“What’s Vol Got to Do With It”

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Theme

An important step in the Long-Run Risks literature à la Bansal and Yaron (2004)

"A calibrated, generalized Long-Run Risks model generates a variance premium with time variation and return predictability that is consistent with the data..."

Theme

My discussion

Seems early to claim total victory: difficulty in explaining the term structure of predictability (R^2) associated with variance risk premium

Outline

1. Variance Risk Premium: Definition and Stylized Facts
2. The Model's Quantitative Performance
3. Time-Varying Risk Aversion: A Missing Ingredient?

Variance Risk Premium

Definition: variance swap

A long variance swap pays the difference between realized variance over the life of the contract, $RV_{t,T}$, and a variance swap rate, $SW_{t,T}$:

$$(RV_{t,T} - SW_{t,T})\, L$$

in which L: the notional dollar amount per unit of variance

A swap has a market value of zero at entry:

$$SW_{t,T} = E_t^{\mathcal{Q}}\left[RV_{t,T}\right]$$

in which $\mathcal{Q}$ is the risk-neutral measure

Variance Risk Premium

Definition: risk premium as the difference between the forward and expected spot prices

Under the physical measure:

$$SW_{t,T} = \frac{E_t\left[M_{t,T} RV_{t,T}\right]}{E_t\left[M_{t,T}\right]} = E_t\left[RV_{t,T}\right] + \frac{Cov_t\left[M_{t,T}, RV_{t,T}\right]}{E_t\left[M_{t,T}\right]}$$

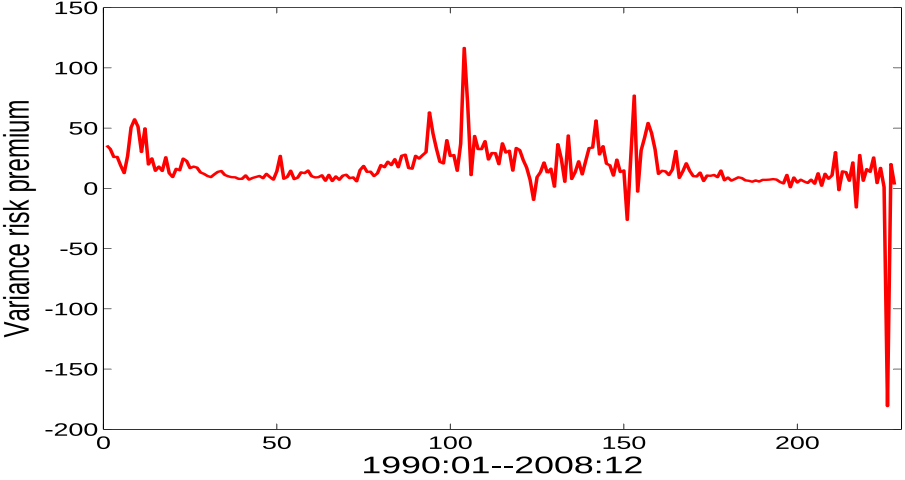
Let VP_t denote the variance risk premium (known at t):

$$VP_t = SW_{t,T} - E_t\left[RV_{t,T}\right] = \frac{Cov_t\left[M_{t,T}, RV_{t,T}\right]}{E_t\left[M_{t,T}\right]} > 0$$

