Guest Lecture:

The Rational Expectations Approach to the Cross Section of Returns

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Theme

- Applying rational expectations economics to the cross section of returns

Roadmap

- Facts
- Equilibrium cross section of returns
- The value premium
- Anomalies
- Future

The-Big-Picture

- Anomalies:

  Observed characteristic-return relations in the cross section not predicted or captured by current asset pricing models

- Rational expectations versus behavioral finance:

$$\overbrace{r_{t+1}}^{\textbf{Realized return}} = \underbrace{\mathrm{E}_t[r_{t+1}]}_{\textbf{Expected return}} + \overbrace{\epsilon_{t+1}}^{\textbf{Abnormal return}}$$

- I apply rational expectations economics to derive and test expected-return models

Facts: Empirical Asset Pricing

- The value anomaly

$$(\text{Market/Book equity})_t \uparrow \quad \Rightarrow \quad \overline{r}_{t+1} \downarrow \qquad \text{Stronger in small firms}$$

Fama and French (1992, 1993); Lakonishok, Shleifer, and Vishny (1994)

- The investment anomaly

$$(\text{Investment/Asset})_t \uparrow \quad \Rightarrow \quad \overline{r}_{t+1} \downarrow$$

Titman, Wei, and Xie (2004); Anderson and Garcia-Feijóo (2005); Polk and Sapienza (2005); Xing (2005)

Facts: Empirical Corporate Finance

- The seasoned-equity-offering underperformance anomaly

$$(\text{Seasoned equity/Asset})_t \uparrow \quad \Rightarrow \quad \overline{r}_{t+1} \downarrow \qquad \text{Stronger in small firms}$$

Loughran and Ritter (1995); Spiess and Affleck-Graves (1995)

- The payout anomaly

$$(\text{Payout/Asset})_t \uparrow \quad \Rightarrow \quad \overline{r}_{t+1} \uparrow \qquad \text{Stronger in value firms}$$

Lakonishok and Vermaelen (1990); Ikenberry, Lakonishok, and Vermaelen (1995); Michaely, Thaler, and Womack (1995)

Facts: Capital Markets Research in Accounting

- The post-earnings-announcement drift

$$(\text{Earnings surprise})_t \uparrow \quad \Rightarrow \quad \overline{r}_{t+1} \uparrow \qquad \text{Stronger in small firms}$$

Ball and Brown (1968); Bernard and Thomas (1989, 1990)

- The profitability/expected-profitability anomalies

$$\begin{array}{rcl} \text{Profitability}_t \uparrow & \Rightarrow & \overline{r}_{t+1} \uparrow \\ \text{E}_t[\text{Profitability}_{t+1}] \uparrow & \Rightarrow & \overline{r}_{t+1} \uparrow \end{array} \qquad \text{Stronger in small firms}$$

Haugen and Baker (1996); Frankel and Lee (1998); Piotroski (2000); Cohen, Gompers, and Vuolteenaho (2002); Fama and French (2005)

Objective

- *Prescott (2004, Prize lecture): "Prior to the transformation, macroeconomics was largely separate from the rest of economics. Indeed, some considered the study of macroeconomics fundamentally different and thought there was no hope of integrating macroeconomics with the rest of economics, that is, with neoclassical economics."*

- Empirical finance is like macro 60 years ago:

  Koopmans (1947): "Measurement without theory"

- Time for **the Lucas-Prescott neoclassical transformation** in finance

Methodology

- Only covariances matter in efficient markets. Daniel and Titman (1997)
- Surprise — **characteristics** can be sufficient statistics of expected returns!

Risk	⟷	Expected returns	⟷	Characteristics
$r_{ft} + \beta_{Mt}\lambda_{Mt}$	$=$	$\mathrm{E}_t[r^S_{t+1}]$	$=$	$\mathrm{E}_t[r^I_{t+1}]$
Consumption-based asset pricing				Investment-based asset pricing

- $\beta_{Mt} = \frac{\mathrm{E}_t[r^I_{t+1}] - r_{ft}}{\lambda_{Mt}}$ — relative measurement errors

Equilibrium Cross Section of Returns

- Gomes, Kogan, and Zhang (2003, Journal of Political Economy)
- The first DSGE model with the cross section of returns
- Using the Kydland-Prescott (1982) methodology, we find:
  - The cross-sectional predictability associated with size and book-to-market
  - The predictability subsists after controlling for empirical estimates of beta
  - Empirical success of size and book-to-market can be consistent with the CAPM

