Stable Economic Depreciation Neutral Replacement Decisions

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ABSTRACT

We give a method for computing depreciation schedules that reflect the underlying economic value of equipment. In the presence of taxes, these economic depreciation schedules depend on several factors, including the depreciation schedules imposed by government. If the resulting economic depreciation schedule offers quicker write-offs than the imposed one, the firm could argue that, based on economic rationale, the imposed schedule should be changed to equal the economic schedule. We show the existence of an imposed schedule that equals the resulting economic schedule, and hence is termed “stable”. We also show that stable depreciation schedules are tax neutral (for a range of tax rates) in that the replacement decision is not changed by the tax system.

1. DEPRECIATION ACCOUNTING

For tax purposes, the cost of purchasing durable assets is spread over the economic life of the asset rather than treating such costs as immediate expenses when they occur. The rationale for doing so is that, since the benefits of owning such assets are spread over their economic life, their costs should be similarly distributed. In the case of physical plant and equipment, these costs are called depreciation. The financial (or tax) accounting concept of depreciation is described in a bulletin of the American Institute of Certified Public Accountants [2] as follows:

“Depreciation accounting is a system of accounting which aims to distribute the cost or other basic value of tangible capital assets, less salvage (if any), over the estimated useful life of the unit (which may be a group of assets) in a systematic and rational manner. It is a process of allocation, not of valuation. Depreciation for the year is the portion of the total charge under such a system that is allocated to the year. Although the allocation may properly take into account occurrences during the year, it is not intended to be a measurement of the effect of all such occurrences.”
There are a variety of methods for allocating costs that are "systematic and rational" [5]. In the United States, the most common methods have been influenced by changes in income tax laws and regulations, specifically by changes in 1934, 1954, 1962, 1971, 1981, and 1986. Depreciation accounting methods may be classified as follows:

1. Methods designed to give a greater write-off in the early years of life rather than in the final years.
2. Methods designed to give a uniform write-off throughout the entire service life.
3. Methods designed to give a smaller write-off in the early years of life than in the final years.
4. Methods that do not fit in categories (1) - (3).

In class (1), there are the declining-balance method, the sum-of-the-years-digits method, and certain multiple-straight-line-methods. The straight-line method is in class (2), while the sinking-fund method is in class (3). The ACRS method falls in-between the cracks, i.e., is in class (4).

This interpretation of accounting is not appropriate for all accounting purposes, however. The American Accounting Association [1] states that:

"the primary function of accounting is to accumulate and communicate information essential to an understanding of the activities of an enterprise...".

Accounting to characterize the financial position of the firm, in order to assist in the decision-making process, is termed managerial accounting. Unlike financial (or tax) accounting, managerial accounting views depreciation as a process of valuation based on endogenous economic rationale, rather than as a simple process of allocation. Ideally, a depreciation schedule should reflect the actual decline in economic value of an asset, so that its book value always equals its economic value.

In this paper we put forth a precise definition of economic depreciation, describe how the necessary computations can be made in the context of a specific machine replacement example, and discuss the implications for tax policy and further research.
2. ECONOMIC DEPRECIATION

An economic interpretation of depreciation requires that the depreciation allowance in any year equal the change in the economic value to the firm during that year, regardless of how that value change may have come about. Of course, economic value is a concept with many meanings. In a classic treatise, Bonbright [4] argues that the two most important and useful interpretations of economic value are market value and value to the owner.

Market value refers to the price at which a capital asset can be sold in a second-hand market. However, this is not necessarily an appropriate concept to use in defining economic depreciation. The services provided by ownership of an asset may be of great financial importance to the owner, but the asset might command an insignificant price if resold. For example, a second-hand market for telecommunications equipment is almost non-existent [3]. Thus, the economic value to a particular owner is crucial in the valuation of capital assets.

Value to the owner may be defined as the dollar amount just sufficient to compensate the owner if he were deprived of the services of the asset. This value will depend on capital cost, operating and maintenance costs, salvage values (market prices), discount rates, tax rates, future replacement options, and the length of the optimal replacement cycle. This value will not exceed the monetary amount for which the owner could replace the asset with the best available substitute. And, the value to the owner will not be less than the market price. Ideally, the economic value of a vintage asset is the price at which a firm would be indifferent between keeping the asset or replacing it with a like asset [14].

