Transparency Masters for

An Introduction to Simulation Using GPSS/H

Chapter 16:
Statistical Experiments with Models of Competing Alternatives
Chapter 16:
Statistical Experiments with Models of Competing Alternatives

Overall Objective:
Estimate Which Alternative Has the Largest or Smallest Expected Value of an Output Variable

- Comparison of **Exactly Two** Alternatives Based on **Paired Independent** Observations of the Alternatives
  *(Uncorrelated Paired-t Comparison of Two Alternatives)*
- A Numeric Example: The Use of Uncorrelated Pairs to Compare FCFS vs. SPT Service Orders at a Tool-Crib
- Comparison of Two Alternatives Based on **Unpaired Independent** Observations of the Alternatives
  *(Comments on Three Other Statistical Methodologies)*

- Comparison of **Exactly Two** Alternatives Based on **Paired Dependent** Observations of the Alternatives
  *(Positively Correlated Paired-t Comparison of Two Alternatives)*
- A Numeric Example: The Use of Correlated Pairs to Compare the FCFS vs. SPT Tool-Crib Alternatives

- Selecting the **Probable** Best from **Two or More** Alternatives
  *(The Methodology of Dudewicz and Dalal)*
- Case Study 16A: Selecting the Probable Best from Four Proposed Production Systems
Descriptive Examples of Two Competing Alternatives

- Which of two service orders at a tool crib, FCFS or SPT, will minimize the average number of mechanics waiting for service at the tool crib?

- Which of two alternatives minimizes the average ship residence time at a particular harbor: widening the mouth of the harbor to permit two ships to move through the harbor's mouth simultaneously, or providing one more tugboat at the harbor?

- If the objective is to minimize the average level of work-in-process at a particular job shop, is it better to add one more Group 2 machine, or one more Group 4 machine?

- Which of these two alternatives will best balance crane and worker utilization in a particular system in which workers sometimes use cranes:
  - 2 cranes for 6 workers, or
  - 3 cranes for 8 workers?

- For a given reorder quantity, which of two alternative reorder points in an inventory control system will best balance the cost of stock-on-hand vs. the opportunity cost of being out of stock?
A Schematic of Independent (Uncorrelated) Paired-Difference Sampling to Make Inferences About the Difference in Expected Values in Two Output Distributions

- n iid Values from the Distribution of X:
  \[ X_1 \rightarrow Y_1 \]
  \[ X_2 \rightarrow Y_2 \]
  \[ \vdots \]
  \[ X_n \rightarrow Y_n \]

- Distribution of the Output Variable X
  Mean: \( \mu_X \); Variance: \( \sigma_X^2 \)

- Distribution of the Output Variable Y
  Mean: \( \mu_Y \); Variance: \( \sigma_Y^2 \)

- Paired Differences:
  \[ D_1 \]
  \[ D_2 \]
  \[ \vdots \]
  \[ D_n \]

- Distribution of the Mean Paired Difference \( \bar{D} \)
  Mean: \( \mu_{\bar{D}} = \mu_X - \mu_Y \); Variance: \( \sigma_{\bar{D}}^2 = (\sigma_X^2 + \sigma_Y^2)/n \)

- Mean: \( \bar{D} \); Variance: \( S_{\bar{D}}^2 \)
Requirements for Use of the Independent Paired-Difference Methodology

Small Sample Sizes

• The mean of the paired differences must come from a normal (or approximately normal) distribution
  
  (Otherwise, use of the t statistic isn't justified)

• One way to satisfy this requirement is for the two output distributions to be normal
  
  (If the output variables are based on an average (such as Average Queue Contents), then by the Central Limit Theorem the output distributions will tend to be normal)

• But the output distributions do not have to be normal
  
  (In fact, the mean of the paired differences might come from an approximately normal distribution even if the output distributions are heavily skewed. As Law and Kelton point out, "troublesome skewness (if present) in the output distributions should be largely ameliorated upon subtraction (assuming the two output distributions are skewed in the same direction)." 1990, p. 587)

• Equality of variances in the output distributions is not an issue

Large Sample Sizes

• By the Central Limit Theorem, the average of the paired differences will tend to be normally distributed for sample sizes ≥ 30, no matter what form the output distributions take

• Equality of variances in the output distributions is not an issue
### Avg. No. of Waiting Mechanics for the FCFS vs. SPT Service Orders
**Basis: Uncorrelated Pairs**
*(Table 16.1)*

<table>
<thead>
<tr>
<th>Replication Number</th>
<th>Average Waiting Line Lengths</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FCFS</td>
<td>SPT</td>
</tr>
<tr>
<td>1</td>
<td>3.826</td>
<td>1.326</td>
</tr>
<tr>
<td>2</td>
<td>1.529</td>
<td>3.668</td>
</tr>
<tr>
<td>3</td>
<td>1.805</td>
<td>0.737</td>
</tr>
<tr>
<td>4</td>
<td>4.140</td>
<td>1.619</td>
</tr>
<tr>
<td>5</td>
<td>2.116</td>
<td>3.572</td>
</tr>
<tr>
<td>6</td>
<td>5.010</td>
<td>1.290</td>
</tr>
<tr>
<td>7</td>
<td>5.411</td>
<td>2.723</td>
</tr>
<tr>
<td>8</td>
<td>1.974</td>
<td>2.229</td>
</tr>
<tr>
<td>9</td>
<td>6.585</td>
<td>1.395</td>
</tr>
<tr>
<td>10</td>
<td>2.550</td>
<td>1.346</td>
</tr>
</tbody>
</table>

Sample Mean: 1.504

Sample Standard Deviation: 2.293

80% Confidence Interval: [0.50, 2.50]

90% Confidence Interval: [0.18, 2.83]

95% Confidence Interval: [-0.14, 3.14]
Model Used to Produce the SPT Tool Crib Observations in Table 16.1

SIMULATE A Tool Crib System (SPT Service Order)
  Base Time Unit: 1 Second

Compiler Directives (INTEGER)

INTEGER &I &I is a DO-loop index

Control Statements (RMULT)

