

A Better Three-Factor Model That Explains More Anomalies

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A new workhorse factor structure for cross-sectional returns:

$$E[r_j] - r_f = \beta_{MKT}^j E[r_{MKT}] + \beta_{INV}^j E[r_{INV}] + \beta_{ROA}^j E[r_{ROA}]$$

in which:

- r_{MKT} : the market factor
- r_{INV} : the investment factor
- r_{ROA} : the *ROA* factor

Outline

- 1 Evidence
- 2 Story
- 3 Interpretation

Evidence

Properties of r_{INV} and r_{ROA} , 1/1972–12/2006, 420 months

r_{INV} : Low-minus-high I/A from the 2×3 sort on size and (annual changes in PPE and inventories)/lagged total assets, annual rebalancing

r_{ROA} : High-minus-low ROA from the 2×3 sort on size and quarterly earnings/one-quarter-lagged total assets, monthly rebalancing

	Mean	α	α_{FF}
r_{INV}	0.43	0.51	0.33
[t]	4.75	6.12	4.23
r_{ROA}	0.96	1.05	1.01
[t]	5.10	5.61	5.60

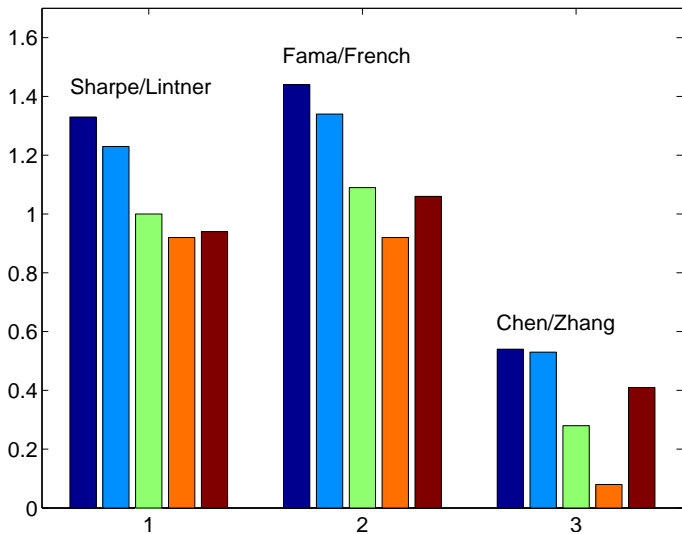
Evidence

The 25 size and momentum portfolios, 1/1972–12/2006

	Loser	3	Winner	W-L	Loser	3	Winner	W-L
	Average excess returns				α			
Small	-0.04	0.80	1.21	1.25	-0.59	0.40	0.73	1.33
3	0.03	0.58	0.98	0.95	-0.51	0.18	0.48	1.00
Big	-0.22	0.29	0.68	0.90	-0.69	-0.07	0.25	0.94
	α_{FF}				α_q			
Small	-0.93	-0.05	0.51	1.44	0.38	0.49	0.92	0.54
3	-0.62	-0.17	0.47	1.09	0.35	0.12	0.63	0.28
Big	-0.60	-0.05	0.46	1.06	-0.10	-0.10	0.31	0.41

Evidence

The winner-minus-loser portfolios across five size quintiles, 1/1972–12/2006



Evidence

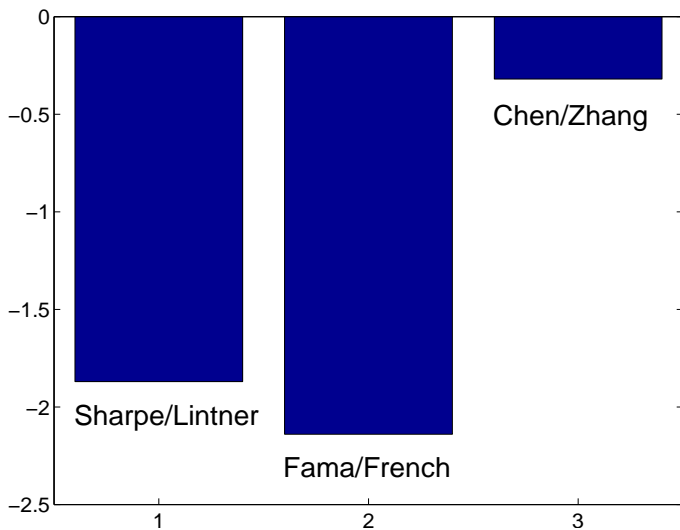
The Campbell-Hilscher-Szilagyi (2008) F -prob deciles, 1/1972–12/2006

	Low	5	High	H-L	GRS
Mean	1.03	0.72	-0.35	-1.38	
α	0.39	0.01	-1.48	-1.87	3.01
α_{FF}	0.39	-0.01	-1.75	-2.14	4.75
α_q	0.19	0.13	-0.13	-0.32	1.78

“p. 2923: [This result] is a challenge to standard models of rational asset pricing in which the structure of the economy is stable and well understood by investors.”

