Intangible Assets and Cross-Sectional Stock Returns: Evidence from Structural Estimation∗

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Abstract

We augment a $q$-theory model with intangible assets where investments in intangible assets incur adjustment costs and the accumulation of intangible assets leads to investment-specific technological change. The key parameters of the model are estimated using cross-sectional return data and the Generalized Method of Moments (GMM). Our results show that the intangible-assets-augmented $q$-theory model explains significantly better the value premium and the return differences among firms with different R&D intensities than the $q$-theory model with only tangible assets. Adjustment cost is estimated to be convex in intangible investments and is a more important determinant of cross-sectional stock returns than the investment-specific technological change effect. The magnitude of the adjustment costs of intangible investments is much larger than that of tangible investments, implying higher difficulty of rapid accumulation of intangible assets compared to tangible assets. This finding indicates that having a superior history of investments in intangible asset is crucial for firms to sustain their comparative advantages and provides a rationale for the higher persistence of R&D investments than the persistence of tangible investments observed in the data.

JEL Classification: G12, E21, D24, O31, O32

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1 Introduction

Intangible assets have been widely recognized as the driving force for productivity growth and are largely responsible for the difference between a firm’s book value and market value of assets. A large literature has been devoted to study the characteristics of intangible assets and how intangible assets affect the growth of the economy. However, less attention has been paid to the impact of intangible assets on stock returns, with the exception of Chan, Lakonishock, and Sougiannis (2001) among others. At the same time, to our knowledge, all the studies on the features of intangible assets use either aggregate or firm-level production data, neglecting the potentially more reliable financial data. In this paper, we build a \( q \)-theory model with both tangible and intangible assets, where investments in both types of assets incur adjustment costs and the accumulation of intangible assets leads to investment-specific technological change. We derive and test the impacts of intangible assets on cross-sectional stock returns using the Generalized Method of Moments (GMM). Moreover, we quantify various features of intangible assets based on the estimation of the intangible-assets-augmented \( q \)-theory model using stock return data.

Our research interest is two-folded. First, we investigate whether intangible asset is an important determinant of firm’s stock returns. Since the market price of a firm reflects the value of both tangible and intangible assets, we naturally expect an intangible-assets-augmented \( q \)-theory to explain stock returns better than the \( q \)-theory with only tangible assets. Especially, the augmented \( q \)-theory should do a better job in explaining return differences among firms that have different book-to-market ratios, i.e., the value effect, because the amount of intangible assets that a firm has relative to tangible assets is likely to be the main determinant of the firm’s book-to-market ratio. For the same reason, we expect the intangible-assets-augmented \( q \)-theory model to better explain the cross-sectional return differences among firms with different R&D intensities. Our empirical results indeed show that by adding intangible assets to the \( q \)-theory model, we are able to significantly improve the explanatory power of the model in the aforementioned two areas, especially the value effect.

Our second research interest is to infer the characteristics of intangible investments and intangible assets from financial data. By estimating the model parameters using the GMM, we are able to quantify the economic significances of the two key features of intangible asset/capital that are incorporated in our model. First, the accumulation of intangible assets leads to investment-specific technological change. Greenwood, Hercowitz, and Krusell (1997) show that a calibrated time series of technological changes plays an important role in explaining the postwar U.S. productivity growth. In their model, technological advances increase the productivity of tangible investments in
producing new capitals and reduce the price of tangible assets over time. Since the direct impact of such a technological change is on the productivity of investments, it is called investment-specific technological change (ISTC). Huffman (2007) models firm’s investment in intangible assets and endogenizes the investment-specific technological changes. We adopt his approach in our model.

The second feature of intangible assets that we explore in this paper is the accumulation process of intangible assets. High adjustment costs prevent rapid accumulation of intangible assets and allow firms that have superior investment histories to sustain their competitive advantages. The magnitude of the adjustment costs of intangible investments hence shapes the evolution of the market structure of an industry and determines the persistence of its incumbents’ profitability. Incorporating the adjustment costs of intangible investments into the $q$-theory model allows us to study its impact on cross-sectional stock returns and at the same time to quantify its magnitude using return data, which is likely to have less measurement errors than the production data.

We estimate three $q$-theory models: (1) a $q$-theory model with only tangible assets and two parameters to estimate (the Q2 model); (2) an intangible-assets-augmented $q$-theory model with investment-specific technological change. We name it the Q4_ISTC model and estimate four parameters. (3) an intangible-assets-augmented $q$-theory model with adjustment costs of intangible investments. We name it the Q4_AC model and estimate four parameters. In all three models, we assume quadratic adjustment costs for tangible investments.

We derive a firm’s levered investment returns for all three models and estimate the parameters of the models using three sets of testing portfolios: ten book-to-market ($B/M$) portfolios, ten R&D-to-intangible-assets ($I_u/K_u$) portfolios, and ten investment-to-capital ($I/K$) portfolios. We use research and development (R&D) expenses as a proxy for intangible investments due to data availability. Our results show that the adjustment costs of intangible investments have a larger impact on asset returns than the ISTC effect of intangible assets. The Q4_AC model explains the value effect significantly better than other models, including the CAPM, the Fama-French 3-factor model, Q2 model and Q4_ISTC model. Portfolios sorted on the $I_u/K_u$ ratio show large return spreads and the Q4_AC again explains significant better than other models. For portfolios sorted on the $I/K$ ratio, adding intangible assets in the model seems to improve the explanatory power of the model much less.

Moreover, in both models with intangible assets, we find that the adjustment cost of intangible investments per unit of investment ratio is at least one and a half times larger than that of tangible investments. Our finding supports the argument that it is more difficult to accumulate intangible assets rapidly than tangible assets. For a firm to sustain its competitive advantage over its rivals,
it is crucial to maintain a persistent investment flow in intangible assets. Our finding provides an explanation to the empirical finding that the persistence of R&D expenses is three times larger than the persistence of capital investment (Bloom (2007)).

Our paper contributes to the literature that tries to explain cross-sectional returns using investment-based \(q\)-theory model, pioneered by Cochrane (2001), by introducing intangible assets to the traditional \(q\)-theory model. We follow the methodology in Liu, Whited, and Zhang (2009), who estimate a structural \(q\)-theory model with only tangible assets. Belo, Xue, and Zhang (2010) explore the importance of the curvature of the tangible investment adjustment costs in explaining the cross-sectional returns. We show that intangible assets play an important role in explaining value premium and R&D related cross-sectional return patterns. There is a literature that specifically studies the impact of R&D on stock returns. Hsu (2009) shows that technological innovations increase risk premium at the aggregate level using patent data and R&D data. Chen, Lakonishok, and Sougiannis (2001) documents the positive relation between R&D intensity and stock returns. Li (2007) shows that such a positive relation mainly exists among R&D intensive firms. Lin (2009) tries to explain this relation using a dynamic model with investment-specific technological change. Our paper adds the adjustment costs of intangible assets to the model and shows, through the GMM estimations, that it is the most important component for explaining asset returns.

Our paper also contributes to the literature that studies the distinctive features of intangible assets. The macroeconomics literature focuses on how the investment-specific technological change affects productivity growth, e.g., Greenwood, Hercowitz, and Krusell (1997), while a large literature in organization science and evolutionary economics is devoted to understand how the accumulation process of intangible assets shapes the structure of an industry and the survival rate of new entrants into the industry, e.g., Knott, Bryce and Posen (2003). Previous studies use aggregate and firm-level production data and to our knowledge, our paper is the first one to investigate the characteristics of intangible assets using asset return data. Our results show that the adjustment costs of intangible investments is a main determinant of firm’s stock returns, while the investment-specific technological changes due to investment in intangible assets are less important. Moreover, the adjustment costs of intangible assets are estimated to be much higher than the adjustment costs of tangible assets.

The paper is organized as follows. Section 2 sets up the model. Section 3 derives the investment return. Section 4 describes the three models that we estimate. Section 5 constructs the data set and the testing portfolios. Section 6 describes the empirical tests and the estimation results. Section 7 concludes. Proof of the proposition is given in the Appendix.
2 The Model Setup

Assume that firm faces infinite horizon and the time is discrete. Firm’s production requires both tangible and intangible capital/assets in addition to non-capital input. Let $y_{jt}$ denotes the revenue of firm $j$ at time $t$

$$y_{jt} = e^{X_t} \left[ (k^m_{jt})^\gamma (k^u_{jt})^{1-\gamma} \right]^{\alpha} (L_{jt})^{1-\alpha}$$

where $k^m_{jt}$ is the capital stock of tangible assets, $k^u_{jt}$ is the capital stock of intangible assets, and $X_t$ is the exogenous productivity shock. Without loss of generality, let $L_{jt}$ be the composite non-capital factor input and assume that firm $j$ is a price taker in the input market. $\alpha$ is the capital (including both tangible and intangible) share of total output, and $\gamma$ is the elasticity of substitution between tangible and intangible assets. We assume constant-return-to-scale Cobb-Douglas production function. The accumulations of tangible assets and intangible assets follow

$$k^u_{jt+1} = (1 - \delta_{u,jt})k^u_{jt} + i^u_{jt}$$
$$k^m_{jt+1} = (1 - \delta_{m,jt})k^m_{jt} + \Theta(i^m_{jt}, k^u_{jt})$$

where $i^m_{jt}$ and $i^u_{jt}$ are the investments made by firm $j$ at time $t$ in tangible assets and intangible assets, respectively, and $\delta_{m,jt}$ and $\delta_{u,jt}$ are the corresponding depreciation rates. Both tangible and intangible investments are produced using final outputs. $\Theta$ is the production function of the new tangible assets and defined as

$$\Theta_{jt} \equiv \Theta(i^m_{jt}, k^u_{jt}) = \left[ a_1 (i^m_{jt})^\xi + a_2 (k^u_{jt})^\xi \right]^{1/\xi}$$

with positive constants $a_1$ and $a_2$. When $0 < \xi < 1$, the capital production function displays the following properties:

$$\frac{\partial \Theta_{jt}}{\partial i^m_{jt}} > 0 \quad \frac{\partial \Theta_{jt}}{\partial k^u_{jt}} > 0 \quad \frac{\partial^2 \Theta_{jt}}{\partial i^m_{jt} \partial k^u_{jt}} > 0 .$$

The third inequality implies that the productivity of the tangible investments increases with the stock of the intangible assets that firm $j$ has accumulated in the past. The production function of tangible assets can be rewritten as

$$\Theta_{jt} = i^m_{jt} \left[ a_1 + a_2 \left( \frac{k^u_{jt}}{i^m_{jt}} \right)^\xi \right]^{1/\xi} \equiv i^m_{jt} Q^m_t$$
where $Q^m_t$ is the amount of new tangible assets that can be produced from one unit of investment at time $t$

$$Q^m_t = \left[a_1 + a_2 \left( \frac{k^u_{jt}}{i^m_{jt}} \right) \xi \right]^{1/\xi}.$$ 

The time series of $Q^m_t$ represents the investment-specific technological changes. We can see that as firm accumulates more intangible assets, tangible investment is more productive in generating new capital, or equivalently, the effective price of the new tangible capital, $1/Q^m_t$, decreases. Our formulation of the production of tangible assets captures the intuition that technology changes make the new equipment either less expensive or more productive than the old equipment. Gordon (1990) constructs a price index for quality-adjusted equipment relative to consumption goods. Incorporating such an index into a calibrated general equilibrium model, Greenwood, Hercowitz, and Krusell (1997) show that the investment-specific technological change accounts for the major part of post-war U.S. growth. Following Huffman (2007) and Lin (2009), our formulation of tangible assets production function endogenizes the technological changes as a result of firm’s investments in intangible assets.

