Central bank intervention and the intraday process of price formation in the currency markets

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ABSTRACT

We propose a novel theory of the impact of sterilized spot interventions on the microstructure of currency markets that focuses on their liquidity. We analyze the effectiveness of intervention operations in a model of sequential trading in which i) a rational Central Bank faces a trade-off between policy motives and wealth maximization; ii) currency dealers’ sole objective is to provide immediacy at a cost while maintaining a driftless expected foreign currency position; and iii) adverse selection, inventory, signaling, and portfolio balance considerations are absent by assumption. In this setting, and consistent with available empirical evidence, we find that i) the mere likelihood of a future intervention—even if expected, non-secret, and uninformative—is sufficient to generate endogenous effects on exchange rate levels, to increase exchange rate volatility, and to impact bid-ask spreads; and ii) these effects are exacerbated by the intensity of dealership competition, the extent of the Central Bank’s policy trade-off, and the credibility of its threat of future actions.

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1. Introduction

The foreign exchange (forex) market is one of the most active financial markets in terms of volume, frequency, and intensity of trading (e.g., Bank for International Settlements, 2008). The recent availability of high-frequency data has stimulated significant interest in its microstructure. However, with few notable exceptions (e.g., Bossaerts and Hillion, 1991; Naranjo and Nimalendran, 2000; Lyons, 2001; Evans and Lyons, 2005), the empirical analysis of this data has not been preceded by theoretical
investigations of the potential impact of the many institutional features specific to the forex market on its functioning. This paper contributes to closing this gap. We focus on one of those features—the presence of a rational, but not necessarily profit-maximizing Central Bank (CB)—and derive from its inclusion novel implications for the process of price formation in currency markets.

Two sets of observations about CB interventions guide our effort. First, many macroeconomics textbooks describe the exchange rate as an intermediate target of monetary policy: CBs choose levels (or bands of fluctuation) for the domestic currency compatible with the “ultimate” trade-off of monetary policy, between sustainable economic growth (the “output gap”) and moderate inflation (e.g., Lewis, 1995; Taylor, 1995). CBs may also serve less stringent agendas, under pressure from political power, interest lobbies, etc. In both cases, their actions may not be motivated by pure profit. There is anecdotal and empirical evidence that policy objectives and wealth maximization often collide (Taylor, 1982; Neely, 2000). CBs may also take positions in the currency markets for purely speculative motives, as in the frequently cited example of Bank Negara, the Malaysian CB, in the early 1990s (e.g., Brown, 2001).

This potential (and often ignored) trade-off between policy goals and costs (or profits) may affect the impact of CB interventions on exchange rate dynamics and on the liquidity of the forex market.

Second, while a consensus has emerged in the economic literature (e.g., Adams and Henderson, 1983) that unsterilized interventions influence the exchange rate through the traditional channels of monetary policy, both the effectiveness of sterilized interventions—i.e., those accompanied by offsetting actions on the domestic monetary base—and their impact on forex market liquidity remain controversial (Sarno and Taylor, 2001). Within the macroeconomic approach, sterilized interventions may affect the exchange rate through either of two channels, portfolio balance and signaling. According to the portfolio channel (Branson, 1983, 1984), interventions altering the relative supply of foreign assets influence the exchange rate when domestic and foreign assets are imperfect substitutes. According to the second channel (Mussa, 1981; Bhattacharya and Weller, 1997; Vitale, 1999), interventions influence the exchange rate by conveying information on policy intentions and/or macroeconomic fundamentals. Many empirical studies of the portfolio channel (e.g., Edison, 1993; Payne and Vitale, 2003; Pasquariello, 2007) find its effects on exchange rates either small and short-lived (particularly in the 1970s and 1980s) or economically and statistically insignificant, despite the important imperfect substitutability documented by Evans and Lyons (2005). There is stronger supporting evidence for the signaling channel, especially when interventions are secret and unannounced (Dominguez, 1992; Kaminsky and Lewis, 1996; Payne and Vitale, 2003; Pasquariello, 2007). Extant studies suggest that, in those circumstances, the resulting dissipation of information and adverse selection may increase exchange rate volatility and widen bid-ask spreads (Vitale, 1999; Naranjo and Nimalendran, 2000; Chari, 2007). Yet, this type of intervention is rather infrequent (e.g., Dominguez, 1998, 2003; Fischer and Zurlinden, 1999). Recent empirical research also reports that even expected, non-secret, and announced interventions have large effects on currency returns, return volatility, bid-ask spreads, and trading intensity, albeit often inconsistent with those predicated by information theory (Dominguez, 1998, 2003, 2006; Payne and Vitale, 2003; Pasquariello, 2007).

Motivated by these considerations, in this paper we propose an alternative theory of the impact of CB interventions on the intraday process of price formation in the currency markets that focuses on their liquidity. This theory illustrates that both the temporary and persistent impact of CB interventions on prices, volatility, and transaction costs in the presence of imperfect substitutability may be related to the special role of forex dealers as liquidity providers. To concentrate on this role, we construct a stylized currency market in which trading occurs sequentially. The market is populated by a continuum of risk-averse investors and an occasionally active CB facing a trade-off between policy and wealth-preservation motives—i.e., accounting for the expected cost of its pursuit of a short-term target level for the exchange rate (e.g., Bhattacharya and Weller, 1997; Vitale, 1999). The likelihood of each order arrival to be from any of the investors or the CB is exogenous, but their trades are endogenously determined in equilibrium. CB interventions are also non-secret, sterilized, uninformative, and unrelated to the relative supply of foreign assets. These assumptions rule out portfolio balance and signaling effects in our framework. Finally, we model forex dealers as risk-neutral, pure market makers providing immediacy, in the spirit of Garman (1976), Brock and Kleidon (1992), and Saar (2000a, 2000b). In particular, we impose that, at each round of trading, they stand ready to buy and sell by setting quotes and spreads that maximize the expected instantaneous compensation for their services.
they can extract from the order flow while maintaining a driftless expected position in the foreign currency. Hence, our forex dealers are indifferent to the value of the exchange rate at the end of the economy (the investors’ concern), given their sole objective of facilitating trading.

Our theory generates novel macro and micro implications of the presence of an active monetary authority in the currency markets. To begin with, we show that the mere possibility of a CB intervention is sufficient to induce forex dealers to revise the quotes they post and the spread they charge to risk-averse investors. Intuitively, the positive likelihood of future CB intervention provides dealers with both an opportunity and a threat. Potential CB trades are additional order flow from which to extract rents. They also represent additional liquidity demand to satisfy, possibly creating an imbalance in the dealers’ expected position in the foreign currency. At each round of trading, the dealers accommodate this potential pressure, while still profiting from their role of liquidity providers, by revising their bid and ask quotes in the direction of the intervention. The revision serves a dual purpose: to elicit the investors to take the other side of the CB trade, and to reduce the expected magnitude of potential CB trades by pushing the foreign currency closer to its policy target. In equilibrium, CB intervention—albeit non-secret and uninformative—not only is effective in influencing the level of the exchange rate but also increases its unconditional volatility. These effects are either temporary or persistent depending exclusively on the likelihood of future CB actions.

We further show that sign and magnitude of the impact of CB trades on bid-ask spreads, as well as on equilibrium prices and price volatility, are crucially related to the degree of dealership competition in the forex market. A monopolist dealer is able to adjust his quotes symmetrically in the presence of an active CB, hence preserving his marginal expected rent from investors’ order flow (i.e., the spread), since he is the sole provider of liquidity in the economy. Perfectly competitive dealers instead have to transfer any additional expected positive or negative rent from the potential presence of a CB onto the population of investors. In equilibrium, the ensuing more significant, and often asymmetric revisions in quotes translate into either wider or tighter spreads (depending on the existing demand for foreign assets), greater effectiveness of CB intervention, and higher unconditional price volatility. The relationship between the extent of dealers’ market power and the effectiveness of CB interventions has never been explored by the economic and financial literature, and is consistent with recent empirical evidence on the importance of dealership competition for forex bid-ask spreads (Huang and Masulis, 1999; Pasquariello, 2007).