Note also that the economic depreciation schedule is a fair depreciation schedule in that depreciation allowances are neither larger nor smaller than the actual decline in value of the asset. As such, we could view such a schedule as the equilibrium of a negotiation between firm and government. A schedule that writes off the cost more quickly than the economic schedule penalizes the government while a schedule that writes off the cost less quickly than the economic schedule penalizes the firm.

There have been a number of papers discussing an economic theory of depreciation. Hotelling [7] considered the problem of determining depreciation schedules by devising a formula for the value of old machinery based upon revenues generated as well as operating and maintenance costs. Unfortunately, his results contain errors, as described in [9]. More recently, Hulten and
Wykoff [8] defined depreciation to be the decline in shadow price due to age, but are not specific about what they mean by shadow price. A particularly interesting piece of work has been done by Bergh et al. [3] at Bell Communications Research (Bellcore), discussing an economic model for depreciation of equipment in the telecommunications industry. The motivation of this work is the observation that telecommunications equipment typically becomes technologically obsolete long before reaching its service life. The Bellcore effort attempts to capture this observation in a model of depreciation so that telecommunications companies may have a rational basis for going before regulatory agencies and arguing for changes in the way their equipment is depreciated. Finally, Jones, Zydiak, and Hopp [9] use infinite-horizon linear programming to develop prices that measure the economic value of owning vintage machinery. These well-defined and computable prices are then used to define depreciation schedules. In this paper, we examine the implications of this model for a theory of economic depreciation. In particular, we study the interplay between tax rates and optimum replacement cycles and hence depreciation schedules.

3. STABLE DEPRECIATION AND NEUTRAL REPLACEMENT DECISIONS

In this section we study a specific model of machine replacement and from it draw conclusions about economic depreciation.

Depreciation Model

We consider a classical machine replacement problem. A new machine is purchased. It ages up to some point and is then replaced with an identical new machine. We assume that the machine must be replaced if it reaches n years old (an obsolescence assumption) and that the sequence of replacement cycles goes on indefinitely. The costs and credits associated with the machines are assumed to be independent of the year in which they occur. See Jones, Zydiak, and Hopp [10] for a treatment of this model in which these costs and credits are time-varying, thereby allowing the inclusion of issues such as technological innovation. Maintenance and operating costs are assumed to occur at the end of the year, while purchase costs and salvage credits occur at the beginning of the year. Finally, we assume an infinite planning horizon with the objective of minimizing the infinite sum of discounted costs. The infinite-horizon assumption is motivated by the work of Henderson and Quandt [6] and Samuelson [13] who show that optimizing over a finite-horizon leads to the wrong answer. Further, it
should be noted parenthetically that the infinite-horizon model yields the same economic life as does the classical method of defining the economic life to be the length of the replacement cycle that minimizes annual equivalent costs. This paper is a sequel to previous work [9] in which we introduce an infinite-horizon optimization model, not to provide a more complicated procedure for computing economic life, but rather because it allows us to compute dual variable values that can be used to define depreciation schedules.

We model this problem by defining the following parameters:

\[ p = \text{the purchase price of a new machine;} \]
\[ s_j = \text{the salvage value of a } j \text{ year old machine } (j=1,...,n); \]
\[ m_j = \text{the annual maintenance and operating cost of a } j \text{ year old machine } (j=1,...,n); \]
\[ \delta = 1/(1+i), \text{ where } i \text{ is the annual discount rate (}\delta \text{ is known as the discount factor).} \]

Let us define the following

\[ \ell = \arg \min_{1 \leq k \leq n} \left\{ \frac{p-\delta^k s_k + \delta^m_1 + \delta^2 m_2 + \ldots + \delta^k m_k}{(1-\delta^k)} \right\}. \quad (1) \]
\[ \tilde{y} = \frac{(1-\delta)}{(1-\delta^\ell)} \left\{ p-\delta^\ell s_\ell + \delta^m_1 + \delta^2 m_2 + \ldots + \delta^\ell m_\ell \right\}. \quad (2) \]

The economic life of the machine is defined to be the value of \( k(1 \leq k \leq n) \) that results in the lowest net present value of the corresponding infinite cost stream, and hence \( \ell \) is the economic life. In turn, \( \tilde{y} \) is the annual equivalent cost corresponding to that net present value. In a profit-maximizing firm, the marginal economic value of the services provided by machine ownership will equal the marginal cost of operating the machine. Hence, \( \tilde{y} \) can be interpreted as the annual economic value of the services provided by machine ownership.