Both investments in tangible assets and in intangible assets incur adjustment costs

$$\Phi^m_{jt} \equiv \Phi^m \left(i^m_{jt}, k^m_{jt} \right) = \frac{a}{2} \left( \frac{i^m_{jt}}{k^m_{jt}} \right)^{\rho} k^m_{jt}$$

$$\Phi^u_{jt} \equiv \Phi^u \left(i^u_{jt}, k^u_{jt} \right) = \frac{b}{2} \left( \frac{i^u_{jt}}{k^u_{jt}} \right)^{\psi} k^u_{jt}$$

If $\rho$ and $\psi$ being larger than 1, the adjustments costs are convex in the investment ratios:

$$\frac{\partial \Phi^m_{jt}}{\partial \left( i^m_{jt} / k^m_{jt} \right)} > 0; \quad \frac{\partial^2 \Phi^m_{jt}}{\partial \left( i^m_{jt} / k^m_{jt} \right)^2} > 0; \quad \frac{\partial \Phi^u_{jt}}{\partial \left( i^u_{jt} / k^u_{jt} \right)} > 0; \quad \frac{\partial^2 \Phi^u_{jt}}{\partial \left( i^u_{jt} / k^u_{jt} \right)^2} > 0.$$ 

In all three models that we estimate, we set $\rho$ equal to two to be consistent with the literature and estimate the value of $\psi$.

Firms are allowed to have both equity and debt financing. Assume that there are no external financing costs. Following Hennessy and Whited (2007) and Liu, Whited, and Zhang (2009), we assume that firm issues one-period debt. The debt outstanding at the beginning of period $t$ is $B^t_{jt}$ with the gross required return $r^B_{jt}$. At the end of period $t$, firm $j$ issues new debt $B^t_{jt+1}$. The net cash flow accrued to the shareholders of firm $j$ at period $t$ is

$$D^S_{jt} = (1-\tau_{jt}) \left( y_{jt} - \omega_t L_{jt} - \Phi^m_{jt} - \Phi^u_{jt} - i^m_{jt} - i^u_{jt} + \tau_{jt} \delta_{m,jt} k^m_{jt} - [1 + (r^B_t - 1)(1-\tau_t)] B_{jt} + B^t_{jt+1}.$$
where \( \varpi_t \) is the price on non-capital input and \( \tau_{jt} \) is the corporate tax rate on firm \( j \) at time \( t \).

3 Investment Return

We solve the maximization problem of a representative firm \( j \) and write its investment return as a function of firm’s observable characteristics. To simplify the notation, we omit subscript \( j \) in all the equations where no ambiguity is present.

**Proposition 1.** Firm’s investment return \( r_{t+1}^I \), defined as

\[
r_{t+1}^I = \left\{ (1 - \tau_{t+1}) \frac{\gamma_{t+1}}{k_{t+1}^m} + \tau_{t+1} \delta_m - (1 - \tau_{t+1}) \Phi_{k,t+1}^m + (1 - \delta_m) \frac{1 + (1 - \tau_{t+1}) \Phi_{k,t+1}^m}{\Theta_{t,t+1}} \right\} \\
+ \left\{ 1 + (1 - \tau_{t+1}) \Phi_{k,t+1}^m \right\} \left( \frac{\Theta_{k,t+1}}{\Theta_{t,t+1}} \right) - (1 - \tau_{t+1}) \Phi_{k,t+1}^u + (1 - \delta_u) (1 - \tau_{t+1}) (1 + \Phi_{u,t+1}^u) \right\} \\
\times \left( \frac{k_{t+1}^u}{k_{t+1}^m} \right) \left\{ 1 + (1 - \tau_{t+1}) \Phi_{k,t+1}^m \right\} + (1 - \tau_{t}) \left( 1 + \Phi_{u,t+1}^u \right) \left( \frac{k_{t+1}^u}{k_{t+1}^m} \right),
\]

satisfies

\[ E_t [M_{t+1} r_{t+1}^I] = 1 \]

where \( M_{t+1} \) is the stochastic discount factor from \( t \) to \( t + 1 \). \( r_{t+1}^I \) is equal to the weighted average of the return on firm’s equity and the after-tax return on its debt,

\[ r_{t+1}^I = (1 - w_t) r_{t+1}^S + w_t r_{t+1}^{Ba}, \]

where \( w_t \) is the ratio of debt value to firm value at the end of period \( t \)

\[ w_t = \frac{B_{t+1}}{P_t - D_t^S + B_{t+1}}, \]

\( r_{t+1}^S \) is stock return from period \( t \) to \( t + 1 \)

\[ r_{t+1}^S = \frac{P_{t+1}}{P_t - D_t^S}, \]

\( r_{t+1}^{Ba} \) is the after-tax return on debt

\[ r_{t+1}^{Ba} = r_{t+1}^B - \tau_{t+1} \left( r_{t+1}^B - 1 \right), \]
and $\Phi_{*}^{m}$ is the derivative of the adjustment cost function of tangible assets w.r.t. variable $*$. Similar definitions for $\Phi_{*}^{u}$ and $\Theta_{*}$. 

**Proof.** See Appendix A.

To understand the economics behind Proposition 1, we decompose firm’s investment return into two components: the return on tangible assets $r_{I,m}^{t+1}$, defined as

$$
\begin{align*}
\left. \frac{\left(1 - \tau_{t+1}\right) \left[ \alpha \gamma y_{t+1} - \Phi_{k,t+1}^{m} \right] + \tau_{t+1} \delta_{m} \right] \Theta_{i,t} + (1 - \delta_{m}) \left[ 1 + (1 - \tau_{t+1}) \Phi_{i,t+1}^{m} \right] \left( \frac{\Theta_{i,t}}{\Theta_{i,t+1}} \right) \\
\left. 1 + (1 - \tau_{t}) \Phi_{i,t}^{m} \right] 
\end{align*}
\tag{7}
$$

and the return on intangible assets $r_{I,u}^{t+1}$, defined as

$$
\begin{align*}
\left. (1 - \tau_{t+1}) \left[ (1 - \alpha) \frac{\gamma y_{t+1}}{k_{t+1}} - \Phi_{u,k,t+1}^{m} \right] + (1 - \delta_{u}) (1 - \tau_{t+1}) \left( 1 + \Phi_{i,t+1}^{u} \right) \\
+ \left. \left[ 1 + (1 - \tau_{t+1}) \Phi_{i,t+1}^{m} \right] \left( \frac{\Theta_{k,t+1}}{\Theta_{i,t+1}} \right) \right] \\
\left. (1 - \tau_{t}) \left( 1 + \Phi_{i,t}^{u} \right) \right] 
\end{align*}
\tag{8}
$$

Portfolio theory tells us that firm’s investment return should be a weighted average of its investment return on tangible assets and its investment return on intangible assets, with the weights being the ratio of the market value of tangible assets to total firm value and the ratio of the market value of intangible assets to total firm value, respectively. The proof is straightforward and is provided in Appendix A.

For both tangible and intangible investment returns, the return from $t$ to $t + 1$ is a ratio of marginal benefit at $t + 1$ of one more unit of investment made at $t$ to its marginal costs at time $t$. The marginal benefits include not only the marginal free cash flow at $t + 1$, but also the marginal continuation value at time $t + 1$. The marginal costs include the price of one unit of investment, which is normalized to one for both types of investments, and the marginal adjustment costs of investment.

For tangible investments, the denominator in equation (7) is the cost of one more unit of tangible investment at time $t$. The first term in the numerator is the marginal cash flow generated at time $t + 1$ from the incremental tangible capital, including the marginal revenue, marginal investment adjustment costs, and the marginal tax benefits from capital deprecation. Note that the complementary effect from intangible capital on tangible investment return is reflected in the term $\Theta_{i,t}$, which is the marginal productivity of tangible investments. It is the amount of new capital produced from one unit of investment made at time $t$, which not only depends on the current tangible capital level and tangible investments, but also the amount of accumulated
intangible capital. The higher the intangible capital the firm has, the higher the return of tangible investments.

The second term in the numerator of equation (7) is the incremental continuation value from $\Theta_{i,t}$ unit of new capital. The marginal continuation value is the shadow price of one unit of tangible capital at time $t + 1$, which as shown in Appendix A can be written as

$$q_{t+1}^m = \frac{(1 - \tau_{t+1}) \left(1 + \Phi_{i,t+1}^m\right)}{\Theta_{i,t+1}},$$

times $(1 - \delta_m)\Theta_{i,t}$, the undepreciated incremental capital from one unit of tangible investment made at time $t$. Because of the convexity of the adjustment costs function of tangible investments, higher investment ratio at time $t + 1$ means a lower value of tangible assets. Higher level of accumulated intangible assets also lowers the shadow price of the tangible assets because new capital gets cheaper as the technology advances.

We can see that technological changes have conflicting effects on the return of tangible assets. On one hand, technological advances increases the productivity of tangible investment, which raises its return; on the other hand, it reduces the price of new capital and hence the marginal continuation value of tangible investments, leading to a lower tangible investment return.

Similarly, the return on intangible assets in equation (8) is a ratio of marginal benefits of intangible investments at time $t + 1$ to its marginal costs at time $t$. The marginal costs, i.e., the numerator, include the price of investment goods itself, which is normalized to one, and the corresponding adjustment costs. The marginal productivity of intangible investment in producing intangible assets is one as opposed to $\Theta_{i,t}$ in equation (7) for the return of tangible investments.

The numerator of equation (8) is the marginal benefits at time $t + 1$ of investment made at time $t$. The first term is the marginal cash flow generated at time $t + 1$, including the marginal revenue and marginal adjustment costs. The marginal continuation value has two parts as incorporated in the second term and the third term. The second term is the value of the incremental intangible assets itself, equal to the shadow price of intangible assets at $t + 1$, given by

$$q_{t+1}^u = (1 - \tau_t) \left(1 + \Phi_{i,t}^u\right),$$

times the undepreciated incremental intangible capital $1 - \delta_u$. In addition, the new intangible capital from intangible investments increases the amount of tangible assets that per unit of tangible investment can produce at $t + 1$. The third term reflects the value of that incremental tangible assets.
Finally, firm’s investment return also depends on the relative weights of tangible assets and intangible assets that the firm has. If a firm has more intangibles assets, its return on intangible assets will have larger impact on the overall return of the firm and vice versa. The similar argument applies to tangible assets. In general, the higher the ratio of intangible assets to tangible assets, the more important the return of intangible investment and *vice versa*.

