Contracting on Credit Ratings: Adding Value to Public Information*

Christine A. Parlour† Uday Rajan‡

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†Haas School of Business, U.C. Berkeley; parlour@berkeley.edu
‡Stephen M. Ross School of Business, University of Michigan; urajan@umich.edu.
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Abstract

We provide a novel interpretation of the role of credit ratings when contracts between investors and portfolio managers are incomplete. In our model, a credit rating on a risky bond provides a verifiable signal about an unverifiable state. The price of the bond also provides information about the state. We show that a credit rating can be valuable in a contract if the price is sufficiently noisy and the rating itself is sufficiently precise. When ratings reveal negative information, highly precise ratings should be used to prohibit the manager from buying the risky bond. When both rating precision and the severity of the agency problem are moderate, the optimal contract is a wage contract that benchmarks the reward of the manager to the return on the risky bond. We compare and contrast the implications of our contracting view of ratings to those of the usual information view of ratings.
1 Introduction

“For almost a century, credit rating agencies have been providing opinions on the creditworthiness of issuers of securities and their financial obligations.”

Annette L. Nazareth; Director, U.S. SEC; Congressional testimony, April 2, 2003.

“Unlike other types of opinions, such as, for example, those provided by doctors or lawyers, credit rating opinions are not intended to be a prognosis or recommendation.”

“What Credit Ratings Are & Are Not,” Standard & Poor’s web site.

Credit rating agencies and regulators routinely describe ratings as opinions, rather than summaries of proprietary information. Despite this, most academic literature characterizes a rating as an informative signal about a security. Further, market participants use and react to credit ratings. The latter is especially surprising in cases such as sovereign bonds or insured municipal bonds, for which it is difficult to claim that the rating agency possesses information not already known to market participants. Yet, market prices of such bonds and related instruments also react to rating changes.1

In this paper, we posit a novel explanation for the existence of this (seemingly) redundant information aggregation and reporting: When contracts are incomplete, the use of ratings allows market participants to write better contracts. We develop the implications of this idea in the context of delegated portfolio management. Our aim is twofold: first, to examine how credit ratings should be used in contracting between an investor and portfolio manager, and second, to understand the implications for market observables and for policy of the “contracting view” as opposed to the “information view” of credit ratings.

To make our point starkly, we consider an extreme scenario in which credit ratings communicate no new information about the asset or issuer. Our model features an investor (a principal) who hires a manager (an agent) to make an investment decision for her. There

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1For example, Brooks, et al. (2004) find that downgrades of sovereign debt adversely affect both the level of the domestic stock market and the exchange rate for the country’s currency.
are two states, high and low, and two feasible actions: hold a risky bond or a riskless asset. The investor prefers to hold the risky bond in the high state and the riskless bond in the low state. The action preferred by the manager depends both on the offered contract and on the realization of a stochastic private benefit. In contrast to many contracting frameworks, the size of the private perquisites the manager can extract are unrealized (and so unknown to both parties) at the time the contract is written. The potential inefficiency is that, due to these private benefits, the investor and manager may end up preferring different state-contingent actions. The state is not directly verifiable, so contracts are incomplete. In this setting, we interpret a credit rating as a verifiable, and therefore contractible, signal about the state. The precision of the signal captures the accuracy of the rating.

The investor offers a contract in which the manager is paid a wage based on an array of contracting variables: the portfolio return, and the cash flow, credit rating and price of the risky bond. In addition, in contrast to a standard moral hazard model, the investor can also simply prohibit the manager from investing in the risky asset. The ability of the principal to both impose an ex ante restriction on the agent’s feasible set as well as to induce a preferred action through a compensation contract is a novel theoretical feature of our model. In addition, we demonstrate how multiple variables should be jointly used at both the ex ante and ex post stages (i.e., both before the manager chooses an action and after the outcome is realized) in determining the optimal contract.

Importantly, in our model, the price of the risky bond communicates some information about the state, and the contract can be contingent on this price. Whether the credit rating should be used in setting contract terms therefore depends critically on its marginal informativeness, beyond what the investor can infer from the price of the risky bond.

The price in our model is a proxy for all ex ante contractible information (including other public signals such as macro-economic or issuer-specific variables) other than the credit rating itself. Given the wide availability of public information, what makes credit ratings on bonds especially valuable as contractible signals? A distinguishing feature of bonds is that they specify a promised stream of future cash flows. The beliefs of different investors over
a bond’s cash flows therefore focus on the same set of events, prominent among which is whether the bond will or will not default. The simplicity of the default event is conducive to generating a signal (i.e., a credit rating) about it. In contrast, other public information (including the price) combines information about cash flows and about discount rates, where the latter in turn depend on more complicated objects such as investor preferences and limits to arbitrage.

In our model, there is no conflict of interest between an investor and a manager in the high state. Thus, if the rating on the risky bond is good and the price also indicates the state is likely to be high, it is optimal to set wages to zero and to let the manager choose freely.2 Conversely, if the combination of a bad rating and a low price leads to a sufficiently pessimistic belief over states, she prohibits the manager from investing in the risky bond. At intermediate beliefs, the investor lets the manager choose an action, but tries to induce the correct one in each state with a wage contract. Each of the wage and prohibitive contracts has its own costs; a direct compensation cost with the former, and forgoing the high-state return on the risky bond with the latter. The optimal contract balances these two costs.