We define the economic value of owning a \( j \) year old machine \((j=1,...,\ell)\) to be \( \tilde{y}_j \), where

\[ \tilde{y}_j = \begin{cases} s_\ell, & j=\ell \\ \tilde{y} - \delta m_{j+1} + \delta \tilde{y}_{j+1}, & j=1,...,\ell-1 \end{cases} \quad (3) \]
Defined in this way, the $\tilde{y}_j$ values represent dual (shadow) prices in an infinite-horizon linear programming model of the machine replacement problem [9]. These prices can also be motivated intuitively. By definition of $\ell$, an $\ell$ year old machine should always be replaced. Thus, the value of an $\ell$ year old machine should be $s_\ell$. On the other hand, a $j$ year old machine ($1 \leq j < \ell - 1$) should be kept for another year. But $\tilde{y} - \delta m_{j+1}$ is the net value to the firm of operating the machine for a year plus $\delta \tilde{y}_{j+1}$ is the discounted value of owning the machine one year later.

Accordingly, we define our economic depreciation schedule, $\bar{d}_j$, where $\bar{d}_j$ denotes the depreciation during year $j$ of an economic life, as follows:

$$
\bar{d}_j = \begin{cases} 
p - \tilde{y}_1, & j = 1 
\tilde{y}_{j-1} - \tilde{y}_j, & j = 2, \ldots, \ell
\end{cases}
$$

Using the LP model, we can show that straight-line depreciation occurs if and only if maintenance costs are linearly increasing with a slope of $(1-\delta)(p-s_\ell)/\delta$ [9]. Other maintenance costs can lead to depreciation schedules in categories (1), (3), and (4) in Section 1. Using the approach in [9], we can identify maintenance cost profiles leading to declining-balance or sum-of-the-years-digits depreciation schedules. However, unlike the straight-line case, these depreciation schedules do not imply maintenance cost functions of any particular form.

To illustrate these concepts, suppose the discount rate is equal to 0.9, $p=200,000$, and $n=25$. Suppose the maintenance costs are linear and are described by $m_t = m_1 + (t-1)h$, $t=1, \ldots, n$, where $m_1 = 0$ and $h$ is the slope parameter.

Finally, let the salvage values be described by

$$
s_t = \begin{cases} 
0.96 s_{t-1}, & t = 1, \ldots, 6 
0.10 s_{t-1}, & t = 7, \ldots, 25
\end{cases}, \text{ where } s_0 = p.
$$

In this example, straight-line depreciation results if $h = 804.60$. It is simple to show that depreciation is monotonically decreasing if the slope is larger than
Figure 1
Monotonic and Straight-Line Depreciation

Figure 2
Non-Monotonic Depreciation
this number and monotonically increasing in the opposite case. Figure 1 shows plots of depreciation schedules with yearly depreciation plotted on the vertical axis and year plotted on the horizontal axis for different values of \( h \). In each of these instances the optimal replacement cycle is 6 years.

Not all depreciation schedules used in practice are monotonic, however. The ACRS schedules, for example, rise at first and then decline. The next example, illustrated in Figure 2, shows that depreciation schedules similar in shape to the ACRS schedules can arise if maintenance costs are nonlinear. In this case the discount rate is \( 0.9, p = 200,000, n = 25, s_0 = p, s_t = 0.7s_{t-1}, t=1,...,n \), \( m_1 = 5000 \), and \( m_t = 1.1m_{t-1}, t=2,...,n \). For this example, the optimal replacement cycle turns out to be 21 years.

