

# Investment-Based Expected Stock Returns

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## Abstract

We derive and test  $q$ -theory implications for cross-sectional stock returns. Under constant returns to scale, stock returns equal levered investment returns, which are tied directly to firm characteristics. When we use GMM to match average levered investment returns to average observed stock returns, the model captures the average stock returns of portfolios sorted by earnings surprises, book-to-market equity, and capital investment. When we try to match expected returns and return variances simultaneously, the variances predicted in the model are largely comparable to those observed in the data. However, the resulting expected return errors are large.

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# 1 Introduction

We use the  $q$ -theory of investment to derive and test predictions for the cross section of stock returns. Under constant returns to scale, stock returns equal levered investment returns, which are directly tied to firm characteristics via the conditions for optimal investment. We use Generalized Methods of Moments (GMM) to match means and variances of levered investment returns with those of stock returns. We conduct the GMM tests using data on portfolios sorted by earnings surprises, book-to-market equity, and capital investment, which are firm characteristics tied closely to cross sectional patterns in returns. We also compare the performance of the  $q$ -theory model with the performance of traditional asset pricing models such as the Capital Asset Pricing Model (CAPM), the Fama-French (1993) three-factor model, and the standard consumption-CAPM with power utility.

When matching the average returns of the testing portfolios, the  $q$ -theory model outperforms the traditional models. We estimate a mean absolute error of 0.7% per annum for ten equal-weighted portfolios sorted by earnings surprises. This error is lower than those from the CAPM, 5.7%, the Fama-French model, 4.0%, and the standard consumption-CAPM, 3.6%. The error for the return on the portfolio that is long on stocks with high earnings surprises and short on stocks with low earnings surprises (high-minus-low earnings surprise portfolio) is  $-0.4\%$  per annum. This error is negligible compared to the errors of 12.6% from the CAPM, 14.1% from the Fama-French model, and 13.4% from the standard consumption-CAPM. Similarly, the  $q$ -theory model produces an error for the high-minus-low book-to-market portfolio of only 1.21% per annum, which is smaller than 18.6% from the CAPM, 7.3% from the Fama-French model, and 12.3% from the standard consumption-CAPM. Finally, the  $q$ -theory model produces an error for the high-minus-low capital investment portfolio of  $-0.5\%$  per annum, which is smaller than the error of  $-6.3\%$  from the CAPM,  $-6.3\%$  from the Fama-French model, and  $-8.4\%$  from the standard consumption-CAPM.

When we use the  $q$ -theory model to match the average returns and variances of the testing portfolios simultaneously, the variances predicted by the model are largely comparable to stock return variances. The average stock return volatility across the earnings surprise portfolios is 21.1% per annum, which is close to the average levered investment return volatility of 20.4%. The average realized and predicted volatilities also are close for the book-to-market portfolios: 25.0% versus 23.6%, and for the capital investment portfolios: 24.8% versus 24.4%. However, the model falls short in two ways. First, while we find no discernible relation between volatilities and firm characteristics in the data, the model predicts a positive relation between volatilities and book-to-market.

Second, the resulting expected return errors vary systematically with earnings surprises and capital investment, and are comparable in magnitude to those from the traditional models.

Although  $q$ -theory originates in Brainard and Tobin (1968) and Tobin (1969), our work is built more directly on Cochrane (1991), who first uses  $q$ -theory to study stock market returns, as well as on Cochrane (1996), who uses aggregate investment returns to parameterize the stochastic discount factor in cross-sectional tests. Several more recent articles model cross-sectional returns based on firms' dynamic optimization problems (e.g., Berk, Green, and Naik (1999) and Zhang (2005)). We differ by doing structural estimation of closed-form Euler equations. Our work is also connected to the literature that estimates investment Euler equations using aggregate or firm level investment data (e.g., Shapiro (1986) and Whited (1992)). Our work differs because we use this framework to study cross-sectional returns rather than investment dynamics or financing constraints. Most important, our  $q$ -theory approach to understanding cross-sectional returns represents a fundamental departure from the traditional consumption-based approach (e.g., Hansen and Singleton (1982) and Lettau and Ludvigson (2001)) in that we do not make any preference assumptions.

## 2 The Model of the Firms

Time is discrete and the horizon infinite. Firms use capital and costlessly adjustable inputs to produce a homogeneous output. These latter inputs are chosen each period to maximize operating profits, defined as revenues minus the expenditures on these inputs. Taking operating profits as given, firms choose optimal investment to maximize the market value of equity.

Let  $\Pi(K_{it}, X_{it})$  denote the maximized operating profits of firm  $i$  at time  $t$ . The profit function depends on capital,  $K_{it}$ , and a vector of exogenous aggregate and firm-specific shocks,  $X_{it}$ . We assume that firm  $i$  has a Cobb-Douglas production function with constant returns to scale. The assumption of constant returns means that  $\Pi(K_{it}, X_{it}) = K_{it} \partial \Pi(K_{it}, X_{it}) / \partial K_{it}$ . The Cobb-Douglas functional form means that the marginal product of capital is given by  $\partial \Pi(K_{it}, X_{it}) / \partial K_{it} = \alpha Y_{it} / K_{it}$ , in which  $\alpha > 0$  is capital's share and  $Y_{it}$  is sales. This parametrization assumes that shocks to operating profits,  $X_{it}$ , are reflected in sales.

End-of-period capital equals investment plus beginning-of-period capital net of depreciation:  $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$ , in which capital depreciates at an exogenous proportional rate of  $\delta_{it}$ , which is firm-specific and time-varying. Firms incur adjustment costs when investing. The adjustment cost function, denoted  $\Phi(I_{it}, K_{it})$ , is increasing and convex in  $I_{it}$ , decreasing in  $K_{it}$ , and exhibits

constant returns to scale in  $I_{it}$  and  $K_{it}$ :  $\Phi(I_{it}, K_{it}) = I_{it}\partial\Phi(I_{it}, K_{it})/\partial I_{it} + K_{it}\partial\Phi(I_{it}, K_{it})/\partial K_{it}$ . We use a standard quadratic functional form:  $\Phi(I_{it}, K_{it}) = (a/2)(I_{it}/K_{it})^2 K_{it}$ , in which  $a > 0$ .

Firms can finance investment with debt. We follow Hennessy and Whited (2007) and model only one-period debt. At the beginning of time  $t$ , firm  $i$  can issue an amount of debt, denoted  $B_{it+1}$ , which must be repaid at the beginning of period  $t+1$ . The gross corporate bond return on  $B_{it}$ , denoted  $r_{it}^B$ , can vary across firms and over time. Taxable corporate profits equal operating profits less capital depreciation, adjustment costs, and interest expenses:  $\Pi(K_{it}, X_{it}) - \delta_{it}K_{it} - \Phi(I_{it}, K_{it}) - (r_{it}^B - 1)B_{it}$ , in which adjustment costs are expensed, consistent with treating them as forgone operating profits. Let  $\tau_t$  denote the corporate tax rate at time  $t$ . The payout of firm  $i$  equals:

$$D_{it} \equiv (1 - \tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it})] - I_{it} + B_{it+1} - r_{it}^B B_{it} + \tau_t \delta_{it} K_{it} + \tau_t (r_{it}^B - 1) B_{it}, \quad (1)$$

in which  $\tau_t \delta_{it} K_{it}$  is the depreciation tax shield and  $\tau_t (r_{it}^B - 1) B_{it}$  is the interest tax shield.

Let  $M_{t+1}$  be the stochastic discount factor from  $t$  to  $t+1$ , which is correlated with the aggregate component of  $X_{it+1}$ . We can formulate the cum-dividend market value of equity as follows:

$$V_{it} \equiv \max_{\{I_{it+s}, K_{it+s+1}, B_{it+s+1}\}_{s=0}^{\infty}} E_t \left[ \sum_{s=0}^{\infty} M_{t+s} D_{it+s} \right], \quad (2)$$

subject to a transversality condition that prevents firms from borrowing an infinite amount to distribute to shareholders:  $\lim_{T \rightarrow \infty} E_t [M_{t+T} B_{it+T+1}] = 0$ .

**Proposition 1.** Firms' value-maximization implies that  $E_t [M_{t+1} r_{it+1}^I] = 1$ , in which  $r_{it+1}^I$  is the investment return, defined as:

$$r_{it+1}^I \equiv \frac{(1 - \tau_{t+1}) \left[ \alpha \frac{Y_{it+1}}{K_{it+1}} + \frac{a}{2} \left( \frac{I_{it+1}}{K_{it+1}} \right)^2 \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1}) a \left( \frac{I_{it+1}}{K_{it+1}} \right) \right]}{1 + (1 - \tau_t) a \left( \frac{I_{it}}{K_{it}} \right)}. \quad (3)$$

Define the after-tax corporate bond return as  $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$ , then  $E_t [M_{t+1} r_{it+1}^{Ba}] = 1$ . Define  $P_{it} \equiv V_{it} - D_{it}$  as the ex-dividend equity value,  $r_{it+1}^S \equiv (P_{it+1} + D_{it+1})/P_{it}$  as the stock return, and  $w_{it} \equiv B_{it+1}/(P_{it} + B_{it+1})$  as the market leverage, then the investment return is the weighted average of the stock return and the after-tax corporate bond return:

$$r_{it+1}^I = w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^S. \quad (4)$$

**Proof.** See Appendix A. ■

The investment return in equation (3) is the ratio of the marginal benefit of investment at time  $t + 1$  to the marginal cost of investment at  $t$ . Define marginal  $q$  as the discounted present value of the future marginal profits from investing in one additional unit of capital (see equation (A2) in Appendix A). Optimality means that the marginal cost of investment equals the marginal  $q$ . In the numerator of equation (3) the term  $(1 - \tau_{t+1})\alpha Y_{it+1}/K_{it+1}$  is the marginal after-tax profit produced by an additional unit of capital, the term  $(1 - \tau_{t+1})(a/2)(I_{it+1}/K_{it+1})^2$  is the marginal after-tax reduction in adjustment costs, the term  $\tau_{t+1}\delta_{it+1}$  is the marginal depreciation tax shield, and the last term in the numerator is the marginal continuation value of the extra unit of capital net of depreciation. In addition, the first term in brackets in the numerator divided by the denominator is analogous to a dividend yield. The second term in brackets in the numerator divided by the denominator is analogous to a capital gain because this ratio is the growth rate of marginal  $q$ .

Equation (4) is exactly the weighted average cost of capital in corporate finance. Without leverage, this equation reduces to the equivalence between stock and investment returns, a relation first established by Cochrane (1991). This relation is an algebraic restatement of the equivalence between marginal  $q$  and average  $q$  from Hayashi (1982). Solving for  $r_{it+1}^S$  from equation (4) gives:

$$r_{it+1}^S = r_{it+1}^{Iw} \equiv \frac{r_{it+1}^I - w_{it} r_{it+1}^{Ba}}{1 - w_{it}}, \quad (5)$$

in which  $r_{it+1}^{Iw}$  is the levered investment return.

### 3 Econometric Methodology

#### 3.1 Moments for GMM Estimation and Tests

To examine whether cross-sectional variation in average stock returns matches cross-sectional variation in firm characteristics, we test the ex-ante restriction implied by equation (5): expected stock returns equal expected levered investment returns,

$$E [r_{it+1}^S - r_{it+1}^{Iw}] = 0. \quad (6)$$

To examine whether the  $q$ -theory model can reproduce empirically plausible stock return volatilities, we also test whether stock return variances equal levered investment return variances:

$$E \left[ (r_{it+1}^S - E [r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E [r_{it+1}^{Iw}])^2 \right] = 0. \quad (7)$$

As noted by Cochrane (1991), taken literally, equation (5) says that levered investment returns

equal stock returns for every stock, every period, and every state of the world. Because no choice of parameters can satisfy these conditions, equation (5) is rejected at any level of significance. However, we can test the weaker conditions in equations (6) and (7), after adding statistical assumptions about the errors that invalidate these two moment conditions (model errors). These errors arise because of either measurement or specification issues. For example, components of investment returns such as the capital stock are difficult to measure; adjustment costs might not be quadratic; and the marginal product of capital might not be proportional to the sales-to-capital ratio.

