank Wiring Room and Monastery cases, in this example the relations are social institutions — markets. A single market is defined as all exchanges involving a specific healthcare service generally recognized and categorized by the healthcare industry. The following five services (i.e., relations) were used:

Maternity — All cases with a principal obstetrics diagnosis. Gynecological cases not related to pregnancy or childbirth are not included.
Pediatrics — All patients under age 15, except those with a principal obstetrics or psychiatric diagnosis (excludes neonatal cases).
Psychiatry — All cases with a principal psychiatric diagnosis, excluding alcoholism, drug dependence, and mental retardation (these are assigned to pediatrics or adult medical/surgical services, depending on age).
Adult Medical/Surgical Services (*Ages 15–64*) — All cases not falling into categories 1–3 and the patient's age is 15–64.
Adult Medical/Surgical Services (*Ages 65+*) — All cases not falling into categories 1–4.

These types of tie are abbreviated as M, PD, PS, AMS1, and AMS2, respectively.

All in-patient discharges for the eight towns during a year period are included, excluding live births and fetal deaths.

The three-dimensional data structure (the "market box") for this local economy forms a 5 × 8 × 15 array (markets by towns by hospitals).* Participation in the various hospital markets, represented

* Because data such as these contain potentially sensitive information, the actual market box cannot be shown and the hospitals and communities cannot be identified.

by each (k, i, j) entry in the market box, is operationalized as the total number of patients from a specific town discharged from a specific hospital in a given year period.

Three-Dimensional Blockmodel — To identify groups of structurally equivalent hospitals, CONCOR was applied to the columns of the market box, resulting in the following bipartite division:

Aggregate Seller 1: H1-H4, H6, H8, H10, and H12.
Aggregate Seller 2: H5, H7, H9, H11, and H13-H15.

The first aggregate was split twice, yielding the following:

Aggregate Seller 1a: H1, H2, H3, and H6.
Aggregate Seller 1b: H8 and H12.
Aggregate Seller 1c: H4 and H10.

Structurally equivalent subgroups of towns were identified by applying CONCOR to the rows of the market box. The bipartite division is:

Aggregate Buyer 1: T1, T3, T5 (H1's hometown), and T6.
Aggregate Buyer 2: T2, T4, T7, and T8.

Each aggregate buyer above was split once, yielding:

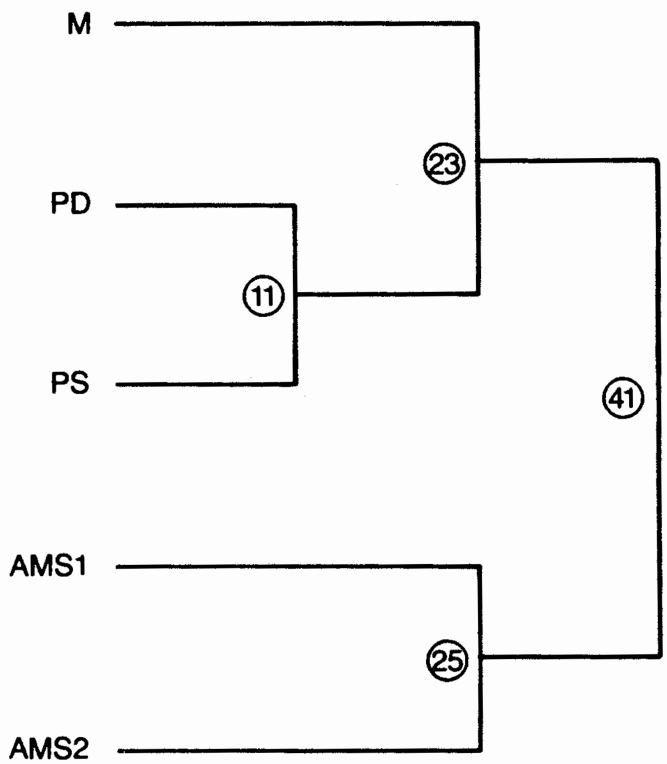
Aggregate Buyer 1a: T1 and T6 (west of T5, closest to urban area).
Aggregate Buyer 1b: T3 and T5 (the focal area and a contiguous neighbor south-southwest).
Aggregate Buyer 2a: T2 and T7 (east of T5).
Aggregate Buyer 2b: T4 and T8 (farthest east of T5).