RMULT 150000 set RN1 initial position = 150000

Model Segment 1 (Type 1 Mechanics)

GENERATE 420,360,,,5 Type 1 mechanics arrive
QUEUE TOOLWAIT start TOOLWAIT Queue membership
SEIZE CLERK request/capture the clerk
DEPART TOOLWAIT end TOOLWAIT Queue membership
ADVANCE 300,90 service time
RELEASE CLERK free the clerk
TERMINATE 0 leave the tool crib area

Model Segment 2 (Type 2 Mechanics)

GENERATE 360,240,,,10 Type 2 mechanics arrive
QUEUE TOOLWAIT start TOOLWAIT Queue membership
SEIZE CLERK request/capture the clerk
DEPART TOOLWAIT end TOOLWAIT Queue membership
ADVANCE 100,30 service time
RELEASE CLERK free the clerk
TERMINATE 0 leave the tool crib area
Continuation of the Model Used to Produce the SPT Tool Crib Observations in Table 16.1

* ************************** Model Segment 3 (Run-Control Xact) ************************** *
* GENERATE 28800 control Xact comes after 8 hours *
* TERMINATE 1 reduce the TC value by 1, ending Xact movement *
* *
* Run-Control Statements ************************************************************ *
* DO &I=1,10,1 control for 10 replications *
* START 1 start the &I-th replication *
* PUTPIC LINES=3,FILE=SYSPRINT,(&I) *
0-

The Postsimulation Report above is for replication *.  

* CLEAR clear for the next replication *
* ENDDO proceed to the next value of &I *
* END end of Model-File execution
Hypothesis Testing
(Context: Expected Number of Mechanics Waiting at the Tool Crib)

- Suppose we hypothesize that for the FCFS service order, the expected number of mechanics waiting at the tool crib is **equal to** that for the SPT service order, i.e.

\[ \mu_{FCFS} = \mu_{SPT} \]

- Call this the **Null Hypothesis**, \( H_0 \)

- The **alternative** is that \( \mu_{FCFS} \neq \mu_{SPT} \)

- Call this the **Alternative Hypothesis**, \( H_a \)

- Either \( H_0 \) is true, or \( H_a \) is true

- In general, we can distinguish like this in hypothesis testing between

  **Two Alternative "States of Nature"**
  and
  **Two Alternative "Conclusions"**

we might reach about the state of nature, as shown in the 2 X 2 table on the next frame
The Two Alternative States of Nature

vs.

The Two Alternative Conclusions

<table>
<thead>
<tr>
<th>Alternative States of Nature</th>
<th>Ho is True</th>
<th>Ha is True</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ho is True</td>
<td>Correct Decision</td>
<td>Type II Error (Probability $\beta$)</td>
</tr>
<tr>
<td>Ha is True</td>
<td>Type I Error (Probability $\alpha$)</td>
<td>Correct Decision</td>
</tr>
</tbody>
</table>
An Interpretation of the Table 16.1 Confidence Intervals in the Context of Hypothesis Testing

• Consider the hypothesis structure:

  \( H_0: \mu_X = \mu_Y \) (e.g., \( \mu_X - \mu_Y = 0 \))

  \( H_a: \mu_X \neq \mu_Y \) (e.g., \( \mu_X - \mu_Y \neq 0 \))

If a P\% confidence interval for \( \mu_X - \mu_Y \) excludes 0.0, then this is equivalent to rejecting \( H_0 \)

(or failing to accept \( H_0 \))

with the corresponding probability of a Type I error set at:

\[ \alpha = \frac{(100-P)}{100} \]

• Example from Table 16.1 (where the output variable is the average number of mechanics waiting to go into service at a tool crib):

  \( H_0: \mu_{FCFS} = \mu_{SPT} \)

  \( H_a: \mu_{FCFS} \neq \mu_{SPT} \)

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>Confidence Interval</th>
<th>( \alpha )</th>
<th>Reject (Fail to Accept)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>[0.50, 2.50]</td>
<td>0.20</td>
<td>Yes</td>
</tr>
<tr>
<td>90%</td>
<td>[0.18, 2.83]</td>
<td>0.10</td>
<td>Yes</td>
</tr>
<tr>
<td>95%</td>
<td>[-0.14, 3.14]</td>
<td>0.05</td>
<td>No</td>
</tr>
</tbody>
</table>
Another Interpretation of the Table 16.1 Confidence Intervals in the Context of Hypothesis Testing

• If a confidence interval for a difference in expected values excludes 0.0, we reject
  
  (with a corresponding probability of a Type I error)

  the hypothesis that the expected values are equal

• But what about the direction of the inequality???

• Consider the hypothesis structure:
  
  $H_0: \mu_X \leq \mu_Y$
  
  $H_a: \mu_X > \mu_Y$

  If a P% confidence interval for $\mu_X - \mu_Y$

    sits entirely above 0.0,

  then this is equivalent to rejecting $H_0$ with

  the probability of a Type I error set at:

  $\alpha = [(100-P)/100]/2$
Interpretation of the Table 16.1 Confidence Intervals with Respect to $H_0: \mu_{\text{FCFS}} \leq \mu_{\text{SPT}}$

- We have seen that for the hypothesis structure:
  
  $H_0: \mu_X \leq \mu_Y$
  
  $H_a: \mu_X > \mu_Y$

  if a P% confidence interval for $\mu_X - \mu_Y$ sits entirely above 0.0,

  then this is equivalent to rejecting $H_0$ with the probability of a Type I error set at:

  $$\alpha = \frac{(100-P)}{100}/2$$

- Applying this idea in the context of Table 16.1:

  $H_0: \mu_{\text{FCFS}} \leq \mu_{\text{SPT}}$
  
  $H_a: \mu_{\text{FCFS}} > \mu_{\text{SPT}}$

<table>
<thead>
<tr>
<th>Confidence Level</th>
<th>Confidence Interval</th>
<th>$\alpha$</th>
<th>Reject $H_0$?</th>
</tr>
</thead>
<tbody>
<tr>
<td>80%</td>
<td>[0.50, 2.50]</td>
<td>0.10</td>
<td>Yes</td>
</tr>
<tr>
<td>90%</td>
<td>[0.18, 2.83]</td>
<td>0.05</td>
<td>Yes</td>
</tr>
<tr>
<td>95%</td>
<td>[-0.14, 3.14]</td>
<td>0.025</td>
<td>No</td>
</tr>
</tbody>
</table>

- We conclude there is strong evidence to indicate that SPT is the superior service order if the objective is to minimize the average number of waiting mechanics
Some Other Methodologies for Comparing Two Alternatives Based on Independent Observations

- Key Questions About the Two Output Distributions:
  Must the Output Distributions be Normal?
  Must the Variances be Equal?
  Must the Samples be of Equal Size?