The paper is organized as follows. Section 2 describes our basic economy. We derive its equilibrium in the presence of a CB in Section 3. Section 4 concludes. Proofs are in the Appendix.

2. The basic model

Currency markets share with most financial markets the attribute that investors do not necessarily hit dealers’ quotes with orders at the same time. Hence, uncertainty regarding the composition of the investors’ population affects the way dealers set prices. In this section we develop a basic model for the sequential arrival of orders to the forex market under symmetric information and either monopoly or perfect dealership competition.

2.1. Assets

There are two assets in the economy, a riskless bond paying $R \geq 1$ units of the domestic currency (the numeraire, e.g., U.S. dollars, USD) and a riskless bond paying $R_F \geq 1$ units of the foreign currency (e.g., British pounds, GBP) at time $T$. The dollar payoff at time $T$ of the GBP-denominated bond, $F$, is uncertain because so is $S_T$, the amount of USD necessary to buy one GBP at time $T$ (USDGBP), i.e., $F = R_F S_T$. W.l.o.g., we let $R_F = 1$. The long-term exchange rate $F$ is normally distributed with mean $\mu_F$ and variance $\sigma_F^2$. We assume that both $\mu_F$ and $\sigma_F^2$ are known to all players, i.e., symmetric information about the intrinsic value of the risky asset.

2.2. Agents and trading

There are three categories of market participants: A continuum of risk-averse investors, risk-neutral market makers (MMs), and a rational CB. The forex market opens at time $t = 0$, and trading occurs at
each discrete interval $t = 1, 2, \ldots, T < T$. The interval $[0, T]$ should be interpreted as a short period of time, e.g., one trading day. Intraday interest rates are assumed to be zero. At the beginning of each trading interval $t$, only one agent is randomly selected to trade from the pool described above, as in many models of sequential trading. There is a constant probability $1 - l$ known to all market participants, of an investor (the CB) to be selected. The parameter $l$ can also be interpreted as the CB’s relative weight in the population of forex traders. Empirical evidence suggests this weight to be generally small, if measured with respect to either the total daily number of trades or the total daily public trading volume in major currency markets (e.g., Dominguez and Frankel, 1993; Fischer and Zurlinden, 1999).

### 2.2.1. Investors

Investors are dollar-based, risk-averse, and of types 1 and 2 depending on their initial endowment of risky GBP-denominated bills, $X_1$ and $X_2$. We impose that $X_1 > X_2$, i.e., that type 1 investors are potentially net sellers, and type 2 investors potentially net buyers of GBP. As is common in sequential trading models (see O’Hara, 1995), we further assume that a fraction $q$ of investors (known to all agents) is of type 1; hence, $q$ is the exogenous probability that an incoming investor’s order is from a type 1 trader. Each investor, acting competitively, takes market prices $S_t$ as given, maximizes the expected CARA utility of his final wealth with the same degree of risk-aversion $\alpha$, and trades only once. The resulting optimal net demand for GBP by an investor of type $i$, $X_{i,t}$, is given by (Grossman and Stiglitz, 1980).

\[
X_{i,t} = \frac{1}{\pi} \left( f - S_t \right) - X_i,
\]

where $\pi = (aoR^2)/R$ characterizes the elasticity of investors’ demand for risky assets, i.e., the less than perfect substitutability of domestic and foreign currency (Evans and Lyons, 2005).

### 2.2.2. Dealers

As discussed in the Introduction, most extant literature analyzes the impact of CB intervention on exchange rate dynamics in the presence of information asymmetry and/or inventory considerations. Within this literature, it is commonly assumed that forex dealers, albeit constantly providing liquidity, have similar horizons and preferences as those of the investors (e.g., Vitale, 1999; Naranjo and Nimalendran, 2000). Yet, this assumption may not represent an accurate description of the nature of dealerships in the fast-paced currency markets, where anecdotal evidence suggests that dealers concentrate on extracting rents from the order flow, rather than maximizing the terminal value of their positions. Alternatively, Evans and Lyons (2005) model forex dealers’ objective functions as bound by desired inventory positions and stringent capital constraints. In their micro portfolio balance framework, risk-averse dealers pass intraday positions due to exogenous foreign currency supply shocks (the CB intervention) first to other dealers and eventually to the risk-averse public of investors to avoid carrying overnight inventory. When studying actual transactions for the largest spot market, Deutsche mark/U.S. dollar, over four months in 1996, Evans and Lyons (2005) find that the public demand for foreign currency is less than perfectly elastic. Yet, recent empirical tests using both informative and uninformative CB transactions (Payne and Vitale, 2003; Pasquariello, 2007) provide little or no support for the notion that portfolio balance considerations may explain intraday and daily dynamics of exchange rate and bid-ask spreads in proximity of CB interventions.

In this paper, we explore the possibility that the temporary and persistent impact of CB interventions on the process of price formation in the forex market in the presence of imperfect substitutability may be related to the special role of forex dealers as liquidity providers—i.e., as pure providers of immediacy to investors and CBs. We model this role parsimoniously by assuming that our risk-neutral MMs are dollar-based, financially unconstrained, unhindered by order-processing costs, and post quotes maximizing their expected instantaneous net dollar profit $\Pi_t$ per unit of time,

\[
E[\Pi_t] = (1-l)qX_{1,t}S_{1,t} + (1-l)(1-q)X_{2,t}S_{2,t} + dX_t^{CB} S_t^{CB},\]
subject to the constraint that their instantaneous GBP inventory position \(-Z_t\) has no expected drift per unit of time:

\[
E[Z_t] = (1 - l)qX_{1,t} + (1 - l)(1 - q)X_{2,t} + lX_{CB}^t = 0, \tag{3}
\]

where \(X_{CB}^t\) is the CB order and \(S_{i,t}\) and \(S_{CB}^t\) are the prices quoted to investors of type \(i\) and the CB, respectively. In the Appendix, we specify conditions on the model's parameters such that type 2 investors are always net buyers hitting the dealers' ask quotes \(S_{2,t}\) and type 1 investors are always net sellers hitting the dealers' bid quotes \(S_{1,t}\). We then interpret any wedge between the price at which the dealers are willing to sell GBP to type 2 investors and the price at which the dealers are willing to buy GBP from type 1 investors, \(S_{2,t} - S_{1,t}\), as a bid-ask spread. We further impose that there is no interdealer trading and that each incoming order (including the CB’s) is split evenly among all dealers quoting the same price. The former assumption does not allow for “hot potato” trading (Lyons, 1997; Evans and Lyons, 2005; Cao et al., 2006), while the latter restricts the CB trades in this paper to be announced (with probability \(l\)) and non-secret (i.e., non-anonymous), contrary to Naranjo and Nimalendran (2000) and Evans and Lyons (2005).\(^1\) This specification—similar in spirit to Garman (1976); Brock and Kleidon (1992), and Saar (2000a, 2000b)—implies that, at each point in time \(t\), before the next incoming order arrives, MMs post binding quotes maximizing the expected dollar revenues from selling GBP to the public or the CB minus the expected dollar cost of buying GBP from the public or the CB subject to the constraint that the expected amount of GBP bought and sold is the same.

Eq. (2) imposes that MMs profit exclusively from the expected instantaneous difference between ask and bid prices for GBP-denominated bonds, regardless of their liquidation value; hence, MMs charge a bid-ask spread exclusively as compensation for providing liquidity. Accordingly, Eq. (3) requires the MMs’ expected instantaneous GBP inventory resulting from their posted quotes before a trade occurs to have no drift. At the beginning of each trading interval \(t\), the dealers’ quotes \(S_{i,t}\) and \(S_{CB}^t\) represent potential (or reservation) prices, i.e., quotes at which they are willing to buy a certain amount (depth), if a type 2 investor arrives, to sell a certain amount, if a type 1 investor arrives, and to trade a certain amount with a CB, if the monetary authority intervenes. It is this declaration of intents by the dealers that constitutes the bid and offer quotes, as in most sequential trade models. When an order hits the dealers’ screens, the investor optimizes (based on his utility function) taking the MMs’ price as given, and submits his order; then the MMs execute it at the quoted price. As we will soon see, in equilibrium MMs’ quotes are “regret-free,” since they generate order sizes equal to the MMs’ depths accompanying them. Yet, at each point in time \(t\), the only equilibrium price is going to be the one resulting from the most recent transaction. We define the expected transaction price with investors as \(S_t = qS_{1,t} + (1 - q)S_{2,t}\).