Repeated application of CONCOR to the five market-matrices of the market box results in the tree diagram shown in Figure 3. The sets of structurally equivalent markets used to form three-dimensional blockmodels are (M, PD, PS) and (AMS1, AMS2). The matrix of submatrix densities and the corresponding image for the three-dimensional blockmodel are:

SUBMATRIX DENSITIES				3-D IMAGE			
248.3	95.2	47.8	67.8	1	1	1	1
1176.8	70.7	85.5	9.6	1	1	1	0
212.0	31.8	32.2	13.4	1	0	0	0
48.0	10.5	7.2	27.2	1	0	0	0
53.4	1.3	1.0	42.8	1	0	0	1
306.1	1.5	4.3	11.4	1	0	0	0

FIGURE 3 Hospital economy — tree diagram and diameter matrix.

a. Tree Diagram of 5 Relations



b. Diameter Matrix

	M	PD	PS	AMS1	AMS2
M	0				
PD	23	0			
PS	20	11	0		
AMS1	41	38	34	0	
AMS2	34	30	27	25	0

KEY

○ = Umax* (maximum diameter of two relations in subset, standardized)

50.9	.3	0	10.0	1 0 0 0
12.8	0	0	10.4	0 0 0 0

(A cutoff of half the average submatrix density was used to form the blockmodel image.)

Interpretation — As argued in Baker (1984c), geography and proximity serve to discriminate among the aggregate sellers (AS) and aggregate buyers (AB) in this local economy. For example, AS1a is comprised of the focal hospital and its two immediate neighbors, which occupy the same township (T5); AS1c is comprised of two hospitals from the urban center. Some of the large, urban hospitals, however, are mixed with the local hospitals; this indicates the draw of large hospitals which can overcome the friction of space.

At the bipartite level, the rule of structural equivalence does not locate groups of towns that are obviously related to the geographical groups. However, these groups follow an east-west division, drawn through H1's hometown, T5. Subdividing each of these groups reveals this even more clearly, as noted in the parenthetical descriptions in the preceding section.

The five relations are all positively correlated with one another, from a low of r = .37 (M and AMS1) to a high of r = .99 (PD and PS). The correlations within each structurally equivalent set are strikingly high: in the Maternity/Pediatrics/Psychiatry set, all correlations are greater than or equal to .90; in the Adult Medical/Surgical set, the correlation is .91. The lower correlations all occur *between* these two sets. These patterns are strong indications that the bipartite split should be used for collapsing relations.

Collapsing the two Adult Medical/Surgical relations makes immediate substantive sense: these both include very similar services. Collapsing maternity and pediatrics also makes sense, since these involve services primarily for mothers and children. The inclusion of Psychiatry in this set is difficult to explain without qualitative data. The unexpected discovery of its strong structural equivalence with Maternity and Pediatrics clearly indicates an important direction for further research.

In the terminology of Baker (1984c), each set of collapsed market-relations is called a "supermarket." A supermarket contains markets that are based on very similar patterns of exchange.*

* The structural equivalence of Maternity, Pediatrics, and Psychiatry does not imply that these *services* are the same. It only implies that the *markets* for these services are similarly structured.

The three-dimensional blockmodel image shows a clear pattern: overall, Aggregate Seller 1a is the principal provider of healthcare services in the economy, connected by bonds in both supermarkets to every aggregate buyer except one. However, this aggregate seller does not dominate the aggregate buyer containing its own township (AB1b), at least in the first supermarket — this aggregate buyer obtains services from all provider groups except one. Of all the aggregate buyers, AB1a is the least dependent on any single provider group: this aggregate buyer obtains services from all providers in the first supermarket, and from two out of four in the second supermarket. Aggregate Buyer 2a is the most dependent on a single provider group in this economy; it is connected only to AB1a. Note that this group of buyers is located *east* of the area, far from the urban hospitals; similarly, AB2b is located the farthest from the urban hospitals, and it is connected by only a single bond in this economy.

Overall, the three-dimensional image indicates a core structure with some competitive diffusion. Aggregate Seller 1a dominates the entire economy (it occupies the core). But in Supermarket 1 its dominance is diffused by competition from the other groups of providers. Its dominance in the second supermarket is much sharper. (Note that a perfectly competitive economy would result in an "amorphous" image — bonds everywhere [Baker 1984c]). By this operational definition of competitive structure, Supermarket 1 exhibits more competition than Supermarket 2).

4. CONCLUSION

This paper developed and explored the concept of three-dimensional blockmodels. It extends the classic blockmodel approach which reduces the three-dimensional data structure of multigraph networks to a two-way analysis. Three-dimensional blockmodeling applies the rule of structural equivalence to all three dimensions of a network with multiple relations. In this approach, relations can be grouped and collapsed according to their degree of structural equivalence in a way that is directly analogous to collapsing structurally equivalent actors into blocks.