- Method 1: Classical Two-Sample t

  Small sample sizes are involved
  \((n_1 < 30 \text{ and/or } n_2 < 30)\)

  The output distributions must be normal
  The variances must be equal
  The samples need not be of equal size
  (But if \(n_x = n_y\), equality of variances is relatively unimportant)

- Method 2: The Welch Confidence Interval

  A Variation of the Classical Two-Sample t
  (Small Sample Sizes Are Involved)

  The output distributions must be normal
  The variances need not be equal
  The samples need not be of equal size
  (But the solution is only approximate, not exact)

- Method 3: Classical Two-Sample z

  The Two-Sample t, but for large samples
  \((n_1 \geq 30 \text{ and } n_2 \geq 30)\)

  The output distributions need not be normal
  The variances need not be equal
  The samples need not be of equal size
An Example of Positively Correlated Paired Differences
(Source: McClave and Benson, 1988, Page 455)

"A paired difference experiment is conducted to compare the starting salaries of male and female college graduates who find jobs. Pairs are formed by choosing a male and a female with the same major and similar grade-point averages. Suppose a random sample of ten pairs is formed in this manner and the starting annual salary of each person is recorded. The results are shown in the following table. Test to see whether there is evidence that the mean starting salary for males exceeds that for females."

<table>
<thead>
<tr>
<th>Pair</th>
<th>Male</th>
<th>Female</th>
<th>Difference (Male - Female)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$8,300</td>
<td>$7,800</td>
<td>$500</td>
</tr>
<tr>
<td>2</td>
<td>10,5000</td>
<td>10,600</td>
<td>-100</td>
</tr>
<tr>
<td>3</td>
<td>9,400</td>
<td>8,800</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>7,500</td>
<td>7,500</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>12,500</td>
<td>11,600</td>
<td>900</td>
</tr>
<tr>
<td>6</td>
<td>6,800</td>
<td>7,000</td>
<td>-200</td>
</tr>
<tr>
<td>7</td>
<td>8,500</td>
<td>8,200</td>
<td>300</td>
</tr>
<tr>
<td>8</td>
<td>10,200</td>
<td>9,100</td>
<td>1,100</td>
</tr>
<tr>
<td>9</td>
<td>7,400</td>
<td>7,200</td>
<td>200</td>
</tr>
<tr>
<td>10</td>
<td>8,200</td>
<td>7,500</td>
<td>700</td>
</tr>
</tbody>
</table>

A 95% confidence interval for "male - female" is

[$89, 711$]

so with a 0.05 probability of a Type I error we reject the null hypothesis that there is no difference in expected starting salaries.
The Nature of Positive Correlation

- **Positive Correlation** Between Two Variables \( X \) and \( Y \)

If

\[ X_i > X_{\text{average}} \quad \text{and} \quad Y_i > Y_{\text{average}} \]

tend to occur together, and

\[ X_i < X_{\text{average}} \quad \text{and} \quad Y_i < Y_{\text{average}} \]

tend to occur together, then a plot of \((X,Y)\) pair will look something like this (where the case of positive linear correlation is shown):

![Positive Linear Correlation](image)
A Schematic of Positively-Correlated Paired-Difference Sampling to Make Inferences About the Difference in Expected Values in Two Output Distributions

- **n iid Values from the Distribution of X**
  - $X_1 \rightarrow Y_1$
  - $X_2 \rightarrow Y_2$
  - ...
  - $X_n \rightarrow Y_n$

- **n Corresponding Positively Correlated Values from the Distribution of Y**

- **Distribution of the Output Variable X**
  - Mean: $\mu_X$; Variance: $\sigma_X^2$

- **Distribution of the Output Variable Y**
  - Mean: $\mu_Y$; Variance: $\sigma_Y^2$

- **Paired Differences**
  - $D_1$
  - $D_2$
  - ...
  - $D_n$

- **Distribution of the Mean Paired Difference $\bar{D}$**
  - Mean: $\mu_\bar{D} = \mu_X - \mu_Y$; Variance: $\frac{\sigma_\bar{D}^2}{D} = \frac{(\sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y)}{n}$

- **Distribution of the Paired Differences**
  - Mean: $\mu_D = \mu_X - \mu_Y$; Variance: $S_D^2$
The Mathematics of Variance Reduction
(Context: Using Positively-Correlated Paired Differences to Estimate the Difference in Expected Values of Two Random Variables)

Let $X$ and $Y$ be random variables with variances $\text{Var}(X)$ and $\text{Var}(Y)$, and let $a$ and $b$ be constants; then:

$$\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab \rho \sigma_X \sigma_Y$$

where $\rho$ is the correlation coefficient.

To formulate the $\text{Var}(X - Y)$, let $a = 1$ and $b = -1$:

$$\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y) - 2 \rho \sigma_X \sigma_Y$$

If samples from the two populations are independent, then $\rho = 0$, and

$$\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$$

If samples from the two populations are positively correlated, then $\rho > 0$, and

$$\text{Var}(X - Y) < \text{Var}(X) + \text{Var}(Y)$$
Producing Random Days for the FCFS Tool Crib, then Reproducing Them *Identically* for the SPT Tool Crib

- On any given day at the tool crib, the average number of mechanics in the waiting line depends on *five* factors:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Service Order</td>
<td></td>
</tr>
<tr>
<td>2. Exact arrival time of the 1st, 2nd, 3rd, etc. Type 1 mechanic</td>
<td></td>
</tr>
<tr>
<td>3. Exact service time of the 1st, 2nd, 3rd, etc. Type 1 mechanic</td>
<td></td>
</tr>
<tr>
<td>4. Exact arrival time of the 1st, 2nd, 3rd, etc. Type 2 mechanic</td>
<td></td>
</tr>
<tr>
<td>5. Exact service time of the 1st, 2nd, 3rd, etc. Type 2 mechanic</td>
<td></td>
</tr>
</tbody>
</table>

- In forming positively correlated pairs for FCFS vs. SPT at the tool crib, the objective is to *isolate the influence of service order* on the expected number of mechanics in the waiting line.