### 2.3. The benchmark equilibrium

In the economy described above, dealers extract rents from the order flow as a compensation for providing liquidity services. Clearly, dealers’ market power is crucial in determining their ability to do so. Several market microstructure models relate bid-ask spread determination to dealer competition.\(^2\) Bessembinder (1994), Bollerslev and Melvin (1994), and Huang and Masulis (1999) document that the intensity of dealer competition has a significant impact on bid-ask spreads in the spot forex market. Yet, neither this relation in the presence of CB interventions nor the potential impact of dealer competition on their effectiveness have been previously investigated. This issue may not be trivial, since the level of competition in the currency markets is highly time-varying (Huang and Masulis, 1999) and geographically heterogeneous. For example, for most exchange rates of developed

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1. Dominguez and Frankel (1993) list several reasons why CBs may on occasion prefer to conduct secret interventions. Nonetheless, according to Dominguez (2003, p. 28), the increasing use of electronic trading systems by CBs in executing their operations (e.g., Neely, 2000) “suggests that they are less interested in keeping operations secret.” Anecdotal and empirical evidence also indicates that CBs routinely trade with multiple dealers at the same time to increase the visibility of their actions (e.g., Dominguez, 1998; Fischer and Zurlinden, 1999).

countries, competition among MMs is intense and may exert a downward pressure on their compensation. However, for some other exchange rates, especially in emerging economies, there are often very few agents making the market on a regular basis and for significant trade sizes.

To investigate this issue, we consider two extreme scenarios of monopolistic and competitive dealership. At the beginning of each trading interval $t$, the monopolistic dealer sets quotes that maximize Eq. (2) subject to Eq. (3). Competitive dealers set identical quotes such that, before an order arrives, both $E[P_t] = 0$ (by virtue of Bertrand competition) and $E[Z_t] = 0$ for each MM (Glosten and Milgrom, 1985; Madhavan, 1992). We start by solving the problem of both the monopolist and the competitive MMs under the assumption that $l = 0$. Proposition 1 summarizes the equilibrium of the resulting benchmark economy in the absence of an active CB.

**Proposition 1.** When $l = 0$, the competitive ($^{C}$) MMs’ reservation bid and ask quotes at time $t$, before an investor’s order arrives, are

$$S_{1,t} = S_{2,t} = ^{C}S_{t} = \frac{f}{R} - \pi X^{*}, \quad (4)$$

where $X^{*} = qX_1 + (1 - q)X_2$. Therefore, $S_{2,t} - S_{1,t} = 0$. The monopolist ($^{M}$) MM’s reservation bid and ask quotes and expected transaction price are

$$S_{1,t} = \left(\frac{f}{R} - \pi X^{*}\right) - \frac{\pi(1 - q)}{2}(X_1 - X_2), \quad (5)$$

$$S_{2,t} = \left(\frac{f}{R} - \pi X^{*}\right) + \frac{\pi q}{2}(X_1 - X_2), \quad (6)$$

$$^{M}S_{t} = \frac{f}{R} - \pi X^{*} = ^{C}S_{t}, \quad (7)$$

implying the following absolute and proportional spread, respectively,

$$S_{2,t} - S_{1,t} = \frac{\pi}{2}(X_1 - X_2), \quad (8)$$

$$P_{St} = \frac{S_{2,t} - S_{1,t}}{^{M}S_{t}} = \pi \left(\frac{X_1 - X_2}{\frac{f}{R} - \pi X^{*}}\right) \quad (9)$$

The expected competitive transaction price $^{C}_{St}$ corresponds to the price we would observe if all investors arrived at the same time in a competitive market, i.e., to the market-clearing price. The price of GBP-denominated bonds is equal to their discounted future payoff minus a risk adjustment factor to induce risk-averse, dollar-based investors to hold GBP assets. No spread emerges in this case because of perfect competition among MMs in this otherwise frictionless forex market. Vice versa, a monopolist liquidity provider uses his market power to extract positive rents from investors by charging a higher ask to net buyers and by paying a lower bid to net sellers. A positive spread ensues, consistent with the evidence in Huang and Masulis (1999). The expected transaction price $^{M}_{St}$ is nevertheless equal to $^{C}_{St}$. These results do not depend on the relative magnitude of the investors’ endowments of GBP, but only on whether there is trading (i.e., risk-sharing) in this economy ($X_1 \neq X_2$).

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3. $P_{St}$ is computed using $^{M}_{St}$, rather than the conventional mid-quote, because the probability that an investor’s order is of type 1 ($q$) is public information in this economy.
3. Central bank intervention

Monetary authorities frequently intervene in the forex market, to manage otherwise free-floating exchange rates, to comply with international currency agreements, to serve macroeconomic agendas, or as a result of domestic political pressure. In some circumstances, CBs also act in pursuit of purely speculative motives. Policy and wealth-preservation (or speculative) motives may be conflicting. For instance, suppose that a CB believes, based on superior information, the USD is fundamentally overvalued ($S_t < f$). A sudden devaluation, however, could create excessive inflationary pressures. To attenuate those pressures, the CB could set an intermediate target level for the exchange rate, $S$, between $S_t$ and $f$, and sell some amounts of GBP to prevent the USDGBP from breaking its current trend too rapidly toward its long-term value. Given its knowledge of $f$, the CB action, if effective on $S_t$, would not be profit-maximizing (as buying GBP would instead be) and could lead to a reduction of its expected future wealth.

In this paper, we model this potential trade-off parsimoniously by assuming that at time $t = 0$ a price-taking CB (e.g., Fischer, 2004) makes an ex ante optimal, binding commitment to the following rule: At the beginning of any trading interval $t$, if called to trade (with probability $l$), it will choose the net amount of foreign (domestic) currency to buy or sell, $X_t^{CB} (B_t^{CB})$, that minimizes the loss function

$$L(S, \lambda) = \left[ S_t^* - S \right]^2 - \lambda E \left[ W_t^{CB} \right],$$

in which $W_t^{CB} = R(B_t^{CB} + B_t) + F(X_t^{CB} + RES_t)$, subject to the budget constraint

$$B_t^{CB} = -S_t^* X_t^{CB},$$

where $RES_t = RES_t^M + X_t^{CB}$ and $B_t = B_t^M + B_t^{CB}$ are the endowments of GBP and USD held by the CB at time $t$ before intervening, and $t \in (0, t]$ is when the most recent past intervention occurred. All parameters of the CB’s loss function are known to all market participants.

The specification of Eq. (10) is similar in spirit to Stein (1989), Bhattacharya and Weller (1997), and Vitale (1999). The first component measures policy motives by the squared distance between $S_t^*$ and $S_t$, the expected exchange rate from trading between MMIs and investors at time $t$ (see Section 2.2.2), and the target $S$. The second component incorporates wealth-preservation motives: Interventions are costly when CB actions are unprofitable from a speculative perspective. The parameter $\lambda \geq 0$ controls for the relevance of the ensuing potential trade-off between policy and speculation in $L(S, \lambda)$, and can be interpreted as a measure of the CB’s commitment to stabilize the exchange rate around $S$. The budget constraint in Eq. (11) imposes that each GBP trade be accompanied by an open-market trade in the opposite direction. In addition, both $f$ and $S_t^*$ are independent of any CB action (see Section 2.1). These assumptions imply that, in our setting, interventions are always sterilized.