- The model is not neoclassical, however

- Market-clearing requires **aggregation** across the cross section of returns

- We assume all firms have equal amount of growth options

- This assumption allows analytical aggregation

- But:

  - Value and growth firms have same amount of growth options

  - Growth firms are more profitable, but cannot invest more

  - Growth firms pay out more, and have long cash-flow duration than value firms

- Counterfactual economic mechanism underlying the simulated value premium

The Value Premium

- Zhang (2005, Journal of Finance)
- Relax the equal-growth assumption, but sacrifices general equilibrium

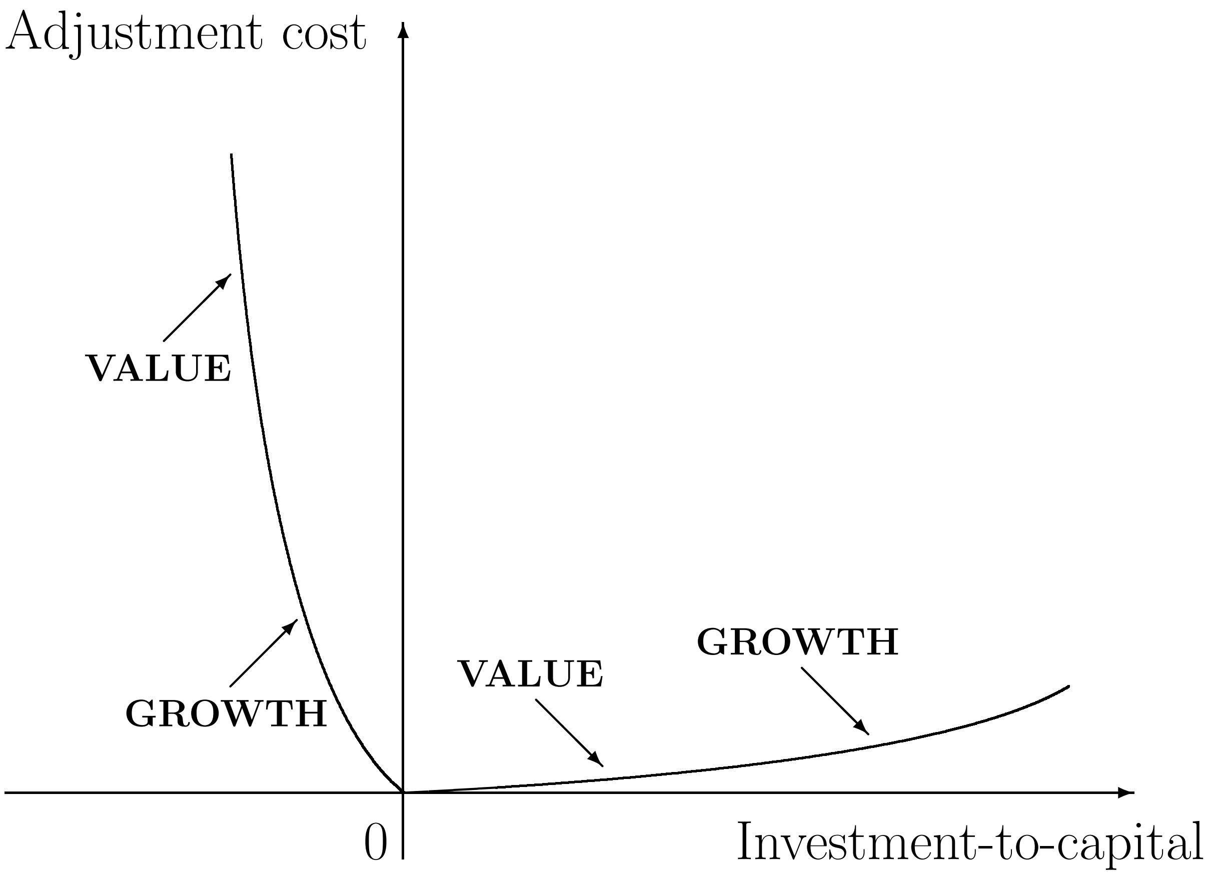
Contribution

- A neoclassical explanation of the value premium:
  - Asymmetry causes countercyclical risk of value-minus-growth strategy
  - Countercyclical price of risk interacts with and propagates the effect
- Endogenous cross section of returns with aggregate uncertainty

Intuition

- Adjustment costs increase risk in a production economy. Jermann (1998):
  - Capital adjustment helps the firms smooth cash flow streams
  - Adjustment costs are the offsetting force of changing capital
- In the data and in the model, high book-to-market (value) signals sustained low profitability, and low book-to-market (growth) signals sustained high profitability