Specifically, we define the model errors from the moment conditions as:

$$e_i^q \equiv E_T [r_{it+1}^S - r_{it+1}^{Iw}], \quad (8)$$

$$e_i^{\sigma^2} \equiv E_T \left[ (r_{it+1}^S - E_T [r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E_T [r_{it+1}^{Iw}])^2 \right], \quad (9)$$

in which  $E_T[\cdot]$  is the sample mean of the series in brackets. We call  $e_i^q$  the expected return error and  $e_i^{\sigma^2}$  the variance error. Both errors are assumed to have a mean of zero. While recognizing that measurement and specification errors, unlike forecast errors, do not necessarily have a zero mean, we note that this simple assumption underlies most Euler equation tests.<sup>1</sup>

We estimate the parameters  $a$  and  $\alpha$  using GMM to minimize a weighted average of  $e_i^q$  or a weighted average of both  $e_i^q$  and  $e_i^{\sigma^2}$ . We use the identity weighting matrix in one-stage GMM. By weighting all the moments equally, the identity matrix preserves the economic structure of the testing assets (e.g., Cochrane (1996)). After all, we choose testing assets precisely because the underlying characteristics are economically important in providing a wide cross-sectional spread in average stock returns. The identity weighting matrix also gives potentially more robust, albeit less efficient, estimates. The estimates from second-stage GMM are similar to the first-stage estimates. To conduct inferences, we nevertheless need to calculate the optimal weighting matrix. We use a standard Bartlett kernel with a window length of five. The results are insensitive to the window length. To test whether all (or a subset of) model errors are jointly zero, we use a  $\chi^2$  test from Hansen (1982, Lemma 4.1). Appendix B provides additional econometric details.

We conduct the GMM estimation and tests at the portfolio level. We use portfolios because the

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<sup>1</sup>Cochrane (1991, p. 220) articulates this point as follows: “The consumption-based model suffers from the same problems: unobserved preference shocks, components of consumption that enter nonseparably in the utility function (for example, the service flow from durables), and measurement error all contribute to the error term, and there is no reason to expect these errors to obey the orthogonality restrictions that the forecast error obeys. Empirical work on consumption-based models focuses on the forecast error since it has so many useful properties, but the importance in practice of these other sources of error may be part of the reason for its empirical difficulties.”

stylized facts in cross-sectional returns can always be represented at the portfolio level (e.g., Fama and French (1993)). As such, the usage of portfolios befits our economic question. The portfolio approach also has the advantage that portfolio investment data are smooth, whereas firm level investment data are lumpy because of nonconvex adjustment costs (e.g., Whited (1998)). Thomas (2002) shows that aggregation substantially reduces the effect of lumpy investment in equilibrium business cycle models. Hall (2004) also shows that nonconvexities are not important for estimating investment Euler equations at the industry level.

## 3.2 Data

We construct annual levered investment returns to match with annual stock returns. Our sample of firm-level data is from the Center for Research in Security Prices (CRSP) monthly stock file and the annual and quarterly 2005 Standard and Poor’s Compustat industrial files. We select our sample by first deleting any firm-year observations with missing data or for which total assets, the gross capital stock, debt, or sales are either zero or negative. We include only firms with fiscal yearend in December. Firms with primary SIC classifications between 4900 and 4999 or between 6000 and 6999 are omitted because  $q$ -theory is unlikely to be applicable to regulated or financial firms.

### 3.2.1 Portfolio Definitions

We use 30 testing portfolios: ten Standardized Unexpected Earnings (SUE) portfolios as in Chan, Jegadeesh, and Lakonishok (1996), ten book-to-market (B/M) portfolios as in Fama and French (1993), and ten corporate investment (CI) portfolios as in Titman, Wei, and Xie (2004). SUE is a measure of earnings surprises or shocks to earnings, B/M is the ratio of accounting value of equity divided by the market value of equity, and CI is a measure of firm-level capital investment. The relations of stock returns with SUE and B/M represent what are arguably the two most important stylized facts in the cross section of returns (e.g., Fama (1998)). We use the CI portfolios because our framework characterizes optimal investment behavior. We equal-weight portfolio returns because equal-weighted returns are harder for asset pricing models to capture than value-weighted returns (e.g., Fama (1998)). Our basic results are similar if we value-weight portfolio returns.

*Ten SUE Portfolios.* Following Chan, Jegadeesh, and Lakonishok (1996), we define SUE as the change in quarterly earnings (Compustat quarterly item 8) per share from its value four quarters ago divided by the standard deviation of the change in quarterly earnings over the prior eight quarters. We rank all stocks by their most recent SUEs at the beginning of each month  $t$  and assign all

the stocks to one of ten portfolios using NYSE breakpoints. We calculate average monthly returns over the holding period from month  $t + 1$  to  $t + 6$ . The sample is from January 1972 to December 2005. The starting point is restricted by the availability of quarterly earnings data.

*Ten B/M Portfolios.* Following Fama and French (1993), we sort all stocks at the end of June of year  $t$  into ten groups based on NYSE breakpoints for B/M. The sorting variable for June of year  $t$  is book equity for the fiscal year ending in calendar year  $t-1$  divided by the market value of common equity for December of year  $t-1$ . Book equity is common equity (Compustat annual item 60) plus balance sheet deferred tax (item 74). The market value of common equity is the closing price per share (item 199) times the number of common shares outstanding (item 25). We calculate equal-weighted annual returns from July of year  $t$  to June of year  $t + 1$  for the resulting portfolios, which are rebalanced at the end of each June. The sample is from January 1963 to December 2005.

*Ten CI Portfolios.* Following Titman, Wei, and Xie (2004), we define  $CI_{t-1}$ , the sorting variable in the portfolio formation year  $t$ , as  $CE_{t-1}/[(CE_{t-2}+CE_{t-3}+CE_{t-4})/3]$ , in which  $CE_{t-1}$  is capital expenditures (Compustat annual item 128) scaled by sales (item 12) for the fiscal year ending in calendar year  $t-1$ . The prior three-year moving average of CE aims to measure the benchmark investment level. At the end of June of year  $t$  we sort all stocks on  $CI_{t-1}$  into ten portfolios using breakpoints based on NYSE, Amex, and Nasdaq stocks. Equal-weighted annual portfolio returns are calculated from July of year  $t$  to June of year  $t+1$ . The sample is from January 1963 to December 2005.

### 3.2.2 Variable Measurement

*Capital, Investment, Output, Debt, Leverage, and Depreciation.* The capital stock,  $K_{it}$ , is gross property, plant, and equipment (Compustat annual item 7), and investment,  $I_{it}$ , is capital expenditures minus sales of property, plant, and equipment (the difference between items 128 and 107). We set sales of property, plant, and equipment to be zero when item 107 is missing. Our basic results are similar when we measure the capital stock as the net property, plant, and equipment (item 8) or investment as item 128. Output,  $Y_{it}$ , is sales (item 12), and total debt,  $B_{it}$ , is long-term debt (item 9) plus short term debt (item 34). Our basic results are similar when we use the Bernanke and Campbell (1988) algorithm to convert the book value of debt into the market value of debt. We measure market leverage as the ratio of total debt to the sum of total debt and the market value of equity. The depreciation rate,  $\delta_{it}$ , is the amount of depreciation (item 14) divided by capital stock.

Both stock and flow variables in Compustat are recorded at the end of year  $t$ . However, the model requires stock variables subscripted  $t$  to be measured at the beginning of year  $t$  and flow

variables subscripted  $t$  to be measured over the course of year  $t$ . We take, for example, for the year 1993 any beginning-of-period stock variable (such as  $K_{i1993}$ ) from the 1992 balance sheet and any flow variable measured over the year (such as  $I_{i1993}$ ) from the 1993 income or cash flow statement.

We follow Fama and French (1995) in aggregating firm-specific characteristics to portfolio-level characteristics:  $Y_{it+1}/K_{it+1}$  is the sum of sales in year  $t+1$  for all the firms in portfolio  $i$  formed in June of year  $t$  divided by the sum of capital stocks at the beginning of  $t+1$  for the same firms;  $I_{it+1}/K_{it+1}$  in the numerator of  $r_{it+1}^I$  is the sum of investment in year  $t+1$  for all the firms in portfolio  $i$  formed in June of year  $t$  divided by the sum of capital stocks at the beginning of  $t+1$  for the same firms;  $I_{it}/K_{it}$  in the denominator of  $r_{it+1}^I$  is the sum of investment in year  $t$  for all the firms in portfolio  $i$  formed in June of year  $t$  divided by the sum of capital stocks at the beginning of year  $t$  for the same firms; and  $\delta_{it+1}$  is the total amount of depreciation for all the firms in portfolio  $i$  formed in June of year  $t$  divided by the sum of capital stocks at the beginning of  $t+1$  for the same firms.

*Corporate Bond Returns.* Firm-level corporate bond data are rather limited, and few or none of the firms in several portfolios have corporate bond ratings. To construct bond returns,  $r_{it+1}^B$ , for firms without bond ratings, we follow Blume, Lim, and MacKinlay's (1998) approach for imputing bond ratings not available in Compustat. First, we estimate an ordered probit model that relates categories of credit ratings to observed explanatory variables. We estimate the model using all the firms that have data on credit ratings (Compustat annual item 280). Second, we use the fitted value to calculate the cutoff value for each rating. Third, for firms without credit ratings we estimate their credit scores using the coefficients estimated from the ordered probit model and impute bond ratings by applying the cutoff values for the different credit ratings. Finally, we assign the corporate bond returns for a given credit rating from Ibbotson Associates as the corporate bond returns to all the firms with the same credit rating.

The explanatory variables in the ordered probit model are interest coverage defined as the ratio of operating income after depreciation (Compustat annual item 178) plus interest expense (item 15) to interest expense, the operating margin as the ratio of operating income before depreciation (item 13) to sales (item 12), long-term leverage as the ratio of long-term debt (item 9) to assets (item 6), total leverage as the ratio of long-term debt plus debt in current liabilities (item 34) plus short-term borrowing (item 104) to assets, and the natural log of the market value of equity deflated to 1973 by the Consumer Price Index (item 24 times item 25). Following Blume, Lim, and MacKinlay (1998), we also include the market beta and residual volatility from the market

regression. For each calendar year we estimate the beta and residual volatility for each firm with at least 200 daily returns. Daily stock returns and value-weighted market returns are from CRSP. We adjust for nonsynchronous trading with one leading and one lagged value of the market return.

*The Corporate Tax Rate.* We measure  $\tau_t$  as the statutory corporate income tax rate. From 1963 to 2005, the tax rate is on average 42.3%. The statutory rate starts at around 50% in the beginning years of our sample, drops from 46% to 40% in 1987 and further to 34% in 1988, and stays at that level afterward. The source is the Commerce Clearing House, annual publications.<sup>2</sup>

### 3.2.3 Timing Alignment

To match levered investment returns with stock returns, we need to align their timing. As noted, we use the Fama-French portfolio approach in forming B/M and CI portfolios at the end of June of each year  $t$ . Portfolio stock returns are calculated from July of year  $t$  to June of year  $t+1$ . To calculate the matching investment returns, we use stock variables at the beginning of years  $t$  and  $t+1$  and flow variables for the years  $t$  and  $t+1$ . As such, the timing of the investment returns approximately matches with the timing of stock returns. Appendix C contains further details including the timing for the monthly rebalanced SUE portfolios and for the after-tax corporate bond returns.