We also impose that $l$, the likelihood of a CB intervention to take place at any trading interval $t$, is exogenous. Intuitively, the frequency parameter $l$ can be interpreted as the result of a prior policy decision accompanying the establishment at time $t = 0$ of a rule for “occasional” CB actions. This assumption, common in the literature (e.g., Naranjo and Nimalendran, 2000; Evans and Lyons, 2005), is plausible over the short period of time (e.g., one trading day) that the interval $[0, T]$ is meant to represent in our model and makes the problem of the monetary authority and the analysis of its impact on quotes and spreads more tractable. However, we do not impose an exogenous relationship between CB trades and the exchange rate (e.g., Bossaerts and Hillion, 1991; Naranjo and Nimalendran, 2000). The market friction allowing interventions to be potentially effective in our economy ensues

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4 Because we interpret the interval $[0, T]$ as a short period of time, we abstract from the issue of a CB being unable to pursue this policy because of lack of reserves by assuming the initial endowments $RES_0$ and $B_0$ are large enough so that $RES_t > 0$ and $B_t > 0$ before and after an intervention.

5 The CB’s loss function can be generalized to the case of a target band of fluctuation $(S_L, S_H)$ for the currency by specifying the policy component in Eq. (10) as $|S_t^* - (1/2)(S_L + S_H)|^2$.

from the constraint that $E[Z_t] = 0$ (Eq. (3)), the definition of $S_t^*$ (Section 2.2.2), and the investors’ optimal net demand for GBP (Eq. (11)): The CB conjectures that $S_t$ is equal to

$$S_t^* = \frac{f}{R} - \pi X_t^* + \pi L X_t^C B,$$

(12)

where $l = 1/(1-\lambda)$. Hence, for a given $l > 0$, the ex ante optimal intervention schedule $X_t^{CB}$ for each price level $S_t^*$ minimizes $L(S, \lambda)$ subject to Eq. (12) and the wealth constraint of Eq. (11). We have the following result:

**Proposition 2.** The ex ante optimal demand function for the CB is

$$X_t^{CB} = \frac{1}{\pi l} \left[ S_t - \left( \frac{f}{R} - \pi X_t^* \right) + \frac{\lambda R}{2\pi l} \left( \frac{f}{R} - S_t^* \right) \right].$$

(13)

The CB’s commitment to the policy rule of Proposition 2 is essential to the analysis because of time consistency considerations, since it has no incentive to trade GBP with the MMs ex post, i.e., when called to the market. Therefore, ex ante, the other players would have no reason to believe that $l > 0$, unless the CB precommits to the above rule, making its actions unavoidable ex post, thus credible ex ante. These commitments, either through legal arrangements or other procedures, are very common, e.g., in the form of constitutions of the monetary authorities (Persson and Tabellini, 1990). According to Eq. (13), the CB needs to buy (sell) GBP, to push the expected transaction rate closer to the target level, if the difference between $S_t$ and the competitive market-clearing price when $l = 0$ ($(f/R) - \pi X^*$) is positive (negative). Under the assumption that $(R_t f/R) < 1$ (see Section 2.1), the CB chases the trend if $S_t > (f/R) - \pi X^*$, attempting to induce a faster depreciation of $S_t^*$ toward its long-term fundamental value, by buying GBP$^6$; the CB instead leans against the wind if $S_t < (f/R) - \pi X^*$, attempting to resist $S_t^*$’s long-term trend, by selling GBP$^9$. In addition, the CB buys more (sells fewer) GBP if the expected net present value (NPV) in dollars of that investment, $(f/R) - S_t^{CB}$, is higher. The amount of GBP bought (or sold) by the CB also depends on the trade-off between policy and speculative motives. This trade-off is, not surprisingly, highest when the CB is trying to lean against the wind, in doing so reducing its expected future wealth.

Ceteris paribus for $S_t^{CB}$, the absolute magnitude of $X_t^{CB}$ depends on both its exogenous frequency ($l$) and the elasticity of investors’ demand for GBP of Eq. (1) via $\pi$. Intuitively, a lower $l$ makes the threat of intervention less significant for the dealers, and a bigger necessary to move $S_t^*$ toward $S^*$. Further, if investors are more risk-averse (higher $\alpha$), if there is more uncertainty surrounding the long-term exchange rate $F$ (higher $\sigma_F$), or if investors have a higher expected endowment of GBP (higher $X^*$), their demand for GBP is less elastic. Hence, a bigger positive (negative) CB intervention is necessary to push $S_t^*$ toward a high (low) $S^*$.

3.1. Intervention and price formation

In the stylized forex market of Section 2, symmetric sharing of information about the liquidation value of GBP-denominated bonds bars information effects of CB interventions—the signaling channel and adverse selection at the dealer level—while the portfolio balance channel is also ruled out by construction—since the total outstanding relative supply of domestic and outside foreign assets plays

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7 After the CB is called to trade at the beginning of the trading interval $t$, targeting $S_t$ is superfluous since no investor transaction is going to take place over that interval, while wealth maximization is pointless since both the MMs and the risk-neutral CB know the parameters of the distribution of $F$; hence, $X_t^{MM} = (f/R)$ and $X_t^{CB} = 0$.

8 If $S_t > (f/R)$, the CB rides the wave, i.e., aggressively pursues a depreciation of the dollar beyond the long-term risk-neutral rate $(f/R)$. These competitive devaluations, conducted with the purpose of remedying balance of payments problems, are explicitly prohibited by the IMF Article 4, Section 1.

9 Alternatively, if $(R_t f/R) > 1$ inasmuch as, when $l = 0$, the dollar is weaker than its long-term expected value ($S_t > E[S_t^*] = f$), the CB would chase the trend if $S_t < (f/R) - \pi X^*$ and lean against the wind if $S_t > (f/R) - \pi X^*$.

10 Consistently, Naranjo and Nimalendran (2000) report that the Bundesbank’s interventions are more frequent than the Federal Reserve’s, but smaller in absolute dollar size.
no role in our model and dealers are unencumbered by end-of-day flat-inventory constraints. We are interested in determining the potential impact of CB trades on the process of price formation in such otherwise frictionless market due to liquidity considerations. To that purpose, we solve for its equilibrium quotes and bid-ask spreads under monopoly and dealership competition in the presence of occasional CB interventions \((l > 0)\). We then compare those quotes and spreads to the benchmark \((l = 0)\) equilibrium of Proposition 1.

We construct the equilibrium in three steps. First, we assume that dealers can always distinguish whether the incoming order is from an investor or the CB, can conjecture the investor’s type from the size and sign of his order, and use this knowledge to formulate their reservation prices for each potential arrival.\(^{11}\) We further assume the CB conjectures that \(S_t^*\) of Eq. (12) is the expected transaction price in its loss function \(L(S, \lambda)\) of Eq. (10). Second, we compute type 1 and type 2 investors’ optimal demands for GBP and the optimal CB intervention at \(S_{1,t}, S_{2,t},\) and \(S_t^CB\). Third, we show that the resulting investors’ orders, given those prices, are different, and that \(S_t^*\) of Eq. (12) is equal to its definition in Section 2.2.2, \(qS_{1,t} + (1 - q)S_{2,t}\), confirming the MMs’ and CB’s conjectures.