**Stable Depreciation Schedules and Neutral Replacement Decisions**

Up to this point we have ignored the effect of taxes on the replacement decisions. To incorporate taxes into our analysis, we define \( \tau \) to be the income tax rate and \( \hat{\tau} \) to be the long-term capital gains tax rate. In addition, we assume that taxes on our capital investment are based on some imposed depreciation schedule \( d=[d_1,...,d_n] \) where \( d_j \geq 0 \) for all \( j=1,...,n \) and \( \sum_{j=1}^{n} d_j \leq p \).

This imposed depreciation schedule need not be the economic depreciation schedule we have defined; it could be some schedule enforced by the government like the ACRS schedule. We do not consider investment tax credits here. Such credits could have easily been added to the following discussion but their inclusion would have served only to complicate the exposition.

Under these conditions, the amount of income taxes paid in year \( j \) is \( \tau \) times the taxable income in year \( j \) less \( (d_j+m_j) \). Thus, the tax bill is reduced by \( \tau(d_j+m_j) \) and the effective after–tax cost in year \( j \) is \( \hat{m}_j(d) = m_j - \tau(d_j+m_j) = (1-\tau)m_j - \tau d_j \).

If the firm sells the machine in year \( j \), taxes are paid, at the rate \( \hat{\tau} \), on the capital gain of \( s_j - (p - \sum_{k=1}^{j} d_k) \). Therefore, the effective after–tax credit for selling in year \( j \) is
\[ \hat{s}_j(d) = s_j - \hat{\tau}[s_j - (p - \sum_{k=1}^{j} d_k)] = (1-\hat{\tau})s_j + \hat{\tau}(p - \sum_{k=1}^{j} d_k). \]

Hence, the economic life is given by:

\[ \ell(d) = \text{arg} \min_{1 \leq k \leq n} \frac{\{p - \delta_k \hat{s}_k(d) + \delta_m l_k(d) + \delta^2 m_{2k}(d) + \ldots + \delta^k m_{k}(d)\}}{(1 - \delta_k)}. \]  

(5)

Hereafter, \( \ell(d) \) will be denoted by \( \ell \). In addition, the annual equivalent cost, \( \tilde{y}_\ell(d) \), is described by:

\[ \tilde{y}_\ell(d) = \frac{(1-\delta)}{(1-\delta \ell)} \{p - \delta \hat{\ell} \hat{s}_\ell(d) + \hat{\delta} m_{1}(d) + \hat{\delta}^2 m_{2}(d) + \ldots + \hat{\delta}^k m_{k}(d)\}. \]  

(6)

Expressions (5) and (6) are modifications of expressions (1) and (2), reflecting the effect of the tax rates \( \tau, \hat{\tau} \), and the imposed depreciation schedule \( d \) on the replacement decision under the assumption that the replacement decision is based on after-tax cash flows. Note that the economic depreciation schedule, \( \tilde{d} \), will depend upon the imposed depreciation schedule, \( d \).

We now consider the issue of fairness of the depreciation schedule. If a firm could choose its depreciation schedule or bargain with the government for it, the firm would opt for a faster write-off of the purchase so as to realize earlier tax savings. In fact, Bellcore [3] has devised an economic theory of depreciation for telecommunications equipment so as to have a rational basis for negotiating with regulatory agencies for depreciation schedules with shorter lengths. In general, utilities can go before government authorities to argue for changes in tax structure, etc. So, a firm could argue for depreciation based on economic rationale if it was in their interest to do so. Given a set of tax rates and an imposed depreciation schedule \( d \), the firm can go through the same approach as before and determine the economic depreciation schedule \( \tilde{d}(d) \). Then, if \( \tilde{d}(d) \) is more favorable to the firm, the firm could argue for this schedule. Can the government impose a depreciation schedule that is stable in the sense that neither the firm nor the government can argue for a change based on economic grounds? In other words, is there an imposed schedule \( d \) identical to the resulting economic schedule, \( \tilde{d}(d) \)?
To address this question, we first compute the shadow prices \( \breve{y}_\ell(d)_j \), for \( j = 1, \ldots, \ell \), representing the value of owning a \( j \) year old machine when the tax rates are \( \tau \) and \( \hat{\tau} \) and the imposed depreciation schedule is \( d \). This results in the following modification of expression (3):

\[
\breve{y}_\ell(d)_j = \begin{cases} 
\hat{s}_\ell(d), & j = \ell \\
\breve{y}_\ell(d) - \delta_m j_1 (d) + \delta \breve{y}_\ell(d)_j + 1, & j = 1, \ldots, \ell - 1
\end{cases}
\]