4 Test Design and Econometric Methodology

To investigate the importance of each feature of intangible assets in explaining cross-sectional stock returns, we construct and estimate three $q$-theory models. We use the $q$-theory model with only tangible assets in Liu, Whited, and Zhang (2009) as the benchmark model. Following their specification, we set $\rho = 2$ so that the adjustment cost is quadratic in tangible investments. We call this benchmark the Q2 model since there are only two parameters, $a$ and $\alpha$, to be estimated.

In addition, we construct two types of intangible-assets-augmented $q$-theory models. The first one, named as the Q4 ISTC model, investigates the importance of the investment-specific technological change (ISTC). We shut down the effects from the adjustment costs of intangible assets by setting parameter $b$ to be zero. We normalize $a_1$ to be one to reduce the number of estimated parameters. There are four parameters to be estimated: $a$, $\alpha$, $a_2$, and $\xi$. If ISTC is an important factor for stock returns, we should find $a_2$ to be significantly different from zero. For the productivity of tangible investment to be increasing with the amount of intangible assets, we need $a_2$ to be positive and $\xi$ to lie between zero and one.

The second intangible-assets-augmented $q$-theory model is named as Q4 AC and investigates the importance of the adjustment costs of intangible assets. We set $a_2$ to be zero to shut down the ISTC effect. The value of $\xi$ becomes irrelevant and we set it to be one. Same as the Q4 ISTC model, we need to estimate four parameters: $a$, $\alpha$, $b$, and $\psi$. The value of $b$ indicates the magnitude of the adjustment costs of intangible investments as the value of $a$ indicates the magnitude of the adjustment costs of tangible investments. If $\psi$ is larger than one, the adjustment costs are convex in the intangible investments, implying that intangible investments become increasingly costly as firm tries to grow its intangible assets rapidly. By comparing the magnitude of $a$ and $b$, we can infer which type of asset, intangible or tangible, is more difficult to accumulate rapidly and hence is more crucial for the sustainability of a firm’s comparative advantage against its rivals.

The impact of intangible assets on stock returns and through which effect are gauged by the specific model’s ability to explain cross-sectional stock returns. Define the levered investment
return as
\[ r_{t+1}^{Iw} = \frac{r_{t+1}^I - w_t r_{t+1}^{Ba}}{1 - w_t} \]

Proposition 1 implies that for any firm, at any period, and in any state of the world, its stock return and its levered investment return should be the same, i.e.,
\[ r_{t+1}^S = r_{t+1}^{Iw} = \frac{r_{t+1}^I - w_t r_{t+1}^{Ba}}{1 - w_t}. \] (9)

Based on equation (6), firm’s levered investment can be calculated using the observed firm characteristics, which is the model predicted stock return. The difference between the predicted and the observed stock returns is used to measure the performance of the model. Moreover, we estimate the set of model parameters that minimizes this difference. For the reasons explained in Liu, Whited, and Zhang (2009), we use portfolio returns to test our models in stead of individual firm returns.

We follow the methodology in Liu, Whited, and Zhang (2009) and use one-stage GMM to estimate the models. For each portfolio \( i \), we define the model error \( e_i \) from the following moment condition
\[ e_i = \mathbb{E}_T [r_{it+1}^S - r_{it+1}^{Iw}] \] (10)
where \( r_{it+1}^S \) is the observed stock return series of portfolio \( i \), \( r_{it+1}^{Iw} \) is derived from firm characteristics using equation (6), and \( \mathbb{E}_T \) is the sample mean of the series in the bracket. Both measurement errors and specification errors contribute to the model error \( e_i \). We estimate the aforementioned parameters for each model using one-stage GMM with identity matrix to minimize the equally weighted average of \( e_i \) using a set of testing portfolios.\(^1\)

5 Data

Our sample of firm level data is from COMPUSTAT for accounting variables and CRSP for stock return variables. The sample period is between 1975 and 2007. We start from 1975 because firms have discretion on how to report their R&D expenses prior to 1974. The accounting treatment of R&D becomes standard after 1974 when FASB issued SFAS No.2 to require the full expending of R&D outlays in financial reports of public firms. The financial firms (SIC code between 6000 and 6999) and regulated utilities (SIC code between 4900 and 4999) are excluded from our sample.

We use thirty testing portfolios: ten book-to-market (B/M) portfolios, ten investment-to-

\(^1\)Liu, Whited, and Zhang (2009) provides details on how to conduct the tests.
capital \((I/K)\) portfolios, and ten R&D-to-intangible-assets \((Iu/Ku)\) portfolios, where \(B\) refers to the book value of tangible assets, \(M\) refers to the market value of the firm, \(I\) refers to tangible investments, \(K\) refers to the level of tangible assets, \(Iu\) refers to intangible investments proxied by R&D expenses, and \(Ku\) refers to the level of intangible assets.

To construct the ten \(B/M\) portfolios, we follow Fama and French (1993). In June of year \(t\), we sort all the stocks into ten portfolios by their book-to-market ratios. The book value of equity (Compustat Data 60) is measured at the fiscal year ending in calendar year \(t - 1\) and market value of equity is measured at December of calendar year \(t - 1\) and the data is from CRSP. The breakpoints are based on the NYSE firms only. We hold the equal-weighted portfolios from July to next June and record the buy-and-hold annual returns.

We form ten \(I/K\) portfolios using investment (Data 128 - Data 107) to capital (Data 8) ratio as the sorting variable. Investment is from the fiscal year ending in calendar year \(t - 1\), while capital is measured at the beginning of the same fiscal year. We construct ten \(Iu/Ku\) portfolios similarly. R&D (Data 46) is commonly used to proxy for intangible investments in the literature due to data availability reason (Lev (2001)). To measure the level of intangible assets, we assume a 20\% annual depreciation rate for intangible assets and consider only the intangible investments from the past five year. Lev and Sougiannis (1996) estimate the impact of the current and past R&D expenses on earnings. They show that the horizon of the impact varies across industries from five years to nine years. We take a low end of five years in order to keep as many observations as possible. Following Chan, Lakonishok, and Sougiannis (2001), we calculate the level of intangible assets as follows:

\[
Ku_{t-1} = R&D_{t-2} + 0.8 R&D_{t-3} + 0.8^2 R&D_{t-4} + 0.8^3 R&D_{t-5} + 0.8^4 R&D_{t-6}.
\]

For each firm in the holding portfolios, we record its accounting variables in order to construct the investment return based on equation (6). Tangible investments, intangible investments, and capital and intangible assets are constructed as mentioned above. Depreciation rate of tangible asset is estimated to be the mean of depreciation (Data 14) to capital (Data 8) ratio over time series for each portfolio. Output is proxied by sales (Data 12) and total debt is the sum of short term debt (Data 34) and long term debt (Data 9). We follow Liu, Whited, and Zhang (2009) for the time alignment. The flow variables reflecting the economic behavior over one fiscal year are measured at the beginning of fiscal year while the stock variables are measured at the beginning of fiscal year end.
6 Empirical Results

6.1 Summary statistics on portfolio returns

Table 1 reports summary statistics of returns for all thirty testing portfolios. We report the means of the portfolio returns and the model errors (the intercepts) of the CAPM model and the Fama-French 3-factor model with their associated $t$-statistics. Panel A reports the results for the ten $B/M$ portfolios. The annual return is monotonically increasing with the sorting variable, the $B/M$ ratio. The value premium (the spread between the highest $B/M$ firms and the lowest $B/M$ firms) is 19.26% per annum. The CAPM model cannot explain the value premium. Five out of the ten $B/M$ portfolios have significant non-zero alphas and the alpha of the high-minus-low portfolio is 22% with $t$-statistics being 5.43. Neither can the Fama-French 3-factor model fully explain the value premium. The Fama-French alpha for the high-minus-low portfolios is 8.41% and remains statistically significant with $t$-statistic being 2.33.

Panel B reports the results for the ten $Iu/Ku$ portfolios. Since we need the past five year’s data to construct $Ku$, the portfolio returns start from 1980. We find that the portfolio returns are decreasing with the $Iu/Ku$ ratio and the return spread across portfolios is economically and statistically significant. The average annual return spread between the highest $Iu/Ku$ firms and the lowest $Iu/Ku$ firms is $-11.09\%$. The CAPM alpha of the high-minus-low portfolio is $-11.60\%$ per annum with $t$-statistics being $-3.36$ and the Fama-French alpha is $-5.52\%$ per annum with $t$-statistics being $-1.20$. To our best knowledge, our paper are the first one to document the aforementioned results on the $Iu/Ku$ portfolios, which are seemingly the opposite of what the previous literature finds, i.e., the positive R&D-return relation. However, this discrepancy is due to the different sorting variable used in previous studies. Chan, Lakonishok, and Sougiannis (2001) and Li (2009), among others, use R&D scaled by the market value of equity, i.e., the $Iu/(M-B)$ ratio, as the sorting variable and find the portfolio returns are increasing with $Iu/(M-B)$. Since the $Iu/(M-B)$ ratio can be decomposed into a product including both $Iu/Ku$ and $B/M$ ratios, we suspect that their results reflect the mixture of the R&D investment effect and the book-to-market effect, with both effects are opposite to each other. To ensure that our result on the $Iu/Ku$ portfolios is not due to the sample differences, we replicate their tests within our sample and find similar patterns.\footnote{We also need $Ku$ to construct investment return for other portfolios. When $Ku$ is not part of the sorting variable, the measurement errors should be spread out across ten portfolios and be less of a problem for the $B/M$ and $I/K$ portfolios. We thus report results with 1975 being the starting year for the $B/M$ and $I/K$ portfolios in order to keep more observations.}

\footnote{The replicated results are not reported here and are available upon request.}
Panel C reports the results for the ten $I/K$ portfolios. Consistent with the results documented in Titman, Wei, and Xie (2004), firms with high tangible investments have lower returns. The average annual return spread between the highest $I/K$ firms and the lowest $I/K$ firms is $-19.06\%$. The CAPM alpha of the high-minus-low portfolio is $-19.09\%$ per annum with $t$-statistic being $-5.03$ and the Fama-French alpha is $-6.05\%$ per annum with $t$-statistic being $-1.57$.

In summary, all three sets of testing portfolios show statistically and economically significant return spreads, which cannot be fully explained by either the CAPM model or the Fama-French 3-factor model. More important, with comparable sorting variables, $Iu/Ku$ and $I/K$, the relation between intangible investments and stock returns and the relation between tangible investments and stock returns are strikingly similar, opposite of what the previous literature claims.