### 3.1.1. The monopolist dealer

**Proposition 3** When \(l > 0\), the monopolist MM’s reservation bid and ask quotes and expected transaction price at time \(t\), before an investor’s order arrives, are

\[
S_{1,t} = \left(\frac{f}{R} - \pi X^*\right) - \frac{\pi (1 - q)}{2}(X_1 - X_2) + \pi L_t^CB
\]

\[
S_{2,t} = \left(\frac{f}{R} - \pi X^*\right) + \frac{\pi q}{2}(X_1 - X_2) + \pi L_t^CB,
\]

\[
M S_t^* = \left(\frac{f}{R} - \pi X^*\right) + \frac{\pi L}{2\pi L + \lambda IR}(S - \frac{f}{R}) + \frac{\pi}{2}X^*,
\]

while the reservation exchange rate if a CB intervenes at time \(t\) is

\[
S_t^CB = \omega_1\frac{f}{R} + \omega_2S + \omega_3\pi X^*,
\]

where \(\omega_1, \omega_2,\) and \(\omega_3\) are in the Appendix. The absolute spread is the same as when \(l = 0\) (Eq. (8)); the proportional spread is not (Eq. (9)), but

\[
PS_t = \frac{S_{2,t} - S_{1,t}}{MS_t^*} = \pi\left(\frac{X_1 - X_2}{2\left(\frac{f}{R} - \pi X^* + \pi L_t^CB\right)}\right)
\]

**Proposition 3** states that, in equilibrium, the monopolist dealer revises upward (downward) his quotes if there is a positive probability that the CB may intervene at time \(t\) buying (selling) GBP. Although the absolute spread is unchanged, the proportional spread declines (increases). The quote revision is symmetric and given by

\[
\Delta S_t = \Delta S_t^* = \pi L_t^CB = \frac{\pi L}{2\pi L + \lambda IR}(\frac{S - f}{R}) + \frac{\pi}{2}X^*
\]

\(^{11}\) This assumption captures two typical aspects of over-the-counter currency markets: Lack of anonymity and, consequently, price discrimination. For more on the trading relationship between CBs and MMs, see Peiers (1997).
To explain the intuition for these results, assume that it becomes known to the dealer (or the CB announces) that from time $t$ onward, with probability $l > 0$, an order $X_t^{CB} > 0$ might arrive. In this stylized forex spot market, the MM’s sole objective is to maximize his expected instantaneous compensation for providing liquidity, regardless of the direction the currency is taking. The presence of a rational, but not necessarily profit-maximizing CB constitutes additional liquidity demand from which to extract rents. This opportunity, however, comes at a cost: At the benchmark ($l = 0$) prices, the CB’s potential buy order creates an imbalance in the dealer’s expected inventory by adding to the originally flat position a negative drift component. In equilibrium, the MM accommodates this drift pressure, while still profiting from his role of liquidity provider, by setting potential bid and ask quotes eliciting risk-averse investors to take the other side of the CB trade, i.e., to become expected net sellers of GBP. This strategy serves a dual purpose. First, it allows the MM to leave the absolute spread unchanged—i.e., to preserve the absolute expected unit rent the MM is extracting from the population of investors—even in the presence of a CB. Second, it allows the MM to reduce the expected magnitude of CB intervention by pushing $M'_{S_t}$ of Eq. (16) toward $S$. Specifically, in order for the no-instantaneous drift condition (Eq. (3)) to be satisfied, the MM increases his reservation ask quote $S_{1,t}$ (to reduce the size of the expected incoming purchases by investors) and bid quote $S_{2,t}$ (to increase the size of the expected incoming sales by investors). Because at the revised quotes net buyers buy less and net sellers sell more, investors’ net demand is now expected to be a sale. This allows the dealer to offset the drift on his inventory when the CB is expected to be a net buyer of GBP. Consequently, $\Delta S_{1,t} > 0$, the expected transaction price $M'_{S_t}$ increases, and the proportional spread declines. Vice versa, if $X_t^{CB} < 0$, then $\Delta S_{1,t} < 0$, the expected transaction price decreases, and the proportional spread increases.

Thus, according to Proposition 3, even a non-secret, uninformative CB intervention may have an impact on quotes (and transaction costs) in the direction of its expected sign. Eq. (19) further implies that such impact is higher when the coefficient of risk-aversion ($\alpha$) and the volatility of the long-term value of GBP($\sigma^2$) are higher or government action is less likely.12 Intuitively, in those circumstances the monopolist dealer needs a bigger quote revision to eliminate the expected drift in his instantaneous inventory while continuing to extract rents from the augmented order flow. When the intervention actually occurs, there is no additional impact on quotes and spreads since CB trades do not affect investors’ and dealers’ beliefs about $F$ or the relative supply of foreign assets.

3.2. The competitive dealers

We now consider the case of dealers with no market power. Proposition 1 shows that, in this case, no spread arises in our frictionless forex market when $l = 0$. Does that conclusion still hold if $l > 0$? Along the lines of Section 2.3, the equilibrium rates $S_{1,t}$, $S_{2,t}$, and $C_{S_t}^*$ are those which, given investors’ and CB’s optimal demands, satisfy both $E[\Pi_t] = 0$ and $E[Z_t] = 0$. These two restrictions are insufficient to identify three reservation prices; hence, we express the bid and offer quotes as functions of a free variable, $S_t^{CB}$. The following equilibrium ensues:

**Proposition 4.** When $l > 0$, the competitive MMs’ reservation bid and ask quotes and expected transaction price as a function of $S_t^{CB}$, at time $t$, before an investor’s order arrives, are

\[ S_{1,t} = \left( \frac{f}{R} - \pi X^* \right) + \pi L X_t^{CB} - \left[ \frac{\pi(1-q)}{2} (X_1 - X_2) - \frac{\pi}{2} R \right], \]  \hfill (20)

\[ S_{2,t} = \left( \frac{f}{R} - \pi X^* \right) + \pi L X_t^{CB} + \left[ \frac{\pi q}{2} (X_1 - X_2) - \frac{\pi q}{2(1-q)} R \right], \]  \hfill (21)

\[ C_{S_t}^* = \bar{S} + \frac{\lambda R}{2\pi L} \left( \frac{f}{R} - S_t^{CB} \right), \]  \hfill (22)

where $\Gamma > 0$ is in the Appendix. The bid-ask wedge is then

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12 It is easy to show that, for any $S < (f/R)$, $(\partial S_t^{CB}/\partial a) > 0$, $(\partial S_t^{CB}/\partial a^2) > 0$, and $(\partial S_t^{CB}/\partial l) < 0$. 
There exists a price $S^*_t^{CB} = S^{CB(*)}$ such that, for $X_t^{CB} \neq 0$, the above wedge is equal to zero:

$$S^*_t^{CB} = S_{1,t} = S_{2,t} = \left( \frac{2\pi L}{2\pi L + \lambda R} \right) \frac{X_1 - X_2}{C_3} + \left( \frac{\lambda R}{2\pi L + \lambda R} \right) f$$

The equilibrium bid and ask quotes differ from those reported in Proposition 1. Both $S_{1,t}$ and $S_{2,t}$ depend now on three components. The first is $(f/R) - \pi X^{(*)}$, the competitive benchmark exchange rate for $l = 0$ (Eq. (4)). The remaining two are revisions induced by the positive likelihood of the CB arriving, but only one of them, $\pi L X^{CB}$, affects symmetrically both bid and offer quotes. This is the adjustment needed to ensure that $E[Z_t] = 0$ when $X_t^{CB} \neq 0$ and $l > 0$, as in the monopoly scenario. If, for example, $X_t^{CB} < 0$, MMs symmetrically decrease bid and ask prices to achieve two related objectives. First, the resulting lower $S^*_t$ is closer to $\bar{S}$ and induces a smaller intervention. Second, investors buy more GBP than they would if $l = 0$, thus facilitating the dealers’ efforts to have a driftless expected instantaneous inventory.

Most interestingly, $l > 0$ induces a wedge between bid and offer quotes: The bid-ask spread (Eq. (23)) is generally nonzero, unless either $S^*_t^{CB} = S^{CB(*)}$ or $l = 0$. To interpret this result, recall that in our frictionless market the benchmark spread is zero when $l = 0$ (Proposition 1). Therefore, we can think of Eq. (23) as the change in an otherwise positive spread resulting from other market frictions, such as adverse selection, inventory control, order-processing costs, etc. When $l > 0$, a wedge arises because competitive pressure among dealers obliges them to pass all extra (positive or negative) rents from the potential arrival of a CB order onto investors. The elasticity of their demand for GBP then determines whether this effort makes that wedge positive or negative. As $\Gamma > 0$, this wedge is always lower than the absolute spread set by a monopolistic dealer, $(\pi/2)(X_1 - X_2)$ of Eq. (8), because competition erodes the MMs’ ability to be compensated for providing liquidity. Therefore, the more binding is the no-profit condition, $E[Z_t] = 0$, the smaller is the competitive spread with respect to $(\pi/2)(X_1 - X_2)$. Consequently, ceteris paribus for $S^*_t^{CB}$, the terms $\frac{L}{2\pi L}$ and $\frac{\pi q}{2\pi L + q}$ in $S_{1,t}$ and $S_{2,t}$, respectively, can be interpreted as the effect of the binding no-profit condition on the monopolistic quotes of Section 2.3.