## 4 Empirical Results

Section 4.1 reports tests of the CAPM, the Fama-French model, and the standard consumption-CAPM on our portfolios. Section 4.2 reports tests of the  $q$ -theory model in matching expected returns, and Section 4.3 reports tests in matching expected returns and variances simultaneously.

### 4.1 Testing Traditional Asset Pricing Models

To test the CAPM, we regress annual portfolio returns in excess of the risk-free rate on market excess returns. The risk-free rate, denoted  $r_{ft+1}$ , is the annualized return on the one-month Treasury bill from Ibbotson Associates. The regression intercept measures the model error from the CAPM. To test the Fama-French model, we regress annual portfolio excess returns on annual returns of

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<sup>2</sup>We have experimented with firm-specific tax rates using the trichotomous variable approach of Graham (1996). The trichotomous variable is equal to i) the statutory corporate income tax rate if the taxable income defined as pretax income (Compustat annual item 170) minus deferred taxes (item 50) divided by the statutory tax rate is positive and net operating loss carryforward (item 52) is nonpositive; ii) one-half of the statutory rate if one and only one condition in i) is violated; and iii) zero otherwise. The trichotomous variable does not vary much across our testing portfolios. The portfolio-level tax rate is on average 36.0% for the low SUE portfolio, 37.9% for the high SUE portfolio, 34.8% for the low CI portfolio, and 37.4% for the high CI portfolio. The spread across the B/M portfolios is slightly larger: the tax rate is 40.2% in the low B/M portfolio and 35.1% in the high B/M portfolio. As such, we use time-varying but portfolio invariant tax rates for simplicity. The results are largely similar using portfolio-specific tax rates.

the market factor, a size factor, and a book-to-market factor (the factor returns data are from Kenneth French's Web site). The intercept measures the error of the Fama-French model. We also estimate the standard consumption-CAPM with the pricing kernel  $M_{t+1} = \beta(C_{t+1}/C_t)^{-\gamma}$ , in which  $\beta$  is time preference coefficient,  $\gamma$  is risk aversion, and  $C_t$  is annual per capita consumption of nondurables and services from the Bureau of Economic Analysis. We use one-stage GMM with the identity weighting matrix to estimate the moments  $E[M_{t+1}(r_{it+1}^S - r_{ft+1})] = 0$ . We also include  $E[M_{t+1}r_{ft+1}] = 1$  as an additional moment condition to identify  $\beta$ . The error of the standard consumption-CAPM is calculated as  $E_T[M_{t+1}(r_{it+1}^S - r_{ft+1})]/E_T[M_{t+1}]$ .

The SUE, B/M, and CI effects cannot be captured by the traditional models. Panel A of Table 1 shows that from the low SUE to the high SUE portfolio the average return increases monotonically from 10.9% to 23.4% per annum. The portfolio volatilities are largely flat at around 22%. The CAPM error of the high-minus-low SUE portfolio is 12.6% per annum ( $t = 5.5$ ), and the (annualized) mean absolute error, denoted m.a.e., is 5.7%. The Gibbons, Ross, and Shanken (1989, GRS) statistic, which tests the null hypothesis that all the ten individual intercepts are jointly zero, rejects the CAPM. (The intercepts do not add up to zero because we equal-weight the portfolio returns.) The performance of the Fama-French model is similar: the m.a.e. is 4.0% per annum and the GRS test rejects the model. The error of the high-minus-low SUE portfolio from the Fama-French model is 14.1% per annum ( $t = 5.3$ ). The consumption-CAPM error increases from  $-8.1\%$  for the low SUE portfolio to  $5.1\%$  per annum for the high SUE portfolio. Although the errors are not individually significant, probably because of large measurement errors in consumption data, the  $\chi^2$  test rejects the null hypothesis that the pricing errors are jointly zero at the 1% significance level. In addition, the parameter estimates are high: the estimate of the time preference coefficient is 2.8, and the estimate of the risk aversion parameter is 127.6.

Panel B of Table 1 shows that value stocks with high B/M ratios earn higher average stock returns than growth stocks with low B/M ratios, 25.8% versus 8.7% per annum. The difference of 17.1% is significant ( $t = 5.5$ ). There is no discernible relation between B/M and stock return volatility: both the value and the growth portfolios have volatilities around 27%. The CAPM error increases monotonically from  $-4.9\%$  for growth stocks to  $13.7\%$  for value stocks. The average magnitude of the errors is 6.3% per annum and the GRS test strongly rejects the CAPM. Even the Fama-French model fails to capture the equal-weighted returns: the high-minus-low portfolio has an error of 7.3% ( $t = 3.3$ ). The consumption-CAPM error increases from  $-5.4\%$  for growth stocks to

6.9% for value stocks with an average magnitude of 2.4%, and the model is rejected by the  $\chi^2$  test.

From Panel C, high CI stocks earn lower average stock returns than low CI stocks: 15.2% versus 22.1% per annum, and the difference is more than four standard errors from zero. The high-minus-low CI portfolio has an error of  $-6.3\%$  ( $t = -3.9$ ) from the CAPM and an error of  $-6.3\%$  ( $t = -4.0$ ) from the Fama-French model. Both models are rejected by the GRS test. The consumption-CAPM error decreases from 4.0% for the low CI portfolio to  $-4.4\%$  for the high CI portfolio with an average magnitude of 2.4%, and the  $\chi^2$  test rejects the model.

## 4.2 The $q$ -theory Model: Matching Expected Returns

### 4.2.1 Point Estimates and Overall Model Performance

We estimate only two parameters in our parsimonious model: the adjustment cost parameter,  $a$ , and capital's share,  $\alpha$ . Panel A of Table 2 provides estimates of  $\alpha$  ranging from 0.2 to 0.5. These estimates are largely comparable to the approximate 0.3 figure for capital's share in Rotemberg and Woodford (1992). The estimates of  $a$  are not as stable across the different sets of testing portfolios. We find significant estimates of 7.7 and 1.0 for the SUE and CI portfolios, respectively. The estimate is 22.3 for the B/M portfolios but with a high standard error of 25.5. These estimates fall within the wide range of estimates from studies using quantity data. The evidence implies that firm's optimization problem has an interior solution: the positive estimates of  $a$  mean that the adjustment cost function is increasing and convex in  $I_{it}$ .

Panel A of Table 2 also reports two measures of overall model performance: the mean absolute error, m.a.e., and the  $\chi^2$  test. The model does a good job in accounting for the average returns of the ten SUE portfolios. The m.a.e. is 0.7% per annum, which is lower than those from the CAPM, 5.7%, the Fama-French model, 4.0%, and the standard consumption-CAPM, 3.6%. Unlike the traditional models that are rejected using the SUE portfolios, the  $q$ -theory model is not rejected by the  $\chi^2$  test. The overall performance of the model is more modest in capturing the average B/M portfolio returns. Although the model is not formally rejected by the  $\chi^2$  test, the m.a.e. is 2.3% per annum, which is comparable to that from the Fama-French model, 2.8%, and that from the standard consumption-CAPM, 2.4%, but is lower than that from the CAPM, 6.3%. The model does better in pricing the ten CI portfolios. The m.a.e. is 1.5% per annum, which is lower than those from the CAPM, 5.7%, the Fama-French model, 2.2%, and the standard consumption-CAPM, 1.8%. The  $q$ -theory model is again not rejected by the  $\chi^2$  test.

### 4.2.2 Euler Equation Errors

The mean absolute errors and  $\chi^2$  tests only indicate overall model performance. To provide a more complete picture, we report each individual portfolio error,  $e_i^q$ , defined in equation (8), in which levered investment returns are constructed using the estimates from Panel A of Table 2. We also report the  $t$ -statistic, described in Appendix B, testing that an individual error equals zero.

The magnitude of the individual errors varies from 0.1% to 1.7% per annum across ten SUE portfolios, and none of the errors are significant. In particular, Panel A of Table 3 shows that the high-minus-low SUE portfolio has an error of  $-0.4\%$  per annum ( $t = -0.3$ ). This error is negligible compared to the large errors from the traditional models: 12.6% for the CAPM, 14.1% for the Fama-French model, and 13.4% for the standard consumption-CAPM. Figure 1 offers a visual presentation of the fit. Panel A plots the average levered investment returns of the ten SUE portfolios against their average stock returns. If the model performs perfectly, all the observations should lie on the 45-degree line. From Panel A, the scatter plot from the  $q$ -theory model is largely aligned with the 45-degree line. The remaining panels contain analogous plots for the CAPM, the Fama-French model, and the standard consumption-CAPM. In all three cases the scatter plot is largely horizontal, meaning that the traditional models fail to predict the average returns across the SUE portfolios.

Panel A of Table 3 reports large errors for the B/M portfolios in the  $q$ -theory model. The growth portfolio has an error of  $-3.9\%$  per annum, and the value portfolio has an error of  $-2.7\%$ . However, the errors do not vary systematically with B/M. The high-minus-low B/M portfolio only has an error of 1.2%, which is smaller than 18.6% in the CAPM, 7.3% in the Fama-French model, and 12.3% in the standard consumption-CAPM. The scatter plots in Figure 2 show that, although the errors from the  $q$ -theory model are largely similar in magnitude to those from the Fama-French model and the standard consumption-CAPM, the average return spread between the extreme B/M portfolios from the  $q$ -theory model is larger than those from the traditional models.

From Panel A of Table 3, the errors from the CI portfolios are larger than those from the SUE portfolios but are smaller than those from the B/M portfolios. The high-minus-low CI portfolio has an error of  $-0.5\%$  per annum ( $t = -0.4$ ), meaning that the  $q$ -theory model generates a large average return spread across the two extreme CI portfolios. The scatter plot in Panel A of Figure 3 confirms this observation. In contrast, none of the traditional models are able to reproduce the average return spread, as shown in the rest of Figure 3.

### 4.2.3 Economic Mechanisms behind Expected Stock Returns

The intuition behind our estimation results comes from the investment return equation (3) and the levered investment return equation (5). The equations suggest several economic mechanisms that underly the cross-sectional variation of average stock returns, with each mechanism corresponding to a specific component of the investment return. The first component is the marginal benefit of investment, which is primarily the marginal product of capital at  $t+1$  in the numerator of the investment return. The second component is roughly proportional to the growth rate of investment, which corresponds to the “capital gain” component of the investment return: investment-to-capital is an increasing function of marginal  $q$ , denoted  $q_{it}$ , which is related to firm  $i$ 's stock price.

The third economic mechanism works through the component  $I_{it}/K_{it}$  in the denominator of the investment return. Because investment today increases with the net present value of one additional unit of capital, and because the net present value decreases with the cost of capital, a low cost of capital means high net present value and high investment. As such, investment today and average stock returns are negatively correlated. Relatedly, because investment is an increasing function of marginal  $q$ , and because marginal  $q$  is in turn inversely related to book-to-market equity, expected stock returns and book-to-market equity are positively correlated. The fourth component is the rate of depreciation,  $\delta_{it+1}$ . Collecting terms involving  $\delta_{it+1}$  in the numerator of equation (3) yields  $-(1 - \tau_{t+1})[1 + a(I_{it+1}/K_{it+1})]\delta_{it+1}$ , meaning that high rates of depreciation tomorrow imply lower average returns. The fifth component is market leverage: taking the first-order derivative of equation (5) with respect to  $w_{it}$  shows that expected stock returns should increase with market leverage today.

In short, all else equal, firms should earn lower average stock returns if they have high investment-to-capital today, low expected investment growth, low sales-to-capital tomorrow, high rates of depreciation tomorrow, and low market leverage today.