As before, these prices define an economic depreciation schedule, given by:

\[
\bar{d}_\ell(d)_j = \begin{cases} 
p - \breve{y}_\ell(d)_j, & j = 1 \\
\breve{y}_\ell(d)_j - \breve{y}_\ell(d)_j, & j = 2, \ldots, \ell \\
0, & j = \ell + 1, \ldots, n
\end{cases}
\]

\( \bar{d}_\ell(d) \) is a depreciation schedule that represents the economic value of the machine under tax rates of \( \tau \) and \( \hat{\tau} \) and depreciation schedule \( d \). We then define a depreciation schedule \( d^\ast \) to be stable if \( d_\ell(d^\ast) = d^\ast \) for \( \ell = \ell(d^\ast) \). The economic depreciation schedule implied by \( d^\ast \), under tax rates \( \tau \) and \( \hat{\tau} \), is just \( d^\ast \) itself.

Demonstrating the existence of stable depreciation schedules requires some results from the theory of fixed points. As shown in [11], there is a point-to-set correspondence, \( \Phi \), whose fixed points are stable depreciation schedules. (A fixed point of a correspondence is a point \( d^\ast \) such that \( d^\ast \in \Phi(d^\ast) \).) The celebrated Kakutani Fixed Point Theorem [12] guarantees that fixed points of \( \Phi \) (and hence stable depreciation schedules) exist. Stable depreciation schedules can be calculated using any standard fixed point algorithm, and have three simple properties:

1. \( p - s_\ell = \sum_{k=1}^{\ell} d^\ast_k \).
2. \( d^\ast \) does not depend on \( \tau \).
3. \( d^\ast \) is fair.

Property (1) implies that the book value is equal to the salvage value in the replacement year. In turn, this implies that no capital gain or loss occurs at
replacement (property (2)). Property (3) implies that book value equals economic value.

It is also of interest to know whether the economic life for a stable depreciation schedule changes as \( \tau \) changes. This issue is important because of policy implications. If it is assumed that imposed depreciation schedules should be stable (because they are the only ones that are fair), does our tax system distort the economic decision? In other words, does the replacement year decision change if \( \tau \) changes? This is of interest because economists frequently argue, for reasons of economic efficiency, that our tax structure should not distort our economic decisions. For instance, if \( \ell = 10 \) for \( \tau = 0 \), while \( \ell = 20 \) for some \( \tau > 0 \), then the tax system would distort the economic decision by causing less frequent replacements of capital equipment than is economically efficient.

Now, it is clear that the economic decision, assuming the imposition of a stable depreciation schedule, will not change as the tax rate is varied from 0 up to some \( \tau^* \) between 0 and 1. This follows by continuity arguments (all functions are continuous in \( \tau \)). This does not mean that the stable depreciation schedule remains unchanged, only that the year of replacement remains unchanged. In this range, the economic decision is said to be tax neutral. Let \( \tau^* \) denote the maximum neutral income tax rate. If \( \tau^* = 1 \) then the economic life, under a stable depreciation schedule, would be independent of the tax rate and hence any tax rate would be economically neutral. However, as the following numerical example shows, this will not always be the case.