- **Asymmetry**:



- **Time-varying risk** — value is riskier than growth in bad times, and growth is riskier than value in good times, but to a somewhat lesser extent

- **Propagation** effects of countercyclical price of risk
- Strengthen the time-varying risk pattern
- A high average value premium can coexist with a low average risk difference

$$\text{Value Premium} \;=\text{Risk Difference}\;\times\;\text{Price of Risk}$$

	Risk Difference	Price of Risk	Value Premium
good times	slightly negative	low	slightly negative
bad times	high	high	very high
on average	low	average	high

The Model

- The Hopenhayn (1992) IE model augmented with aggregate uncertainty
  - See a recent GE extension by Gala (2005)
- Production function:

$$y_{jt} = e^{(x_t + z_{jt})} k_{jt}^{\alpha}$$

where x_t: aggregate productivity; z_{jt}: idiosyncratic productivity

- Industry demand:

$$e^{p_t} = Y_t^{-\eta}$$

where $0 < \eta < 1$ is the inverse price elasticity of demand.

- Exogenous **stochastic discount factor**:

$$\log M_{t+1} = \log \beta + \gamma(x_t - x_{t+1})$$

$$\gamma = \gamma_0 + \gamma_1(x_t - \bar{x}) \qquad \gamma_1 < 0$$

- Reflects the focus on the firm side, not consumer side

- Suppose a representative agent with isoelastic utility:

$$\left.\begin{array}{rcl} \log M_{t+1} & = & \log \beta + \sigma(c_t - c_{t+1}) \\ c_t & \approx & a + b x_t \end{array}\right\} \quad \Rightarrow \quad \gamma \approx \sigma b$$

- Time-varying risk aversion generates countercyclical market price of risk

- **Value-Maximization** of firms:

$$v(k_t, z_t; x_t, p_t) = \max_{\{i_t\}} \left\{ \overbrace{e^{x_t+z_t+p_t} k_t^\alpha - f - i_t - c(i_t, k_t)}^{\textbf{Current Period Dividend}} + \overbrace{\iint M_{t+1} v(k_{t+1}, z_{t+1}; x_{t+1}, p_{t+1})\, Q_z\,(dz_{t+1}|z_t)\, Q_x\,(dx_{t+1}|x_t)}^{\textbf{Expected Continuation Value}} \right\}$$

$$\text{subject to :} \qquad k_{t+1} = i_t + (1-\delta)k_t$$

- The adjustment-cost function is **asymmetric** and quadratic:

$$c(i_t, k_t) = \frac{\theta_t}{2}\left(\frac{i_t}{k_t}\right)^2 k_t$$

where $\theta^- \geq \theta^+ > 0$ and $\theta_t = \theta^+ \cdot \chi_{\{i_t \geq 0\}} + \theta^- \cdot \chi_{\{i_t < 0\}}$

- **Aggregation**

- The law of motion of the cross-sectional distribution of firms, μ_t, is:

$$\mu_{t+1}(\Theta; x_{t+1}) = T(\Theta, (k_t, z_t); x_t)\mu_t(k_t, z_t; x_t)$$

where

$$T(\Theta, (k_t, z_t); x_t) \equiv \iint \chi_{\{(k_{t+1}, z_{t+1}) \in \Theta\}} \, Q_z(dz_{t+1}|z_t) Q_x(dx_{t+1}|x_t)$$

- Industry output:

$$Y_t \equiv \iint y(k_t, z_t; x_t) \, \mu_t \, (dk, dz; x_t)$$

- **Equilibrium**

- A recursive competitive equilibrium is composed of: (i) a law of motion for the cross-sectional distribution of firms; (ii) a value function and a set of investment decision rules; and (iii) an output price, such that:

- Optimality conditions hold.

- Consistency conditions hold.

- Market clears

- **Solution**
- The endogenous state variable, p_t, depends upon the firm distribution, μ_t
- Use approximate aggregation: Krusell and Smith (1998):

$$p_{t+1} = a_1 + a_2\, p_t + a_3\, (x_t - \overline{x})$$

- Solve the firms' problem by the value function iteration method.
- Simulate the economy with 5,000 firms for 12,000 monthly periods.
- Use the data in the stationary region to update the coefficients a_1, a_2, and a_3.
- Check convergence; if yes, go to next step; otherwise go back to step 2.
- Check goodness-of-fit; if yes, done; otherwise try a different specification.