Evidence

The high-minus-low F -prob alpha, 1/1972–12/2006



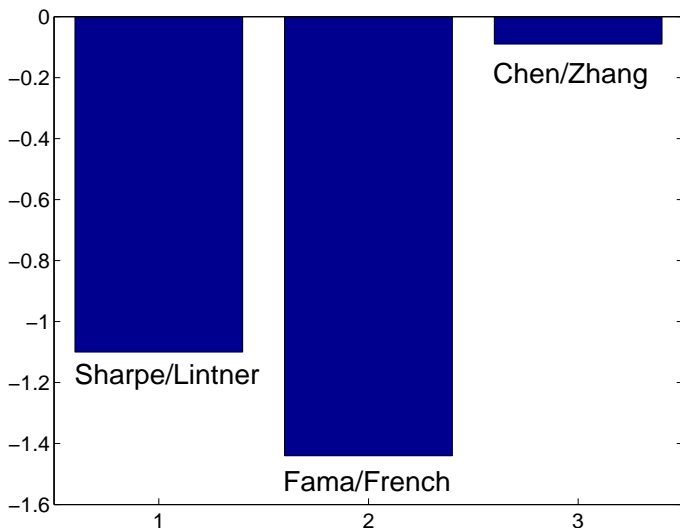
Evidence

The Ohlson's (1980) O -score deciles, 1/1972–12/2006

	Low	5	High	H-L	GRS
Mean	0.48	0.50	-0.44	-0.92	
α	-0.04	0.00	-1.14	-1.10	2.49
α_{FF}	0.12	-0.24	-1.32	-1.44	6.33
α_q	0.02	0.02	-0.07	-0.09	1.10

Evidence

The high-minus-low O -score alpha, 1/1972–12/2006



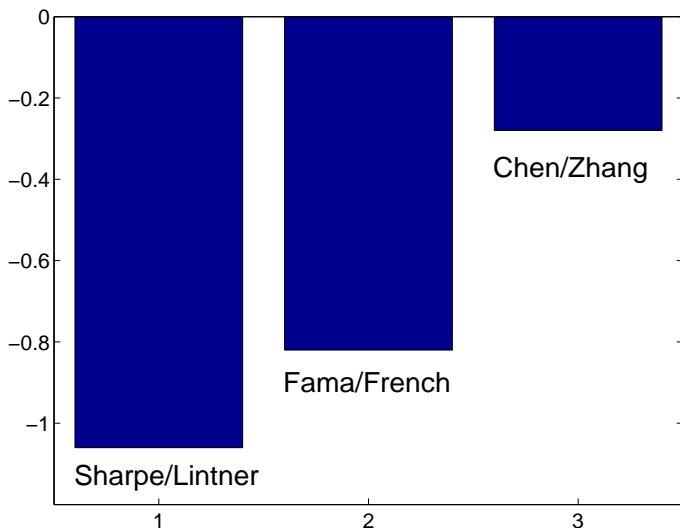
Evidence

The net stock issues deciles, 1/1972–12/2006

	Low	5	High	H-L	<i>GRS</i>
Mean	1.00	0.82	0.16	-0.84	
α	0.42	0.17	-0.64	-1.06	3.97
α_{FF}	0.22	0.13	-0.59	-0.82	3.10
α_q	0.09	0.24	-0.19	-0.28	2.60