3.3. A further look at the effectiveness of intervention

The model of Sections 2 and 3 provides a novel explanation of the impact of CB interventions on the process of price formation in the currency markets in the absence of information and portfolio balance effects, one based on the role of forex dealers as providers of liquidity to those markets. Propositions 3 and 4 show that such role is sufficient to induce quote and spread revisions in the presence of occasional CB interventions even in an otherwise frictionless market in which their potential portfolio balance and signaling effects are ruled out by construction. Intuitively, this occurs because these revisions are necessary for the MMs to maximize their expected compensation for providing immediacy while ensuring that their expected position in the foreign currency is flat at each round of trading. We also show that the degree of competition among MMs affects the extent to which MMs can extract rents from the order flow by charging a wedge between ask and bid prices.

The impact of CB intervention on exchange rate levels also depends on the degree of market power held by the currency dealers. We use the equilibrium expected transaction prices induced by $l > 0$ in the case of a monopolist MM (of Eq. (16)) or of dealers competition (of Eq. (22)) to measure the relative effectiveness of interventions under the two regimes as $EM_t = (M S^*_t - \bar{S})^2 - (C S^*_t - \bar{S})^2$. Positive values for $EM_t$ indicate that the threat of the arrival of CB trades pushes $S^*_t$ closer to $\bar{S}$ in the competitive scenario of Section 3.2. We then have the following result:

**Proposition 5.** CB interventions are maximally effective when $S_t^{CB} = (f/R)$. CB interventions are more effective for competitive MMs if $\lambda = 0$. When instead $\lambda > 0$, the same is true only if the absolute NPV of currency trading for the CB is “small” and/or $\lambda$ is “small.”

---

13 The apparent arbitrage opportunity offered by a negative wedge cannot be exploited by the investors, since they can trade with the dealers only once.
According to Proposition 5, interventions tend to be more effective when dealers’ market power is minimal. When market power is significant, the MMs’ attempt to maximize their expected compensation for providing liquidity services prevents interventions from being fully effective: The monopolist dealer does not adjust his quotes completely, in order to extract some rents from the CB or to pass most of the costs of its intervention onto investors. Consequently, absolute $X_t^B$ is smaller and the intervention is less effective. Competition instead induces the dealers to transfer expected rents from CB actions fully to investors. Hence, quotes’ revisions are more substantial. Speculative motives also affect the impact of interventions on the exchange rate. Proposition 5 further indicates that when $\lambda = 0$ intervention is always more effective in the competitive scenario since the CB—acting as a pure price-manipulator indifferent to any cost—offers the monopolist MM more opportunities to extract rents from the order flow by not fully adjusting $S_t$ toward $\mathcal{S}$.\(^{14}\) When $\lambda > 0$, the reverse might be true (and $EM_t < 0$) if wealth-preservation is “very important,” because it is then “very profitable” to trade currencies (if GBP trading is a positive NPV decision) and/or the CB’s loss function is “very sensitive” to its final wealth $W_{T}^{CB}$. Empirical evidence on the relationship between the effectiveness of intervention and dealers’ market power is scarce and mostly anecdotal. There is nonetheless a consensus that smaller, less frequent, and less effective interventions are observed for less intermediated exchange rates of emerging economies (Chancellor, 2000; Moloney, 2000; Brown, 2001).

There is much greater evidence that CB interventions significantly affect exchange rate volatility, regardless of their effectiveness. According to extant literature (e.g., Lewis, 1989; Dominguez, 1998; Pasquariello, 2007), interventions can be linked to exchange rate volatility through their information content and the degree of efficiency of the forex market. Our paper suggests an alternative link between CB interventions and exchange rate volatility, operating through the MMs’ role as liquidity providers. From Proposition 1, the unconditional variance of the transaction price in every round of trading in the absence of CB interventions is decreasing in the intensity of dealerships competition. Eq. (4) implies that incoming type 2 and type 1 investors face identical quotes $S_{1,t} = S_{2,t} = (f/R) − \pi X^{*}$ under dealership competition; hence, $\text{var}(C_{S_t}) = 0$. The monopolist MM's ability to extract rents from investors’ order flow (Eqs. (5) and (6)) implies a positive unconditional variance,

$$\text{var}(M_{S_t}) = \frac{\pi}{4}(1-q)(\overline{X}_1 - \overline{X}_2)^2,$$

when $l = 0$. In the presence of occasional CB interventions ($l > 0$), the unconditional variance of the transaction price is unchanged (i.e., equal to Eq. (25)) when the latter is set by a monopolistic dealer ($S_{1,t}$ and $S_{2,t}$ of Eqs. (14) and (15)), and equal to

$$\text{var}(C_{S_t}) = \frac{\pi}{4}(1-q)(\overline{X}_1 - \overline{X}_2)^2 + \frac{\pi^2 q}{4} \rho^2 - \frac{\pi^2 q}{2} \rho(i\overline{X}_1 - \overline{X}_2))0,$$

when the latter is set under dealership competition ($S_{1,t}$ and $S_{2,t}$ of Eqs. (20) and (21), and any $S_{l,M}^{CB} = S_{l,M}^{CB}(\cdot)$ of Eq. (24)). As in Proposition 4, $\text{var}(C_{S_t})$ depends on three terms. The first one is $\text{var}(M_{S_t})$, the unconditional price variance when quotes are set by a monopolistic dealer; the latter two depend on the MMs’ ability to be compensated for providing liquidity to investors while facing an active CB ($l > 0$) and a binding no-profit constraint ($\Gamma > 0$). Hence, in our model, a positive probability of an incoming CB trade, even if sterilized, uninformative, and non-secret, is sufficient to increase unconditional exchange rate volatility, yet only when dealers compete for the incoming trade and only for as long as the threat of a future CB intervention is present. This occurs because, when $l > 0$, competitive MMs push $S_{1,t}$ and $S_{2,t}$ asymmetrically toward $\mathcal{S}$ to reduce the expected magnitude of the CB’s endogenous demand for GBP, i.e., to attenuate the CB’s endogenous demand for liquidity.\(^{15}\)

\(^{14}\) Consistently, we also show in the Appendix that if $\lambda$ is “small” or there is no trade-off between wealth-preservation and policy motives, the CB is always better off when dealers compete for the incoming trade, i.e., $l_{D}(\mathcal{S}, \lambda) < l_{M}(\mathcal{S}, \lambda)$.

\(^{15}\) The resulting unconditional price volatility of Eq. (26) is higher or lower than under monopolistic dealership (Eq. (25)) depending on the parameters controlling for the absolute magnitude of the potential CB intervention ($\lambda$, $l$, $\mathcal{S}$, and $S_{l,M}^{CB}$), i.e., on whether $l > 2(1-q)(\overline{X}_1 - \overline{X}_2)$.
Consistently, Dominguez (1998, 2006); Chari (2007), and Pasquariello (2007) provide evidence that CB interventions often lead to greater daily and intraday exchange rate volatility even when non-secret and with explicit exchange rate targets.