6.2 Summary statistics on firm characteristics

Table 2 reports the summary statistics of firm characteristics across the $B/M$, $Iu/Ku$, and $I/K$ portfolios in Panel A, B, and C, respectively. Across the ten $B/M$ portfolios, growth firms have higher current and future $I/K$ ratios and lower $I/K$ growth rates, compared to value firms. The spreads on current and future $I/K$ ratios and $I/K$ growth rate are $0.08$ ($t$-statistic = 8.67), $0.07$ ($t$-statistic = 9.12), and $0.04$ ($t$-statistic = 1.21), respectively. Once again, we find striking similarity between tangible investments and intangible investments. Growth firms have higher current and future $Iu/Ku$ ratios and lower $Iu/Ku$ growth rates, compared to value firms. The spreads on current and future $Iu/Ku$ ratios and $Iu/Ku$ growth rate are $0.09$ ($t$-statistic = 8.96), $0.10$ ($t$-statistic = 10.52), and $0.03$ ($t$-statistic = 1.41) respectively, all of which are larger than the spreads on tangible investment related variables. Moreover, growth firms have significantly larger $Ku/K$ ratios than value firms with a spread of $0.17$ ($t$-statistic = 11.93). It indicates that growth firms accumulate larger amount of intangible assets relative to tangible assets, compared to value firms. Other notable firm characteristics that are different between growth and value firms are the leverage and the sales-to-capital ($Y/K$) ratios. Growth firms on average have significantly lower leverage ratios and significantly higher $Y/K$ ratios, compared to value firms.

Across the ten $Iu/Ku$ portfolios, we observe a strong positive correlation between firms’ decisions on intangible investments and decisions on tangible investments. As the sorting variable, the $Iu/Ku$ ratio, goes up, both firm’s current and future $I/K$ ratios rise while the growth rates of both $Iu/Ku$ and $I/K$ drop. Across the ten $Iu/Ku$ portfolios, the spreads on current and future $I/K$ ratios and $I/K$ growth rate are $0.08$ ($t$-statistic = 8.27), $0.06$ ($t$-statistic = 6.47), and $0.11$ ($t$-statistic = 1.36), respectively. The spreads on current and future $Iu/Ku$ ratios and $Iu/Ku$
growth rate are 0.41 \((t\text{-statistic} = 16.10)\), 0.26 \((t\text{-statistic} = 15.66)\), and 0.36 \((t\text{-statistic} = 8.41)\), respectively. Not surprisingly, high \(Iu/Ku\) firms have higher \(Ku/K\) ratios than low \(Iu/Ku\) firms with a spread of 0.18 \((t\text{-statistic} = 4.68)\). In addition, high \(lu/ku\) firms have higher leverage ratios and higher \(Y/K\) ratios than low \(Iu/Ku\) firms, with spreads of 0.14 \((t\text{-statistic} = 10.82)\) and 1.08 \((t\text{-statistic} = 4.38)\).

As expected, from the ten \(I/K\) portfolios, we observe the identical patterns on the \(I/K\) and \(Iu/Ku\) ratios, the growth rates of the \(I/K\) and \(Iu/Ku\) ratios, the leverage ratio, and the \(Y/K\) ratio as those of the ten \(Iu/Ku\) portfolios. It is worthwhile to point out that high \(I/K\) firms have higher, instead of lower \(Ku/K\) ratios than low \(I/K\) firms with a spread of 0.09 \((t\text{-statistic} = 6.80)\). There are two possible explanations: (1) firms that invest more in tangible assets have sufficiently higher levels of intangible assets to begin with; (2) as firms invest more in tangible assets, they also invest more in intangible assets, which is indeed what we see in the data.

To summarize, we observe significantly large differences on intangible assets related firm characteristics across the \(B/M\) portfolios, which underscores the important role of intangible assets in explaining the value premium. Moreover, we find that firms’ decisions on intangible investments is positively correlated with their decisions on intangible investments and high level of intangible assets is likely the driving force behind high future tangible investments. Next, we turn to the structural estimations to test the impacts of the intangible assets on stock returns and to quantify the economic significances of the ISTC effect and the adjustment cost effect of intangible assets as formulated in Section 2.

### 6.3 Parameter Estimations

Using one-stage GMM with the identity weighting matrix, we estimate three \(q\)-theory models: a \(q\)-theory model with only tangible assets (the Q2 model), an intangible-assets-augmented \(q\)-theory model with investment-specific technological change (the Q4 ISTC model), and an intangible-assets-augmented \(q\)-theory model with adjustment costs of intangible investments (the Q4 ISTC model). The parsimonious Q2 model is the same as in Liu, Whited, and Zhang (2009). The two parameters to be estimated are the capital-to-output share, \(\alpha\), and the tangible investment adjustment cost parameter, \(a\). In the Q4 ISTC model, we add two more parameters: \(a_2\) and \(\xi\). For the Q4 AC model, we add two different parameters: \(b\) and \(\psi\).

Using the ten \(B/M\) portfolios, we estimate \(a\) to be 31.30 and \(\alpha\) to be 0.60 in the Q2 model. However, neither parameters are significant. These results are consistent with those reported in Liu, Whited, and Zhang (2009). The average absolute pricing error (a.a.p.e.) is 3.27% per annum,
which is much smaller than that of the CAPM model (22.27%) but slightly larger than that of the Fama-French 3-factor model (2.67%). Adding the ISTC effect to the Q2 model improves its performance only marginally with the a.a.p.e. dropping to 3.04%. The magnitude of $a_2$ is statistically insignificant with a point estimation of zero. The curvature parameter $\xi$ is irrelevant when $a_2$ is zero. Therefore, both the zero value of $a_2$ and the marginal improvement on a.a.p.e. indicate little or no ISTC effect.

Adding adjustment costs of intangible investments significantly improves the explanatory power of the model and reduces the a.a.p.e. to 0.85%, more than 70% reduction from that of the Q2 model. The Q4_AC model estimates $b$ to be 27.66 and $\psi$ to be 1.44 with $t$-statistics being 2.24 and 7.20. The point estimation of $a$ drops to $-1.07$ with $t$-statistic being $-0.26$. $\alpha$ is 0.37 with $t$-statistic being 3.36. Our results show that the adjustment costs of intangible investments play a crucial role in explaining the value premium while the ISTC effect does not. The estimation of $\psi$ implies that the adjustment costs of intangible investments are convex. Moreover, the much larger magnitude of $b$, compared to $a$, implies that it is more difficult to accumulate intangible assets rapidly than tangible assets.

Panel B reports the parameter estimations using the ten $Iu/Ku$ portfolios. For the Q2 model, $a$ is 13.66 with $t$-statistic being 1.27 and $\alpha$ is 0.38 with $t$-statistic being 3.17. The average absolute pricing error of the Q2 model is 1.77, which is smaller than that of the CAPM model (5.81) and that of the Fama-French 3-factor model (3.76). Similar to what we find using the $B/M$ portfolios, adding ISTC effect of intangible assets to the Q2 model does not improve the model performance. The a.a.p.e. barely decreases and the point estimate of $a_2$ is zero and insignificant. On the contrary, adding the adjustment costs of intangible assets reduce the a.a.p.e. significantly to 0.87, a reduction of more than 50% from the Q2 model. The Q4_AC model estimates $b$ to be 3.80 with $t$-statistic being 1.97 and $\psi$ to be 3.10 with $t$-statistic being 2.25. The adjustment cost parameter for the tangible investments $a$ is estimated to be 2.21 and statistically insignificant. Consistent with the finding using the $B/M$ portfolios, adjustment costs of intangible investments are convex and are larger in magnitude than the adjustment costs of tangible investments.

Finally, we test the three models using the ten $I/K$ portfolios and the results are presented in Panel C. The Q2 model estimates $a$ to be 1.86 and $\alpha$ to be 0.24 with $t$-statistics being 2.91 and 13.63, respectively. The a.a.p.e. is 1.22% per year, smaller than those from the CAPM model and the Fama-French 3-factor model. Adding the ISTC effect once again shows little improvement on the model performance based on the a.a.p.e. and $a_2$ is estimated to be zero and insignificant. Adding the adjustment costs of intangible investments reduces the a.a.p.e. of the Q2 model by 30%. The estimate of $b$ and $\psi$ are 3.77 and 2.81 with $t$-statistics being 0.32 and 0.88, respectively.
The magnitude of $a$ is 1.47 with $t$-statistic being 1.46. Based on the point estimates, we get the same conclusion as from the $B/M$ and $I/K$ portfolios. However, none of these parameters are statistically significant.

In summary, the parameter estimations from all three sets of testing portfolios produce consistent results: (1) adjustment costs of intangible assets are convex and are larger than the adjustment costs of tangible assets; (2) the adjustment cost effect of intangible assets is more crucial than the ISTC effect in explaining the cross-sectional stock returns.

### 6.4 Model pricing errors

In Table 4, We report the model pricing errors of all three $q$-theory models for each individual portfolio. Recall the definition of the pricing error $e_i$ for portfolio $i$ in equation (10):

$$e_i \equiv E_T \left[ r_{it+1}^S - r_{it+1}^{lw} \right],$$

in which $E_T[\cdot]$ denotes the sample mean, $r^S_{it+1}$ is the realized return while $r^{lw}_{it+1}$ is the predicted return constructed using equation (6) with the parameters reported in Table 3. The economic meaning of $e_i$ is analogous to the alphas in the factor model regressions, representing the unexplained part of portfolio returns by the model.

For the ten $B/M$ portfolios, the Q4-AC model outperforms all the other models that we test. Six out of ten portfolios have their pricing errors less than 1% per annum and the largest pricing error is $-2.58\%$. In comparison, the pricing errors of the Q2 model range from $-5.82\%$ to $4.3\%$ and $-5.68\%$ to $3.99\%$ for the Q4-ISTC model. From Table 1, we know that the pricing errors of the CAPM model and the Fama-French 3-factor model are even larger. The pricing errors for the high-minus-low $B/M$ portfolio are statistically insignificant for all three $q$-theory models, with $1.22\%$ for the Q2 model, $2.11\%$ for the Q4-ISTC model, and $-0.46\%$ for the Q4-AC model, all of which are much smaller than the $19.26\%$ for the CAPM model and the $2.67\%$ for the Fama-French 3-factor model.

To provide a visual representation of the model performance, we plot the predicted returns from the CAPM model, the Fama-French 3-factor model, and the three $q$-theory models against the average realized returns for the ten $B/M$ portfolios in Figure 1. For the CAPM model, the scatter plots are largely horizontal, suggesting low or no explanatory power. The Fama-French 3-factor model shows a significant improvement as the plots are more aligned on the 45-degree lines. The scatters from the Q2 model and the Q4-ISTC model look almost identical, indicating little improvement by adding the ISTC effect of intangible assets to the Q2 model. On the other
hand, adding the adjustment costs of intangible assets improves the explanatory power of the Q2 model significantly. The scatters from the Q4_AC model line up along the 45-degree line almost perfectly. In particular, the points representing the two extreme B/M portfolios fall exactly on the 45-degree line.