Three-dimensional blockmodels are important for both method and theory. It is often argued that social structure is seldom defined accurately by a single relation; in fact, one of the principal advantages of blockmodeling techniques over sociometric techniques is their ability to block multigraph data simultaneously. But reducing a three-way representation of social structure to a two-way analysis obscures the view of social structure from the perspective of the patterns of underlying relations.

Three-dimensional blockmodeling reduces the complexity of social structure further than the conventional two-way approach. Models built in this way are more parsimonious representations of social structure than two-dimensional blockmodels.

The three-dimensional approach offers some practical benefits. The rule of structural equivalence (weaker form) allows one to work directly with empirical data at the individual level, instead of image-level matrices or cleaned and modified data (Schwartz and Sprinzen 1984). Direct application to the data also provides a measure of the *degree* of the equivalence of relations at various levels of fineness. Other advantages include the ability to easily handle binary and nonbinary data and the ability to model two populations with multiple relations (or three populations with a single relation).

By explicitly focusing on the patterns of relations that underlie a social structure, three-dimensional blockmodeling aids in understanding the content of positions and patterns among positions in a social structure. For example, it helps to reveal the mutually reinforcing relations in a structure. Discovering such relations is important for many theoretical problems. In the study of international trade and the structure of the world system, for example, analyzing interlocking among commodity types is essential for understanding the interdependencies of nations as expressed in their exchanges of raw materials and finished goods (e.g., Breiger 1981; Snyder and Kick 1979).*

Three dimensional blockmodeling can also be used to study questions of economic dualism. The hypothesis of a "dual labor market economy," for instance, can be operationalized as a particular three-dimensional blockmodel (Baker 1984c). And, as the third case study demonstrated, the three-dimensional approach can be used to study the interrelationships of social structure and of social institutions.

Further research should involve three different areas. Three-dimensional blockmodeling should be applied to more empirical social

* Breiger (1981:369) examined the zero-order correlations of the trade matrices used in his study. The manner in which these correlations were calculated (see Breiger 1981: 377 [fn. 16]) is the same as that used in this paper. Though this is the first step in the application of CONCOR, Breiger (1981) did not take the remaining steps in identifying structurally equivalent relations and developing three-dimensional blockmodels.

structures to judge the utility of the approach. Algorithms other than CONCOR should be tested as means to assess the structural equivalence of relations. And the structural equivalence approach to the analysis of role structures should be further explored (see Appendix).

APPENDIX

A Structural Equivalence Approach to Role Structures

The purpose of this appendix is to explore the use of the rule of structural equivalence to analyze the pattern of relations among relations. Traditionally, relations among relations have been analyzed by using the algebra of semigroups on blockmodel images (e.g., Boorman and White 1976). Although the algebra of semigroups can be used with individual-level data (Bonacich and McConaghy 1980), the presence of idiosyncratic ties often makes it impractical to do so. In response to this problem, Schwartz and Sprinzen (1984) have developed an approach for cleaning and modifying network data, and generating algebraic equations directly from the (modified) individual-level matrices.

The structural equivalence approach can also be used to identify equations among relations, with results consistent with the results of other approaches. However, this approach can be applied directly to the individual-level matrices *without* prior data cleaning or modification.

I apply the structural equivalence approach to the Liking (Friends) and Antagonism (Enemies) relations from the Bank Wiring Room, along with their compounds of length four or less. The length of a compound tie is defined as the number of generators that are multiplied to form the tie (e.g., FEF has a length of three, EFEF has a length of four). Compounds were formed directly from the raw individual-level Liking and Antagonism relations, using Boolean matrix multiplication.

With the structural equivalence approach, it is not necessary to specify *any* equations among relations prior to analysis. However, to compare the results of this analysis with those of others, I hypothesized that the structural equivalence approach would identify the same equations as obtained in the semigroup of the three-block images of the Bank Wiring Room (Schwartz and Sprinzen 1984:130). There are 10 unique compound relations in the three-block semigroup, one more than in the two-block semigroup (Schwartz and Sprinzen 1984:114).