- This sets up the need to determine characteristics 2, 3, 4 and 5 (above) at random for an FCFS day, then reproduce these characteristics *identically* for the paired SPT day.

- This is accomplished by using *Common Random Numbers (CRN)*
Block Diagram for Case Study 9A
Two-Block Run-Control Segment Not Shown
(SPT Service Order ... Figure 9A.1)

Model Segment 1
(Type 1 Mechanics)

GENERATE
420, 360, 360, 5

QUEUE

TOOLWAIT

SEIZE

▲ CLERK

DEPART

TOOLWAIT

ADVANCE
300, 90

RELEASE

CLERK ▼

TERMINATE
0

Model Segment 2
(Type 2 Mechanics)

GENERATE
360, 240, 360, 10

QUEUE

TOOLWAIT

SEIZE

▲ CLERK

DEPART

TOOLWAIT

ADVANCE
100, 30

RELEASE

CLERK ▼

TERMINATE
0
### Facility and Queue Reports, SPT Service Order

<table>
<thead>
<tr>
<th>FACILITY</th>
<th>TOTAL</th>
<th>AVAIL</th>
<th>UNAVL</th>
<th>ENTRIES</th>
<th>AVERAGE TIME/XACT</th>
<th>CURRENT STATUS</th>
<th>PERCENT</th>
<th>SEIZING XACT</th>
<th>PREEMPTING XACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLERK</td>
<td>.903</td>
<td>142</td>
<td></td>
<td>183.097</td>
<td>100.0</td>
<td>145</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>QUEUE</th>
<th>MAXIMUM CONTENTS</th>
<th>AVERAGE CONTENTS</th>
<th>TOTAL ENTRIES</th>
<th>ZERO ENTRIES</th>
<th>PERCENT ZEROS</th>
<th>AVERAGE TIME/UNIT</th>
<th>$AVERAGE TIME/UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOOLWAIT</td>
<td>6</td>
<td>1.501</td>
<td>146</td>
<td>26</td>
<td>17.8</td>
<td>296.093</td>
<td>360.246</td>
</tr>
</tbody>
</table>

### Facility and Queue Reports, FCFS Service Order

<table>
<thead>
<tr>
<th>FACILITY</th>
<th>TOTAL</th>
<th>AVAIL</th>
<th>UNAVL</th>
<th>ENTRIES</th>
<th>AVERAGE TIME/XACT</th>
<th>CURRENT STATUS</th>
<th>PERCENT</th>
<th>SEIZING XACT</th>
<th>PREEMPTING XACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLERK</td>
<td>.894</td>
<td>141</td>
<td></td>
<td>182.690</td>
<td>100.0</td>
<td>141</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>QUEUE</th>
<th>MAXIMUM CONTENTS</th>
<th>AVERAGE CONTENTS</th>
<th>TOTAL ENTRIES</th>
<th>ZERO ENTRIES</th>
<th>PERCENT ZEROS</th>
<th>AVERAGE TIME/UNIT</th>
<th>$AVERAGE TIME/UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOOLWAIT</td>
<td>6</td>
<td>1.705</td>
<td>142</td>
<td>24</td>
<td>16.9</td>
<td>345.867</td>
<td>416.213</td>
</tr>
</tbody>
</table>
Common Random Numbers (CRN)

- Consider the sequence of values sampled from a given input distribution

  (such as the interarrival times of Type 1 mechanics;
  or the service times of Type 1 mechanics;
  or the interarrival times of Type 2 mechanics;
  or the service times of Type 2 mechanics)

  in a replication for Alternative 1

    (such as FCFS)

- If the **same sequence** of values is sampled from the same input distribution in a replication for Alternative 2

    (such as SPT)

  then **common random numbers** are said to be in use in the two replications

- It is this use of common random numbers which leads to positive correlation in pairs of replications in simulation
Operationalizing the Use of Common Random Numbers

- Two steps are involved in bringing about the use of common random numbers:

1. Dedicated 0-1 Uniform RNGs Are Used

   Each input distribution has its own 0-1 uniform RNG dedicated to it for exclusive use in sampling from that distribution

   (this satisfies the requirement that the values sampled from input distributions be independent of service order)

2. Unique Initial RNG Positions Are Computed/Recomputed for Each of the Two Replications in a Pair

   Unique Initial Positions of each RNG are calculated at the beginning of each FCFS replication and then are recomputed identically at the beginning of each paired SPT replication

   (this satisfies the requirement that corresponding RNG Initial Positions be Identical at the beginning of each of the two replications making up a pair and be unique to that replication pair)
SIMULATE Figure 16.1 of Chapter 16
* Common Random Numbers
* with FCFS vs. SPT Alternatives at the Tool Crib
* Base Time Unit: 1 Second

******************************************************************************
* Compiler Directives (INTEGER; OPERCOL; UNLIST)  *
******************************************************************************
*
  INTEGER  &I            &I is a DO-loop index
  INTEGER  &TYPE2PR      Type 2 mechanic Priority Level
  OPERCOL  30            scan to column 30 for A Operands
  UNLIST   CSECHO        don't echo Control Statements
*

******************************************************************************
* Control Statements (FUNCTIONs)  *
******************************************************************************
*
  TYPE1IAT FUNCTION    RN1,C2    Type 1 interarrival time
0,60/1,780
*
  TYPE1ST FUNCTION     RN2,C2    Type 1 service time
0,210/1,390
*
  TYPE2IAT FUNCTION    RN3,C2    Type 2 interarrival time
0,120/1,600
*
  TYPE2ST FUNCTION     RN4,C2    Type 2 service time
0,70/1,130
Continuation of the Figure 16.1 Model