Both the contracting and information views imply that bond prices will react to rating changes, and that managers have a higher gross alpha when rating precision is lower. However, because they have very different policy implications, it is critical to distinguish between them empirically. If credit ratings contain new information, their release mitigates information asymmetry in the economy. This should both diminish a manager’s ability to earn an alpha and reduce adverse selection for the relevant stock. However, such changes should not occur under the contracting view of ratings. Further, in our model, reliance on ratings increases when the rating is more precise. Thus, in the contracting view, changes in credit ratings have a bigger impact on bond turnover when ratings are precise, whereas the link between precision and turnover is unclear in the information view. Finally, if ratings contain new information, their existence makes it easier for individuals to construct their own portfolios, which should lead to a reduction in delegation. By contrast, under the contracting view,

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2 A zero wage is just a normalization in our model, and corresponds in a given period to the standard management fee based on assets under management received by fund managers.
credit ratings reduce the cost of writing contracts with managers, which should increase the demand for portfolio management services.

In the information view, increasing ratings precision leads to more efficient investment, and therefore typically increases welfare. In contrast, in our framework, increased precision of ratings can lead to a lower surplus in the transaction between the investor and the manager, because the manager’s payoff is reduced when he is prohibited from investing in the risky asset. Further, the widespread use of ratings can lead to bond returns being volatile even when fundamentals are fixed, as long as the rating is a noisy signal of the state.

Credit ratings are widely used in the economy. Mutual funds, pension funds, and insurance companies commonly have internal restrictions and investment policies that require minimum credit ratings on investments, akin to the prohibitive contract in our model. Prominent examples are provided by CalPERS, which manages about $300 billion in assets and is the largest public pension fund in the US, and Norges Bank Government Global Pension Fund, which has just over $1 trillion in assets. Indirect evidence for the widespread use of contractual investment mandates is provided by Chen, et al. (2014), who demonstrate a price change in affected bonds following 2005 re-labeling of which split-rated bonds were eligible for index inclusion by Lehman Brothers. Credit ratings are also used to rule out potential counterparties in some transactions.

The wage contracts in our model reward the manager for performance relative to a benchmark (the realized payoff on the risky bond), and also specify that the wage should depend on the credit rating of the bond. In particular, the manager is paid more for outperforming the benchmark when the rating is bad, as compared to when the rating is good. As the credit

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3The CalPERS Total Investment Fund Policy establishes minimum credit ratings for different kinds of bonds in the various investment programs, and the Global Fixed Income Program policy document states that the portfolio formed under the Credit Enhancement Program will maintain an average rating of single A or higher. Both documents are available at [https://www.calpers.ca.gov/page/investments/about-investment-office/policies](https://www.calpers.ca.gov/page/investments/about-investment-office/policies).

4On the fixed income side, it is required that all debt instruments the fund buys have a credit rating, and that high-yield bonds constitute no more than 5% of the value of the bond portfolio. See [https://www.nbim.no/en/transparency/submissions-to-ministry/2017/bonds-in-the-government-pension-fund-global/](https://www.nbim.no/en/transparency/submissions-to-ministry/2017/bonds-in-the-government-pension-fund-global/).

rating in our model is correlated with the likelihood the risky bond will default, our contract essentially specifies that the reward to the manager should be adjusted for the risk of the portfolio.

Our focus on the use of non-informative credit ratings to mitigate contracting frictions is novel. Other work on non-informative ratings includes Boot, Milbourn, and Schmeits (2006), who present a framework in which a firm’s funding costs depend on the market’s beliefs about the type of project being chosen. The credit rating agency, by providing a rating, allows infinitesimal investors to coordinate on particular beliefs when multiple equilibria are possible. Manso (2013) considers how a credit rating might have real effects, in a model with multiple equilibria and self-fulfilling beliefs. In his framework, changes in a firm’s credit rating affects its ability to raise capital, which then reinforce the original rating.

Much of the work in the literature considers credit ratings that communicate new information about the firm to the market, as well as frictions in the rating process that lead to noisy or inflated ratings. Several papers comment on the drawbacks of regulators or investors relying on ratings. For example, Opp, Opp and Harris (2013) illustrate how the use of ratings by regulators leads to rating inflation, and so may have pernicious effects. Kartasheva and Yilmez (2013) consider the optimal precision of ratings, and find in their model that efficiency is enhanced by reducing the reliance of regulation on credit ratings. Donaldson and Piacentino (2018) consider an environment in which the first-best outcome can be achieved by contracts that do not rely on credit ratings, and show that investment mandates based on ratings lead to inefficiency.

Our work provides a counter-perspective by focusing on the positive role of ratings in contracts. We also abstract away from frictions in the process of producing and reporting ratings. Many such frictions have been pointed out in the literature, building on the work of Lizzeri (1999) on certification intermediaries. Frictions in the rating process include rating inflation by the credit rating agency in a desire to capture high fees (Fulghieri, Strobl, and Xia, 2014), the breakdown of reputation as a disciplining device when flow income from new transactions is high (Mathis, McAndrews, and Rochet, 2009), and various inefficiencies
stemming from rating shopping (Skreta and Veldkamp, 2009; Bolton, Freixas, and Shapiro, 2012; and Sangiorgi and Spatt, 2017). Goldstein and Huang (2017) consider the effect of such frictions on firm investment, and show that the existence of informative ratings sometimes reduces social welfare. In our model, introducing frictions into the rating process will necessarily reduce the precision of the rating. However, if these frictions are not too severe, the optimal contract remains contingent on the rating.

The core of our framework is inspired by some aspects of the model of Aghion and Bolton (1992), who present an incomplete contracting model with a principal and an agent in which states are observable, but not verifiable. In our framework, the credit rating is a verifiable signal, potentially improving efficiency in the contracting relationship.

We introduce our model in Section 2. In Section 3, we demonstrate the optimal contract for a single investor-manager pair, holding the price of the risky bond as fixed for each state and credit rating. We then examine how the contract varies with the precision of the credit rating and the price of the risky bond in Section 4. We provide some implications of our findings in Section 5. All proofs appear in the appendix.