The "extra" element in the 3-block case in FE^2F; in the two-block case, $FEEF = F$. Including ties of length four or less, the three-block semigroup expresses equality equations among 26 primitive and compound relations (ties of length five are also shown in Schwartz and Sprinzen). The equality equations among these 26 relations are shown below:

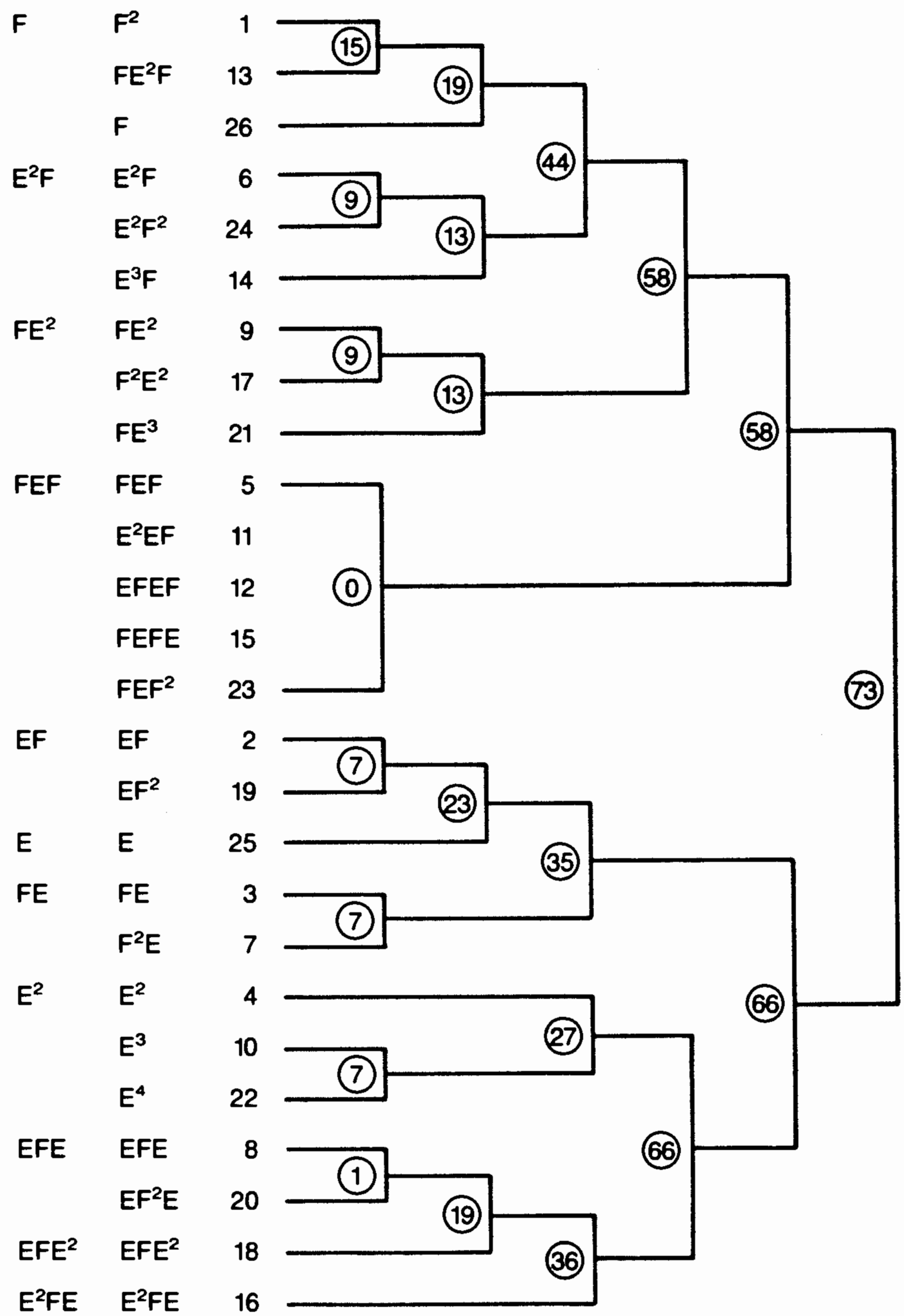
1) $F = F^2$
2) $EF = EF^2$
3) $FE = F^2E$
4) $E^2 = E^3 = E^4$
5) $FEF = F^2EF = EFEF = FEFE = FEF^2$
6) $E^2F = E^2F^2 = E^3F$
7) $EFE = EF^2E$
8) $FE^2 = F^2E^2 = FE^3$
9) $FE^2F = FE^2F$
10) $E^2FE = E^2FE$
11) $EFE^2 = EFE^2$
12) $E = E$

CONCOR was applied to the 26 relations, resulting in the tree diagram shown in Figure A1. The diameter matrix is presented in Figure A2. In the application of CONCOR and the calculation of diameters, whenever a primitive relation was compared with a compound relation, the diagonals were excluded (because reflexive ties in Liking and Antagonism were not allowed). The diameter matrix is expressed in percentages because the total number of possible ties when a diagonal is excluded is lower (182) than when a diagonal is included (196).