### 4.2.4 Expected Returns Accounting

To understand our estimation results, Table 4 presents averages of the different components of levered investment returns across testing portfolios. From Panel A, the average  $I_{it}/K_{it}$ ,  $\delta_{it+1}$ , and the bond returns,  $r_{it+1}^B$ , are largely flat across ten SUE portfolios. The average  $(I_{it+1}/K_{it+1}) / (I_{it}/K_{it})$  (future investment growth) and  $Y_{it+1}/K_{it+1}$  both increase from the low SUE portfolio to the high SUE portfolio, going in the right direction to capture average stock returns. However, going in the wrong direction, market leverage decreases from the low SUE portfolio to the high SUE portfolio.

For ten B/M portfolios,  $I_{it}/K_{it}$  decreases from 0.2 to 0.1 per annum from the low to the high B/M portfolio. The low B/M firms also have lower market leverage (0.1 versus 0.5) than the high B/M firms. Both characteristics go in the right direction to match average stock returns. However, going in the wrong direction, the low B/M firms have higher average  $Y_{it+1}/K_{it+1}$  (2.0 versus 1.4) than the high B/M firms. The depreciation rate, corporate bond returns, and investment growth are largely flat. Sorting on CI produces a spread in  $I_{it}/K_{it}$  of 0.1. Compared to the high CI firms, the low CI firms have higher future investment growth (1.3 versus 0.8) and higher market leverage (0.4 versus 0.3). All three patterns go in the right direction to match expected stock returns.

The observed patterns in characteristics shed light on the differences in the parameter estimates across the different sets of portfolios. Intuitively, GMM fits the model to the data by minimizing the differences between average investment returns and average stock returns. If the cross-sectional variation in the main components of the investment returns (the sales-to-capital ratio, the investment-to-capital ratio, and investment growth) is not matched in the same way with the cross-sectional variation in average stock returns across the different sets of portfolios, our estimation necessarily produces different parameter estimates. As noted, the sales-to-capital ratio goes in the right direction to match the expected returns of the SUE portfolios, but goes in the wrong direction to match the expected returns of the B/M portfolios. The different estimates imply different economic mechanisms underlying the cross-section of expected returns across the different sets of portfolios.

To quantify the role of each component of the investment return in matching expected returns, we conduct the following accounting exercises. We set a given component equal to its cross-sectional average in each year. We then use the parameter estimates in Panel A of Table 2 to reconstruct levered investment returns, while keeping all the other characteristics unchanged. In the case of investment growth we hold constant the capital gain component of the investment return, which is given by  $[1 + (1 - \tau_{t+1})a(I_{it+1}/K_{it+1})] / [1 + (1 - \tau_t)a(I_{it}/K_{it})] = q_{it+1}/q_{it}$ . We focus on the resulting change in the magnitude of the expected return errors: a large change would suggest that the component in question is quantitatively important.

Panel B of Table 4 reports several insights. First, the most important component for the SUE portfolio returns is  $q_{it+1}/q_{it}$ : eliminating its cross-sectional variation makes the  $q$ -theory model underpredict the average stock return of the high-minus-low SUE portfolio by 8.9% per annum. In contrast, this error is only  $-0.4\%$  in the benchmark estimation. Without the cross-sectional variation of  $Y_{it+1}/K_{it+1}$ , the error of the high-minus-low SUE portfolio becomes 4.3%. Second, investment

and leverage are both important for the B/M portfolios. Fixing  $I_{it}/K_{it}$  to its cross-sectional average produces an error of 90.2% per annum for the high-minus-low B/M portfolio. This huge error reflects the large estimate of the parameter  $a$  for the B/M portfolios. Setting  $w_{it}$  to its cross-sectional average produces an error of 11.6% for the high-minus-low B/M portfolio. The terms  $Y_{it+1}/K_{it+1}$  and  $q_{it+1}/q_{it}$  are less important. Third, the dominating force in driving the average stock returns across the CI portfolios is  $I_{it}/K_{it}$ . Eliminating its cross-sectional variation gives rise to an error of  $-8.5\%$  per annum for the high-minus-low CI portfolio. Fixing  $q_{it+1}/q_{it}$  produces a large error of 4.6%, and fixing  $w_{it}$  produces an error of 2.7% per annum. The effect of  $Y_{it+1}/K_{it+1}$  is negligible.

### 4.3 Matching Expected Returns and Variances Simultaneously

#### 4.3.1 Point Estimates and Overall Model Performance

Panel B of Table 2 reports the point estimates and overall model performance when we use the  $q$ -theory model to match both the expected returns and variances of the testing portfolios. Capital's share,  $\alpha$ , is estimated from 0.4 to 0.6, and all estimates are significant. The estimates of the adjustment cost parameter,  $a$ , are on average higher than those reported in Panel A. The estimates are 11.5 and 16.2 for the B/M and CI portfolios, and both are significant. The estimate of  $a$  for the SUE portfolios is 28.9, but with a large standard error of 16.3.

As explained in Erickson and Whited (2000), it can be misleading to interpret the parameter  $a$  in terms of adjustment costs or speeds. We follow their suggestion of gauging the economic magnitude of this parameter in terms of the elasticity of investment with respect to marginal  $q$ . Evaluated at the sample mean, this elasticity is given by  $1/a$  times the ratio of the mean of  $q_{it}$  to the mean of  $I_{it}/K_{it}$ . The estimates in Panel B imply elasticities that range from 0.4 to 0.7. A similar inelastic response of 0.1 is implied by the estimate of  $a$  for the B/M portfolios in Panel A. However, the implied elasticity for the SUE portfolios is greater than one, and that for the CI portfolios is over ten. Although this last estimate seems large, the others fall in a reasonable range between zero and 1.3. The general inference is that investment responds to  $q$  inelastically.

Panel B of Table 2 reports three tests of overall model performance.  $\chi^2_{(2)}$  is the  $\chi^2$  test that all the variance errors are jointly zero,  $\chi^2_{(1)}$  is the  $\chi^2$  test that all the expected return errors are jointly zero, and the statistic labeled  $\chi^2$  tests that all the model errors are jointly zero. The  $\chi^2_{(2)}$  tests do not reject the model, and the mean absolute variance errors, denoted m.a.e.(2), are small. To better interpret their economic magnitude, we use the parameter estimates from Panel B of Table 2 to calculate the average levered investment return volatility (instead of variance). At 20.4%, this

average predicted volatility is close to the average realized volatility, 21.1%, across the ten SUE portfolios. For the ten B/M portfolios, the average stock return volatility is 25.0%, and the average levered investment return volatility is 23.6%. Finally, for the ten CI portfolios the average stock return volatility is 24.8%, and their average levered investment return volatility is 24.4%.

Cochrane (1991) reports that the aggregate investment return volatility is only about 60% of the value-weighted stock market volatility. Our results complement Cochrane’s in several ways. First, we account for leverage, while Cochrane does not. Second, we use portfolios as testing assets, in which firm-specific shocks are unlikely to be diversified away entirely, while Cochrane studies the stock market portfolio. Third, we formally choose parameters to match variances, while Cochrane calibrates his parameters to match expected returns exactly but allows variances to vary.

Although the  $\chi^2_{(1)}$  tests on the expected return errors do not reject the model, the mean absolute expected return errors, denoted m.a.e.(1), are large. The m.a.e.(1) for the SUE portfolios is 3.5% per annum, up from 0.7% when matching only expected returns. The m.a.e.(1) for the B/M portfolios increases from 2.3% to 2.6%, while that for the CI portfolios goes up from 1.5% to 2.2%. This increase is to be expected because we are asking more of the model by matching more moments.

#### 4.3.2 Euler Equation Errors

Panel B of Table 3 reports individual variance errors, defined as in equation (9), and expected return errors, defined as in equation (8), in which levered investment returns,  $r_{it+1}^{Iw}$ , are constructed using the estimates from Panel B of Table 2. The  $t$ -statistics of the errors, described in Appendix B, are calculated using the variance-covariance matrix from one-stage GMM.

Panel B of Table 3 shows that the magnitude of the variance errors is small relative to stock return variances. Most variance errors are insignificant. The left panels in Figure 4 plot levered investment return volatilities against stock return volatilities for the testing portfolios. To facilitate interpretation, we plot volatilities instead of variances. The points in the scatter plot are generally aligned with the 45-degree line. However, while there is no discernible relation between stock return volatilities and the characteristics in the data, the model predicts a negative relation between levered investment return volatilities and SUE (Panel A) and a positive relation between the predicted volatilities and B/M (Panel C). Panel B of Table 3 also shows that the variance errors increase with SUE and decrease with B/M. The difference in the variance errors is 7.6/100 ( $t = 1.8$ ) between the high and low SUE portfolios and is  $-20/100$  ( $t = -2.4$ ) between the high and low B/M portfolios.

Panel B of Table 3 shows that the expected return errors vary systematically with SUE, increasing from  $-7.0\%$  per annum for the low SUE portfolio to  $5.4\%$  for the high SUE portfolio. The difference of  $12.4\%$  ( $t = 2.5$ ) is similar in magnitude to those from the traditional models. Panel B of Figure 4 plots the average levered investment returns against the average stock returns. The pattern is largely horizontal, similar to those from the traditional models.

The expected return errors for the B/M portfolios in Panel B also are larger than those in Panel A from matching only expected returns. However, the model still predicts an average return spread of  $11.3\%$  per annum between the extreme B/M portfolios. The expected return error for the high-minus-low B/M portfolio is  $5.9\%$  per annum in the  $q$ -theory model, which is lower than  $7.3\%$  from the Fama-French model. The CAPM and the standard consumption-CAPM produce even higher errors,  $18.6\%$  and  $12.3\%$ , respectively. The  $q$ -theory model's performance in reproducing the average returns of the CI portfolios deteriorates to the same level as in the traditional models. The difference in the expected return errors between the extreme CI portfolios is  $-6.6\%$ , which is similar to those from the CAPM and the Fama-French model. From Panel F of Figure 4, the scatter plots of average returns from the  $q$ -theory model are largely horizontal.

The evidence shows that the  $q$ -theory model does a poor job of matching expected returns and variances simultaneously in the SUE and CI portfolios but a somewhat better job in the B/M portfolios. For the SUE and CI portfolios, when we only match expected returns, the predicted investment return variances are lower than observed stock return variances because investment and output are not as volatile as stock returns. As such, to minimize model errors, the joint estimation of expected returns and variances produces empirically plausible variances by picking large estimates of the adjustment cost parameter and of capital's share. These large estimates in turn cannot produce small expected return errors. For the B/M portfolios, when we only match expected returns, the predicted variances are no longer low because of the high adjustment cost parameter estimate required to match expected returns. As such, the mean absolute expected return error does not deteriorate as much as it does in the case of the SUE and CI portfolios when we do the joint estimation.

### 4.3.3 A Correlation Puzzle

As noted, equation (5), taken literally, predicts that stock returns should equal levered investment returns at every data point. We have so far examined the first and second moments of returns that are the focus of much work in financial economics. We can explore yet another, even stronger prediction of the model: stock returns should be perfectly correlated with levered investment returns.

Table 5 reports that the contemporaneous time series correlations between stock and levered investment returns are weakly negative, while those between one-period-lagged stock returns and levered investment returns are positive. When we pool all the observations in the SUE portfolios together, the contemporaneous correlation is  $-0.1$ , which is significant at the 5% level. However, the correlation between one-period-lagged stock returns and levered investment returns is  $0.2$ , which is significant at the 1% level. Replacing levered investment returns with investment growth yields similar results, meaning that the correlations are insensitive to the investment return specifications.