* 
*******************************************************************************
* Model Segment 1 (Type 1 Mechanics)  *
*******************************************************************************
* GENERATE  FN(TYPE1IAT),,,5  Type 1 mechanics arrive  
            (Priority Level = 5)
QUEUE TOOLWAIT  start TOOLWAIT Queue membership
SEIZE CLERK  request/capture the clerk
DEPART TOOLWAIT  end TOOLWAIT Queue membership
ADVANCE FN(TYPE1ST)  service time
RELEASE CLERK  free the clerk
TERMINATE 0  leave the tool crib area

*******************************************************************************
* Model Segment 2 (Type 2 Mechanics)  *
*******************************************************************************
* GENERATE  FN(TYPE2IAT),,,_&TYPE2PR  Type 2 mechanics arrive  
            (Priority Level = &TYPE2PR)
QUEUE TOOLWAIT  start TOOLWAIT Queue membership
SEIZE CLERK  request/capture the clerk
DEPART TOOLWAIT  end TOOLWAIT Queue membership
ADVANCE FN(TYPE2ST)  service time
RELEASE CLERK  free the clerk
TERMINATE 0  leave the tool crib area

*******************************************************************************
* Model Segment 3 (Run-Control Xact)  *
*******************************************************************************
* GENERATE  28800  control Xact comes after 8 hours
TERMINATE  1  reduce the TC value by 1,  
            ending Xact movement
Conclusion of the Figure 16.1 Model

* Run-Control Statements *
********************************************************************************
* DO &TYPE2PR=5,10,5 FCFS case first, then SPT case *
* DO &I=1,10,1 10 replications for current case *
* RMULT 99000+1000*&I, _ &I-th RN1 for current case 
  199000+1000*&I, _ RN2 
  299000+1000*&I, _ RN3 
  399000+1000*&I RN4
* START 1 start &I-th current-case replication
* IF &TYPE2PR=5 true => provide FCFS labeling
* PUTPIC LINES=3,FILE=SYSPRINT,(&I)
The report above is for replication * for the FCFS service order
* ELSE else, provide SPT labeling
* PUTPIC LINES=3,FILE=SYSPRINT,(&I)
The report above is for replication * for the SPT service order
* ENDF end of the if
* CLEAR clear for the next replication
* ENDDO next value of &I
* ENDDO next case
* END end of model-file execution
### Avg. No. of Waiting Mechanics for the FCFS vs. SPT Service Orders

**Basis:** Correlated Pairs

*(Table 16.2)*

<table>
<thead>
<tr>
<th>Replication Number</th>
<th>Average Waiting Line Lengths</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FCFS</td>
<td>SPT</td>
</tr>
<tr>
<td>1</td>
<td>1.219</td>
<td>0.973</td>
</tr>
<tr>
<td>2</td>
<td>3.065</td>
<td>2.125</td>
</tr>
<tr>
<td>3</td>
<td>1.057</td>
<td>0.856</td>
</tr>
<tr>
<td>4</td>
<td>3.124</td>
<td>2.244</td>
</tr>
<tr>
<td>5</td>
<td>5.084</td>
<td>3.612</td>
</tr>
<tr>
<td>6</td>
<td>8.654</td>
<td>5.878</td>
</tr>
<tr>
<td>7</td>
<td>2.638</td>
<td>1.888</td>
</tr>
<tr>
<td>8</td>
<td>0.745</td>
<td>0.635</td>
</tr>
<tr>
<td>9</td>
<td>1.592</td>
<td>1.216</td>
</tr>
<tr>
<td>10</td>
<td>2.405</td>
<td>1.710</td>
</tr>
</tbody>
</table>

**Sample Mean:** 0.845

**Sample Standard Deviation:** 0.796

80% Confidence Interval: [0.498, 1.192]

90% Confidence Interval: [0.384, 1.306]

95% Confidence Interval: [0.276, 1.414]

98% Confidence Interval: [0.135, 1.555]

99% Confidence Interval: [0.024, 1.662]
### Avg. No. of Waiting Mechanics for the FCFS vs. SPT Service Orders
*(Basis: Uncorrelated Pairs)*
*(Table 16.1)*

<table>
<thead>
<tr>
<th>Replication Number</th>
<th>Average Waiting Line Lengths</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FCFS</td>
<td>SPT</td>
</tr>
<tr>
<td>1</td>
<td>3.826</td>
<td>1.326</td>
</tr>
<tr>
<td>2</td>
<td>1.529</td>
<td>3.668</td>
</tr>
<tr>
<td>3</td>
<td>1.805</td>
<td>0.737</td>
</tr>
<tr>
<td>4</td>
<td>4.140</td>
<td>1.619</td>
</tr>
<tr>
<td>5</td>
<td>2.116</td>
<td>3.572</td>
</tr>
<tr>
<td>6</td>
<td>5.010</td>
<td>1.290</td>
</tr>
<tr>
<td>7</td>
<td>5.411</td>
<td>2.723</td>
</tr>
<tr>
<td>8</td>
<td>1.974</td>
<td>2.229</td>
</tr>
<tr>
<td>9</td>
<td>6.585</td>
<td>1.395</td>
</tr>
<tr>
<td>10</td>
<td>2.550</td>
<td>1.346</td>
</tr>
</tbody>
</table>

Sample Mean: 1.504

Sample Standard Deviation: 2.293

80% Confidence Interval: [0.50, 2.50]

90% Confidence Interval: [0.18, 2.83]

95% Confidence Interval: [-0.14, 3.14]
Descriptive Examples of 3 or More Competing Alternatives

Alternatives Involving Resource Types and Levels

- Should the material-handling devices in a manufacturing system take the form of automated guided vehicles, or of human-operated vehicles, or of conveyors, or of some combination of these?