2 Model

We consider optimal contracting between a portfolio manager and an investor. As ratings are typically used in setting terms for external transactions, we have in mind an external investor who delegates to a portfolio manager. However, it is worth noting that credit ratings were widely used in the US before the recent rise in delegated portfolio management. Of course, a moral hazard problem exists internal to an organization as well, for example between those saving for retirement and a bond portfolio manager at a pension fund, and our model also applies there.

There are two assets, a risky bond and a risk-free one. The investor–manager relationship continues over four dates, $t = 1, \ldots, 4$. The contract is signed at date 1, information is released at date 2, the manager chooses an asset at date 3, and payoffs are realized at date $t = 4$. 
Figure 1 shows the sequence of events in the model.

At date 1, the investor offers the manager a contract to manage a $1 investment. The contract specifies both feasible actions for the manager at date 3 and a compensation (or wage) at the final date 4. For simplicity, we assume that the manager invests the entire $1 in either the risky asset (action $a_y$) or in the risk-free asset (action $a_f$). The wage at $t = 4$ can be conditioned on the following information: the risky bond’s credit rating, price, and cash flow, as well as the investor’s realized portfolio value.

At $t = 2$, three pieces of information become available. First, one of two possible states, high ($h$) or low ($\ell$), occurs. Critically, even though both parties know the state, it is not verifiable, and so not directly contractible. The state affects the cash flow distribution of the risky bond at date 4. Let $y$ denote the realized cash flow on the bond.

The investor suffers a private disutility $\delta > 0$ from holding the risky bond in state $\ell$, and values the cash flows from each unit of the bond at $y - \delta$. As the risky bond has a higher default probability in this state, $\delta$ may be interpreted as a reduced-form way to capture risk aversion on the part of the investor. Alternatively, relative to other traders in the market, in state $\ell$ the investor is at a disadvantage in securing favorable terms in a bankruptcy negotiation. The disutility $\delta$ ensures that the investor’s preferred asset is state-dependent. As will be clear below, if they made their own decisions, the investor would like to hold the risky bond in state $h$ and the riskless bond in state $\ell$. 

Figure 1: Sequence of Events
The second piece of information available at date 2 is a contractible signal \( \sigma \), in the form of a credit rating on the risky bond. The rating is correlated with the state, but we do not explicitly model its source. Specifically, the rating takes on one of two values, \( g \) or \( b \), and is potentially informative, with \( \text{Prob}(\sigma = g \mid s = h) = \text{Prob}(\sigma = b \mid s = \ell) = \psi \geq \frac{1}{2} \). Thus, the rating is completely uninformative if \( \psi = \frac{1}{2} \), and perfectly informative if \( \psi = 1 \). We refer to \( \psi \) as the precision of the rating.

The third piece of information at date 2 relates to the degree of conflict between the manager and investor. The manager obtains a private benefit \( m \) from having the investor hold the risky bond in state \( \ell \). The private benefit corresponds to transfers that he obtains from a sell-side firm if he places the risky bond in an investor’s portfolio. The private benefit is random, and is drawn from a uniform distribution with support \([0, M]\). As is customary, the private benefit is not verifiable, so cannot be part of the contract.

Conflicts of interest between investors and portfolio managers have been extensively documented.\(^6\) More specifically, fixed income markets are not centralized, so it is difficult for investors to track and evaluate the cost of trades by managers. Mark-ups for trading bonds are both large and heterogeneous across both asset classes and types of investors.\(^7\) High and opaque mark-ups for bonds are a source of rents accruing to the broker-dealer, and it is reasonable to think that some of these rents will in turn translate into extra compensation for the portfolio manager who trades the bond. Such transfers represent the private benefit \( m \) in our model.

At \( t = 3 \), the portfolio manager chooses an action from his feasible set. The riskless bond delivers a cash flow of $1 per unit at date 4. We normalize the risk-free rate to zero, so the price of the riskless bond at date 3 is equal to 1. The price of the risky bond at \( t = 3 \) is given by \( p(s, \sigma) = E_s y - \epsilon_s \), where \( E_s y \) is the expected cash flow on the risky bond in state \( s \) and

\(^6\)For example, Egan, Matvos and Seru (2017) report that on average 7% of financial advisors have misconduct records, and the number is as high as 15% at large firms.

\( \epsilon_s \) is a noise term that reflects randomness in supply. The noise can potentially depend on the state \( s \) and the rating \( \sigma \). For convenience, we assume that for each \( s \) and \( \sigma \), \( \epsilon_s \sim F_\sigma[0, \bar{\epsilon}] \), where \( \bar{\epsilon} > 0 \). That is, the range of the noise term is invariant to \( s \) and \( \sigma \), although the distribution over this range, \( F_\sigma \), may depend on \( s \) and \( \sigma \). Note that the price of the risky bond is generally less than the expected cash flow, reflecting the fact that it has a higher return than the risk-less asset. Let \( f_\sigma \) be the density function of \( \epsilon_s \).

At \( t = 4 \), both bonds mature. A manager who invested in the riskless bond delivers a portfolio value of $1. A manager who invested in the risky bond purchases \( \frac{1}{p} \) units of the bond, and so delivers a random portfolio value \( \frac{y}{p} \). The cash flow \( y \) is drawn from a distribution \( \tilde{G}_s \) with density \( \tilde{g}_s \). The risky bond has a promised cash flow of $1, and repays fully with probability \( g_s(1) \), which we denote as \( g_s \). The bond is more likely to make its promised payment of $1 in state \( h \) (i.e., is less likely to default in state \( h \)), so that \( g_h > g_\ell \). If the bond defaults, its cash flow is less than 1. Conditional on default, the cash flow \( y \) is uninformative about the state. That is, the conditional distribution given \( y < 1 \) is invariant across state. We further assume that the conditional distribution is atomless, so that the only mass point of \( \tilde{G}_s \) is at the cash flow of 1.