Panel B of Table 4 reports the results for the ten Iu/Ku portfolios. The Q2_AC model again significantly outperforms the Q2 model and the Q4 ISTC model and we observe little difference between the latter two models. The pricing errors range from −2.18% to 1.24% for the Q4_AC model, while from −3.71% to 4.88% for the Q2 model and from −3.89% to 4.74% for the Q4 ISTC. The pricing error for the high-minus-low portfolio is −1.80% for the Q2 model, −2.71% for the Q4 ISTC model, and 0.66% for the Q4_AC model, all of which are statistically insignificant and much smaller than the alphas for the CAPM model and the Fame-French 3-factor model in Table 1. Figure 2 visualizes the performances of the three \( q \)-theory models as well as the CAPM model and the Fama-French 3-factor model and deliver the same conclusions.

Finally, Panel C of Table 4 reports the results for the ten I/K portfolios. The \( q \)-theory models outperform the traditional asset pricing models shown in Table 1. However, there is little improvement from adding either the ISTC effect or the adjustment cost effect of intangible assets to the Q2-model. The high-minus-low portfolio has pricing error of −1.8% for the Q2 model, 1.63% for the Q4 ISTC model, and 1.47% for the Q4 AC model, none of which is statistically significant. The scatter plots in Figure 3 confirm the conclusion drawn from the estimations of the pricing errors, suggesting that intangible assets do not play an important role in explaining the return differences among the I/K portfolios.

Consistent with the conclusion from the estimations of the model parameters in the previous subsection, the results on the pricing errors indicate that the adjustment cost effect of intangible assets play an important role in explaining the value premium and the return differences among the Iu/Ku portfolios, but not for the I/K portfolios. The ISTC effect has no added explaining power for either set of the testing portfolios.

### 6.5 Comparative static analysis

Equation (6) and (9) show the relation between a firm’s stock returns and its observable characteristics as the results of shareholder value maximization. In this subsection, we conduct a comparative static analysis on how different components in equation (6) affect stock returns.

The time \( t \) investment ratios, both tangible and intangible, have negative effects on the stock return from \( t \) to \( t+1 \) because they increase the marginal costs of investments due to the convexity of
the adjustment costs. The higher the investment costs, the lower the return per unit of investment. The time \( t + 1 \) sales-to-assets ratio has positive effects on returns from \( t \) to \( t + 1 \) of both tangible and intangible investments because higher sales-to-assets ratio means higher marginal revenue for both types of investments. Time \( t + 1 \) investment ratio has negative effects on the stock return from \( t \) to \( t + 1 \) because higher investment ratios lead to lower shadow prices of both types of capital at \( t + 1 \) and reduce the marginal continuation value of investments made at time \( t \).

Note that we cannot make causality arguments between stock returns and firm characteristics and the above comparative analysis is only valid in the sense of “ceteris paribus”. All the components in equation (6), including firm characteristics and stock returns, are endogenously determined in the model and the only exogeneous factors in the model are productivity shocks, which are unobservable. We can see from Table 2 that all the firm characteristics vary across the testing portfolios at the same time. The return differences among those portfolios eventually depend on which of the aforementioned effects dominants. We conduct a quantitative analysis to shed light on the relative importance of each component to the returns of the testing portfolios.

We implement five experiments and the results are reported in Table 5. We take away the cross-sectional variations of \( I/K, Iu/Ku, Ku/K, Y/K \) and \( w \) one at a time. If certain characteristic is important in generating the cross-sectional return differences, we would observe the model performance deteriorates when the corresponding variable is set at its cross-sectional average. The more crucial the characteristic, the greater the reduction in the model performance. As previous results suggest that the Q4_AC model does the best job in explaining the cross-sectional returns, we focus on the Q4_AC model in the comparative static experiments.

For the ten \( B/M \) portfolios, the most important component is the intangible-to-tangible-assets ratio, \( Ku/K \). Taking away the cross-sectional variation of \( Ku/K \) increases the pricing error of the high-minus-low portfolio from 0.46% to 22.22%. The average absolute pricing error increases from 0.85% to 6.90%. This result confirms our intuition that intangible assets contribute largely to the difference between the book value and the market value of a firm and naturally play an important role in explaining the value premium. The second most important component is the sales-to-assets ratio, \( Y/K \). The \( Y/K \) ratio captures the unobservable firm idiosyncratic and systematic productivity shocks and, not surprisingly, is crucial to the cross-sectional return spreads.

For the \( Iu/Ku \) portfolios, the most important component is \( Y/K \), followed by the \( Iu/Ku \) and the \( Ku/K \) ratios. For the \( I/K \) portfolios, the most important component is \( Y/K \), followed by the \( I/K \) and the \( Ku/K \) ratios. Again, it is not surprising that the \( Y/K \) ratio has high explanatory power for cross-sectional returns due to the aforementioned reason. In addition to
the corresponding sorting variables of the $Iu/Ku$ and the $I/K$ portfolios, the $Ku/K$ ratio remains important to both sets of portfolios.

7 Conclusion

Intangible assets have become increasingly important for firm’s survival and prosperity since 1980s. The literature has emphasized two important features of intangible assets. One effect is the adjustment cost of intangible assets, e.g., Bloom (2007) among others; the other effect is on the investment-specific technologic change. In this paper, we examine the impact of intangible assets on asset returns and at the same time quantify the importance of those two effects of the intangible assets using financial data.

Using the methodology developed in Liu, Whited, and Zhang (2009), we ask whether an intangible-assets-augmented $q$-theory model can explain cross-sectional returns better than the model with only tangible assets. Moreover, the parameter estimations of the model allow us to quantify various features of intangible assets. We use three sets of testing portfolios, ten book-to-market portfolios, ten R&D-to-intangible-assets portfolios, and ten investment-to-capital portfolios, to test three $q$-theory models: a two-parameter model with only tangible assets (the Q2 model), a four-parameter intangible-assets-augmented model with investment-specific technologic change (the Q4 ISTC model), and a four-parameter intangible-assets-augmented model with adjustment costs of intangible investments (the Q4 AC model).

The summary of our findings is as follows. First, by adding the adjustment costs of intangible assets to the Q2 model, we are able to explain the value premium and the return differences among the $Iu/Ku$ portfolios significantly better than other models. On the other hand, the ISTC effect of intangible assets seems to have no explanatory power for any set of the testing portfolios. Second, we provide empirical evidence for the convexity of the adjustment costs of intangible investments based on the financial data and the structural estimations of the intangible-assets-augmented $q$-theory models. Third, our structural estimations show that the adjustment costs of per unit intangible investment is at least one and a half times larger than that of tangible investments. This finding provides an explanation for the higher persistence in R&D investments than the persistence of tangible investments observed in the data. It also implies that by persistently investing in intangible assets, firms can deter competitions from their rivals more effectively because of the higher difficulty to rapidly accumulate intangible assets.

Last but not least, we document that the R&D intensity, when measured as the $Iu/Ku$ ratio,
is negatively related to stock returns, which resembles the relation between stock returns and tangible investment intensity, measured as the $I/K$ ratio. This finding is opposite to the previous argument in the literature that the R&D intensity, measured as R&D investments scaled by firm’s market value, is positively related to stock returns. We suspect that the discrepancy is due to the value effect or/and the leverage effect. Our results call for a more careful interpretation of the return patterns associated with R&D investments.

Although we do not find evidence supporting the importance of the investment-specific technological change effect on cross-sectional stock returns, we caution the generalization of our results to the aggregate level. Due to the spill-over effect of intangible assets, one might find a larger and detectable impact of intangible assets on the price and quality of tangible assets and the productivity growth at the aggregate level and over the longer horizon.
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Appendix

A Proof of Proposition 1

Shareholder’s maximization problem can be written as

\[
P_t \equiv P(k^m_t, k^u_t, B_t, X_t) = \max_{\{i^m_t, i^u_t, k^m_{t+1}, k^u_{t+1}, L_t\}} \left\{ D^S_t + \mathbb{E}_t [M_{t+1} P_{t+1}] \right\}
\]  

(11)

where \( P_t \) is the cum-dividend equity value of the firm, \( D^S_t \) is the cash flow to shareholders at time \( t \). If \( D^S_t \) is positive, firm pays out dividends; if \( D^S_t \) is negative, firm issues equity. \( D^S_t \) can be written as

\[
D^S_t = (1 - \tau_t) [y_t - \varpi_t L_t - i^u_t - \Phi^m_t - \Phi^u_t] - \tau_t \delta_m k^m_t + B_{t+1} - [r^B_t - (r^B_t - 1) \tau_t] B_t
\]

and the maximization is subject to

\[
q^u_t : k^u_{t+1} = (1 - \delta_u) k^u_t + i^u_t
\]

(12)

\[
q^m_t : k^m_{t+1} = (1 - \delta_m) k^m_t + \Theta (i^m_t, k^u_t)
\]

(13)

where \( r^B_t \) is the gross required return on debt, \( B_t \) is the beginning-of-the-period debt outstanding at time \( t \), and \( B_{t+1} \) is the end-of-the-period debt outstanding at time \( t \). \( q^u_t \) and \( q^m_t \) are the lagrangian multipliers and can be interpreted as the shadow prices for tangible and intangible assets at time \( t \), respectively. Since firm is a price taker in the input market, input price \( w_t \) is exogenously given.

Lemma 1. Ex-dividend firm value \( V_t \) is given by

\[
V_t \equiv P_t - D^S_t + B_{t+1} = q^u_t k^u_{t+1} + q^m_t k^m_{t+1}
\]

(14)
Proof: The first order conditions of shareholder’s maximization problem are

\[ i_t^u : \quad q_t^u = (1 - \tau_t) \left( 1 + \Phi_{i,t}^u \right) \quad (15) \]
\[ i_t^m : \quad 1 + (1 - \tau_t)\Phi_{i,t}^m = \Theta_{i,t} q_t^m \quad (16) \]
\[ L_t : \quad \omega_t = y_{L,t} \]
\[ k_{t+1}^u : \quad q_t^u = \mathbb{E}_t [M_{t+1} V_{k^u,t+1}] \]
\[ k_{t+1}^m : \quad q_t^m = \mathbb{E}_t [M_{t+1} V_{k^m,t+1}] \]
\[ B_{t+1} : \quad 1 = \mathbb{E}_t [M_{t+1} [r_{t+1}^B - (r_{t+1}^B - 1)\tau_{t+1}]] \]

where \( V_{k^u,t+1} \) is the derivative of the value function w.r.t. \( k_{t+1}^u \) and similar definition for \( V_{k^m,t+1} \) and \( y_{L,t} \). It’s straightforward to show that the adjustment function, production function of new tangible assets, and the production function satisfy constant-return-to-scale, i.e.,

\[ \Phi_{i,t}^m = \Phi_{k^m,t} k_{t:t}^m + \Phi_{i,t} i_{t:t}^m \]
\[ \Phi_{i,t}^u = \Phi_{k^u,t} k_{t:t}^u + \Phi_{i,t} i_{t:t}^u \]
\[ \Theta_{t} = \Theta_{k^u,t} k_{t:t}^u + \Theta_{i,t} i_{t:t}^m \]
\[ y_{t} = y_{k^m,t} k_{t:t}^m + y_{k^u,t} k_{t:t}^u + y_{L,t} L_{t} \]