- Should capacity at a harbor be increased by adding tugboats, or by installing faster unloading equipment at one or more berths (if so, at how many berths), or by increasing the number of berths (if so, how many and of which types), or by some combination of the above?

- How many repairpeople and how many backup machines should we have to respond to machine failures?

Alternatives in Control Settings and Procedures

- In an inventory control system in which reorder point and reorder quantity must be determined for a type of item, at what value should the reorder point be set, and should the reorder quantity be constant (if so, at what value should it be set?) or should it depend on the level of stock-on-hand at the time reordering takes place (if so, how should the reorder quantity be computed)?
Flow Schematic for Case Study 16A
(Figure 16A.1)

Spare machine ready for use

Ten machines in productive use

No machines waiting to be repaired

Failed machine being repaired
The Situation in Case Study 16A

- The objective is to operate 10 machines 8 hours per day, 5 days per week (thereby achieving 400 machine operating hours/week)

- 10 machine operators are on the payroll; 11 machines of the type in question are owned 1 machine repairperson is on the payroll

- The machines are subject to random failure (machine running time until failure is 200±100 hours; time to repair a failed machine is 24±8 hours; only about 320 machining hours per week are being achieved now)

- Management's available options:
  - Lease 0 or 1 more machines, and/or
  - Hire 0 or 1 more repairperson
    (see the next frame)

- Cost structure:
  - Leasing cost: $100 per 8-hour workday
  - Repairperson cost: $120 per 8-hour workday
  - Opportunity cost: $18 per machining hour

- The Problem:
  Which option should management choose if the objective is to minimize the estimated average daily cost of operation?
# A Summary of the Options Open to Management in Case Study 16A

<table>
<thead>
<tr>
<th>Number of Repairpeople</th>
<th>Number of Leased Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

16-30
### Table of Definitions
for Case Study 16A

(Table 16A.1)

<table>
<thead>
<tr>
<th>GPSS/H Entity</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transactions</strong></td>
<td></td>
</tr>
<tr>
<td>Model Segment 1</td>
<td>Machines (the 10 owned machines in productive use initially)</td>
</tr>
<tr>
<td>Model Segment 2</td>
<td>Machines (the leased machine, if any, initially waiting to be needed;)</td>
</tr>
<tr>
<td>Model Segment 3</td>
<td>Machines (the 1 owned machine initially waiting to be needed; this machine is</td>
</tr>
<tr>
<td></td>
<td>eventually joined by the owned machines, and the leased machine, if there is</td>
</tr>
<tr>
<td></td>
<td>one; all the machines then loop repeatedly through this segment)</td>
</tr>
<tr>
<td>Model Segment 4</td>
<td>A run-control Xact</td>
</tr>
<tr>
<td><strong>Ampervariables</strong></td>
<td></td>
</tr>
<tr>
<td>FIXERS</td>
<td>the number of repairpeople (1 or 2)</td>
</tr>
<tr>
<td>I</td>
<td>the replication counter</td>
</tr>
<tr>
<td>LEASED</td>
<td>the number of leased machines (0 or 1)</td>
</tr>
<tr>
<td><strong>Storages</strong></td>
<td></td>
</tr>
<tr>
<td>FIXSHOP</td>
<td>The Storage which models the repairperson or people</td>
</tr>
<tr>
<td>OPRATORS</td>
<td>The Storage which models the 10 machine operators</td>
</tr>
</tbody>
</table>
Block Diagram for Case Study 16A
(Figure 16A.2)

Model Segment 1
(Leased Machine, if Any)

1. GENERATE
   - , 1, LEASED
   - provide LEASED machines
     (introduced at time 1)

2. TRANSFER
   - (REPEAT)
   - transfer to mainline logic

Model Segment 2
(Machines in Productive Use Initially)

1. GENERATE
   - 0, 10
   - provide 10 of the owned machines

2. ENTER
   - (OPERATORS)
   - capture an operator
     (no delay)

3. ADVANCE
   - 150, 140
   - remaining lifetime until failure

4. TRANSFER
   - (BROKEN)
   - transfer to mainline logic

Model Segment 3
(Mainline Logic)

1. GENERATE
   - 1, 1
   - provide the other owned machine
     (introduced at time 1)

2. ENTER
   - (OPERATORS)
   - request/capture an operator
   - running time until failure

3. ADVANCE
   - 200, 100
   - broken; let the operator go

4. LEAVE
   - (OPERATORS)
   - request/capture a repairperson

5. FIXSHOP
   - repair time

6. LEAVE
   - repaired; let the repairperson go

7. TRANSFER
   - (REPEAT)
   - play it again, Sam
Model File for Case Study 16A
(Figure 16A.3)

SIMULATE
* Case Study 16A
* Production System
* Base Time Unit: 1 Hour
***************************************************************
* Compiler Directives (INTEGER; OPERCOL; UNLIST) *
***************************************************************
*
INTEGER &FIXERS number of repair people
INTEGER &I &I is a DO-loop index
INTEGER &LEASED number of leased machines
OPERCOL 30 scan up to column 30 for the
* start of the Operands field
*
UNLIST CSECHO no control-statement echoes
*
***************************************************************
* Control Statements (STORAGE) *
***************************************************************
*
OPRATORS STORAGE 10 10 people to operate machines
*
***************************************************************
* Model Segment 1 (Leased Machines, if Any) *
***************************************************************
*
GENERATE ,1,&LEASED provide &LEASED machines
* (introduced at time 1)
TRANSFER ,REPEAT transfer to mainline logic
*
***************************************************************
* Model Segment 2 (Machines in Productive Use Initially) *
***************************************************************
*
GENERATE 0,,10 provide 10 of the owned machines
ENTER OPRATORS capture an operator (no delay)
ADVANCE 150,140 remaining lifetime until failure
TRANSFER ,BROKEN transfer to mainline logic

16-33
Continuation of the Fig. 16A.3 Model

* Model Segment 3 (Mainline Logic) *

* GENERATE ,,1,1 provide the other owned machine (introduced at time 1) *
* REPEAT ENTER OPRATORS request/capture an operator ADVANCE 200,100 running time until failure BROKEN LEAVE OPRATORS broken; let the operator go ENTER FIXSHOP request/capture a repairperson ADVANCE 24,8 repair time LEAVE FIXSHOP repaired; let the repairperson go TRANSFER ,,REPEAT play it again, Sam *