The payoff to the investor from this relationship is the portfolio value at date 4 less the wage paid to the manager. The payoff to the manager is the sum of the wage and any private benefit he may garner. The manager enjoys limited liability, so the wage in any outcome is non-negative. His reservation utility is zero, so any contract that satisfies limited liability is also individually rational. Both investor and manager are risk-neutral; as the risk-free rate is equal to zero, there is no discounting.

We assume that the investor cannot directly invest in the risky bond on her own. Implicitly, the cost of direct investing is too high for her. This cost may be interpreted as either the opportunity cost of time for the investor or the direct cost of access to certain securities.\(^8\)

In equilibrium, the investor offers an optimal contract to the manager, which prescribes both a wage function and feasible actions. Let \( V \) denote the value of the portfolio at date 4.

\(^8\)For example, under SEC Rule 144A, only qualified institutional buyers may purchase certain private securities.
As the investment at date 3 is $1, V is also the gross return on the portfolio. Let $w(V, y | \sigma, p)$ be the wage paid to the manager when the price of the risky bond is $p$, the credit rating on the bond is $\sigma$, the cash flow of the risky bond is $y$, and the portfolio value at date 4 is $V$. Let $w = \{w(V, y | \sigma, p)\}_{\sigma \in \{g, b\}, \in [0,1], p, V}$. Similarly, let $A(\sigma, p) \subseteq \{a_y, a_f\}$ denote the feasible action set for the manager, given $p$ and $\sigma$, and let $A = \{A(\sigma, p)\}_{\sigma \in \{g, b\}, p}$. A contract offered by investor $i$ is denoted by $\{w, A\}_i$.

Going forward, we distinguish between three kinds of contracts. A “null contract” is defined to be a contract in which there is no restriction on action and no wage is offered; that is, $A(\sigma, p) = \{a_y, a_f\}$ for each $\sigma$ and $p$ and $w(V, y | \sigma, p) = 0$ for each combination of $V, y, \sigma$, and $p$. A “wage contract” is a contract in which again no restriction is placed on manager action but the wage is sometimes strictly positive, so that $A(\sigma, p) = \{a_y, a_f\}$ for each $\sigma$ and $p$, but $w(V, y | \sigma, p) > 0$ for some $(V, y, \sigma, p)$. Finally, a “prohibitive contract” is one in which the manager is prohibited from holding the risky bond, so that $A(\sigma, p) = \{a_f\}$. It is immediate that in such a contract it cannot be optimal to offer a strictly positive wage, so that we must have $w(V, y | \sigma, p) = 0$ in each case.\(^9\)

### 2.1 Manager Best Response

Consider the best response of the manager, given a contract $(w, A)$. Recall that the manager takes an action knowing the state $s$, the price of the risky bond, $p$, and the rating $\sigma$. He can purchase $\frac{1}{p}$ units of the risky bond; as each unit pays out $y$ at date 4, the value of the portfolio is then $V = \frac{y}{p}$, and his wage is $w(\frac{y}{p}, y | \sigma, p)$.

Two points are worth nothing here. First, for any feasible price $p$, the event $\frac{y}{p} = 1$, or $y = p$, has zero measure. Therefore, the portfolio value essentially reveals whether the manager purchased the risky or the risk-free bond. Second, if the risky bond defaults, so that $y < 1$, the actual cash flow conditional on default is uninformative about the state. Therefore, when the risky bond defaults, the wage will not depend on the realized cash flow.

\(^9\)Note that, as the manager is inclined to buy the risky bond in any situation in which wages are zero, there is no need for the investor to consider a contract that forces the manager to buy the risky bond, or equivalently, prohibits the purchase of the riskless bond. Thus, in considering a prohibitive contract, we restrict attention to the contract that prohibits investment in the risky bond.
y. That is, the wage function can then be thought of in terms of two cases: a wage paid when
the risky bond defaults, and a wage paid when the risky bond repays fully.

Thus, we have two actions and two contingencies to base the wage on. That is, at a
given price and rating, the wage contract can be represented by the following four numbers:
\( w_d^f(\sigma, p) \) and \( w_d^r(\sigma, p) \), the wage paid when the manager invests in the risk-free bond (indicated
by the superscript \( f \)) and the risky bond defaults or fully repays, respectively; \( w_y^r(\sigma, p) \), the
wage paid when the manager invests in the risky bond and the risky bond repays fully; and
\( w_y^d(\sigma, p) \), the wage paid when the manager invests in the risky bond and the risky bond
defaults.

It follows that the manager’s expected wage if he buys the risky bond is

\[
\hat{w}_s^y \overset{d}{=} E_s w_y(y \mid \sigma, p) = g_s w_y^r(\sigma, p) + (1 - g_s) w_y^d(\sigma, p).
\] (1)

Conversely, if he buys the risk-free bond, his expected wage is

\[
\hat{w}_s^f \overset{d}{=} E_s w_f(1 \mid \sigma, p) = g_s w_f^r(\sigma, p) + (1 - g_s) w_f^d(\sigma, p).
\] (2)

For convenience, we suppress the dependence of \( \hat{w}_s^y \) and \( \hat{w}_s^f \) on \( \sigma \) and \( p \).

In state \( h \), the manager buys the risky bond if \( \hat{w}_h^y \geq \hat{w}_h^f \), and the riskfree bond otherwise
(as usual, we assume that when indifferent, the manager takes the action preferred by the
investor). In state \( s \), the manager buys the risk-free bond if \( m \leq \hat{w}_s^f - \hat{w}_s^y \), and the risky
bond otherwise.

### 2.2 Fully informative prices

We now determine the contract the investor should offer. As mentioned earlier, the optimal
contract will depend on the investor’s belief over state given the price and rating. We begin
by observing that in some cases, the price of the bond at date 3 is fully informative about
the state, and so trivially there is no need for the contract to depend on the rating.