The derivatives of the investment adjustment costs w.r.t. the investments and the asset levels, both tangible and intangible, are given by

\[ \Phi_{i,t}^m = \frac{a\rho}{2} \left( \frac{i_{t:t}^m}{k_{t:t}^m} \right)^{\rho - 1} \]
\[ \Phi_{k,t}^m = \frac{a(1 - \rho)}{2} \left( \frac{i_{t:t}^m}{k_{t:t}^m} \right)^{\rho} \]
\[ \Phi_{i,t}^u = \frac{b\psi}{2} \left( \frac{i_{t:t}^u}{k_{t:t}^u} \right)^{\psi - 1} \]
\[ \Phi_{k,t}^u = \frac{b(1 - \psi)}{2} \left( \frac{i_{t:t}^u}{k_{t:t}^u} \right)^{\psi} \]

and the partial derivatives of the capital production function \( \Theta \) w.r.t. investment and asset level.
are given by

\[ \Theta_{i,t} = a_1 + a_2 \left( \frac{i_t^m}{k_t^u} \right)^{-\xi} \]

\[ \Theta_{k,t} = a_2 \left( a_1 \left( \frac{i_t^m}{k_t^u} \right)^{\xi} + a_2 \right)^{-\xi} \]

and the derivatives of the value function w.r.t. both tangible assets $k_t^m$ and intangible assets $k_t^u$ are given by

\[ V_{k^m,t} = (1 - \tau_t) \left( y_{k^m,t} - \Phi_{k,t+1}^m \right) + \tau_t \delta_m + q_t^m (1 - \delta_m) \]

\[ V_{k^u,t} = (1 - \tau_t) y_{k^u,t} + q_t^u (1 - \delta_u) + q_t^m \Theta_{k,t} \]

From the first order conditions, we can write the right hand side of equation (14) as

\[ q_t^u k_{i+1}^u + q_t^m k_{i+1}^m = E_t \left[ M_{t+1} \left( V_{k^m,t+1}^m k_{t+1}^m + V_{k^u,t+1}^u k_{t+1}^u \right) \right] \]

Define a function $\Omega_{t+1}$ as

\[ \Omega_{t+1} = V_{k^m,t+1}^m k_{t+1}^m + V_{k^u,t+1}^u k_{t+1}^u \]

\[ = \left[ (1 - \tau_t) \left( y_{k^m,t} - \Phi_{k,t+1}^m \right) + \tau_t \delta_m + q_t^m (1 - \delta_m) \right] k_{t+1}^m \]

\[ + \left[ (1 - \tau_t) y_{k^u,t} + q_t^u (1 - \delta_u) + q_t^m \Theta_{k,t} \right] k_{t+1}^u \]

Substituting the first order equations into the above equation and using the constant-return-to-
scale property of the production function and the adjustment cost function, we get

$$
\Omega_{t+1} = (1 - \tau_{t+1}) (y_{km,t} k_{i+1}^{lm} + y_{ku,t+1} k_{i+1}^{lu} - \Phi_{k,t+1} k_{i+1}^{lm} - \Phi_{k,t+1} k_{i+1}^{lu}) + \tau_{t+1} \delta_m k_{i+1}^{lm} \\
+ q_{l+1} k_{i+1}^{lm} + q_{l+1} (1 - \delta_m) k_{i+1}^{lm} + q_{l+1} \Theta_{k,t+1} k_{i+1}^{lu} \\
= (1 - \tau_{t+1}) (y_{km,t+1} k_{i+1}^{lm} + y_{ku,t+1} k_{i+1}^{lu} - \Phi_{k,t+1} k_{i+1}^{lm} - \Phi_{k,t+1} k_{i+1}^{lu}) + \tau_{t+1} \delta_m k_{i+1}^{lm} \\
+ q_{l+1} \left( k_{i+1}^{lm} - \Theta_{t+1} \right) + q_{l+1} \left( k_{i+1}^{lu} - \Theta_{t+1} \right) + q_{l+1} \Theta_{k,t+1} k_{i+1}^{lu} \\
= (1 - \tau_{t+1}) (y_{km,t+1} k_{i+1}^{lm} + y_{ku,t+1} k_{i+1}^{lu} - \Phi_{k,t+1} k_{i+1}^{lm} - \Phi_{k,t+1} k_{i+1}^{lu}) + \tau_{t+1} \delta_m k_{i+1}^{lm} \\
- \left[ 1 + (1 - \tau_1) \Phi_{k,t+1}^{lm} \right] \left( \Theta_{t+1} - \Theta_{k,t+1} k_{i+1}^{lm} \right) \\
- \left( 1 - \tau_1 \right) \left( i_{t+1} + q_{l+1} k_{i+1}^{lu} + q_{l+1} k_{i+1}^{lu} \right) \\
= (1 - \tau_{t+1}) \left( y_{t+1} - \omega_{t+1} L_{t+1} - \Phi_{t+1} - \Phi_{t+1} - \tau_{t+1} \delta_m k_{i+1}^{lm} - \tau_{t+1} \delta_m k_{i+1}^{lm} \right) \\
- \left[ 1 + (1 - \tau_1) \Phi_{t+1}^{lm} \right] \left( \Theta_{t+1} - \Theta_{k,t+1} k_{i+1}^{lm} \right) \\
= D_{t+1} + q_{l+1} k_{i+1}^{lm} + q_{l+1} k_{i+1}^{lm}
$$

where $D_{t+1}$ is the free cash flow of the firm at time $t + 1$ and defined as

$$
D_{t+1} = (1 - \tau_{t+1}) \left( y_{t+1} - \omega_{t+1} L_{t+1} - \Phi_{t+1} - \Phi_{t+1} - \tau_{t+1} \delta_m k_{i+1}^{lm} - \tau_{t+1} \delta_m k_{i+1}^{lm} \right)
$$

Hence, the right hand side of equation (14) can be written as

$$
q_{l+1} k_{i+1}^{lm} + q_{l+1} k_{i+1}^{lu} = \mathbb{E}_t \left[ M_{t+1} \left[ D_{t+1} + q_{l+1} k_{i+1}^{lm} + q_{l+1} k_{i+1}^{lu} \right] \right]
= \mathbb{E}_t \left[ \sum_{s=0}^{\infty} M_{t+s} D_{t+s} \right] 
$$

(17)

Firm value $V_t$ is the sum of ex-dividend equity value and debt value, i.e.,

$$
V_t = P_t - D_t^s + B_{t+1} \\
= \mathbb{E}_t \left[ M_{t+1} \left\{ D_t^s + P_{t+1} + \left[ r_{t+1} B_{t+1} (1 - \tau_{t+1}) \right] B_{t+1} \right\} \right]
$$

where the second equation is derived from the first order condition on the optimal debt issuance

$$
B_{t+1} = \mathbb{E}_t \left[ M_{t+1} \left[ r_{t+1} B_{t+1} - r_{t+1} (1 - \tau_{t+1}) \right] B_{t+1} \right].
$$
It’s straightforward to show that

\[ D_{t+1}^S + [r_{t+1}^B - r_{t+1}^B (1 - \tau_{t+1})] B_{t+1} = D_{t+1} + B_{t+2} \]

Therefore, we have

\[ V_t = \mathbb{E}_t [M_{t+1} (D_{t+1} + B_{t+2} + P_{t+1})] = \mathbb{E}_t [M_{t+1} (D_{t+1} + V_{t+1})] . \]

Iterate the above equation, we get

\[ V_t = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} M_{t+s} D_{t+s} \right] \]

which, combined with equation (17), implies that

\[ V_t = q_t^m k_{t+1}^m + q_t^u k_{t+1}^u . \]

Q.E.D.

**Lemma 2.** Define firm’s investment return as

\[ r_{t+1}^I = \frac{D_{t+1} + q_{t+1}^m k_{t+2}^m + q_{t+1}^u k_{t+2}^u}{q_t^m k_{t+1}^m + q_t^u k_{t+1}^u} \]

and \( r_{t+1}^I \) satisfies the following equations:

\[ \mathbb{E}_t [M_{t+1} r_{t+1}^I] = 1 \] (18)

\[ r_{t+1}^I = \varpi_t r_{t+1}^S + (1 - \varpi_t) r_{t+1}^B . \] (19)

**Proof:** Equation (18) is straightforward to prove. From Lemma 1, we know that

\[ q_t^m k_{t+1}^m + q_t^u k_{t+1}^u = \mathbb{E}_t [M_{t+1} \left( D_{t+1} + q_{t+1}^m k_{t+2}^m + q_{t+1}^u k_{t+2}^u \right)] . \]

Divide the right hand side of the equation by the left hand side, we get

\[ 1 = \mathbb{E}_t \left[ \frac{D_{t+1} + q_{t+1}^m k_{t+2}^m + q_{t+1}^u k_{t+2}^u}{q_t^m k_{t+1}^m + q_t^u k_{t+1}^u} \right] = \mathbb{E}_t [M_{t+1} r_{t+1}^I] . \]

We prove equation (19) by three steps:
Step 1: we show that given the level of both tangible and intangible assets and under the optimal choice of non-capital input $L_t^\ast$, for each period $t$,

$$y_t - \varpi_t L_t^\ast = \gamma y_t.$$ 

At any given levels of tangible and intangible assets, the optimal non-capital input $L_t^\ast$ is given by the following maximization problem

$$\max_{\{L_t\}} y_t - \varpi_t L_t$$

subject to the revenue function

$$y_t = e^{X_t \left[ (k_m^t)^\alpha (k_u^t)^{1-\alpha}\right]} (L_t)^{1-\gamma}$$

The FOC w.r.t. $L_t$ gives

$$(1-\gamma)e^{X_t \left[ (k_m^t)^\alpha (k_u^t)^{1-\alpha}\right]} (L_t^\ast)^{-\gamma} = \varpi_t$$

Substitute the above equation into the revenue function, we get that under the optimal hiring of input $L_t^\ast$, the revenue after the input cost is

$$y_t - \varpi_t L_t^\ast = L_t^\ast \left( e^{X_t \left[ (k_m^t)^\alpha (k_u^t)^{1-\alpha}\right]} (L_t^\ast)^{-\gamma} - \varpi_t \right) = \gamma y_t$$

(20)

Step 2: we show that

$$(1 - w_t) r_{t+1}^S + w_t r_{t+1}^{Ba} = \frac{D_{t+1} + q_{t+1}^m k_{t+1}^{m_{t+2}} + q_{t+1}^u k_{t+1}^{u_{t+2}}}{q_{t}^m k_{t+1}^{m_{t+1}} + q_{t}^u k_{t+1}^{u_{t+1}}}. $$

From Lemma 1, we get

$$\frac{D_{t+1} + q_{t+1}^m k_{t+1}^{m_{t+2}} + q_{t+1}^u k_{t+1}^{u_{t+2}}}{q_{t}^m k_{t+1}^{m_{t+1}} + q_{t}^u k_{t+1}^{u_{t+1}}} = \frac{D_{t+1}^S + [r_{t+1}^{B_t} - \tau_{t+1} (r_{t+1}^{B_t} - 1)] B_{t+1} - B_{t+2} + V_{t+1}}{V_t}$$

$$= \frac{D_{t+1}^S + r_{t+1}^{B_t} B_{t+1} - B_{t+2} + P_{t+1} - D_{t+1}^S + B_{t+2}}{V_t}$$

$$= \frac{P_{t+1} + r_{t+1}^{B_t} B_{t+1}}{V_t}$$

$$= (1 - w_t) r_{t+1}^S + w_t r_{t+1}^{Ba}.$$
Step 3: Substitute equation (15), (16), (20), and the accumulation rules of both tangible and intangible assets into equation (19). Divided both the denominator and the nominator by $k_{m,t+1}$, it’s straightforward to get equation (6).