* Model Segment 4 (Run-Control Transaction) *

* GENERATE 1000 1000 hours = 25 working weeks TERMINATE 1 shut off the simulation
Conclusion of the Figure 16A.3 Model

*  
* Run-Control Statements  
*  
* DO &FIXERS=1,2,1 first 1, then 2 repairpeople  
* FIXSHOP STORAGE &FIXERS put FIXSHOP Capacity into effect  
* DO &LEASED=0,1,1 first 0, then 1 leased machine  
* DO &I=1,15,1 15 replications  
* START 1 initiate the &Ith replication  
* PUTFIC LINES=6,FILE=SYSPRINT,(&I,&FIXERS,&LEASED)  

Shown above is Replication Report * for this configuration:

Number of repairpersons:  
Number of leased machines:  

*  
* CLEAR clear for next replication  
* ENDDO next replication  
* ENDDO next number of leased machines  
* ENDDO next number of repairpeople  
* END end of Model-File execution
The Top-Down Order of Statements in a GPSS/H Model File
(Figure 14.2)

(1) A SIMULATE Control Statement

(2) Additional Control Statements (if any)

(3) Block Statements

(4) A START Control Statement

(5) Additional Control Statements (if any)

(6) An END Control Statement
The Steps Followed to Execute a Model File in Batch Mode
(Figure 2.19)

1. Model Compilation and Compiler Report
   - Components of Compiler Report:
     - Source Echo (Enhanced Copy of Model File);
     - Compile-Time Warning and Error Messages;
     - Dictionary;
     - Cross-Reference Listing;
     - Summary of Storage Requirements
   - Compile-Time Error(s), or no "SIMULATE"

2. Control-Statement Execution
   - (Execution-Time Warning Messages might be issued)
   - "START"

3. Transaction Movement
   - (Block-Statement Execution)
   - (Execution-Time Warning Messages might be issued)
   - TC ≤ 0

4. Postsimulation Report
   - Components of Postsimulation Report:
     - Clock time; Block Counts;
     - Facility Report (see Chapter 6);
     - Queue Report (see Chapter 9);
     - Storage Report (see Chapter 11);
     - Random-Numbers Report (see Chapter 14); etc.

5. Computer-Usage Report
   - "END"
   - Stop

6. Error Report
   - Stop
Means and Standard Deviations of the Average Number of Machines in Use in the Case Study 16A First-Stage Sampling (Table 16A.2)

<table>
<thead>
<tr>
<th>Number of Leased Machines</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.9584</td>
<td>8.3601</td>
</tr>
<tr>
<td>0.27484</td>
<td>0.32232</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8.8687</td>
<td>9.4850</td>
</tr>
<tr>
<td>0.054225</td>
<td>0.1243</td>
<td></td>
</tr>
</tbody>
</table>

Legend: Mean, Std. Dev.
Means and Standard Deviations of the Average Daily Costs in the Case Study 16A First-Stage Sampling (Table 16.3)

<table>
<thead>
<tr>
<th>Number of Repairpeople</th>
<th>Number of Leased Machines</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>$413.99</td>
<td>$39.58</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$456.15</td>
<td>$46.41</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>$402.91</td>
<td>$7.81</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>$414.16</td>
<td>$17.90</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Legend:
- Mean
- Std. Dev.
Selecting the Probable Best from k Competing Alternatives, k \geq 2

The Methodology of Dudewicz and Dalal - (D/D)

- Case Study 16A provides an example of four competing alternatives
- The D/D methodology involves two-stage sampling
- These are the steps involved:

1. An identical number of replications (at least 15) is carried out for each alternative
   (This is the first stage of sampling)

2. Various criteria are then applied to compute the number of additional independent replications needed for each alternative
   (This number usually varies from alternative to alternative)

3. The additional replications are then carried out
   (This is the second stage of sampling)

4. For each alternative, the weighted means of the stage one and stage two samples are computed
   (The weights are not simply proportional to the first- and second- stage sample sizes, but are computed by a procedure to be described shortly)

5. The alternative with the smallest or largest weighted sample mean
   (depending on whether the objective is to minimize or maximize the expected value of the output variable)
   is then the winner
The Three Criteria Used to Determine How Many Second-Stage Replications Are Needed in the D/D Methodology

1. The First-Stage Sample Variance

The larger the first-stage sample variance for a given alternative, the larger the size of the second-stage sample for that alternative, other things being equal.

The size of the second-stage sample for an alternative is directly proportional to the first-stage variance for that alternative.

2. The Probability of Making the Correct Selection

The larger the user-specified probability of making the correct selection, the greater the sizes of the second-stage samples.

The second-stage sample sizes are a complex function of the probability of making the correct selection, per a table of \( h_1 \) values soon to be shown.

3. An Indifference Amount

The modeler specifies the amount by which a difference in the expected value of the output variable is unimportant.

For example, if cost is to be minimized and if the minimum cost in the first-stage sampling is about $400/day to $500/day, then an indifference amount might be set at, say, $20/day or $30/day.

The smaller the indifference amount, the larger the sizes of the second-stage samples needed to discriminate among the alternatives, other things being equal.