Denote \( p_s = E_s y - \bar{\epsilon} \) for each \( s = h, \ell \). Then, as shown in Figure 2, there are three
relevant ranges of $p$. If $p \in [p_\ell, p_h)$, it is immediate that the state is $\ell$; that is, $\rho(p \mid \sigma) = 0$. Similarly, if $p \in (E_\ell y, E_h y]$, it is immediate that the state is $h$, so that $\rho(p \mid \sigma) = 1$. Finally, if $p \in [p_h, E_\ell y]$, the state could be either $h$ or $\ell$, with $\rho(p \mid \sigma) \in (0, 1)$.

![Diagram](https://via.placeholder.com/150)

Figure 2: Price range for risky bond

When the price reveals the state to be high, the optimal contract involves zero wages and no restriction on action. Conversely, when the state is low, the optimal contract prevents the agent from investing in the risky asset, and again features zero wages.

**Lemma 1**  
(i) Suppose that $p \in (E_\ell y, E_h y]$. Then, the optimal contract is a null contract; that is, it sets $w(V, y \mid \sigma, p) = 0$ for each $\sigma, p, V, \text{ and } y$, and sets $A(\sigma, p) = \{a_y, a_f\}$ for each $p$ and $\sigma$.

(ii) Suppose that $p \in [p_\ell, p_h)$. Then, the optimal contract is a prohibitive contract; that is, it sets $w(V, y \mid \sigma, p) = 0$ for each $\sigma, p, V, \text{ and } y$, and sets $A(\sigma, p) = \{a_f\}$ for each $p$ and $\sigma$.

### 3 Contracting on Ratings

For the rest of the paper, we consider prices in the range $[p_h, E_\ell y)$. At these prices, the optimal contract depends on the posterior probability of the high state given all information available to the investor. Although the contract is offered at date 1, the action set is contingent on contractible information available at date 3. Because the state is not directly contractible, the optimal contract depends critically on the investor’s posterior belief over state, given the contractible information at date 3. This information consists of the credit rating on the risky
bond (σ) and its price (p), and the relative informativeness of the two variables determines the choice of contract.

It is useful to define an intermediate belief ρ(p | σ) as the probability that the state is h given the price p, before the precision of the credit rating is taken into account. That is

\[
\rho(p | \sigma) = \frac{\phi f_\sigma(\epsilon_h)}{\phi f_\sigma(\epsilon_h) + (1 - \phi) f_\sigma(\epsilon_\ell)}. \tag{3}
\]

Taking into account the precision of the signal, ρ(p | σ) is further updated to the posterior probability of state h, μ(σ, p), where

\[
\mu(g, p) = \frac{\psi \rho(p | g)}{\psi \rho(p | g) + (1 - \psi)(1 - \rho(p | g))}, \tag{4}
\]

\[
\mu(b, p) = \frac{(1 - \psi) \rho(p | b)}{(1 - \psi) \rho(p | b) + \psi(1 - \rho(p | b))}. \tag{5}
\]

Observe that both ρ(·) and μ(·) are exogenous in our model.

We make the following assumptions.

**Assumption 1**

(i) \( \delta > \frac{\bar{\epsilon}}{E_{\bar{y} - \bar{\epsilon}}} \).

(ii) \( 0 \leq g_\ell < g_h < 1 \).

(iii) \( \rho(p | \sigma) \) is continuous and weakly increasing in p for each \( \sigma = g, b \).

Part (i) implies that \( \delta > \frac{E_{\bar{y}} y}{E_{\bar{y} - \bar{\epsilon}}} - 1 \), which ensures that in the low state, the investor always prefers to own the risk-free bond. Note that, in the high state, the price of the risky bond is weakly less than \( E_{h,y} \), so the investor prefers to hold the risky bond. As \( g_s \) is the probability that the risky bond fully repays (i.e., pays off $1) in state s, part (ii) ensures that even in the high state, the bond defaults with strictly positive probability. Part (iii) implies that higher prices are more likely to signal that the state is h.

Our main result in this section is Proposition 1. We show that there are two thresholds for the posterior belief \( \mu(\sigma, p) \) which determine ranges in which each of the prohibitive, wage, and null contracts are optimal. Call these thresholds \( \mu_1 \leq \mu_2 \). Broadly, the prohibitive contract
is optimal when \( \mu(\sigma, p) \in [0, \mu_1] \), the wage contract is optimal when \( \mu(\sigma, p) \in (\mu_1, \mu_2) \), and the null contract is optimal when \( \mu(\sigma, p) \in (\mu_2, 1) \).

To build up to these thresholds, we start by separately considering prohibitive contracts and wage contracts, and then turning to the overall optimal contract.

**Prohibitive and null contracts**

If the manager is forced to invest in the risk-free asset, the portfolio value at date 4 is $1. The payoff to the investor from a prohibitive contract is therefore equal to 1.

Next, consider the null contract. Faced with such a contract, the manager always buys the risky bond. The payoff to the investor from such a contract is then

\[
\Pi^n(\sigma, p) = \mu(\sigma, p) \left( \frac{E_h y}{p} + (1 - \mu(\sigma, p)) \left( \frac{E_f y}{p} - \delta \right) \right).
\] (6)

Let \( R_s(p) = \frac{E_s y}{p} - 1 \) be the expected excess return on the risky asset in state \( s \), given its price \( p \) (recall that the risk-free rate is normalized to zero). Comparing the payoffs to the prohibitive and the null contracts, it follows that the null contract has a higher payoff if and only if \( \mu \geq \frac{\delta - R_f(p)}{\delta + R_h(p) - R_f(p)} \); that is, if the posterior probability of the high state is sufficiently large.