Let \( n = 25 \), \( p = 1500 \), \( \delta = 0.9 \), \( \hat{\tau} = 0.1 \), and \( m_t = m_1 + h(t-1) \), \( t = 1, 2, \ldots, n \), where \( m_1 = 500 \) and \( h = 50 \). Also, let \( s_t = s_1/(1.1^{t-1}) \), \( t = 1, 2, \ldots, n \), where \( s_1 = 800 \). For \( 0 \leq \tau \leq 0.525 \), \( \ell = 7 \). Thus, the replacement decision is neutral for a tax rate up to 52.5%. See Table 1 for a summary of these results. Note that, in this example, the schedules are all monotonically decreasing. Also, the economic life, under the imposition of stable depreciation schedules, increases in this example as \( \tau \) increases. We conjecture that the economic life is a nondecreasing function of \( \tau \) in general. After all, as \( \tau \to 1 \), the government allows the firm to write-off all maintenance costs, thereby effectively subsidizing the firm to keep the machine.
Table 1

\[ \tau \quad \ell \quad d^\tau (\text{stable depreciation schedule}) \]

<table>
<thead>
<tr>
<th>\tau</th>
<th>\ell</th>
<th>\quad (229.79, 208.16, 183.86, 156.56, 125.89, 91.43, 52.72)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>7</td>
<td>\quad (222.39, 203.28, 181.51, 156.72, 128.49, 96.33, 59.71)</td>
</tr>
<tr>
<td>0.3</td>
<td>7</td>
<td>\quad (213.27, 197.12, 178.41, 156.73, 131.61, 102.50, 68.77)</td>
</tr>
<tr>
<td>0.4</td>
<td>7</td>
<td>\quad (201.79, 189.16, 174.19, 156.44, 135.41, 110.49, 80.95)</td>
</tr>
<tr>
<td>0.5</td>
<td>7</td>
<td>\quad (186.94, 178.48, 168.14, 155.30, 140.06, 121.19, 98.12)</td>
</tr>
<tr>
<td>0.525</td>
<td>7</td>
<td>\quad (182.53, 175.22, 166.21, 155.09, 141.37, 124.44, 103.55)</td>
</tr>
<tr>
<td>0.6</td>
<td>8</td>
<td>\quad (164.05, 159.62, 153.97, 146.73, 137.49, 125.69, 110.60, 92.32)</td>
</tr>
<tr>
<td>0.7</td>
<td>9</td>
<td>\quad (132.96, 132.21, 131.18, 129.76, 127.82, 125.16, 121.51, 116.52, 109.67)</td>
</tr>
<tr>
<td>0.8</td>
<td>15</td>
<td>\quad (89.96, 89.93, 89.89, 89.83, 89.74, 89.59, 89.37, 89.02, 88.47, 87.62, 86.29, 84.24, 81.03, 76.05, 68.31)</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

We have studied the interplay between tax rates, optimum replacement decisions, and economic depreciation schedules in the context of a machine replacement problem. We have demonstrated the existence of depreciation schedules that are stable in the sense that neither firm nor government would argue for change based on economic rationale. We have also shown that a range of tax rates exists where the replacement decision, based on stable depreciation schedules, does not change. However, this range does not extend to unity in general. Further note that our model and results can easily be extended to the case where machines are subject to stochastic deterioration. This suggests a concept of expected depreciation. See Jones, Zydiak, and Hopp [10].

Our model addresses one simple class of problems. Other models have been used in more specific situations (e.g., the work done in [3] relative to the telecommunications industry). The advantage of our method is that it can be applied directly (with modifications) to the analysis of depreciation schedules that are set individually for specific types of equipment, e.g., utilities. Our model does not, however, explicitly address many of the institutional issues
arising from specific tax structures. These are important issues, but their inclusion in this paper would obscure the central idea.

In situations where, for reasons of practicality, depreciation schedules are to be set for a wide class of equipment, no imposed schedule can be fair for all firms and all equipment. This is because the costs differ among types of assets and characteristics of firms (like tax position) also differ. Further work is needed to determine "how fair" imposed blanket schedules are and whether defining more categories of equipment is a desirable public policy option. To do this, data pertaining to depreciable assets (capital cost, maintenance costs, etc.) must be collected and methods like those in this paper used to determine depreciation schedules for classes of assets. The objective would be to determine how many classes are needed and how close current schedules are to economic schedules. The resulting knowledge about stable depreciation schedules would help gauge the extent to which accelerated depreciation schedules, such as ACRS, are actually accelerated.

Finally, it should be noted that linear programming is an attractive method for performing computations associated with these issues because it can generate dual prices. Other optimization methods, such as dynamic programming, can also be used in certain problems and may be attractive in certain circumstances.

REFERENCES