Lastly, Propositions 3 and 4 imply that the duration of these effects in our stylized model of sequential currency trading depends crucially on the likelihood of the incoming order being from the CB at each round of trading. When dealers learn that \( l > 0 \), prices and spreads are initially revised to pass the potential incoming order from the CB onto customer order flow from risk-averse investors at a premium. This adjustment is temporary if \( l \) immediately reverts to zero, since additional compensation is no longer needed for new investors. The revision is persistent if the threat of future CB trades is significant and credible (i.e., for as long as \( l > 0 \)), since all incoming investors have to be compensated for possibly having to rebalance their optimal portfolios. This mechanism may help explain numerous anecdotes and recent empirical evidence (e.g., Fratzscher, 2005; Ehrmann and Fratzscher, 2007) of success and failure of uninformative “verbal interventions”—official and unofficial CB announcements in the media—to affect the dynamics of exchange rates.

4. Conclusions

Despite much careful work in the economic and financial literature, there remains an intense theoretical and empirical debate on the channels through which sterilized Central Bank (CB) interventions may affect not only the level of exchange rates—e.g., portfolio balance or signaling—but also the microstructure of the markets where they are traded—e.g., adverse selection or interdealer trading. For instance, mounting evidence suggests that, although the portfolio balance channel performs poorly in direct tests, there is significant imperfect substitutability in the currency markets and even expected, non-secret, or uninformative interventions significantly influence daily and intraday currency prices, price volatility, and bid-ask spreads (e.g., Dominguez, 2003; Evans and Lyons, 2005; Chari, 2007; Pasquariello, 2007). Our study contributes to this debate by proposing a novel theory that links the impact of those interventions on the process of price formation in the forex market to the important role of forex dealers as providers of liquidity.

We develop a model of sequential trading in which i) prices are set by either a monopolist or competitive risk-neutral dealers; ii) those dealers’ main activity is to dispense immediacy to all market participants at a cost (the bid-ask spread); iii) the demand schedule of the monetary authority results endogenously from the optimal resolution of a trade-off between policy and wealth-preservation; and iv) adverse selection, inventory, signaling, and macro and micro portfolio balance considerations are ruled out by construction. In this setting, we show that the mere likelihood of a non-secret and uninformative CB intervention is sufficient to push the equilibrium exchange rate closer to the CB’s policy target, to increase its unconditional volatility, and to induce revisions in the quoted bid-ask spreads—for as long as such threat is credible and the more so the less is forex dealers’ market power. These implications are consistent with the aforementioned empirical evidence on the microstructure of currency markets, as well as with anecdotal evidence on the rapid cycles of success and failure of several interventions by monetary authorities around the world over the last three decades.

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Appendix

Proof of Proposition 1. We construct the equilibrium for both monopoly and dealership competition in three steps, similarly to Saar (2000a, Section 1.2). This procedure is described in greater detail in Section 3. We start by assuming that MMs conjecture the investors’ type from sign and magnitude of their orders and set prices accordingly. The monopolistic MM solves:

\[
\begin{align*}
\max_{S_{1,t}, S_{2,t}} & \quad E[\Pi_t] = qX_{1,t}S_{1,t} + (1 - q)X_{2,t}S_{2,t} \\
\text{s.t.} & \quad E[Z_t] = qX_{1,t} + (1 - q)X_{2,t} = 0 \\
& \quad X_{1,t} = \frac{1}{\pi} \left( \frac{f}{R} - S_{1,t} \right) - X_i \quad i = \{1, 2\}.
\end{align*}
\]

(A-1)

Plugging Eq. (1) into the constraint of Eq. (3) allows us to rewrite Eq. (A-1) in terms of \(S_{2,t}\). It is then easy to show that the ensuing f.o.c. generates the optimal prices \(S_{1,t}\) and \(S_{2,t}\) of Proposition 1 (Eqs. (5) and (6)). We then plug these prices into Eq. (1) to find \(X_{1,t} = -(1/2)(1 - q)(X_1 - X_2)\) and \(X_{2,t} = (q/2)(X_1 - X_2)\). Finally, we verify that the MM’s conjecture—that \(X_1 < X_2\) implies not only that \(X_{1,t} \neq X_{2,t}\), but also that \(X_{1,t} < 0\) and \(X_{2,t} > 0\)—is correct. Bertrand competition among MMs moves prices away from the levels of Eqs. (5) and (6) until \(E[\Pi_t] = 0\) and \(S_{1,t} = S_{2,t}\). Using Eq. (3) for \(l = 0\) to rewrite \(X_{1,t}\) in terms of \(X_{2,t}\), and plugging it into \(E[\Pi_t] = 0\) leads to \((1 - q)X_{2,t}(S_{2,t} - S_{1,t}) = 0\), which, with trading \((X_{2,t} \neq 0\) and \(X_{1,t} \neq 0\)), can be satisfied only by \(S_{1,t} = S_{2,t} = S_t\). Eq. (4) then follows from plugging Eq. (1) into Eq. (3) and solving for \(S_t\). Investors’ optimal orders are then given by \(X_{1,t} = -(1 - q)(X_1 - X_2)\) and \(X_{2,t} = q(X_1 - X_2)\), consistent with the MMs’ initial conjecture about investors’ types.

Proof of Proposition 2. The ex ante optimal, precommitted demand function of the CB, if called to trade at time \(t\), is the solution of the following problem:

\[
\min_{{X_{CB}^t}} L(\bar{S}, \lambda) = \left[ S^*_t - \bar{S} \right]^2 - \lambda E\left(W^t_{CB}\right) \quad \text{s.t.} \quad W^t_{CB} = R\left( B^t_{CB} + \bar{B}_t \right) + F\left(X^t_{CB} + RES_t\right)B^t_{CB} = -S^*_{CB}X^t_{CB},
\]

where \(S^*_t = (f/R) - \pi X^* + \pi LX^t_{CB}\) (Eq. (12)) and \(E(W^t_{CB}) = [R(B^t_{CB} + \bar{B}_t) + f(X^t_{CB} + RES_t)] + (1 - l)[R\bar{B}_t + fRES_t]\), since \(E(F) = f\) and the CB’s demand function in its policy rule has to be ex ante optimal. Eq. (13) then ensues straightforwardly from the F.O.C. of this constrained minimization w.r.t. \(X^t_{CB}\). The s.o.c. for a minimum, \(2\pi^2L^2 > 0\), is always satisfied.

Proof of Proposition 3. The optimal \(S^t_{CB}\) of Eq. (17), with \(\omega_1 = (\lambda^2R^2 - 2\pi^2L^2)/[\lambda R(2\pi L + \lambda IR)], \omega_2 = [2\pi L(2\pi L + \lambda IR)]/[\lambda R(2\pi L + \lambda IR)]\), and \(\omega_3 = \pi L/\lambda IR > 0\), ensues straightforwardly from the f.o.c. of the following problem:

\[
\begin{align*}
\max_{S_{1,t}, S_{2,t}, S^t_{CB}} & \quad (1 - l)qX_{1,t}S_{1,t} + (1 - l)(1 - q)X_{2,t}S_{2,t} + lX^t_{CB}S^t_{CB} \\
\text{s.t.} & \quad E[Z_t] = (1 - l)qX_{1,t} + (1 - l)(1 - q)X_{2,t} + lX^t_{CB} = 0 \\
& \quad X^t_{CB} = \frac{1}{\pi} \left( \frac{f}{R} - \pi X^* + \frac{\lambda IR}{2\pi L + \lambda IR} \right) X^t_{CB} = \frac{f}{\pi} \left( \frac{R}{R} - S_{1,t} \right) - X_i \quad i = \{1, 2\}.
\end{align*}
\]

(A-3)

We assume that model’s parameters are such that \(S^t_{CB} > 0\). Plugging \(S^t_{CB}\) in \(X^t_{CB}\) of Eq. (13), and then the resulting expression into Eqs. (14) and (15) gives

\[
S_{i,t} = \bar{S}\left(\frac{\pi L}{2\pi L + \lambda IR}\right) + f\left(\frac{\pi L + \lambda IR}{2\pi L + \lambda IR}\right) - \pi X^*_i,
\]