Denote \( \mu^p(p) = \frac{\delta - R_f(p)}{\delta + R_h(p) - R_f(p)} \); this is the threshold posterior belief at which the payoffs of the prohibitive and null contracts are equal.

**Lemma 2** Consider \( p \in [p_h, E_f y] \). Then, comparing the null and prohibitive contracts, the null contract has a strictly higher payoff if \( \mu(\sigma, p) > \mu^p(p) \), and the prohibitive contract has a strictly higher payoff if \( \mu(\sigma, p) < \mu^p(p) \).

**Wage contract**

Next, suppose the investor offers the manager a wage contract. Recall from equations (1) and (2) that \( \hat{w}_s^y \) is the expected payoff to the manager from holding the risky asset and \( \hat{w}_s^f \) is the expected payoff from holding the riskless asset in state \( s \).
Suppose that \( \hat{w}^f_\ell - \hat{w}^y_\ell \in [0, M] \). Then, the investor’s payoff from a given wage contract may be written as follows:

\[
\Pi^w(\sigma, p) = \mu(\sigma, p) \left\{ \frac{E_y}{p} - \hat{w}^y_\ell \right\} + (1 - \mu(\sigma, p)) \left\{ \frac{\hat{w}^f_\ell - \hat{w}^y_\ell}{M} (1 - \hat{w}^f_\ell) + \frac{M - (\hat{w}^f_\ell - \hat{w}^y_\ell)}{M} \left( \frac{E_y}{p} - \delta - \hat{w}^y_\ell \right) \right\} \tag{7}
\]

If \( \hat{w}^f_\ell < \hat{w}^y_\ell \), the second line in the above equation reduces to \( (1 - \mu(\sigma, p)) \left\{ \frac{E_y}{p} - \delta - \hat{w}^y_\ell \right\} \).

We show that the optimal wage contract has the feature that the manager is paid only if their investment decision is vindicated ex post; that is, if either they invested in the risky asset and the asset fully repaid, or if they invested in the riskfree asset and the risky asset defaulted.

**Lemma 3** Suppose that \( p \in [p_h, E_y] \) and no restriction is imposed on the manager’s action, and the credit rating is \( \sigma \). Then:

(i) The manager is paid a zero wage if they invest in the riskfree asset and the risky asset fully repays, or if they invest in the risky asset and the risky asset defaults. That is, \( w^f_\ell(\sigma, p) = w^y_d(\sigma, p) = 0 \).

(ii) If the manager invests in the risky asset and the asset fully repays, the manager is paid a wage \( w^y_\ell(\sigma, p) = \frac{1-\gamma_h}{\gamma_h} w^f_d(\sigma, p) \).

(iii) There exists a threshold belief \( \mu^w \) such that, if the manager invests in the riskfree asset and the risky asset defaults, the manager is paid a wage

\[
w^f_d(\sigma, p) = \begin{cases} 
\min \left\{ \frac{1}{2(1-\gamma_h)} \left[ \delta - R_\ell(p) - \frac{1-\gamma_h}{\gamma_h} \left( \frac{\mu(\sigma, p)}{1-\mu(\sigma, p)} + \frac{\gamma_h}{\gamma_h} \right) M \right] , \frac{M}{1-\gamma_h} \right\} & \text{if } \mu(\sigma, p) \leq \mu^w \\
0 & \text{if } \mu(\sigma, p) > \mu^w 
\end{cases} \tag{8}
\]

At the threshold belief \( \mu^w \) in part (iii) of the Lemma, the payoffs of the wage and null contracts are equal. Or, put another way, the optimal wage \( w^f_d \) in the wage contract just reduces to zero.
Intuitively, investing in the risky bond is more likely to be the correct action if the risky bond repays fully, whereas buying the riskless bond is more likely to be the correct action if the risky bond defaults. Part (i) of the Lemma establishes that there is no reason to reward a manager who took an incorrect action. Part (ii) states that the wage $w_y^f$, paid if the manager invested in the risky bond and the risky bond fully pays off, inherits the properties of $w_d^f$, the wage paid for investing in the riskless bond when the risky bond defaults.

Consider part (iii). If the posterior belief $\mu(\sigma, p)$ is relatively high (in particular, above $\psi_1$), we are likely in state $h$. In this case, it is better to offer a null contract (i.e., set all wages to zero) rather than set a positive wage in any scenario. The wages $w_d^f$ and $w_y^f$ are positive if $\mu(\sigma, p) < \mu^w$; i.e., the posterior probability of state $h$ is relatively low.

In this last case, the wages $w_d^f$ and $w_y^f$ are chosen to ensure that the manager buys the risky bond (and so takes the correct action) in state $h$. The wage levels depend on the value of $M$ (the maximal private benefit). When $M$ is relatively low, the second term in the “min” expression in equation (8) is lower than the first term, and we have $w_d^f = \frac{M}{1-\delta}$. Here, $w_d^f$ is sufficiently high that all managers, regardless of their private benefit $m$, choose to hold the riskless bond in state $\ell$. In this region, the wage increases in $M$, and is independent of $\mu$ (and hence of the credit rating $\sigma$), and of $\delta$.