Anomalies

- Zhang (2006, Manuscript)
- A neoclassical theory of asset pricing anomalies with rational expectations

Contribution

- Unify many anomalies using one single, analytical framework
- Propose a theoretically-motivated, easy-to-implement asset pricing test

The Model

- The basic idea: linking characteristics to expected returns analytically
  - Cochrane (1991) and Berk, Green, and Naik (1999)
- Two periods, t and $t+1$
- The operating-profit function, $\Pi(K_t, X_t)$
- Firms invest in period t, produce in t and $t+1$, liquidate at the end of $t+1$
- $K_{t+1} = (1-\delta)K_t + I_t$, adjustment costs $(a/2)\,(I_t/K_t)^2\,K_t$
- Exogenous M_{t+1} correlates with the aggregate component of X_{t+1}

- The market value of the firm

$$\max_{\{I_t\}} \left\{ \underbrace{\Pi(K_t, X_t) - I_t - \frac{a}{2}\left[\frac{I_t}{K_t}\right]^2 K_t}_{\textbf{Payout/Outside equity at period } t} + \mathrm{E}_t\left[M_{t+1}\left[\underbrace{\Pi(K_{t+1}, X_{t+1}) + (1-\delta)K_{t+1}}_{\textbf{Firm value at period } t+1}\right]\right]\right\}$$

- The first-order condition with respect to I_t:

$$\underbrace{1 + a\left[\frac{I_t}{K_t}\right]}_{\textbf{Marginal cost of investment at time } t} = \underbrace{\mathrm{E}_t\left[M_{t+1}\left[\overbrace{\Pi_1(K_{t+1}, X_{t+1}) + (1-\delta)}^{\textbf{Marginal benefit of investment at time } t+1}\right]\right]}_{\textbf{marginal } q \text{ at time } t}$$

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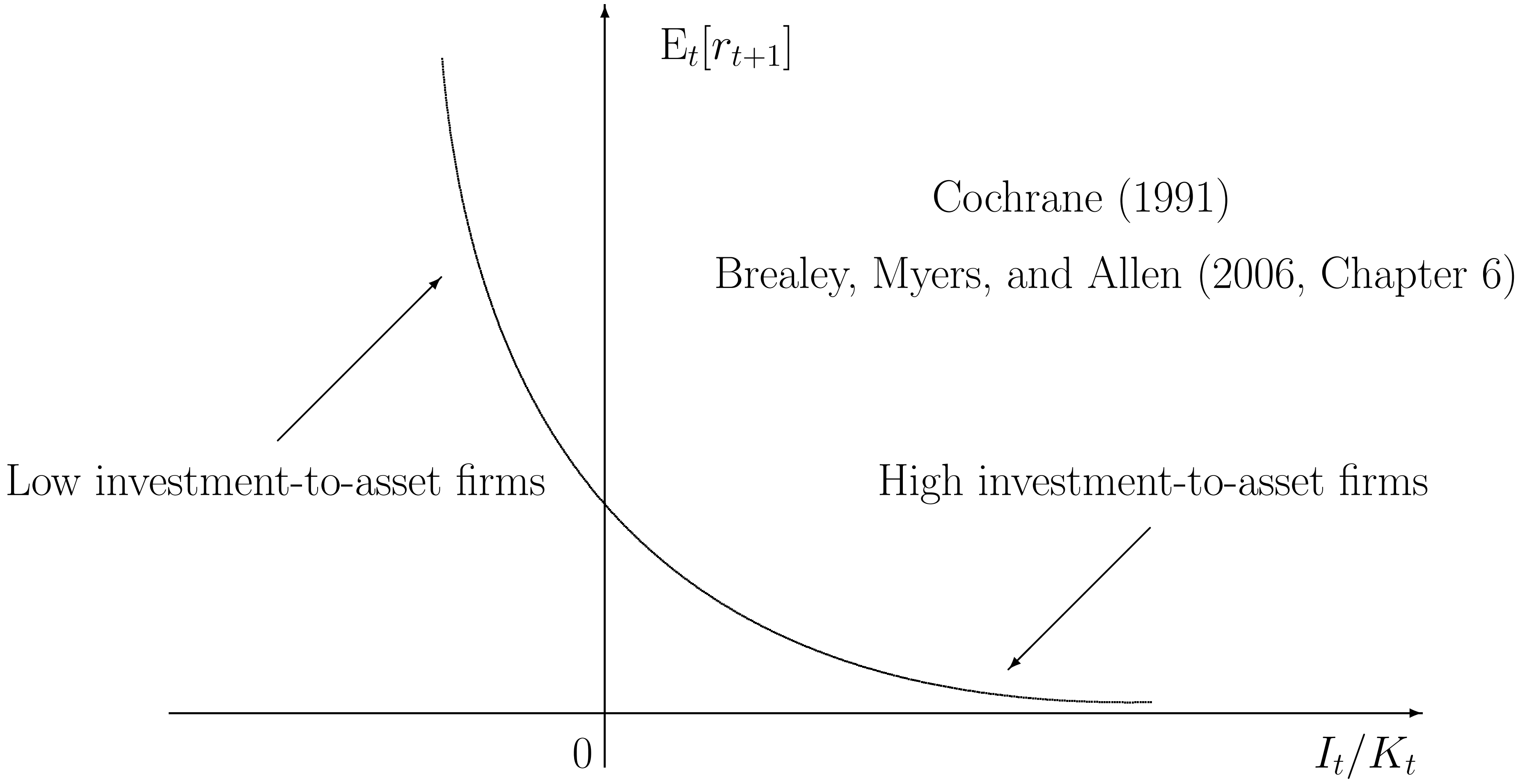
$$\mathrm{E}_t[M_{t+1} r^I_{t+1}] = 1; \qquad \underbrace{r^I_{t+1}}_{\textbf{Investment return}} \equiv \frac{\Pi_1(K_{t+1}, X_{t+1}) + (1-\delta)}{1 + a\,(I_t/K_t)}$$