The variance swap is a hedge against high economic uncertainty

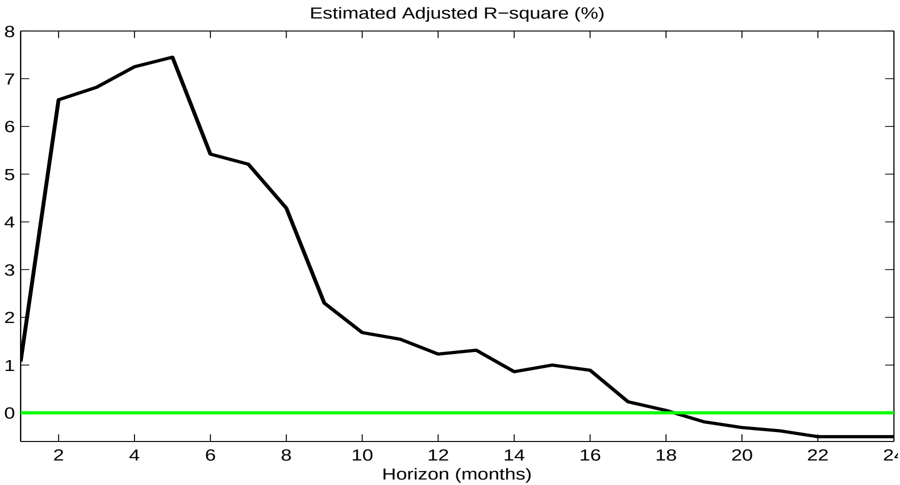
Variance Risk Premium

Data from Hao Zhou's Web site, 1990:01–2008:12, m = 17.07, std = 19.99, $\rho(1) = 0.29$;
$SW_{t,T} = VIX^2$ and $E_t[RV_{t,T}] = RV_t$ (summing squared 5-minute log returns in month t)



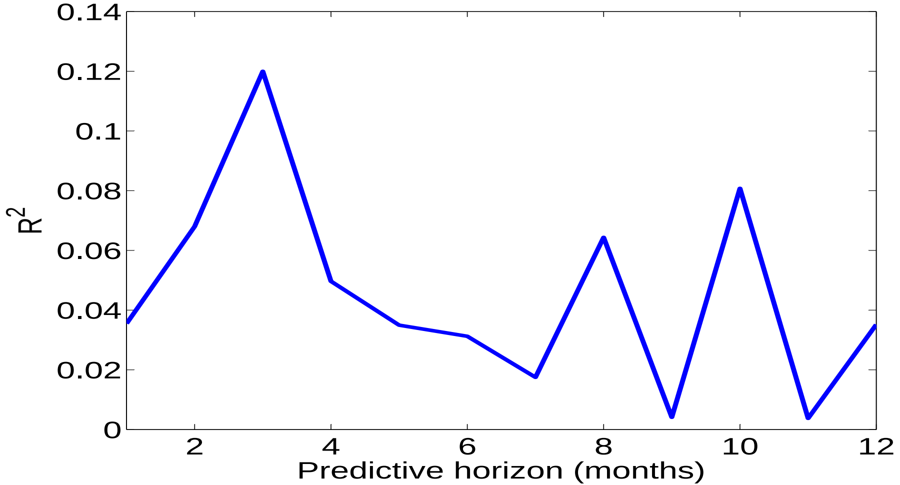
Variance Risk Premium

The term structure of predictive R^2 (1990:01–2007:12): S&P 500 excess returns, overlapping monthly observations, from Bollerslev, Tauchen, and Zhou (2008)



Variance Risk Premium

The term structure of predictive R^2 (1990:01–2008:12): CRSP value-weighted market excess returns, non-overlapping observations, different from traditional predictors!



The Model's Quantitative Performance

The benchmark calibration misses the term structure of predictive R^2 (and $E[VP]$?)

Statistic	Data		Model 5%	Model 50%	Model 95%
Variance Premium					
$\sigma(\text{var}_t(r_m))$	17.18	(2.21)	6.62	23.46	73.23
$AC1(\text{var}_t(r_m))$	0.81	(0.04)	0.66	0.82	0.92
$AC2(\text{var}_t(r_m))$	0.64	(0.08)	0.45	0.67	0.85
$E[VP]$	11.27	(0.93)	4.02	7.57	17.63
$\sigma(VP)$	7.61	(1.08)	3.00	10.65	33.23
$skew(VP)$	2.39	(0.59)	1.84	3.36	5.36
$kurt(VP)$	12.03	(3.30)	6.52	15.74	38.00
$\beta(1)$	0.76	(0.35)	-0.39	0.83	2.63
$R^2(1)$	1.46	(1.52)	0.02	1.94	9.73
$\beta(3)$	0.86	(0.27)	-0.27	0.76	2.09
$R^2(3)$	5.92	(4.67)	0.04	4.21	23.80
$\beta(6)$	0.49	(0.24)	-0.38	0.55	1.68
$R^2(6)$	3.97	(4.74)	0.07	5.66	33.64