However, if $M$ becomes sufficiently high, the first term in the min expression becomes lower than the second term. Now, in state $\ell$, a proportion of managers $\frac{w_d^f-w_y^f}{M}$ hold the riskless bond (i.e., take the correct action), whereas a proportion $1 - \frac{w_d^f-w_y^f}{M}$ hold the risky bond instead, and obtain their private benefit $m$. The wage is decreasing in $M$, increasing in $\delta$, and decreasing in $\mu$. In this region, increasing $w_d^f$ by a small amount has two effects. First, it increases the proportion of managers who purchase the riskless asset in state $\ell$, which is beneficial to the investor. Recall that the private benefit $m$ is uniform over $[0, M]$; the quantitative magnitude of this effect therefore decreases with $M$. Second, (from part (ii) of the Lemma) it leads to an increase in $w_y^f$, to induce the manager to continue to purchase the risky asset in state $h$. This leads to a reduction in the investor’s payoff in state $h$. The optimal level of $w_d^f$ trades off these two effects.
Observe that when \( g_h = 1 \), both terms in the min expression in equation (8) are independent of the credit rating \( \sigma \), and hence of its precision, \( \psi \). When \( g_h = 1 \), a default immediately reveals the state is \( \ell \), and so the credit rating adds no additional information beyond what is available at the time the wage is paid (time 4).

**Overall optimal contract**

We now turn to the overall optimal contract. We identify conditions under which the optimal contract is the prohibitive, wage, or null contract, as the case may be.

The intuition of the optimal contract is as follows. If, taking into account all available information at date 4, the state is almost surely \( \ell \) (i.e., if \( \mu(\sigma, p) \approx 0 \)), the prohibitive contract is optimal. It forces the manager to invest in the risk-free asset, at zero cost to the investor. Conversely, if the state is almost surely \( h \) (i.e., if \( \mu(\sigma, p) \approx 1 \)), the null contract is optimal, as the manager takes the correct action in state \( h \). We show in the next proposition that when the maximal private benefit \( M \) is sufficiently small, there exists a range of \( \mu \) such that the wage contract is optimal.

**Proposition 1** Consider a price \( p \in [p_h, E]\). There exist two thresholds for the probability of the high state, \( \mu_1 \leq \mu_2 \), and a threshold maximal private benefit \( \hat{M}(p, \delta) \) such that:

(a) If the maximal private benefit is sufficiently high (in particular, if \( M \geq \hat{M}(p, \delta) \)), the wage contract is never optimal. The optimal contract is the prohibitive contract if \( \mu(p, \sigma) < \mu_1 \) and the null contract if \( \mu(p, \sigma) > \mu_1 \). In this case, \( \mu_1 = \mu^n \), the belief at which the prohibitive and null contracts have the same payoff.

(b) If the maximal private benefit is sufficiently low (in particular, \( M < \hat{M}(p, \delta) \)), the optimal contract is the prohibitive contract if \( \mu(\sigma, p) < \mu_1 \), the wage contract if \( \mu(\sigma, p) \in (\mu_1, \mu_2) \) and the null contract if \( \mu(p, \sigma) > \mu_2 \). In this case, \( \mu_2 = \mu^w \), the belief at which the wage and null contracts have the same payoff, and \( \mu_1 < \mu^w \).

Figure 3 illustrates the different cases exhibited in Proposition 1. When the maximal private benefit \( M \) is high (as in part (a) of the figure), the null contract is optimal when \( \mu(\cdot) \)
is low and the prohibitive contract is optimal when $\mu(\cdot)$ is high. When $M$ is low (as in part (b)), there exists an intermediate region of $\mu(\cdot)$ at which the wage contract is optimal.

![Figure 3: Optimal contract as posterior probability varies](image)

The three feasible contracts each have their own tradeoffs. The null contract induces the correct action in the high state at zero cost (as the manager holds the risky bond), but the incorrect action in the low state (i.e., the manager continues to hold the risky bond). Conversely, the prohibitive contract induces the correct action in the low state at zero cost (as the manager holds the riskless bond), but the incorrect action in the high state (i.e., the manager continues to hold the riskless bond). The wage contract induces the correct action in state $h$ and some probability of the correct action in state $\ell$, but incurs a direct compensation cost in both states. It is therefore intuitive that when the posterior probability of the high state is large, the null contract is optimal. Conversely, the prohibitive contract must be optimal if the posterior probability of the high state is small.

At intermediate beliefs, depending on the size of the maximal private benefit $M$, the wage contract may be optimal. Recall from the discussion following Lemma 3 that, when $M$ is high, the proportion of managers who take the correct action in state $\ell$ is inversely related to $M$. Consider an extreme case in which $M \rightarrow \infty$. In such a case, at any finite level of wages, the proportion of managers holding the riskless bond in state $\ell$ remains infinitesimal. Thus, the wage contract is completely ineffective. More broadly, even when $M$ is finite, at any given level of wages, the effectiveness of the wage contract (in the sense of inducing the manager to hold the riskless bond in state $\ell$) varies inversely with $M$. Thus, when $M$ is high (as in part (a) of the proposition), the wage contract is never optimal. However, when $M$
is low (part (b)), there is a range of posterior probability $\mu(\cdot)$ at which the wage contract is optimal.

The manager has two sources of rent in the model: the wage and the private benefit, which is earned if they hold the risky bond and the state is low. Comparing across the three contracts, the prohibitive contract eliminates rent from both sources. Under the wage contract, managers with a low private benefit earn rent through the wage, whereas those with a high private benefit consume their side transfers. The null contract lets the manager keep their entire private benefit.

4 Comparative Statics of the Optimal Contract

We now consider how the optimal contract varies with the precision of the credit rating of the risky bond, $\psi$, and the price $p$. Throughout, we focus on prices in the range $(p_h, E_i y)$, to ensure that the prices are not fully informative.

4.1 Effect of Rating Precision

Consider the effect of $\psi$, the precision of the credit rating, on the optimal contract. This is an important comparative static in our model because the precision of the rating differs across assets (such as municipal bonds and mortgage-backed securities). Such differences lead to observable implications that we explore in the next section.