Evidence

The high-minus-low net stock issues alpha, 1/1972–12/2006



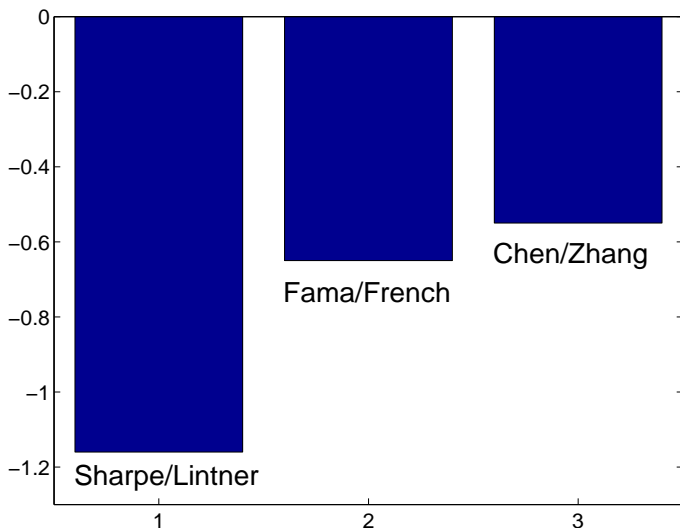
Evidence

The asset growth deciles, 1/1972–12/2006

	Loser	5	High	H-L	GRS
Mean	0.95	0.62	0.16	-0.79	
α	0.39	0.17	-0.53	-0.92	4.33
α_{FF}	0.10	0.05	-0.35	-0.46	2.45
α_q	0.28	0.08	-0.08	-0.37	2.02

Evidence

The high-minus-low asset growth alpha, 1/1972–12/2006



Evidence

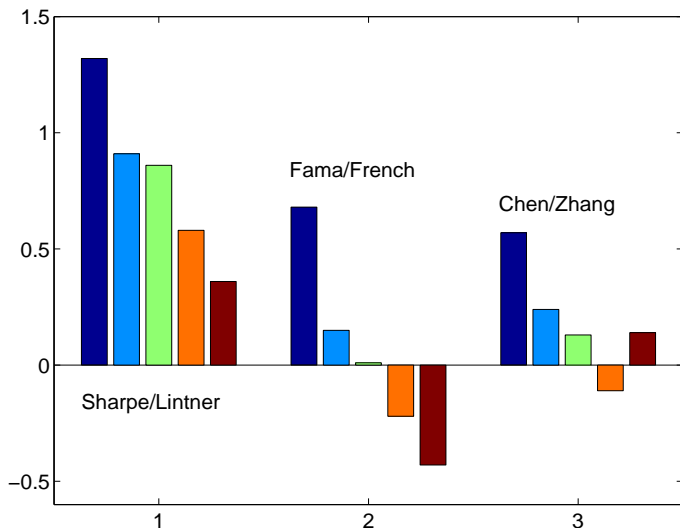
The 25 size and book-to-market portfolios, 1/1972–12/2006

	Low	3	High	H-L	Low	3	High	H-L
	Average excess returns				α			
Small	0.10	0.88	1.19	1.09	-0.63	0.37	0.70	1.32
3	0.41	0.74	1.07	0.66	-0.27	0.27	0.59	0.86
Big	0.40	0.59	0.65	0.25	-0.11	0.16	0.25	0.36
	α_{FF}				α_q			
Small	-0.52	0.09	0.16	0.68	0.08	0.46	0.64	0.57
3	-0.03	-0.12	-0.02	0.01	0.19	0.07	0.31	0.13
Big	0.17	-0.02	-0.26	-0.43	-0.11	-0.04	0.03	0.14

Similar results for earnings-to-price, dividend-to-price, cash flow-to-price, prior 13–60 month returns, five-year sales rank, and assets-to-market equity

Evidence

The value-minus-growth portfolios across five size quintiles, 1/1972–12/2006



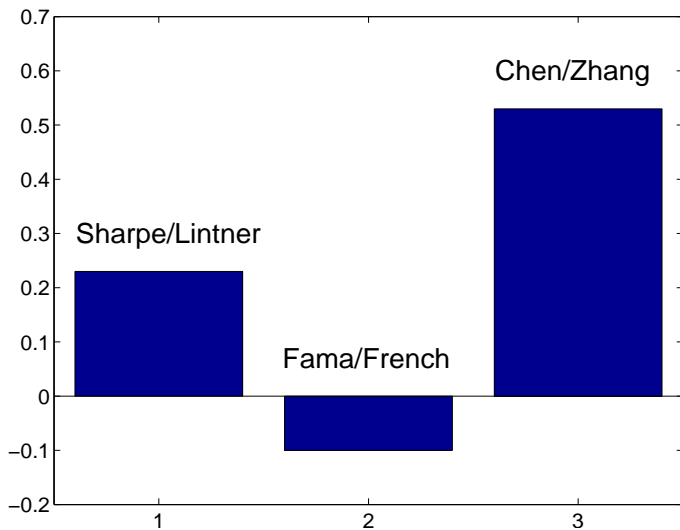
Evidence

The market equity deciles, 1/1972–12/2006

	Small	5	Big	S-B	<i>GRS</i>
Mean	0.73	0.71	0.46	0.28	
α	0.21	0.15	-0.02	0.23	1.79
α_{FF}	-0.04	-0.02	0.06	-0.10	1.82
α_q	0.46	0.29	-0.07	0.53	1.57