Investment lags (lags between the decision to invest and the actual investment expenditure) can temporally shift the correlations between investment growth and stock returns (e.g., Lamont (2000)). Lags prevent firms from adjusting investment immediately in response to discount rate changes. Consider a one-year lag. A discount rate fall in year  $t$  increases investment only in year  $t+1$ . When stock returns rise in year  $t$  (due to the discount rate fall), investment growth rises in year  $t+1$ : lagged stock returns should be positively correlated with investment growth. The discount rate fall in year  $t$  also means low average stock returns in year  $t+1$ , coinciding with high investment growth in year  $t+1$ . As such, the contemporaneous correlation between stock returns and investment growth should be negative. These lead-lag correlations are consistent with the evidence in Table 5.

## 5 Conclusion

We use GMM to estimate a structural model of cross-sectional stock returns derived from the  $q$ -theory of investment. The model is parsimonious with only two parameters. We construct empirical first and second moment conditions based on the  $q$ -theory prediction that stock returns equal levered investment returns, the latter of which can be constructed from firm characteristics. When matching the first moments only, the model captures the average stock returns of portfolios sorted by earnings surprises, book-to-market equity, and capital investment. When matching the first and the second moments simultaneously, the volatilities from the model are empirically plausible, but the resulting expected returns errors are large. Finally, the model also falls short in reproducing the correlation structure between stock returns and investment growth. We conclude that on average portfolios of firms do a good job of aligning investment policies with their costs of capital, and that this alignment drives many stylized facts in cross-sectional returns. However, because we do not parameterize the stochastic discount factor, our work is silent about why average return spreads across characteristics-sorted portfolios are not matched with spreads in covariances empirically.

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## A Proof of Proposition 1

Let  $q_{it}$  be the Lagrangian multiplier associated with  $K_{it+1} = I_{it} + (1 - \delta_{it})K_{it}$ . The optimality conditions with respect to  $I_{it}$ ,  $K_{it+1}$ , and  $B_{it+1}$  from maximizing equation (2) are, respectively,

$$q_{it} = 1 + (1 - \tau_t) \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} \quad (\text{A1})$$

$$q_{it} = E_t \left[ M_{t+1} \left[ (1 - \tau_{t+1}) \left[ \frac{\partial \Pi(K_{it+1}, X_{it+1})}{\partial K_{it+1}} - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) q_{it+1} \right] \right] \quad (\text{A2})$$

$$1 = E_t \left[ M_{t+1} \left[ r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1} \right] \right]. \quad (\text{A3})$$

Equation (A1) equates the marginal purchase and adjustment costs of investing to the marginal benefit,  $q_{it}$ . Equation (A2) is the investment Euler condition, which describes the evolution of  $q_{it}$ . The term  $(1 - \tau_{t+1}) \partial \Pi(K_{it+1}, X_{it+1}) / \partial K_{it+1}$  captures the marginal after-tax profit generated by an additional unit of capital at  $t + 1$ , the term  $-(1 - \tau_{t+1}) \partial \Phi(I_{it+1}, K_{it+1}) / \partial K_{it+1}$  captures the marginal after-tax reduction in adjustment costs, the term  $\tau_{t+1} \delta_{it+1}$  is the marginal depreciation tax shield, and the term  $(1 - \delta_{it+1}) q_{it+1}$  is the marginal continuation value of an extra unit of capital net of depreciation. Discounting these marginal profits of investment dated  $t + 1$  back to  $t$  using the stochastic discount factor yields  $q_{it}$ .

Dividing both sides of equation (A2) by  $q_{it}$  and substituting equation (A1), we obtain  $E_t[M_{t+1} r_{it+1}^I] = 1$ , in which  $r_{it+1}^I$  is the investment return, defined as:

$$r_{it+1}^I \equiv \frac{(1 - \tau_{t+1}) \left[ \frac{\partial \Pi(K_{it+1}, X_{it+1})}{\partial K_{it+1}} - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} \right] + \tau_{t+1} \delta_{it+1} + (1 - \delta_{it+1}) \left[ 1 + (1 - \tau_{t+1}) \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial I_{it+1}} \right]}{1 + (1 - \tau_t) \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}}}. \quad (\text{A4})$$

The investment return is the ratio of the marginal benefit of investment at time  $t + 1$  to the marginal cost of investment at  $t$ . Substituting  $\partial \Pi(K_{it+1}, X_{it+1}) / \partial K_{it+1} = \alpha Y_{it+1} / K_{it+1}$  and  $\Phi(I_{it}, K_{it}) = (a/2)(I_{it}/K_{it})^2 K_{it}$  into equation (A4) yields the investment return equation (3).

Equation (A3) says that  $E_t[M_{t+1} r_{it+1}^B] = 1 + E_t[M_{t+1} (r_{it+1}^B - 1) \tau_{t+1}]$ . Intuitively, because of the tax benefit of debt, the unit price of the pre-tax bond return,  $E_t[M_{t+1} r_{it+1}^B]$ , is higher than one. The difference is precisely the present value of the tax benefit. Because we define the after-tax corporate bond return,  $r_{it+1}^{Ba} \equiv r_{it+1}^B - (r_{it+1}^B - 1) \tau_{t+1}$ , equation (A3) says that the unit price of the after-tax corporate bond return is one:  $E_t[M_{t+1} r_{it+1}^{Ba}] = 1$ .

To prove equation (4), we first show that  $q_{it} K_{it+1} = P_{it} + B_{it+1}$  under constant returns to scale.

We start with  $P_{it} + D_{it} = V_{it}$  and expand  $V_{it}$  using equations (1) and (2):

$$\begin{aligned}
P_{it} + (1 - \tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it}) - r_{it}^B B_{it}] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} = \\
(1 - \tau_t) \left[ \Pi(K_{it}, X_{it}) - \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} I_{it} - \frac{\partial \Phi(I_{it}, K_{it})}{\partial K_{it}} K_{it} - r_{it}^B B_{it} \right] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} \\
- q_{it}(K_{it+1} - (1 - \delta_{it})K_{it} - I_{it}) + E_t[M_{t+1}((1 - \tau_t) \left[ \Pi(K_{it+1}, X_{it+1}) - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial I_{it+1}} I_{it+1} \right. \\
\left. - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} K_{it+1} - r_{it+1}^B B_{it+1} \right] - \tau_{t+1} B_{it+1} - I_{it+1} + B_{it+2} \\
+ \tau_{t+1} \delta_{it+1} K_{it+1} - q_{it+1}(K_{it+2} - (1 - \delta_{it+1})K_{it+1} - I_{it+1}) + \dots ] \tag{A5}
\end{aligned}$$

Recursively substituting equations (A1), (A2), and (A3), and simplifying, we obtain:

$$\begin{aligned}
P_{it} + (1 - \tau_t)[\Pi(K_{it}, X_{it}) - \Phi(I_{it}, K_{it}) - r_{it}^B B_{it}] - \tau_t B_{it} - I_{it} + B_{it+1} + \tau_t \delta_{it} K_{it} = \\
(1 - \tau_t) \left[ \Pi(K_{it}, X_{it}) - \frac{\partial \Phi(I_{it}, K_{it})}{\partial K_{it}} K_{it} - r_{it}^B B_{it} \right] - \tau_t B_{it} + q_{it}(1 - \delta_{it})K_{it} + \tau_t \delta_{it} K_{it} \tag{A6}
\end{aligned}$$

Simplifying further and using the linear homogeneity of  $\Phi(I_{it}, K_{it})$ , we obtain:

$$P_{it} + B_{it+1} = (1 - \tau_t) \frac{\partial \Phi(I_{it}, K_{it})}{\partial I_{it}} I_{it} + I_{it} + q_{it}(1 - \delta_{it})K_{it} = q_{it}K_{it+1} \tag{A7}$$

Finally, we are ready to prove equation (4):

$$\begin{aligned}
w_{it} r_{it+1}^{Ba} + (1 - w_{it}) r_{it+1}^S &= \frac{\left[ \begin{aligned} &(1 - \tau_{t+1}) r_{it+1}^B B_{it+1} + \tau_{t+1} B_{it+1} + P_{it+1} \\ &+ (1 - \tau_{t+1}) [\Pi(K_{it+1}, X_{it+1}) - \Phi(I_{it+1}, K_{it+1}) - r_{it+1}^B B_{it+1}] \\ &\quad - \tau_{t+1} B_{it+1} - I_{it+1} + B_{it+2} + \tau_{t+1} \delta_{it+1} K_{it+1} \end{aligned} \right]}{P_{it} + B_{it+1}} \\
&= \frac{1}{q_{it} K_{it+1}} \left[ \begin{aligned} &q_{it+1}(I_{it+1} + (1 - \delta_{it+1})K_{it+1}) + (1 - \tau_{t+1}) [\Pi(K_{it+1}, X_{it+1}) \\ &\quad - \Phi(I_{it+1}, K_{it+1})] - I_{it+1} + \tau_{t+1} \delta_{it+1} K_{it+1} \end{aligned} \right] \\
&= \frac{q_{it+1}(1 - \delta_{it+1}) + (1 - \tau_{t+1}) \left[ \frac{\partial \Pi(K_{it+1}, X_{it+1})}{\partial K_{it+1}} - \frac{\partial \Phi(I_{it+1}, K_{it+1})}{\partial K_{it+1}} \right] + \tau_{t+1} \delta_{it+1}}{q_{it}} = r_{it+1}^I. \tag{A8}
\end{aligned}$$

## B Estimation Details

Following the standard GMM procedure (e.g., Hansen and Singleton (1982)), we estimate the parameters,  $\mathbf{b} \equiv (a, \alpha)$ , to minimize a weighted combination of the sample moments (8) or (8) and (9). Specifically, let  $\mathbf{g}_T$  be the sample moments. The GMM objective function is a weighted sum of squares of the model errors across assets,  $\mathbf{g}'_T \mathbf{W} \mathbf{g}_T$ , in which we use  $\mathbf{W} = \mathbf{I}$ , the identity matrix. Let  $\mathbf{D} = \partial \mathbf{g}_T / \partial \mathbf{b}$  and  $\mathbf{S}$  a consistent estimate of the variance-covariance matrix of the sample errors  $\mathbf{g}_T$ . We estimate  $\mathbf{S}$  using a standard Bartlett kernel with a window length of five.

The estimate of  $\mathbf{b}$ , denoted  $\hat{\mathbf{b}}$ , is asymptotically normal with variance-covariance matrix:

$$\text{var}(\hat{\mathbf{b}}) = \frac{1}{T} (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1} \mathbf{D}'\mathbf{W}\mathbf{S}\mathbf{W}\mathbf{D} (\mathbf{D}'\mathbf{W}\mathbf{D})^{-1} \quad (\text{B1})$$

To construct standard errors for the model errors on individual portfolios or groups of model errors, we use the variance-covariance matrix for the model errors,  $\mathbf{g}_T$ :

$$\text{var}(\mathbf{g}_T) = \frac{1}{T} [\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}] \mathbf{S} [\mathbf{I} - \mathbf{D}(\mathbf{D}'\mathbf{W}\mathbf{D})^{-1}\mathbf{D}'\mathbf{W}]' \quad (\text{B2})$$

In particular, the  $\chi^2$  test whether all model errors are jointly zero is given by:

$$\mathbf{g}'_T [\text{var}(\mathbf{g}_T)]^+ \mathbf{g}_T \sim \chi^2(\# \text{ moments} - \# \text{ parameters}) \quad (\text{B3})$$

The superscript  $+$  denotes pseudo-inversion.