From Step 2 and Step 3, we conclude that

$$r_{t+1} = (1 - w_t) r_{t+1}^S + w_t r_{t+1}^{Ba}. $$

**Lemma 3.** Firm’s investment return $r_{t+1}^I$ is a value-weighted average of its investment return on tangible assets $r_{t+1}^{Im}$ and investment return on intangible assets $r_{t+1}^{Ju}$, where

$$r_{t+1}^{Im} = \frac{(1 - \tau_{t+1}) \left[ \alpha_{k_{m,t+1}} - \Phi_{i,t+1}^{m} \right] + \tau_{t+1} \delta_m + (1 - \delta_m) \left[ 1 + (1 - \tau_{t+1}) \Phi_{i,t+1}^{m} \right]}{1 + (1 - \tau_t) \Phi_{i,t+1}^{m}} \Bigg/ \Theta_{i,t+1},$$

$$r_{t+1}^{Ju} = \left\{ (1 - \tau_{t+1}) \left[ (1 - \alpha) \gamma_{k_{u,t+1}} \Phi_{k_{u,t+1}}^{u} - \Phi_{i,t+1}^{m} \right] + \left[ 1 + (1 - \tau_{t+1}) \Phi_{i,t+1}^{m} \right] \left( \frac{\Theta_{k,t+1}^{m}}{\Theta_{i,t+1}^{m}} \right) \right\} / \left[ (1 - \tau_t) \left( 1 + \Phi_{i,t+1}^{u} \right) \right]$$

and the weights are the market value of tangible assets and intangible assets, respectively, given by

$$w_{t}^{m} = \frac{q_{t}^{m} k_{t}^{m}}{V_t}, \quad w_{t}^{u} = \frac{q_{t}^{u} k_{t}^{u}}{V_t}.$$

**Proof:** Since $q_{t}^{m}$ is the shadow price of one unit of tangible assets at time $t$, the market value of firm’s tangible assets is $q_{t}^{m} k_{t}^{m}$. Similarly, the market value of firm’s intangible assets is $q_{t}^{u} k_{t}^{u}$. From Lemma 1, we know that $V_t = q_{t}^{m} k_{t}^{m} + q_{t}^{u} k_{t}^{u}$. Hence, the weights $w_{t}^{m}$ and $w_{t}^{u}$ add up to 1. From the FOCs from shareholder’s value maximization in the proof of Lemma 1, we have

$$q_{t}^{m} = (1 - \tau_t) \left( 1 + \Phi_{i,t}^{u} \right),$$

$$q_{t}^{u} = \frac{1 + (1 + \tau_t) \Phi_{i,t}^{m}}{\Theta_{i,t}}.$$

Plug in the above equations, it is straightforward to show that

$$r_{t+1}^I = w_t^{m} r_{t+1}^{Im} + w_t^{u} r_{t+1}^{Ju}.$$

Q.E.D.
Table 1: Descriptive Statistics of Testing Portfolio Returns

For each testing portfolio $i$, we report in annualized percentage the average stock return, $\bar{r}_i^S$, the intercept from the CAPM regression, $\alpha_i^{CAPM}$, and the intercept from the Fama-French 3-factor regression, $\alpha_i^{FF}$. The H-L portfolio is long in the high portfolio and short in the low portfolio. The t-statistics for the model errors are reported in brackets beneath the corresponding errors. a.a.p.e. is the average of the absolute values of the errors for a given set of ten testing portfolios. For the CAPM model and the Fama-French 3-factor model, the $p$-values in brackets in the last column are for the Gibbons, Ross, and Shanken (1989) tests of the null hypothesis that the intercepts for a given set of portfolios are jointly zero. Panel A reports results for ten $B/M$ portfolios, Panel B for $Iu/Ku$ portfolios and Panel C for $I/K$ portfolios.

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Panel A: Ten $B/M$ Portfolios

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Panel B: Ten $Iu/Ku$ portfolios

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Table 2: Summary Statistics of Firm Characteristics

This table reports the averages of future investment-to-capital, $I_{it+1}/K_{it}$, current investment-to-capital, $I_{it}/K_{it}$, investment growth, $(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$, future intangible investment-to-intangible asset, $Iu_{it+1}/Ku_{it+1}$, current intangible investment-to-intangible asset, $Iu_{it}/Ku_{it}$, intangible investment growth, $(Iu_{it+1}/Ku_{it+1})/(Iu_{it}/Ku_{it})$, sales-to-capital, $Y_{it+1}/K_{it+1}$, the depreciation rate, $\delta_{it+1}$, market leverage, $w_{it}$, intangible asset-to-capital, $Ku_{it+1}/K_{it+1}$, and corporate bond returns in annual percent, $r_{it+1}^B$, for all the testing portfolios. The column H–L reports the average differences between high and low portfolios and the column $[t_{H-L}]$ reports the t-statistics for the test that the differences equal zero. Panel A has results for ten $B/M$ portfolios, Panel B for $Iu/Ku$ portfolios and Panel C for $I/K$ portfolios.

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<tr>
<td>Panel A: Ten $B/M$ portfolios</td>
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<tr>
<td>$I_{it+1}/K_{it+1}$</td>
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<td>0.13</td>
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<td>0.10</td>
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<td>[-9.12]</td>
</tr>
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<td>0.11</td>
<td>0.10</td>
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<td>0.09</td>
<td>0.09</td>
<td>-0.08</td>
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</tr>
<tr>
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<td>0.97</td>
<td>0.99</td>
<td>0.97</td>
<td>0.98</td>
<td>0.99</td>
<td>1.01</td>
<td>0.98</td>
<td>0.99</td>
<td>1.01</td>
<td>1.02</td>
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</tr>
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<td>0.38</td>
<td>0.37</td>
<td>0.37</td>
<td>0.36</td>
<td>0.35</td>
<td>0.34</td>
<td>0.35</td>
<td>0.35</td>
<td>-0.32</td>
<td>[-10.52]</td>
</tr>
<tr>
<td>$Iu_{it}/Ku_{it}$</td>
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<td>0.39</td>
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<td>0.36</td>
<td>0.35</td>
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<td>0.35</td>
<td>0.35</td>
<td>-0.34</td>
<td>[-8.96]</td>
</tr>
<tr>
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<td>1.00</td>
<td>1.01</td>
<td>1.02</td>
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<td>1.57</td>
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<td>0.49</td>
<td>0.51</td>
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<tr>
<td>$r_{it+1}^B$</td>
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<td>10.53</td>
<td>10.50</td>
<td>10.56</td>
<td>10.72</td>
<td>10.76</td>
<td>10.79</td>
<td>10.87</td>
<td>11.14</td>
<td>[0.47]</td>
</tr>
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</table>
Table 3: Parameter Estimates and Tests of Overidentification For Different Models

Estimates and tests are from one-stage GMM with an identity weighting matrix. The moment conditions are $E[r_{it+1}^S - r_{it+1}^{lw}] = 0$. $a$ is the adjustment cost parameter for tangible asset, $b$ is the adjustment cost parameter for intangible asset, $a_2$ is the effect of intangible asset to tangible asset, $\psi$ is the power parameter for the intangible asset adjustment cost function, $\alpha$ is capital's share, and $\xi$ is the power parameter for the effect of investment on capital. Their $t$-statistics, denoted as $t$, are reported in brackets beneath the estimates. $\chi^2$ is the statistic from the first-stage GMM that the moment conditions are jointly zero. d.f. is the degrees of freedom, and $p$ is the $p$-value associated with the test. a.a.p.e. is the average absolute value of the model errors, $E_T[r_{it+1}^S - r_{it+1}^{lw}]$, in which $E_T[\cdot]$ is the sample mean of the series in brackets, in annual percent across a given set of testing portfolios. Panel A has the results for $B/M$ portfolios, Panel B for $Iu/Ku$ portfolios, and Panel C for $I/K$ portfolios. For each panel, the results for three models are reported: the Q2 model, the Q4.ISTC model, and the Q4.AC model.

<table>
<thead>
<tr>
<th></th>
<th>Panel A: B/M</th>
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<th>Panel B: Iu/Ku</th>
<th></th>
<th>Panel C: I/K</th>
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</thead>
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<td>Q2</td>
<td>Q4.ISTC</td>
<td>Q4.AC</td>
<td>Q2</td>
<td>Q4.ISTC</td>
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<td>$a$</td>
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<td>11.34</td>
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<tr>
<td>$[t]$</td>
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<td>[-0.26]</td>
<td>[1.27]</td>
<td>[0.88]</td>
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<td>$b$</td>
<td>27.66</td>
<td></td>
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<td>3.80</td>
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</tr>
<tr>
<td>$[t]$</td>
<td>[2.24]</td>
<td></td>
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<td>[1.97]</td>
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</tr>
<tr>
<td>$a_2$</td>
<td>0.00</td>
<td></td>
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<td>0.00</td>
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</tr>
<tr>
<td>$[t]$</td>
<td>[0.00]</td>
<td></td>
<td></td>
<td>[0.00]</td>
<td></td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.44</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$[t]$</td>
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<td>[2.25]</td>
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<tr>
<td>$\alpha$</td>
<td>0.60</td>
<td>0.48</td>
<td>0.37</td>
<td>0.38</td>
<td>0.40</td>
</tr>
<tr>
<td>$[t]$</td>
<td>[0.91]</td>
<td>[1.23]</td>
<td>[3.36]</td>
<td>[3.17]</td>
<td>[2.05]</td>
</tr>
<tr>
<td>$\xi$</td>
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</tr>
<tr>
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<td>3.24</td>
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<td>6</td>
<td>6</td>
<td>8</td>
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<tr>
<td>$p$</td>
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<td>0.55</td>
<td>0.78</td>
<td>0.87</td>
<td>0.78</td>
</tr>
<tr>
<td>a.a.p.e.</td>
<td>3.27</td>
<td>3.04</td>
<td>0.85</td>
<td>1.77</td>
<td>1.72</td>
</tr>
</tbody>
</table>
Table 4: Euler Equation Errors for Different Models

Euler equation errors and t-statistics are from the one-stage GMM estimation with an identity weighting matrix. The moment conditions are $E [ r_{it+1}^S - r_{it+1}^Iw ] = 0$. The mean errors are defined as $e_i = E_T [ r_{it+1}^S - r_{it+1}^Iw ]$, in which $E_T [·]$ is the sample mean of the series in brackets. In the last column we report the difference in the mean errors in annual percent between the high and low portfolios, as well as their t-statistics. Panel A reports results for $B/M$ portfolios, Panel B for $Iu/Ku$ portfolios and Panel C for $I/K$ portfolios. $e^{Q2}$ represents Euler equation error for the Q2 model, $e^{Q4\text{ ISTC}}$ for the Q4 ISTC model, and $e^{Q4\text{ AC}}$ for the Q4 AC model. All the numbers are in percentage and per annum.

