

# “Two Trees”

Cochrane, Longstaff, and Santa-Clara (2007, *RFS*)

Summarized by: Lu Zhang

Michigan Asset Pricing Reading Group  
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# Theme

Interesting asset pricing dynamics in an economy with two i.i.d. Lucas trees

## Model

The representative investor with log utility

$$U_t = E_t \left[ \int_0^{\infty} e^{-\delta\tau} \ln(C_{t+\tau}) d\tau \right]$$

Two trees, each with a dividend stream  $D_i dt$ :

$$\frac{dD_i}{D_i} = \mu dt + \sigma dZ_i$$

$Z_i, i = 1, 2$  uncorrelated

Endowment economy

$$C = D_1 + D_2$$

# Results

A single state variable

$$s = \frac{D_1}{D_1 + D_2} \Rightarrow ds = -2\sigma^2 s(1-s)(s - 1/2) dt + \sigma s(1-s)(dZ_1 - dZ_2)$$

Consumption dynamics

$$\frac{dC}{C} = \mu dt + \sigma s dZ_1 + \sigma(1-s) dZ_2 \Rightarrow \text{Var}_t \left[ \frac{dC}{C} \right] = \sigma^2 [s^2 + (1-s)^2]$$

Real interest rate

$$M_t = \frac{e^{-\delta t}}{C_t} \Rightarrow r = \delta + \mu - \sigma^2 [s^2 + (1-s)^2]$$

# Results

Cont'd

Market portfolio return

$$R_M = (\mu + \delta) dt + \sigma s dZ_1 + \sigma(1 - s) dZ_2$$

Price-dividend ratio

$$\frac{P}{D} = \frac{1}{2\delta s} \left[ 1 + \left( \frac{1-s}{s} \right) \ln(1-s) - \left( \frac{s}{1-s} \right) \ln(s) \right]$$

Similar closed-form solutions to expected (excess) returns and volatility, market betas, cross-correlations, and autocorrelations

# Questions

A *rich* set of nice closed-form results

No quantitative insights, however:

- Solve a more realistic model numerically and match moments (high risk aversion, mean-reverting cash flows)
- Examine how predictions qualitatively differ from Cochrane et al.

p. 3-4: “Expected returns typically rise with a tree’s share of dividends, to attract investors to hold that larger share” but “there are also parameters and regions of the state space in which expected returns decline as functions of the dividend share”

- More specific characterization of the state space: When do we see momentum; when do we see value?

# Questions

Cont'd

p. 4: “Since price-dividend ratios vary despite i.i.d. dividend growth, price-dividend ratios forecast returns in the time series and in the cross section”

- Time series, yes; but a cross-section of two firms? The value premium is an intra-industry effect, see Cohen et al. and Novy-Marx

p. 30-31: “A natural question is, are the effects generated by the model quantitatively important and empirical relevant, and if not, what conclusions should one come to?” “Our first answer is that this is the wrong question. Our aim is theory. . . , not to provide a calibrated model that replicates the full range of asset pricing facts and puzzles.”

!?

# Questions

Cont'd

Extension to a general equilibrium two-sector production economy:

Help address the excess comovement puzzle — returns seem to covary without correlated fundamental shocks

Link the magnitude of excess comovement to production and capital adjustment technology, capital mobility, preferences, etc