- $r^S_{t+1} = r^I_{t+1}$ based on Hayashi (1982)

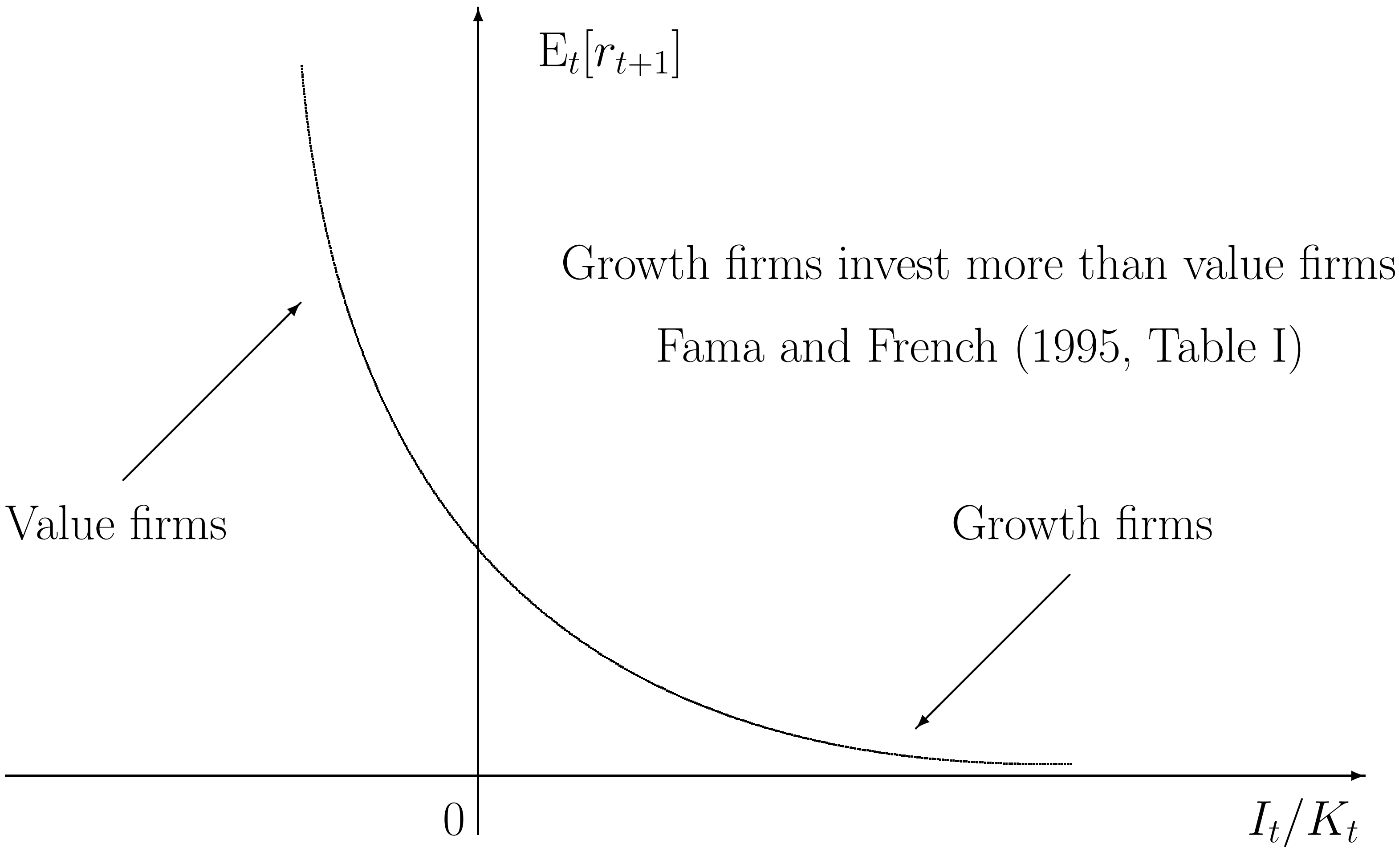
Intuition

- **The investment anomaly:**



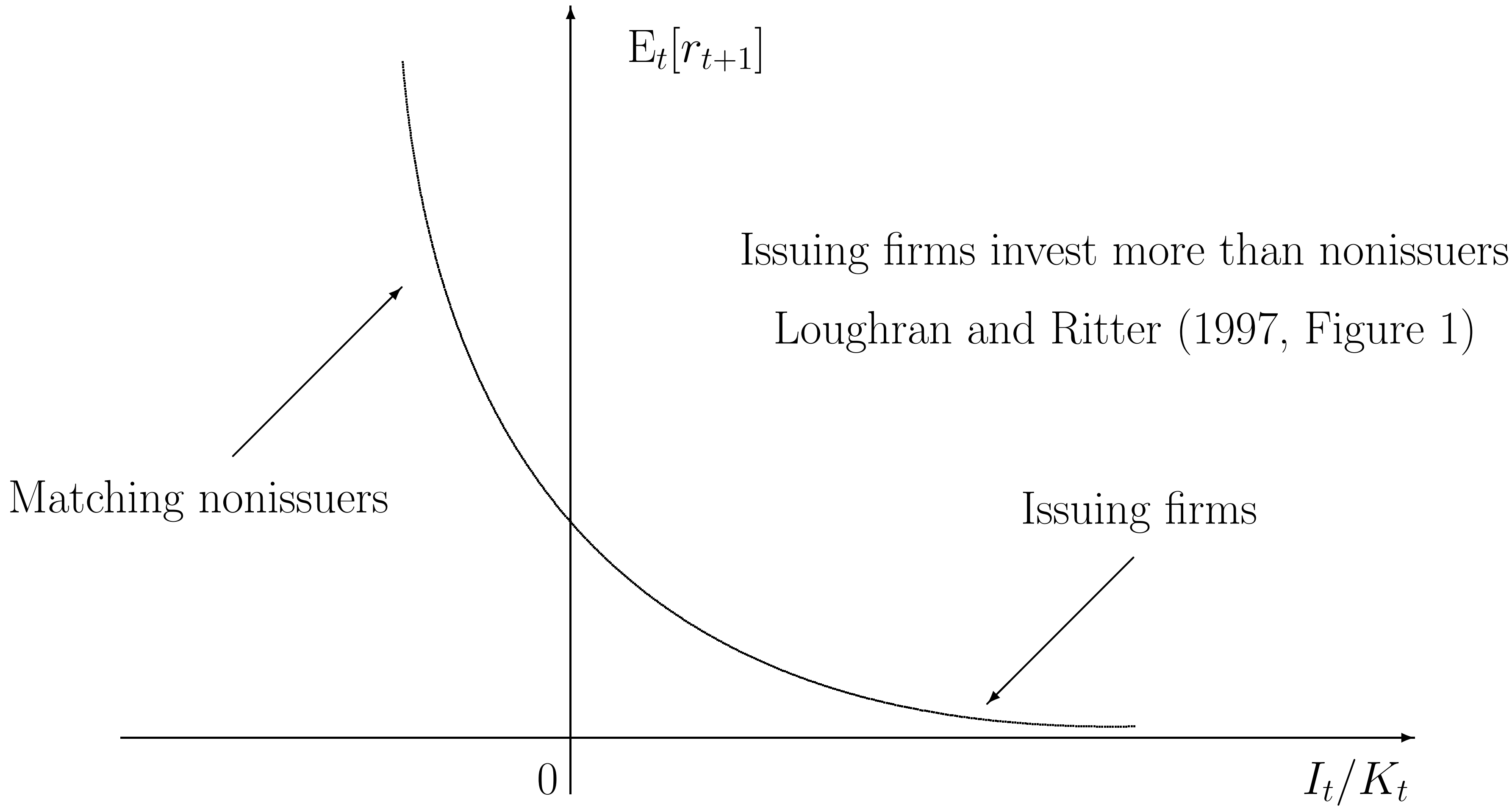
- **The value anomaly:**

$$\underbrace{1 + a(I_t/K_t)}_{\textbf{Marginal cost of investment}} = \overbrace{q_t}^{\textbf{Marginal benefit of investment}} = \underbrace{Q_t}_{\textbf{Market-to-book}}$$