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>High</th>
<th>H−L</th>
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<tbody>
<tr>
<td>$e_i^{Q2}$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$[t]$</td>
<td>−1.84</td>
<td>−1.30</td>
<td>−0.95</td>
<td>1.60</td>
<td>0.53</td>
<td>0.69</td>
<td>1.28</td>
<td>1.31</td>
<td>0.74</td>
<td>−1.61</td>
<td>1.11</td>
</tr>
<tr>
<td>$e_i^{Q4\text{ ISTC}}$</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$[t]$</td>
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<td>−1.02</td>
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<td>0.79</td>
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<td>1.09</td>
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<tr>
<td>$e_i^{Q4\text{ AC}}$</td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>$[t]$</td>
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<td>0.32</td>
<td>−1.66</td>
<td>0.85</td>
<td>0.78</td>
<td>0.99</td>
<td>−0.03</td>
<td>−0.87</td>
<td>0.54</td>
<td>−0.62</td>
<td>−0.38</td>
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</table>

Panel B: Euler equation errors for $Iu/Ku$ portfolios

<table>
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<th>5</th>
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<th>8</th>
<th>9</th>
<th>High</th>
<th>H−L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_i^{Q2}$</td>
<td>−1.92</td>
<td>−0.03</td>
<td>−0.16</td>
<td>1.39</td>
<td>0.76</td>
<td>4.88</td>
<td>2.28</td>
<td>−0.16</td>
<td>−2.45</td>
<td>−3.71</td>
<td>−1.80</td>
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<tr>
<td>$[t]$</td>
<td>−0.60</td>
<td>−0.02</td>
<td>−0.04</td>
<td>0.48</td>
<td>0.38</td>
<td>1.50</td>
<td>1.03</td>
<td>−0.04</td>
<td>−1.11</td>
<td>−1.33</td>
<td>−0.57</td>
</tr>
<tr>
<td>$e_i^{Q4\text{ ISTC}}$</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>$[t]$</td>
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<td>−0.41</td>
<td>−0.09</td>
<td>0.51</td>
<td>0.34</td>
<td>2.44</td>
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<td>0.17</td>
<td>−0.70</td>
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<td>−0.57</td>
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<tr>
<td>$e_i^{Q4\text{ AC}}$</td>
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</tr>
<tr>
<td>$[t]$</td>
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<td>−0.41</td>
<td>0.14</td>
<td>−0.53</td>
<td>−0.62</td>
<td>1.12</td>
<td>0.66</td>
<td>0.62</td>
<td>−1.13</td>
<td>0.93</td>
<td>0.22</td>
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Panel C: Euler equation errors for $I/K$ portfolios

<table>
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<th>8</th>
<th>9</th>
<th>High</th>
<th>H−L</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_i^{Q2}$</td>
<td>−1.92</td>
<td>−0.03</td>
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<td>0.76</td>
<td>4.88</td>
<td>2.28</td>
<td>−0.16</td>
<td>−2.45</td>
<td>−3.71</td>
<td>−1.80</td>
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<td>−0.04</td>
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<td>−0.04</td>
<td>−1.11</td>
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<td>−0.57</td>
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<tr>
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<tr>
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<td>0.75</td>
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<td>−1.05</td>
<td>0.39</td>
<td>0.22</td>
<td>0.01</td>
<td>0.96</td>
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</table>
Table 5: Expected Return Errors from Comparative Static Experiments

This table reports the results from comparative static experiments. For rows denoted by $I_{it+1}/K_{it+1}, I_{it}/K_{it}$, we set $I_{it+1}/K_{it+1}$ for a given portfolio, denoted by $i$, to its cross sectional average value at time $t + 1$ and we set $I_{it}/K_{it}$ for a given portfolio to its cross sectional average value at time $t$. We use parameters reported in Table 3 for the Q4.AC model to reconstruct the expected return. The difference between these reconstructed expected return and the realized return for each portfolio, the high-minus-low portfolio and the average absolute pricing errors are then reported. The results for others rows are designed analogously.

<table>
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<tr>
<th></th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>High</th>
<th>H−L</th>
<th>a.a.p.e.</th>
</tr>
</thead>
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<td><strong>Panel A: Ten $B/M$ portfolios</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{it+1}/K_{it+1}, I_{it}/K_{it}$</td>
<td>0.39</td>
<td>0.58</td>
<td>−2.43</td>
<td>1.21</td>
<td>0.64</td>
<td>1.26</td>
<td>−0.21</td>
<td>−1.35</td>
<td>0.08</td>
<td>−1.34</td>
<td>−1.72</td>
<td>0.95</td>
</tr>
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<td>0.49</td>
<td>−2.20</td>
<td>1.85</td>
<td>0.52</td>
<td>1.69</td>
<td>−0.41</td>
<td>−0.50</td>
<td>−0.42</td>
<td>−0.63</td>
<td>−1.68</td>
<td>0.98</td>
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<td>−3.48</td>
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<td>3.37</td>
<td>6.42</td>
<td>6.50</td>
<td>9.11</td>
<td>11.17</td>
<td>9.95</td>
<td>22.22</td>
<td>6.90</td>
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<tr>
<td>$Y_{it+1}/K_{it+1}$</td>
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<td>0.41</td>
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<td>−4.54</td>
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<td>4.82</td>
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<td>1.99</td>
</tr>
<tr>
<td><strong>Panel B: Ten $I_{u}/K_{u}$ portfolios</strong></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$I_{it+1}/K_{it+1}, I_{it}/K_{it}$</td>
<td>2.35</td>
<td>1.72</td>
<td>1.89</td>
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<td>−0.65</td>
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<td>0.53</td>
<td>1.05</td>
<td>−3.53</td>
<td>−0.26</td>
<td>−2.61</td>
<td>1.25</td>
</tr>
<tr>
<td>$I_{u_{it+1}}/K_{u_{it+1}}, I_{u_{it}}/K_{u_{it}}$</td>
<td>2.21</td>
<td>1.30</td>
<td>1.47</td>
<td>0.27</td>
<td>−0.64</td>
<td>2.09</td>
<td>0.75</td>
<td>−0.16</td>
<td>−6.10</td>
<td>−8.31</td>
<td>−10.51</td>
<td>2.33</td>
</tr>
<tr>
<td>$K_{u_{it+1}}/K_{it+1}$</td>
<td>1.25</td>
<td>2.53</td>
<td>2.97</td>
<td>1.44</td>
<td>0.59</td>
<td>0.42</td>
<td>−0.46</td>
<td>−1.27</td>
<td>−6.05</td>
<td>−4.59</td>
<td>−5.83</td>
<td>2.16</td>
</tr>
<tr>
<td>$Y_{it+1}/K_{it+1}$</td>
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<td>−11.74</td>
<td>−8.71</td>
<td>−6.46</td>
<td>−7.27</td>
<td>−0.77</td>
<td>1.82</td>
<td>4.66</td>
<td>5.16</td>
<td>8.80</td>
<td>15.05</td>
<td>6.16</td>
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<td>$\bar{w}$</td>
<td>1.39</td>
<td>1.50</td>
<td>2.45</td>
<td>0.23</td>
<td>−0.83</td>
<td>1.61</td>
<td>0.04</td>
<td>−0.01</td>
<td>−3.48</td>
<td>−0.04</td>
<td>−1.43</td>
<td>1.16</td>
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<tr>
<td><strong>Panel C: Ten $I/K$ portfolios</strong></td>
<td></td>
<td></td>
<td></td>
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</tr>
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<td>6.50</td>
<td>4.23</td>
<td>4.31</td>
<td>4.09</td>
<td>2.70</td>
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<td>−0.73</td>
<td>−3.78</td>
<td>−12.04</td>
<td>−17.96</td>
<td>4.65</td>
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<td>0.57</td>
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<td>1.96</td>
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<td>$K_{u_{it+1}}/K_{it+1}$</td>
<td>0.91</td>
<td>2.71</td>
<td>1.44</td>
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<td>1.79</td>
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<td>−8.52</td>
<td>−6.09</td>
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<td>−2.07</td>
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<td>4.88</td>
<td>12.31</td>
<td>20.98</td>
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<tr>
<td>$\bar{w}$</td>
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<td>3.26</td>
<td>0.86</td>
<td>0.66</td>
<td>1.65</td>
<td>1.29</td>
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<td>0.58</td>
<td>−0.52</td>
<td>2.43</td>
<td>−0.80</td>
<td>1.70</td>
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Figure 1: Scatter Plots of Average Predicted Return against Average Stock Return for the B/M Portfolios

Average predicted stock returns versus average realized stock returns for the B/M portfolios, using CAPM model, Fama-French model (FF), two-parameter \( q \)-theory (Q2) model, four-parameter intangible-assets-augmented \( q \)-theory model with investment-specific technological changes (Q4\_ISTC) and four-parameter intangible-assets-augmented \( q \)-theory model with intangible investment adjustment costs (Q4\_AC), respectively.
Figure 2: Scatter Plots of Average Predicted Return against Average Stock Return for the Iu/Ku Portfolios

Average predicted stock returns versus average realized stock returns for the Iu/Ku portfolios, using the CAPM model, the Fama-French 3-factor model (FF), two-parameter $q$-theory (Q2) model, four-parameter intangible-assets-augmented $q$-theory model with investment-specific technological changes (Q4 ISTC) and four-parameter intangible-assets-augmented $q$-theory model with intangible investment adjustment costs (Q4 AC), respectively.

Panel A: CAPM Model
Panel B: FF Model
Panel C: Q2 Model
Panel D: Q4 ISTC Model
Panel E: Q4 AC Model
Figure 3: Scatter Plots of Average Predicted Return against Average Stock Return for the I/K Portfolios

Average predicted stock returns versus average realized stock returns for the I/K portfolios, using the CAPM model, the Fama-French model (FF), two-parameter $q$-theory (Q2) model, four-parameter intangible-assets-augmented $q$-theory model with investment-specific technological changes (Q4 ISTC) and four-parameter intangible-assets-augmented $q$-theory model with intangible investment adjustment costs (Q4 AC), respectively.