The sizes of the second-stage samples vary inversely with the square of the indifference amount.
Computation of the Total Sample Size N

\[ N = \max\{n_0 + 1, \lceil (h_1 s / d) 2 \rceil \} \]

N: total sample size for the alternative

n₀: first-stage sample size

N-n₀: second-stage sample size for the alternative

s: the alternative's first-stage sample standard deviation

d: indifference amount

h₁: a number from Table 16.4

*: this value is the same for all alternatives

(ISU page 347)
## Selected Values of $h_1$ for the D/D Procedure

(Table 16.4)

<table>
<thead>
<tr>
<th>Probability of Selecting the Best</th>
<th>First-Stage Sample Size, $n_0$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.90</td>
<td>15</td>
<td>1.93</td>
<td>2.39</td>
<td>2.63</td>
<td>2.81</td>
<td>2.93</td>
<td>3.04</td>
<td>3.12</td>
<td>3.20</td>
</tr>
<tr>
<td>0.90</td>
<td>20</td>
<td>1.90</td>
<td>2.34</td>
<td>2.58</td>
<td>2.75</td>
<td>2.87</td>
<td>2.97</td>
<td>3.05</td>
<td>3.12</td>
</tr>
<tr>
<td>0.90</td>
<td>25</td>
<td>1.88</td>
<td>2.32</td>
<td>2.55</td>
<td>2.72</td>
<td>2.84</td>
<td>2.93</td>
<td>3.01</td>
<td>3.08</td>
</tr>
<tr>
<td>0.90</td>
<td>30</td>
<td>1.87</td>
<td>2.30</td>
<td>2.54</td>
<td>2.69</td>
<td>2.81</td>
<td>2.91</td>
<td>2.98</td>
<td>3.05</td>
</tr>
<tr>
<td>0.95</td>
<td>15</td>
<td>2.50</td>
<td>2.94</td>
<td>3.17</td>
<td>3.34</td>
<td>3.46</td>
<td>3.57</td>
<td>3.65</td>
<td>3.72</td>
</tr>
<tr>
<td>0.95</td>
<td>20</td>
<td>2.46</td>
<td>2.87</td>
<td>3.10</td>
<td>3.26</td>
<td>3.38</td>
<td>3.47</td>
<td>3.55</td>
<td>3.62</td>
</tr>
<tr>
<td>0.95</td>
<td>25</td>
<td>2.43</td>
<td>2.84</td>
<td>3.06</td>
<td>3.21</td>
<td>3.33</td>
<td>3.42</td>
<td>3.59</td>
<td>3.56</td>
</tr>
<tr>
<td>0.95</td>
<td>30</td>
<td>2.41</td>
<td>2.81</td>
<td>3.03</td>
<td>3.18</td>
<td>3.30</td>
<td>3.39</td>
<td>3.46</td>
<td>3.53</td>
</tr>
<tr>
<td>0.99</td>
<td>15</td>
<td>3.18</td>
<td>4.04</td>
<td>4.27</td>
<td>4.43</td>
<td>4.55</td>
<td>4.64</td>
<td>4.73</td>
<td>4.80</td>
</tr>
<tr>
<td>0.99</td>
<td>20</td>
<td>3.10</td>
<td>3.92</td>
<td>4.13</td>
<td>4.28</td>
<td>4.39</td>
<td>4.48</td>
<td>4.55</td>
<td>4.62</td>
</tr>
<tr>
<td>0.99</td>
<td>25</td>
<td>3.06</td>
<td>3.85</td>
<td>4.05</td>
<td>4.20</td>
<td>4.30</td>
<td>4.39</td>
<td>4.46</td>
<td>4.53</td>
</tr>
<tr>
<td>0.99</td>
<td>30</td>
<td>3.03</td>
<td>3.81</td>
<td>4.01</td>
<td>4.14</td>
<td>4.25</td>
<td>4.33</td>
<td>4.40</td>
<td>4.46</td>
</tr>
</tbody>
</table>
Values of $(h_{1s}/d)^2$ and of the Second-Stage Sample Sizes $N - n_0$

in Case Study 16A

(Table 16.5, where $n_0 = 15$, $h_1 = 3.17$ and $d = $10)
Means and Standard Deviations of the Average Number of Machines in Use in the Case Study 16A Second-Stage Sampling (Table 16A.3)

<table>
<thead>
<tr>
<th>Number of Leased Machines</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.8939</td>
<td>8.3390</td>
</tr>
<tr>
<td></td>
<td>0.27051</td>
<td>0.31895</td>
</tr>
<tr>
<td>2</td>
<td>8.938</td>
<td>9.4667</td>
</tr>
<tr>
<td>(undefined)</td>
<td></td>
<td>0.080237</td>
</tr>
</tbody>
</table>

Legend: Mean, Std. Dev.
### Means of the Average Daily Costs in the Case Study 16A Second-Stage Sampling

(Table 16.6)

<table>
<thead>
<tr>
<th>Number of Repairpeople</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$423.28</td>
<td>$459.18</td>
</tr>
<tr>
<td>2</td>
<td>$392.93</td>
<td>$416.80</td>
</tr>
</tbody>
</table>
Computation of the Weight $W_0$

Applied to the First-Stage Samples in the Dudewicz/Dalal Procedure

(Second-Stage Sample Weight $W_1 = 1 - W_0$)

(the computation must be carried out alternative-by-alternative)

(An Intro to Simulation Using GPSS/H, page 348)

$$W_0 = \frac{n_0}{N}[1 + \sqrt{1 - \left(\frac{n_0}{N}\right)[1 - (N - n_0)/(h_1 s/d)^2]}]$$

N: total sample size for the alternative

n_0: first-stage sample size*

s: the alternative's first-stage sample standard deviation

d: indifference amount*

h_1: a number from Table 16.4*

*: the value is the same for all alternatives
Values of the Weights $W_0$ and $W_1$

for the First- and Second-Stage Samples in Case Study 16A

(Table 16.7)

<table>
<thead>
<tr>
<th>Number of Leased Machines</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Repairpeople</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.116</td>
<td>0.082</td>
</tr>
<tr>
<td></td>
<td>0.884</td>
<td>0.918</td>
</tr>
<tr>
<td>2</td>
<td>1.243</td>
<td>0.526</td>
</tr>
<tr>
<td></td>
<td>-0.243</td>
<td>0.474</td>
</tr>
</tbody>
</table>

Legend: $W_0$ $W_1$
The Weighted Mean Daily Costs Resulting from Use of the D/D Procedure in Case Study 16A

(Table 16.8)

<table>
<thead>
<tr>
<th>Number of Repairpeople</th>
<th>Number of Leased Machines</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>$422.20</td>
</tr>
<tr>
<td>2</td>
<td>$405.34</td>
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</tbody>
</table>