## C Details of Timing Alignment

Figure 5 illustrates our timing convention. We use the Fama-French portfolio approach to form the B/M and CI portfolios by sorting stocks at the end of June of each year  $t$  based on characteristics for the fiscal year ending in calendar year  $t-1$ . Portfolio stock returns,  $r_{it+1}^S$ , are calculated from July of year  $t$  to June of year  $t+1$ . To construct the annual investment returns in equation (3),  $r_{it+1}^I$ , we use the tax rate and investment observed at the end of year  $t$  ( $\tau_t$  and  $I_{it}$ ) and other variables at the end of year  $t+1$  ( $\tau_{t+1}$ ,  $Y_{it+1}$ ,  $I_{it+1}$ , and  $\delta_{it+1}$ ). Because stock variables are measured at the beginning of the year, and because flow variables are realized over the course of the year, the investment returns go roughly from the middle of year  $t$  to the middle of year  $t+1$ . As such, the investment return timing largely matches the stock return timing.

The changes in stock composition in a given portfolio from portfolio rebalancing raise further subtleties. In the Fama-French portfolio approach, for the annually rebalanced B/M and CI portfolios, the set of firms in a given portfolio formed in year  $t$  is fixed when we aggregate returns from July of year  $t$  to June of  $t+1$ . The stock composition changes only at the end of June of year  $t+1$  when we rebalance. As such, we fix the set of firms in a given portfolio in the formation year  $t$  when aggregating characteristics, dated both  $t$  and  $t+1$ , across firms in the portfolio. In particular, to construct the numerator of  $r_{it+1}^I$ , we use  $I_{it+1}/K_{it+1}$  from the portfolio formation year  $t$ , which is different from the  $I_{it+1}/K_{it+1}$  from the formation year  $t+1$  used to construct the denominator of  $r_{it+2}^I$ .

The SUE portfolios are initially formed monthly. We time-aggregate monthly returns of the SUE portfolios from July of year  $t$  to June of  $t+1$  to obtain annual returns. Constructing the matching annual investment returns,  $r_{it+1}^I$ , requires care because the composition of the SUE portfolios changes from month to month. First, consider the 12 low SUE portfolios formed in each month from July of year  $t$  to June of  $t+1$ . For each month we calculate portfolio level characteristics by aggregating individual characteristics over the firms in the low SUE portfolio. We use the following

specific characteristics:  $I_{it}$  and  $\tau_t$  observed at the end of year  $t$ ,  $K_{it}$  at the beginning of year  $t$ ,  $K_{it+1}$  at the beginning of  $t+1$ , and  $\tau_{t+1}$ ,  $Y_{it+1}$ ,  $I_{it+1}$ , and  $\delta_{it+1}$  at the end of year  $t+1$ . Because the portfolio composition changes from month to month, these portfolio level characteristics also change from month to month. Accordingly, we average these portfolio characteristics over the 12 monthly low SUE portfolios, and use these averages to construct  $r_{it+1}^I$ , which is in turn matched with the annual  $r_{it+1}^S$  from July of  $t$  to June of  $t+1$ . We then repeat this procedure for the remaining SUE portfolios.

The after-tax corporate bond return,  $r_{it+1}^{Ba}$ , depends on the tax rate and the pre-tax bond return,  $r_{it+1}^B$ , which we measure as the observed corporate bond returns in the data. The timing of  $r_{it+1}^B$  is the same as that of stock returns: after sorting stocks on characteristics for the fiscal year ending in calendar year  $t-1$ , we measure  $r_{it+1}^B$  as the equal-weighted corporate bond return from July of year  $t$  to June of  $t+1$ . However, calculating  $r_{it+1}^{Ba} = r_{it+1}^B - (r_{it+1}^B - 1)\tau_{t+1}$  is less straightforward:  $\tau_{t+1}$  is applicable from January to December of year  $t+1$ , but  $r_{it+1}^B$  is applicable from July of year  $t$  to June of  $t+1$ . We deal with this timing-mismatch by replacing  $\tau_{t+1}$  in the calculation of  $r_{it+1}^{Ba}$  with the average of  $\tau_t$  and  $\tau_{t+1}$  in the data. This timing-mismatch matters little for our results because the tax rate exhibits little time series variation. In particular, we have experimented with time-invariant tax rates in calculating  $r_{it+1}^{Ba}$ , and the results are largely similar.

**Table 1 : Descriptive Statistics of Testing Portfolio Returns**

For testing portfolio  $i$ , we report in annual percent the average stock return,  $\bar{r}_i^S$ , the stock return volatility,  $\sigma_i^S$ , the intercept from the CAPM regression,  $e_i$ , the intercept from the Fama-French three-factor regression,  $e_i^{FF}$ , and the model error from the standard consumption-CAPM,  $e_i^C$ . In each panel we only report results for three (Low, 5, and High) out of ten portfolios to save space. The H-L portfolio is long in the high portfolio and short in the low portfolio. The heteroscedasticity-and-autocorrelation-consistent  $t$ -statistics for the model errors are reported in brackets beneath the corresponding errors. m.a.e. is the mean absolute error in annual percent for a given set of ten testing portfolios. For the CAPM and the Fama-French model, the  $p$ -values in brackets in the last column in each panel are for the Gibbons, Ross, and Shanken (1989) tests of the null hypothesis that the intercepts for a given set of ten portfolios are jointly zero. For the consumption-CAPM the  $p$ -values are for the  $\chi^2$  test from one-stage GMM that the moment restrictions for all ten portfolios are jointly zero. In Panel A for the consumption-CAPM the estimate of the time preference coefficient is  $\beta = 2.8$  with a standard error (ste) of 0.9 and the estimate of risk aversion is  $\gamma = 127.6$  (ste = 54.9). In Panel B  $\beta = 3.3$  (ste = 1.2) and  $\gamma = 142.1$  (ste = 58.5). In Panel C  $\beta = 3.3$  (ste = 1.2) and  $\gamma = 143.3$  (ste = 57.6).

	Low	5	High	H-L	m.a.e.	[ $p$ ]	Low	5	High	H-L	m.a.e.	[ $p$ ]	Low	5	High	H-L	m.a.e.	[ $p$ ]
	Panel A: Ten SUE portfolios						Panel B: Ten B/M portfolios						Panel C: Ten CI portfolios					
$\bar{r}_i^S$	10.9	19.0	23.4	12.5			8.7	17.9	25.8	17.1			22.1	18.1	15.2	-7.0		
$\sigma_i^S$	22.4	22.5	21.1	8.5			27.9	24.9	27.0	20.5			32.4	22.3	26.7	11.4		
$e_i$	-1.7	6.6	10.9	12.6	5.7	[0.0]	-4.9	5.2	13.7	18.6	6.3	[0.0]	8.2	5.9	1.9	-6.3	5.7	[0.0]
[ $t$ ]	[-0.7]	[2.2]	[5.0]	[12.7]			[-1.8]	[2.3]	[3.8]	[6.0]			[2.4]	[2.6]	[0.7]	[-4.5]		
$e_i^{FF}$	-4.6	2.0	9.5	14.1	4.0	[0.0]	-0.5	1.8	6.8	7.3	2.8	[0.0]	6.5	1.5	0.1	-6.3	2.2	[0.0]
[ $t$ ]	[-2.2]	[1.0]	[6.7]	[8.1]			[-0.2]	[1.8]	[2.6]	[2.5]			[2.9]	[1.6]	[0.1]	[-6.5]		
$e_i^C$	-8.1	-0.0	5.3	13.4	3.6	[0.0]	-5.4	0.3	6.9	12.3	2.4	[0.0]	4.0	0.5	-4.3	-8.4	1.8	[0.0]
[ $t$ ]	[-1.3]	[0.0]	[1.4]	[0.6]			[-0.7]	[0.1]	[2.4]	[0.2]			[0.7]	[0.1]	[-0.8]	[-0.4]		

**Table 2 : Parameter Estimates and Tests of Overidentification**

Results are from one-stage GMM with an identity weighting matrix. In Panel A the moment conditions are  $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$ .  $a$  is the adjustment cost parameter and  $\alpha$  is capital's share. Their standard errors, denoted *ste*, are reported in brackets beneath the corresponding estimates.  $\chi^2$  is the statistic that the moment conditions are jointly zero. d.f. is the degrees of freedom, and  $p$  is the  $p$ -value associated with the test. m.a.e. is the mean absolute error in annual percent,  $E_T[r_{it+1}^S - r_{it+1}^{Iw}]$ , in which  $E_T[\cdot]$  is the sample mean of the series in brackets, across a given set of testing portfolios. In Panel B the moment conditions are  $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$  and  $E[(r_{it+1}^S - E[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E[r_{it+1}^{Iw}])^2] = 0$ .  $\chi_{(2)}^2$ , d.f.(2), and  $p(2)$  are the statistic, degrees of freedom, and  $p$ -value for the  $\chi^2$  test that the variance errors, defined as  $E_T[(r_{it+1}^S - E_T[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E_T[r_{it+1}^{Iw}])^2]$ , are jointly zero. m.a.e.(2) is the mean absolute variance error.  $\chi_{(1)}^2$ , d.f.(1), and  $p(1)$  are the statistic, degrees of freedom, and  $p$ -value for the  $\chi^2$  test that the expected return errors are jointly zero. m.a.e.(1) is the mean absolute expected return error in annual percent.  $\chi^2$ , d.f., and  $p$  are the statistic, degrees of freedom, and  $p$ -value of the test that the expected return errors and the variance errors are jointly zero.

Panel A: Matching expected returns				Panel B: Matching expected returns and variances			
	SUE	B/M	CI		SUE	B/M	CI
$a$	7.7	22.3	1.0	$a$	28.9	11.5	16.2
[ste]	[1.7]	[25.5]	[0.3]	[ste]	[16.3]	[4.8]	[5.5]
$\alpha$	0.3	0.5	0.2	$\alpha$	0.6	0.4	0.4
[ste]	[0.0]	[0.3]	[0.0]	[ste]	[0.3]	[0.1]	[0.1]
$\chi^2$	4.4	6.0	6.5	$\chi_{(2)}^2$	5.1	6.2	6.1
d.f.	8	8	8	d.f.(2)	8	8	8
$p$	0.8	0.7	0.6	$p(2)$	0.7	0.6	0.6
m.a.e.	0.7	2.3	1.5	m.a.e.(2) $\times 100$	2.5	4.1	2.2
				$\chi_{(1)}^2$	5.2	4.4	4.8
				d.f.(1)	8	8	8
				$p(1)$	0.7	0.8	0.8
				m.a.e.(1)	3.5	2.6	2.2
				$\chi^2$	5.5	6.2	6.6
				d.f.	18	18	18
				$p$	1.0	1.0	1.0

**Table 3 : Euler Equation Errors**

Results are from one-stage GMM estimation with an identity weighting matrix. In Panel A the moment conditions are  $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$ . The expected return errors are defined as  $e_i^q \equiv E_T[r_{it+1}^S - r_{it+1}^{Iw}]$ , in which  $E_T[\cdot]$  is the sample mean of the series in brackets. In Panel B the moment conditions are  $E[r_{it+1}^S - r_{it+1}^{Iw}] = 0$  and  $E[(r_{it+1}^S - E[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E[r_{it+1}^{Iw}])^2] = 0$ . The variance errors are defined as  $e_i^{\sigma^2} \equiv E_T[(r_{it+1}^S - E_T[r_{it+1}^S])^2 - (r_{it+1}^{Iw} - E_T[r_{it+1}^{Iw}])^2]$ . The expected return errors are defined as in Panel A. In both panels the expected return errors are in annual percent. In each set of ten portfolios we only report results for three (Low, 5, and High) out of the ten portfolios to save space. The column H–L reports the difference in the expected return errors and the difference in the variance errors between portfolios High and Low, as well as their  $t$ -statistics.