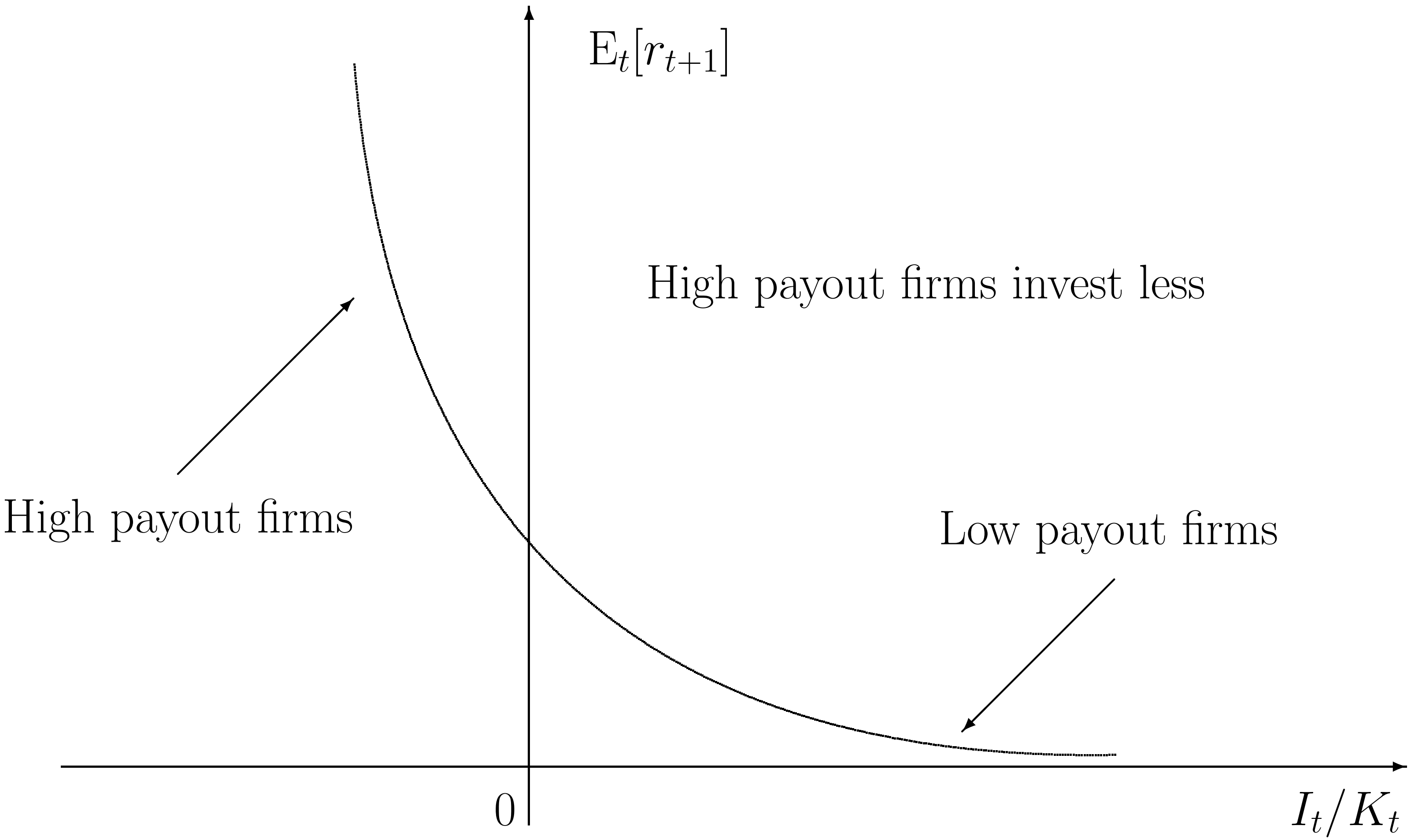
■ **The seasoned-equity-offering anomaly:**

$$\underbrace{\text{Outside equity} + \text{operating profits}}_{\textbf{The sources of funds}} = \underbrace{\text{Investment} + \text{adjustment costs}}_{\textbf{The uses of funds}}$$



- **The payout anomaly:**

$$\underbrace{\text{Operating profits}}_{\textbf{The sources of funds}} = \underbrace{\text{Payout} + \text{investment} + \text{adjustment costs}}_{\textbf{The uses of funds}}$$



Convexity

- **Second-order effects:**

  The value anomaly is stronger in small firms; the payout anomaly is stronger in value firms; the SEO anomaly is stronger in small firms

- The investment-return relation is also convex, $\partial^2 \mathrm{E}_t[r_{t+1}]/\partial(I_t/K_t)^2 > 0$

- By the chain rule, the convexity leads to the second-order effects.