(A-4)

for \(i = \{1, 2\}\). Eqs. (1) and (A-4) imply that \(X_{2,t} - X_{1,t} = (1/2)(X_1 - X_2)\). Hence, as in Saar (2000a), \(X_{1,t} \neq X_{2,t}\) (consistently with the MM’s initial conjecture) unless \(X_1 = X_2\), i.e., unless there is no trading (and risk-sharing). The CB’s initial conjecture \((S_t = (f/R) + \pi X^* + \pi LX^t_{CB}\) in Eq. (12)) is also correct in equilibrium. In fact, it follows from Eq. (A-3) that:
\(qS_{1t} + (1-q)S_{2t} = \left( \frac{f}{R} - \pi X^t \right) + \left[ \left( S - \frac{f}{R} \right) \left( \frac{\pi L}{2\pi L + \lambda IR} \right) + \frac{\pi}{2} \right] \), \hspace{1cm} (A-5)

while plugging Eqs. (17) and (A-4) into Eq. (13) gives

\[ X^t = \frac{1}{\pi L} \left[ \left( S - \frac{f}{R} \right) \left( \frac{\pi L}{2\pi L + \lambda IR} \right) + \frac{\pi}{2} \right] \]

(A-6)

It is then clear that \( S_t = qS_{1t} + (1-q)S_{2t} = (f/R) - \pi X^t + \pi L X^t \), as guessed by the CB. Finally, Eqs. (1) and (A-4) imply that \( X_{2t} > 0 \) and \( X_{1t} < 0 \) if \( X_2 < 2 \left[ \left( f/R \right) / \left( (f/R) / ((2\pi L + \lambda IR) - S) / (2\pi L + \lambda IR) \right) \right] < X_1 \), i.e., if \( X_1 \) is "sufficiently high" and \( X_2 \) is "sufficiently low."

Proof of Proposition 4. Construction of the equilibrium mimics the proof of Proposition 3. However, here we use \( E[Z_t] = 0 \) to express \( S_{1t} \) as a function of \( S_{2t} \) and \( X^t \). Then, plugging Eqs. (1) and (13) in \( E[\Pi_t] = 0 \) generates a quadratic equation with respect to \( S_{2t} \), whose solutions are

\[ S_{2t} = \frac{f}{R} - \pi X^t + \pi L X^t + \frac{\pi q}{R} (X_1 - X_2) \pm \frac{\pi q}{R} \Gamma \]

(A-7)

where \( \Gamma = \left[ (1 - q)^2 \left( X_2 - X_1 - (2\pi L)q \right)^2 - 4 \left( 1 - q \right) / \pi q \right] \left( 1/2 \right) > 0 \), \( A = S + (\lambda IR/2\pi L) \), and \( C = A / \pi q + X_1 - f/\pi R - L X^t \). We choose the solution for \( S_{2t} \) implied by the minus sign in Eq. (A-7) (i.e., Eq. (21)) for it alone reverts to the monopoly (\( \Gamma = 0 \)) ask that in Eq. (6) when \( X_1 > X_2 \). Eq. (20) then ensues. Because of the definition of \( \Gamma \), we impose that

\[ (1 - q)^2 \left( \left( X_2 - X_1 - (2\pi L)q \right)^2 - 4 \left( 1 - q \right) / \pi q \right) \left( 1/2 \right) > 0 \] \hspace{1cm} (A-8)

This condition is satisfied when \( X_1 - X_2 \) is not "too small." Finally, we verify that MMs’ initial conjectures are confirmed in equilibrium. It easily follows from plugging Eqs. (20) and (21) into Eq. (1) that \( X_{1t} \neq X_{2t} \) if \( X_1 - X_2 = (1/1 - q)) \), which is indeed the case when Eq. (A-8) holds and \( X_1 > X_2 \), as assumed in Section 2.2.1. We leave to the reader to verify that the CB’s conjecture about \( S^t \) in Eq. (12) is indeed correct in equilibrium. Now, we search for \( S^t \) such that \( S_{2t} - S_{1t} = 0 \). It is clear from Proposition 1 that this is always the case if \( I = 0 \) and/or \( S^t = 0 \). Hence, when \( I > 0 \), it is possible to find the (extremely high or extremely low) price such that \( S^t = 0 \). Because in this study we focus on the impact of CB interventions on quotes and spreads, \( S^t (\cdot) \) is instead the one price such that \( S_{2t} - S_{1t} = 0 \) but \( S^t \neq 0 \). To find this price, we first observe that Eq. (3) implies that \( q X_{1t} + (1-q)X_2 = -L X^t \). Hence, for \( E[\Pi_t] = 0 \) to hold at a zero spread and \( X^t = 0 \), it has to be true that \( S_1 = S_{2t} = S^t \). Lastly, plugging Eq. (13) in Eq. (12) and solving for \( S^t (\cdot) \) gives Eq. (24). It is easy to verify that, when \( S^t = S^t (\cdot) \), investors and CB’s conjectures are correct in equilibrium. If \( S^t = S^t (\cdot) \), then \( X^t \) are twice the amounts traded in monopoly, hence \( X_{2t} > 0 \) and \( X_{1t} < 0 \) under the same restriction reported in the proof of Proposition 3. If instead \( S^t \neq S^t (\cdot) \), it can be shown that \( X_{2t} > 0 \) and \( X_{1t} < 0 \) when the following restrictions hold:

\[ X_2 + \frac{q}{2} (X_1 - X_2) - \frac{q}{2(1-q)} \Gamma < \frac{1}{\pi} \left[ (f/R - S) - \frac{\lambda IR}{2\pi L} \frac{f/R - S^t}{(f/R - S^t)}} \right] \]

(A-9)

\[ X_1 - \frac{(1-q)}{2} (X_1 - X_2) + \frac{1}{2} \Gamma > \frac{1}{\pi} \left[ (f/R - S) - \frac{\lambda IR}{2\pi L} \frac{f/R - S^t}{(f/R - S^t)}} \right] \]

(A-10)

i.e., again when \( X_1 \) is "sufficiently high" and \( X_2 \) is "sufficiently low." For \( I = 0 \), Eqs. (A-9) and (A-10) reduce to \( X_1 > X_2 \).

Proof of Proposition 5. The first statement of Proposition 5 ensues from \( E[Z_t] = 0 \) implying that \( S^t = S + (\lambda IR/2\pi L) (f/R - S^t) \). If \( \lambda = 0 \) in \( (S, \lambda) \), then \( C S^t = S \) while \( M S^t = (1/2)S + (1/2) \) \( (f/R - \pi X) \). Clearly, \( M S^t = S \) just when \( (f/R) - \pi X^t = S \), i.e., when \( X^t = 0 \). Finally, if we substitute Eqs. (16) and (22) into \( EM_t \), it is easy to see that \( EM_t < 0 \) if, under competitive dealership,
\[(f/R) - C_{st}^B > (4\pi^2L^2/\lambda^2P^2R^2)((f/R) - S)((\pi + \lambda IR)/(2\pi L + \lambda IR)) > (\pi/2)X^*].\]

For reasonable parametrizations of the model, this occurs just for “very small” or “very large” values of \(C_{st}^B\) and/or for “very high” values of \(\lambda\). In those circumstances, the CB resists the monopolist MM’s attempts to maximize profits at its expenses. Additionally, it is possible to show that if \(\lambda\) is “small” or if \((f/R) - S_{st}^B\) is positive when the CB is chasing the trend (but \(C_{st}^B < M_{st}^CB\)) and negative when the CB is leaning against the wind (but \(C_{st}^B > M_{st}^CB\)), Proposition 5 applies to \(M_{st}(S; \lambda)\) versus \(\tilde{E}_{st}(S; \lambda)\) as well. In fact, their difference is given by \(EM_{st} + \lambda(W^r_t - M_{st}W^r_t)\). Moreover, under those conditions, \(C_{st}^B > M_{st}^CB\). It then follows that \(\lambda(W^r_t - M_{st}W^r_t) = \lambda((f - R^s_{st})X_t - (f - R^s_{st})M_{st}^B) > 0\), and so is \(M_{st}(S; \lambda) - C_{st}(S; \lambda)\).

References