As shown at the termini of the tree diagram, CONCOR locates all the equalities among relations, with only a single difference. While the semigroup based on three-block images indicates that $F^2 = F$ (equation 1) and $FE^2F = FE^2F$ (equation 9), the structural equivalence approach indicates that $F^2 = F = FE^2F$ at the third to last level in the tree, and $FE^2F = F^2$ and $F = F$ at the second to last level (and each relation is equal to itself at the last level). Thus, equation 1 is true at the third to last level, but equation 9 is also included. In other words, the structural equivalence approach suggests that "a friend of a friend is a friend" and that "a friend of an enemy's enemy's friend is also a friend."

If equality equations are based on the *two-block* semigroup, then *all*

FIGURE A1 Three diagram for relations in semigroup formed from liking and antagonism relations in bank wiring room.



KEY
○ = Umax (in percentages)

FIGURE A2 Diameter matrix for relations in semigroup formed from liking and antagonism relations in bank wiring room (in percentages).

1	13	26	6	24	14	9	17	21	5	11	12	15	23	2	19	25	3	7	4	10	22	8	20	18	16
0																									
15	0																								
12	19	0																							
33	28	35	0																						
37	34	44	9	0																					
39	34	43	10	13	0																				
33	28	35	49	52	54	0																			
37	34	44	52	51	56	9	0																		
39	34	43	54	56	58	10	13	0																	
25	33	14	42	51	46	42	51	46	0																
25	33	14	42	51	46	42	51	46	0	0															
25	33	14	42	51	46	42	51	46	0	0	0														
25	33	14	42	51	46	42	51	46	0	0	0	0													
25	33	14	42	51	46	42	51	46	0	0	0	0	0												
37	44	27	38	45	34	54	62	58	12	12	12	12	12	0											
42	50	33	38	42	32	59	68	63	17	17	17	17	17	7	0										
43	52	35	53	59	53	53	59	53	21	21	21	21	21	20	23	0									
37	44	27	54	62	58	38	45	34	12	12	12	12	12	23	29	20	0								
42	50	33	59	68	63	38	42	32	17	17	17	17	17	29	35	23	7	0							
52	54	51	39	40	46	39	40	46	54	54	54	54	54	55	56	51	55	56	0						
66	67	69	52	48	47	52	48	47	66	66	66	66	66	59	57	47	59	57	27	0					
67	69	71	51	45	49	51	45	49	73	73	73	73	73	66	64	52	66	64	19	7	0				
33	40	21	49	58	54	49	58	54	8	8	8	8	8	19	25	21	19	25	46	60	66	0			
33	41	21	50	59	54	50	59	54	8	8	8	8	8	20	26	21	20	26	46	59	65	1	0		
47	55	37	47	52	44	64	73	68	22	22	22	22	22	15	13	19	34	40	45	44	51	19	18	0	
47	55	37	64	73	68	47	52	44	22	22	22	23	22	34	40	19	15	13	45	44	51	19	18	36	0

NOTE

Row/Column numbers correspond to numbering of relations in figure A1.

For pairs of relations containing a primitive relation, denominator is 182; for all other pairs, denominator is 196.

equations are located by the structural equivalence approach (FEEF = F in the two-block semigroup). (Note that Schwartz and Sprinzen 1984:129 argue that FEEF = F masks the three-block structure, concealing the distinction between the first two blocks.)

This application of CONCOR suggests that the structural equivalence approach can be used to identify the patterns of relations among relations. The advantages of the approach are (1) the analyst can work directly with the individual-level matrices instead of the image matrices, (2) data cleaning or modification are not necessary, and (3) equality equations are not required prior to analysis. Of course, experimentation on other empirical networks is needed.

Another advantage, which has no direct analog in other approaches, is that the structural equivalence approach permits one to assess the *degree* to which relations differ. The tree diagram implies a hierarchy of equivalence. Each level indicates a different set of equalities among relations, with a set at a lower level included in a set at a higher level. At the top of the tree diagram, all relations are equated, which does not add to our understanding at all. But at just the first split, a pattern begins to emerge which more or less distinguishes two main types of relations: "types of friends" and "types of enemies". As one moves down the tree, the similarity of relations in any set increases (see diameter matrix). Substantive interpretations can be made for each set of structurally equivalent relations at each level.

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