	Low	5	High	H–L	Low	5	High	H–L	Low	5	High	H–L
Panel A: Euler equation errors from matching expected returns												
	Ten SUE portfolios				Ten B/M portfolios				Ten CI portfolios			
$e_i^q$	0.3	1.7	-0.2	-0.4	-3.9	2.4	-2.7	1.2	-1.0	2.7	-1.5	-0.5
$[t]$	[0.6]	[1.7]	[-0.1]	[-0.4]	[-1.8]	[1.4]	[-1.4]	[0.8]	[-0.5]	[1.7]	[-1.2]	[-0.4]
Panel B: Euler equation errors from matching expected returns and variances												
	Ten SUE portfolios				Ten B/M portfolios				Ten CI portfolios			
$e_i^{\sigma^2} \times 100$	-4.5	1.8	3.1	7.6	9.5	1.5	-10.4	-20.0	1.0	3.4	-6.3	-7.3
$[t]$	[-1.9]	[1.0]	[1.5]	[1.8]	[2.4]	[0.5]	[-2.0]	[-2.4]	[0.3]	[1.3]	[-1.8]	[-1.4]
$e_i^q$	-7.0	2.6	5.4	12.4	-6.5	1.7	-0.6	5.9	1.3	3.5	-5.3	-6.6
$[t]$	[-2.2]	[1.9]	[2.0]	[2.5]	[-1.9]	[0.9]	[-0.2]	[1.1]	[0.5]	[2.0]	[-2.0]	[-2.0]

**Table 4 : Expected Returns Accounting**

Panel A reports the averages of investment-to-capital,  $I_{it}/K_{it}$ , future investment growth,  $(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$ , sales-to-capital,  $Y_{it+1}/K_{it+1}$ , the depreciation rate,  $\delta_{it+1}$ , market leverage,  $w_{it}$ , and corporate bond returns in annual percent,  $r_{it+1}^B$ . We only report results for three (Low, 5, and High) out of a given set of ten portfolios to save space. The column H–L reports the average differences between portfolios High and Low and the column  $[t_{H-L}]$  reports the heteroscedasticity-and-autocorrelation-consistent  $t$ -statistics for the test that the differences equal zero. Panel B performs four comparative static experiments denoted  $\overline{I_{it}/K_{it}}$ ,  $\overline{q_{it+1}/q_{it}}$ ,  $\overline{Y_{it+1}/K_{it+1}}$ , and  $\overline{w_{it}}$ , in which  $q_{it+1}/q_{it} = [1 + (1 - \tau_{t+1})a(I_{it+1}/K_{it+1})]/[1 + (1 - \tau_t)a(I_{it}/K_{it})]$ . In the experiment denoted  $\overline{Y_{it+1}/K_{it+1}}$ , we set  $Y_{it+1}/K_{it+1}$  for a given set of ten portfolios, indexed by  $i$ , to be its cross-sectional average in  $t+1$ . We then use the parameters reported in Panel A of Table 2 to reconstruct the levered investment returns, while keeping all the other characteristics unchanged. The other three experiments are designed analogously. We report the expected return errors defined as  $e_i^q \equiv E_T[r_{it+1}^S - r_{it+1}^Iw]$  in annual percent for the testing portfolios, the high-minus-low portfolios, and the mean absolute value of  $e_i^q$  (m.a.e.) across a given set of ten testing portfolios.

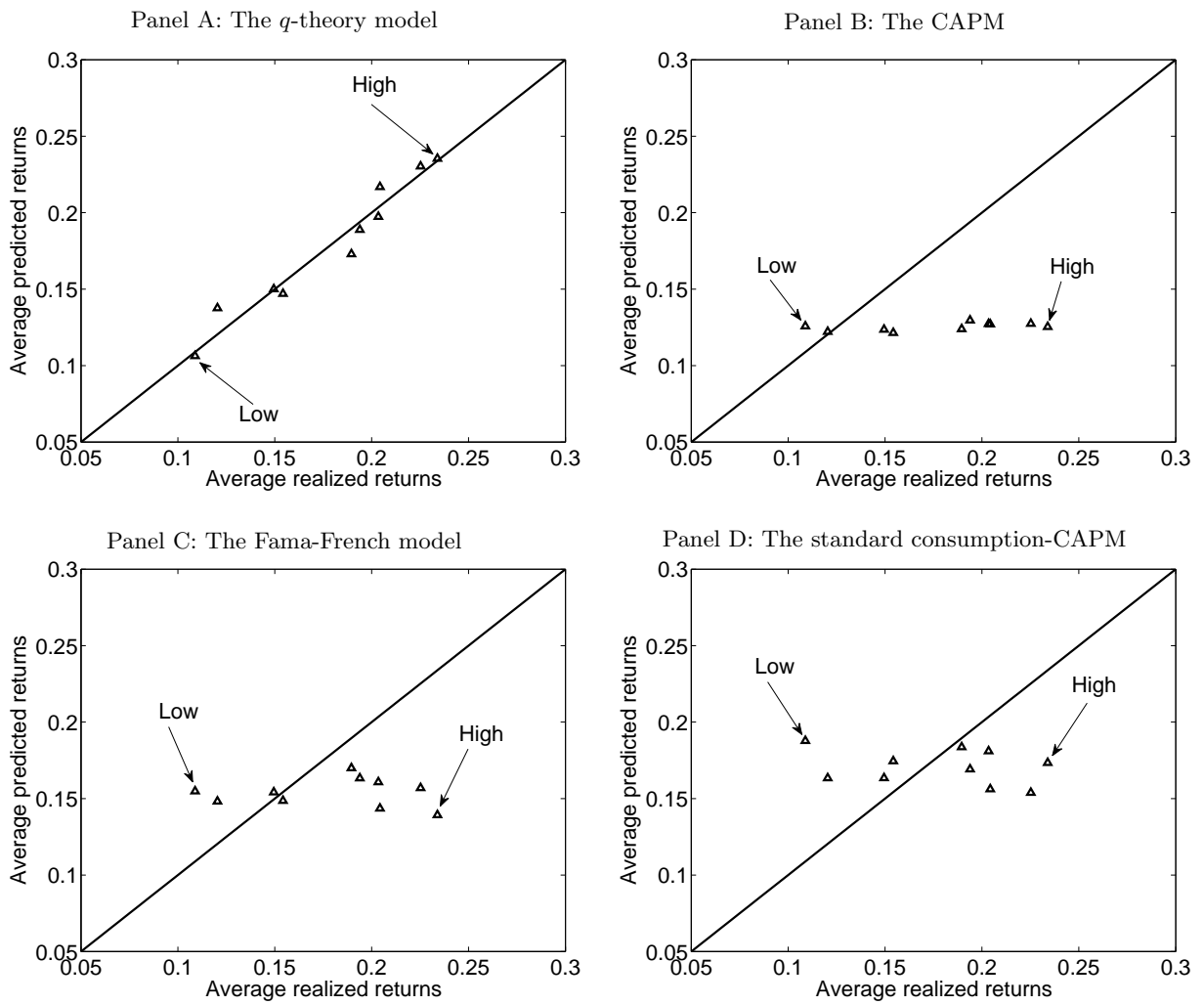
Panel A: Characteristics in levered investment returns															
	Low	5	High	H–L	$[t_{H-L}]$	Low	5	High	H–L	$[t_{H-L}]$	Low	5	High	H–L	$[t_{H-L}]$
	Ten SUE portfolios					Ten B/M portfolios					Ten CI portfolios				
$I_{it}/K_{it}$	0.1	0.1	0.1	0.0	[0.7]	0.2	0.1	0.1	-0.1	[-8.0]	0.1	0.1	0.2	0.1	[11.1]
$(I_{it+1}/K_{it+1})/(I_{it}/K_{it})$	0.9	1.0	1.1	0.2	[4.1]	1.0	1.0	1.0	0.0	[0.7]	1.3	1.0	0.8	-0.4	[-7.2]
$Y_{it+1}/K_{it+1}$	1.5	1.5	1.8	0.3	[5.2]	2.0	1.5	1.4	-0.6	[-6.8]	1.8	1.6	1.9	0.1	[0.4]
$\delta_{it+1}$	0.1	0.1	0.1	0.0	[0.6]	0.1	0.1	0.1	-0.0	[-5.0]	0.1	0.1	0.1	0.0	[-0.5]
$w_{it}$	0.3	0.3	0.2	-0.1	[-5.8]	0.1	0.3	0.5	0.4	[12.4]	0.4	0.3	0.3	-0.1	[-2.6]
$r_{it+1}^B$	9.4	9.8	9.4	-0.1	[-0.3]	8.2	8.1	8.5	0.4	[1.1]	8.5	8.3	8.4	-0.0	[-0.2]
Panel B: Expected return errors from comparative static experiments															
	Low	5	High	H–L	m.a.e.	Low	5	High	H–L	m.a.e.	Low	5	High	H–L	m.a.e.
	Ten SUE portfolios					Ten B/M portfolios					Ten CI portfolios				
$\overline{I_{it}/K_{it}}$	-2.5	4.5	-4.3	-1.8	2.4	-42.1	4.7	48.2	90.2	21.3	2.9	3.5	-5.7	-8.5	2.3
$\overline{q_{it+1}/q_{it}}$	-5.2	1.8	3.6	8.9	2.6	-1.9	2.1	-4.1	-2.1	1.9	0.7	3.0	-3.9	-4.6	1.7
$\overline{Y_{it+1}/K_{it+1}}$	-0.8	0.4	3.5	4.3	1.3	0.2	0.9	-6.3	-6.5	1.9	0.6	-0.4	0.1	-0.5	0.4
$\overline{w_{it}}$	0.1	1.9	-1.5	-1.6	0.9	-6.0	2.2	5.6	11.6	3.6	1.8	2.6	-0.9	-2.7	1.4