Evidence

The small-minus-big alpha, 1/1972–12/2006



Story

A simple q -theory model à la Cochrane (1991) and Liu, Whited, and Zhang (2009)

Two periods, 0 and 1

Firm j 's assets: A_{j0} and A_{j1} , $A_{j1} = I_{j0} + (1 - \delta)A_{j0}$

Firm j 's ROA: Π_{j0} and Π_{j1}

Firm j 's earnings: $\Pi_{j0}A_{j0}$ and $\Pi_{j1}A_{j1}$

Firm j 's adjustment costs: $\frac{a}{2} \left(\frac{I_{j0}}{A_{j0}} \right)^2 A_{j0}$, $a > 0$

Firm j 's discount rate: r_j

Story

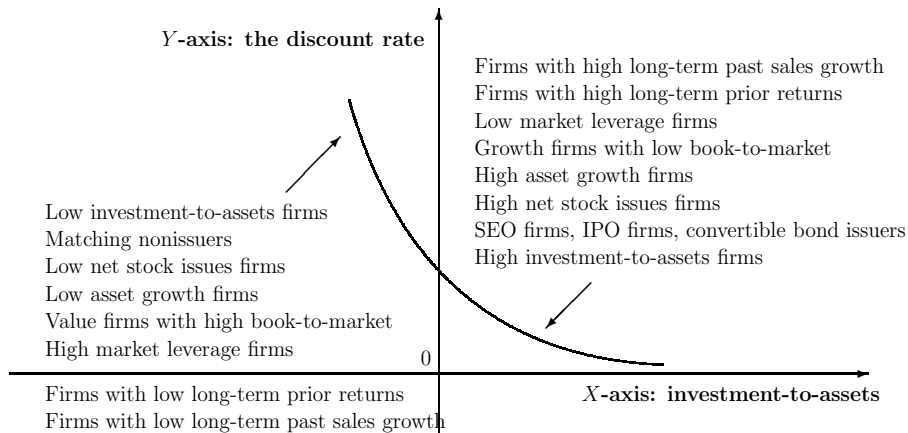
The q -theory drivers of expected returns

$$\begin{aligned} \max_{\{A_{j1}\}} \quad & \Pi_{j0}A_{j0} - [A_{j1} - (1 - \delta)A_{j0}] - \frac{a}{2} \left[\frac{A_{j1}}{A_{j0}} - (1 - \delta) \right]^2 A_{j0} \\ & + \frac{1}{r_j} [\Pi_{j1}A_{j1} + (1 - \delta)A_{j1}]. \end{aligned}$$

$$\text{FOC :} \quad r_j = \frac{\Pi_{j1} + 1 - \delta}{1 + a(I_{j0}/A_{j0})} = \frac{\text{Expected ROA} + 1}{1 + a(I/A)}$$

Story

The investment hypothesis from $(\text{Expected } ROA + 1)/[1 + a(I/A)]$



Story

The *ROA* hypothesis from $(\text{Expected } ROA + 1)/[1 + a(I/A)]$

High *ROA* relative to investment (or *Q*) means high discount rates

- Story for momentum:

Prior returns $\uparrow \Rightarrow$ Expected *ROA* $\uparrow \Rightarrow$ Expected return \uparrow

- Story for distress:

Distress $\uparrow \Rightarrow$ Expected *ROA* $\downarrow \Rightarrow$ Expected return \downarrow

- Story for earnings surprises:

SUE $\uparrow \Rightarrow$ Expected *ROA* $\uparrow \Rightarrow$ Expected return \uparrow

Interpretation

Risk vs. mispricing?

We interpret the new factor model agnostically as a **parsimonious description of cross-sectional returns**

- Loadings on characteristics-based factors are not necessarily risk measures, different from Fama and French (1993, 1996) but consistent with Daniel and Titman (1997)
- Characteristics dominating covariances: **beta measurement errors**, not necessarily mispricing, different from Daniel and Titman (1997)
- Debate irrelevant for practice: if we cannot measure risk, use characteristics instead! An array of characteristics-return relations predicted by q -theory (without mispricing)

Summary

The q -theory factor model

Update Fama and French (1996) with a new three-factor model:

$$E[r^j] - r_f = \beta_{MKT}^j E[r_{MKT}] + \beta_{INV}^j E[r_{INV}] + \beta_{ROA}^j E[r_{ROA}]$$

with a wide range of potential applications

★★★A general purpose data library à la Ken French's is available at:
http://apps.olin.wustl.edu/faculty/chenl/linkfiles/data_equity.html