- The investment-value-SEO-payout anomalies are basically the same phenomenon

The Earnings-Return Relation

- Fama (1998): the "granddaddy" of anomalies

- Earnings + depreciation = Operating cash flow

$$\underbrace{N_{t+1}/K_{t+1}}_{\textbf{Profitability}} + \overbrace{\delta}^{\textbf{The rate of depreciation}} = \underbrace{\Pi_{t+1}/K_{t+1}}_{\textbf{Average product of capital}} = \overbrace{\Pi_1(K_{t+1}, X_{t+1})}^{\textbf{Marginal product of capital}}$$

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$$\mathrm{E}_t[r_{t+1}] = \frac{\overbrace{\mathrm{E}_t[\Pi_1(K_{t+1}, X_{t+1})]}^{\textbf{Expected marginal product of capital}} + 1 - \delta}{1 + a\,(I_t/K_t)} = \frac{\overbrace{\mathrm{E}_t[N_{t+1}/K_{t+1}]}^{\textbf{Expected profitability}} + 1}{Q_t}$$

The expected-profitability anomaly, the loading $1/Q_t$ decreases in size, P_t!

Post-Earnings-Announcement Drift

- Profitability is highly persistent, Fama and French (1995, 2000, 2004)

$$\overbrace{N_{t+1}/K_{t+1}}^{\textbf{Realized profitability}} \quad = \quad \overbrace{\bar{n}(1-\rho_n) \ + \ \rho_n(N_t/K_t)}^{\textbf{Expected profitability}} \quad + \quad \overbrace{\epsilon^n_{t+1}}^{\textbf{Earnings suprise}}$$

- $$\overbrace{\mathrm{E}_t[r_{t+1}] = \frac{1}{Q_t}\left[\rho_n(N_t/K_t) + \bar{n}(1-\rho_n) \ + \ 1\right]}^{\textbf{Profitability}\uparrow \quad \Rightarrow \quad \textbf{Expected profitability}\uparrow \quad \Rightarrow \quad \textbf{Expected return}\uparrow}$$

- $$\overbrace{\mathrm{E}_t[r_{t+1}] = \frac{1}{Q_t}\left[\bar{n}(1-\rho_n)(1+\rho_n) + \rho_n^2(N_{t-1}/K_{t-1}) + \rho_n\epsilon^n_t + 1\right]}^{\textbf{Earnings surprise}\uparrow \quad \Rightarrow \quad \textbf{Profitability}\uparrow \quad \Rightarrow \quad \textbf{Expected return}\uparrow}$$

Test

- The theory implies a new, structural expected-return model

$$\mathrm{E}\left[\left(r^S_{t+1} - \underbrace{\frac{\Pi_1(K_{t+1}, X_{t+1}) + (1-\delta)}{1 + a(I_t/K_t)}}_{\textbf{the investment return}}\right) \bigotimes \mathcal{Z}_t\right] = 0$$

- Add multi-period, operating costs, and leverage
- See Whited and Zhang (2006, Testing the q-theory of anomalies)

Summary

- A neoclassical theory of asset pricing anomalies with rational expectations:

$$\mathrm{E}_t[r_{t+1}] = \frac{\mathrm{E}_t[\Pi_1(K_{t+1}, X_{t+1})] + 1 - \delta}{1 + a\,(I_t/K_t)}$$

  The earnings-return relation via the "cash-flow" channel

  The investment-value-SEO-payout anomalies via the "discount-rate" channel

- A theoretically motivated, easy-to-implement expected-return model

The Future

- The neoclassical transformation of the cross section of returns has only started
- Most of the qualitative analysis needs to be quantified
- Model other corporate decisions such as mergers and acquisitions; debt and corporate savings, research and development; corporate governance; and accruals
- Applying dynamic, quantitative tools in asset pricing and corporate finance
- Corporate finance is even more abysmal than asset pricing
- **The neoclassical transformation: Wave of the future!**