**Table 5 : Correlations**

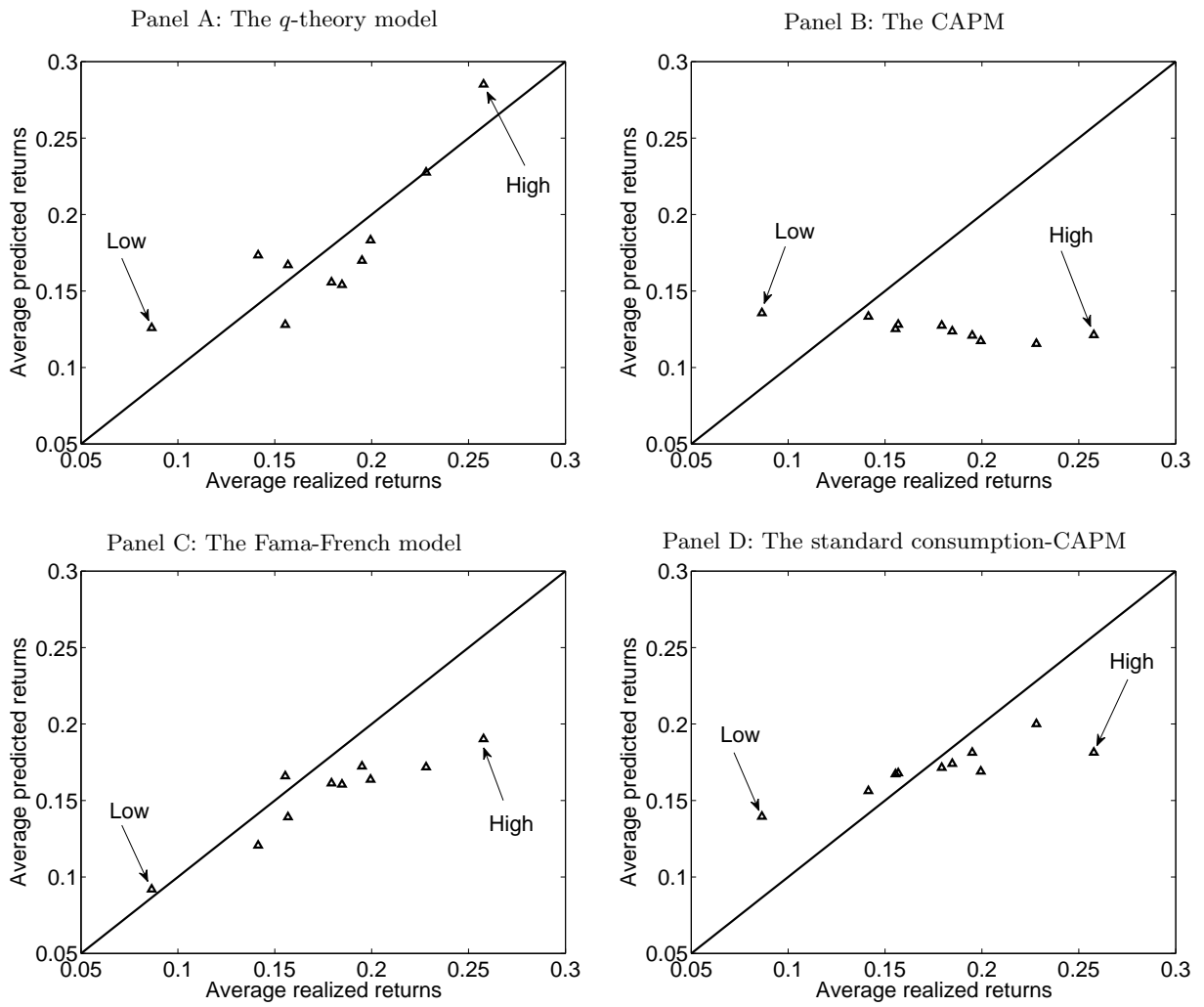
We report time series correlations of stock returns (contemporaneous,  $r_{it+1}^S$ , and one-period-lagged,  $r_{it}^S$ ) with levered investment returns,  $r_{it+1}^{Iw}$ , and with investment growth,  $I_{it+1}/I_{it}$ . In each panel we only report results for three (Low, 5, and High) out of ten portfolios to save space.  $\rho(\cdot, \cdot)$  denotes the correlation between the two series in the parentheses. We report the significance of a given correlation with a star system: 10%, 5%, and 1% significance levels are indicated by one, two, and three stars, respectively. In the last column of each panel, All, we report the correlations and their significance by pooling all the observations for a given set of ten testing portfolios. The levered investment returns are constructed using the parameters in Panel A of Table 2.

	Low	5	High	All	Low	5	High	All	Low	5	High	All
	Panel A: Ten SUE portfolios				Panel B: Ten B/M portfolios				Panel C: Ten CI portfolios			
$\rho(r_{it+1}^S, r_{it+1}^{Iw})$	-0.3	-0.2	-0.3	-0.1**	-0.2	-0.2	-0.1	-0.1**	0.2	-0.3**	-0.3*	-0.1
$\rho(r_{it}^S, r_{it+1}^{Iw})$	0.2	0.0	0.1	0.2***	0.1	0.2	0.3***	0.2***	0.4***	0.2	0.3*	0.2***
$\rho(r_{it+1}^S, I_{it+1}/I_{it})$	-0.3	-0.2	-0.2	-0.1	-0.1	-0.1	-0.1	-0.2***	0.3*	-0.3**	-0.1	-0.0
$\rho(r_{it}^S, I_{it+1}/I_{it})$	0.2	0.0	-0.0	0.1**	0.1	0.2	0.3*	0.1***	0.2	0.1	0.3	0.2***

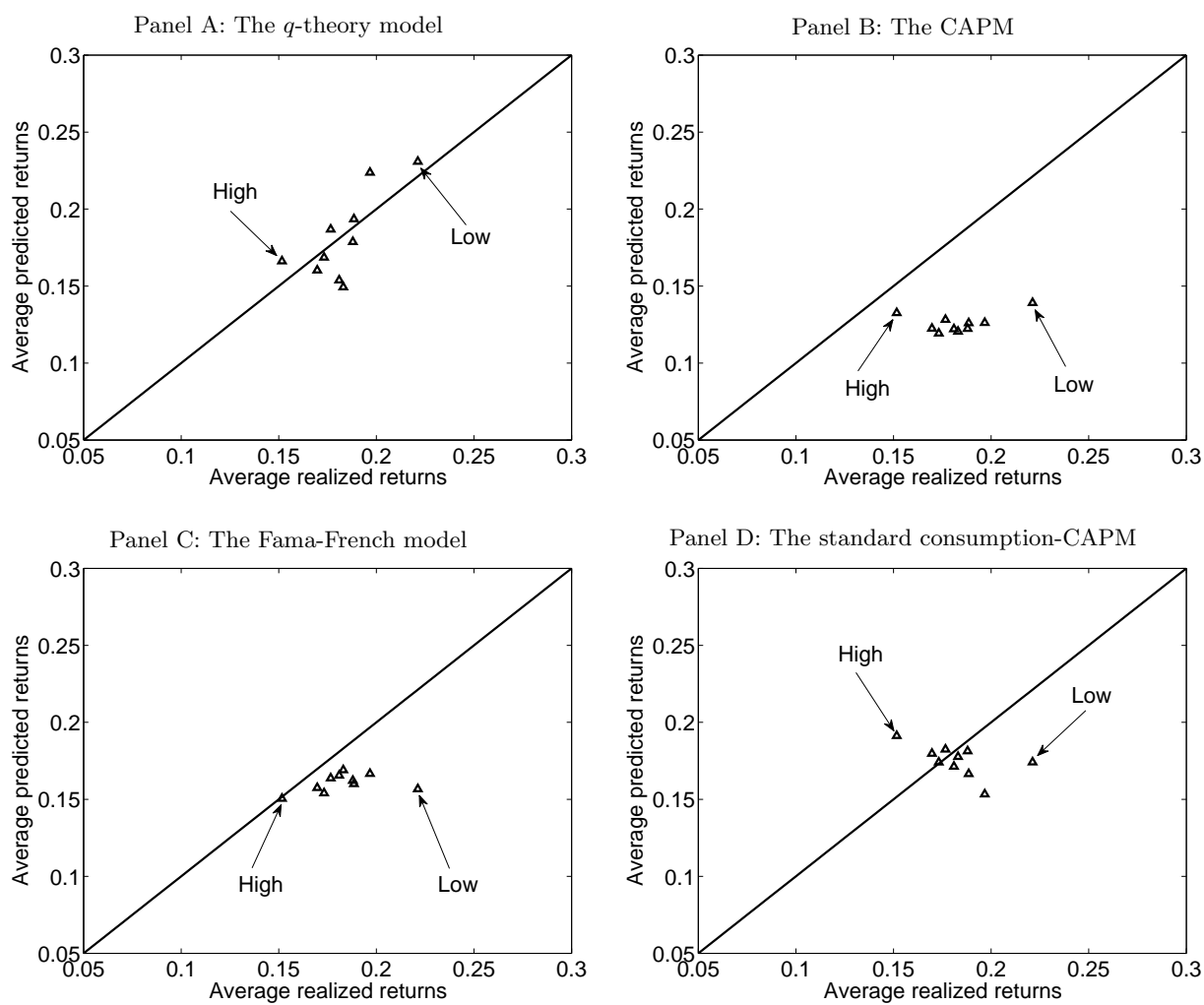
**Figure 1 : Average Predicted Stock Returns versus Average Realized Stock Returns, Ten SUE Portfolios, Matching Only Expected Stock Returns**



**Figure 2 : Average Predicted Stock Returns versus Average Realized Stock Returns, Ten B/M Portfolios, Matching Only Expected Stock Returns**



**Figure 3 : Average Predicted Stock Returns versus Average Realized Stock Returns, Ten CI Portfolios, Matching Only Expected Stock Returns**



**Figure 4 : Predicted Stock Return Volatilities versus Realized Stock Return Volatilities, Average Predicted Stock Returns versus Average Realized Stock Returns, The  $q$ -theory Model, Matching Expected Returns and Variances Simultaneously**

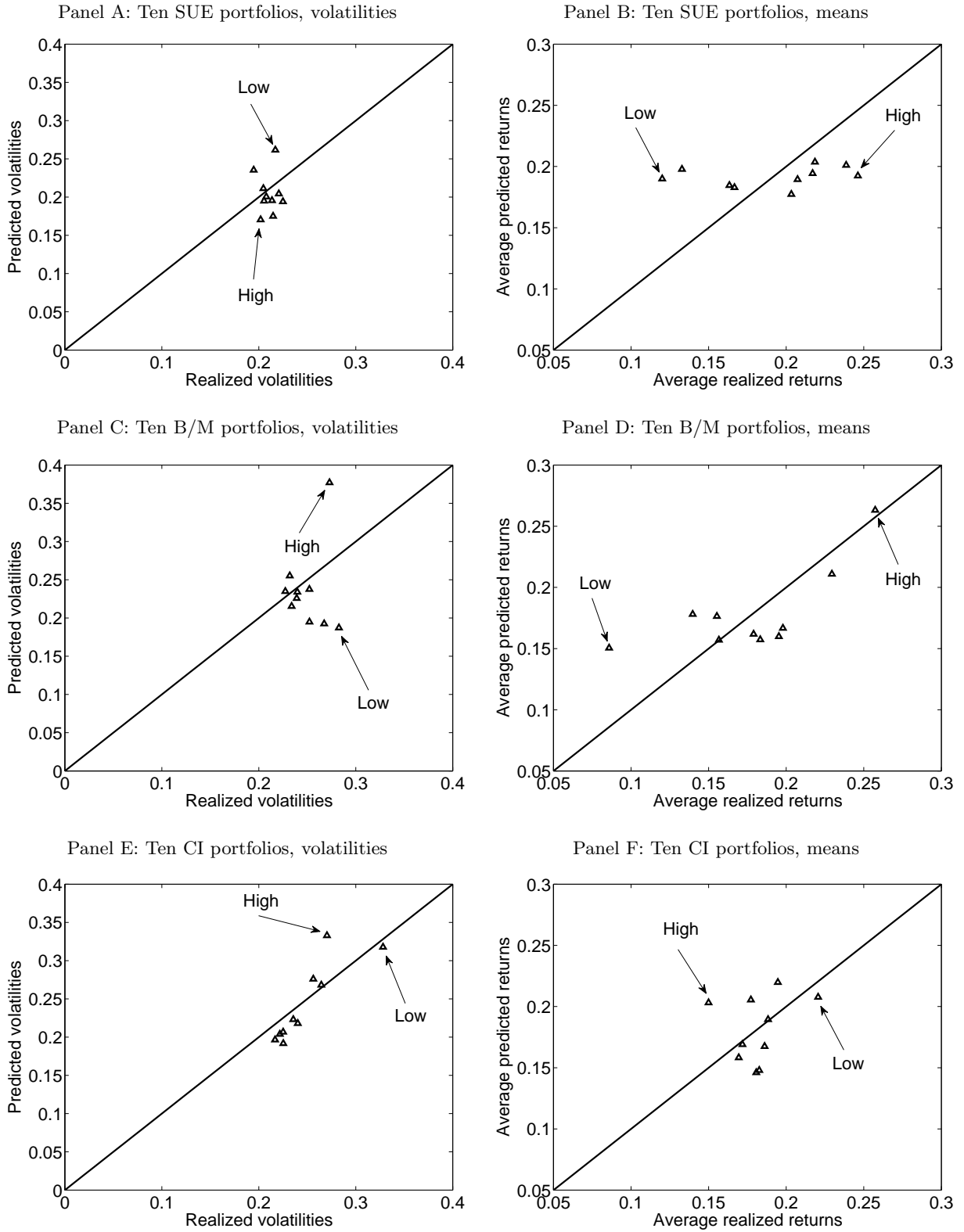


Figure 5: Timing Alignment between Stock Returns and Investment Returns