The Model's Quantitative Performance

The term structure of predictive R^2: similar issues in alternative parameterizations

Statistic	Data		Model 1-A			Model 1-B			Model 1-C		
			5%	50%	95%	5%	50%	95%	5%	50%	95%
Variance Premium											
$\sigma(\text{var}_t(r_m))$	17.18	(2.21)	4.64	6.22	9.44	3.40	10.51	32.21	3.45	4.76	7.15
$AC1(\text{var}_t(r_m))$	0.81	(0.04)	0.79	0.87	0.93	0.65	0.81	0.924	0.81	0.87	0.93
$AC2(\text{var}_t(r_m))$	0.64	(0.08)	0.63	0.75	0.86	0.43	0.66	0.84	0.64	0.75	0.87
$E[VP]$	11.27	(0.93)	0.22	0.30	0.40	0.23	0.41	1.10	0.02	0.02	0.03
$\sigma(VP)$	7.61	(1.08)	0.12	0.17	0.25	0.19	0.59	1.81	0.01	0.01	0.02
$skew(VP)$	2.39	(0.59)	0.32	0.88	1.71	1.79	3.26	5.12	0.36	0.82	1.60
$kurt(VP)$	12.03	(3.30)	2.35	3.33	6.72	6.79	15.22	35.98	2.34	3.26	6.39
$\beta(1)$	0.76	(0.35)	-33.43	7.43	60.41	-14.94	4.31	28.52	-478.07	39.43	604.86
$R^2(1)$	1.46	(1.52)	0.00	0.22	2.42	0.01	0.61	5.89	0.01	0.20	1.96
$\beta(3)$	0.86	(0.27)	-32.28	6.19	62.27	-13.83	3.27	21.25	-485.47	39.43	604.86
$R^2(3)$	5.92	(4.67)	0.00	0.50	7.13	0.01	1.39	10.70	0.01	0.53	5.24
$\beta(6)$	0.49	(0.24)	-33.47	4.44	47.04	-10.89	2.59	16.30	-447.01	29.35	522.27
$R^2(6)$	3.97	(4.74)	0.016	1.11	12.62	0.01	1.74	15.83	0.01	1.04	9.11

The Model's Quantitative Performance

Discussion

Bollerslev, Tauchen, and Zhou (2008) match the term structure of R^2 with $\psi = 2.5$, but without matching the equity premium

Drechsler and Yaron (2008) try to match the level and dynamics of the equity premium and the variance premium (ambitious!)

Tension between matching the level of equity premium and matching the dynamics of variance premium: does the jump in long-run risks make the model overshoot the optimal quarterly horizon of the *VP* predictability?

Time-Varying Risk Aversion

An alternative driving force for variance risk premium: Heston (1993), Bakshi and Kapadia (2003), Bollerslev, Gibson, and Zhou (2008), Nyberg and Wilhelmsson (2008)

Assume power utility and an affine stochastic volatility model:

$$\gamma = -\frac{\lambda}{\rho\sigma} \approx \lambda$$

in which ρ is the correlation between market returns and market volatility

A similar close link exists under time-varying risk aversion

Summary
My discussion

An important step toward understanding an important economic question in an important literature

What drives the short-run predictability of variance risk premium?

A unified framework with time-varying risk aversion and long-run risks? Necessary for evaluating their quantitative importance in driving the dynamics of aggregate risk premiums