The precision of the credit rating affects the optimal contract through its effect on the posterior belief $\mu(\sigma, p)$, given the intermediate belief $\rho(p \mid \sigma)$ (see equations (4) and (5)). The posterior belief $\mu$ lies between 0 and $\rho(p \mid \sigma)$ when the rating is bad, and between $\rho(p \mid \sigma)$ and 1 when the rating is good. Further, this posterior belief depends on the precision of the rating, with $\mu(b, p)$ decreasing in $\psi$ and $\mu(g, p)$ increasing in $\psi$. This observation, in conjunction with Proposition 1 (which describes the optimal contract in terms of $\mu$) allows us to analyze the effect of $\psi$ on the contract. As different ratings (i.e., $b$ or $g$) have a different impact on $\mu$, we consider each rating separately in what follows.
First, consider a bad rating for the risky bond.

**Proposition 2** Suppose that $p \in (p_h, \ell_y)$ and the credit rating of the risky bond is $b$. Then, there exists a threshold rating precision $\psi_1(p)$ such that, when $\psi > \psi_1(p)$, the prohibitive contract is optimal. In addition:

(a) If the maximal private benefit is sufficiently high (in particular, $M > \hat{M}(p, \delta)$), the null contract is optimal when $\psi < \psi_1(p)$. In this case, the wage contract is never optimal.

(b) If the maximal private benefit is sufficiently low (in particular, $M < \hat{M}(p, \delta)$), then:

   (i) If $\rho(p \mid b) > \mu^w$, there exists an additional rating precision threshold, $\psi_2(p) \in (\frac{1}{2}, \psi_1(p))$, such that the wage contract is optimal when $\psi \in (\psi_2(p), \psi_1(p))$ and the null contract is optimal when $\psi < \psi_2(p)$.

   (ii) If $\rho(p \mid b) < \mu^w$, the null contract is never optimal. In this case, if $\rho(p \mid b) > \mu_1$, where $\mu_1$ is the threshold belief in Proposition 1 (b), the wage contract is optimal when $\psi \in [\frac{1}{2}, \psi_1(p))$.

When the credit rating is highly precise (i.e., $\psi > \psi_1(p)$), the prohibitive contract prevails, as a bad rating reveals it is very likely that the state is low. At lower precisions, the optimal contract depends on the maximal private benefit $M$. When $M$ is high (part (a)), the wage contract is too expensive, and is never offered. When $M$ is relatively low (part (b)), moderate precision leads to a wage contract if the intermediate belief $\rho$ is sufficiently high. In this situation, imprecise ratings imply a null contract. If the intermediate belief is low (as in part (b) (ii) of the proposition), the likelihood the state is low is large enough that the null contract cannot be optimal, regardless of the precision of the credit rating. If this belief is extremely low (and, in particular, below the threshold $\mu_1$ identified in Proposition 1 (b)), the prohibitive contract prevails even when the rating is fully imprecise (i.e., when $\psi = \frac{1}{2}$). Otherwise, at low rating precisions, the wage contract is optimal.

Next, consider the effect of rating precision when the risky bond has a good credit rating.
Proposition 3 Suppose that \( p \in (p_h, E, y) \) and the credit rating of the risky bond is \( g \). Then, there exists a rating precision \( \psi_3(p) \) such that for \( \psi > \psi_3(p) \), the null contract is optimal. In addition:

(a) If the maximal private benefit is sufficiently high (in particular, \( M > \hat{M}(p, \delta) \)), the prohibitive contract is optimal when \( \psi < \psi_3(p) \). In this case, the wage contract is never optimal.

(b) If the maximal private benefit is sufficiently low (in particular, \( M < \hat{M}(p, \delta) \)), then:

(i) If \( \rho(p \mid g) < \mu_1 \), where \( \mu_1 \) is the belief threshold identified in Proposition 1 (b), there exists an addition rating precision threshold \( \psi_4(p) \) such that the prohibitive contract is optimal if \( \psi \in [\frac{1}{2}, \psi_4(p)] \) and the wage contract is optimal if \( \psi \in (\psi_4(p), \psi_3(p)) \).

(ii) If \( \rho(p \mid g) > \mu_1 \), the prohibitive contract is never optimal. In this case, if \( \rho(p \mid g) < \mu^w \), the wage contract is optimal when \( \psi \in [\frac{1}{2}, \psi_3(p)] \).

The good rating too may influence the contract when the posterior belief about the high state given the price, \( \rho(g \mid p) \) is sufficiently low. In this case, if the rating is sufficiently noisy, updating to \( \mu(g, p) \) still leads to a low probability of the high state, leaving room for the contract to improve the investor’s payoff. If the rating is very precise, the investor prefers to offer the null contract and let the manager invest in the risky asset.

4.2 Effect of Price

We now consider how the optimal contract changes with \( p \), the price of the security. The comparative statics on this variable allow us to develop observable implications for returns in the next section.

Recall that the intermediate belief \( \rho(p \mid \sigma) \), as defined in equation (3), is a function of the prior probability the state is high, \( \phi \), and the densities of the noise terms in the two states. In Assumption 1 part (iii), we have assumed that \( \rho(p \mid \sigma) \) is weakly increasing in \( p \). When \( p < \underline{p_h} \), the state must be low, so that \( \rho(p \mid \sigma) = 0 \) for prices in this range. Similarly, when
\( p > E, y \), the state must be high, so that \( \rho(p \mid \sigma) = 1 \) for prices in this range. We make the following assumption on \( \rho(p \mid \sigma) \) when prices are partially informative.

**Assumption 2** For each \( \sigma \in \{g, b\} \),

(i) \( \rho(p \mid \sigma) \) is differentiable and strictly increasing for \( p \in (\underline{p}_h, E, y) \).

(ii) For \( z = \mu^n(p), \mu^w(p), \) and \( \mu_1(p) \), where \( \mu_1(p) \) is the threshold belief from Proposition 1 part (b), \( \rho(p \mid \sigma) = z \) implies that \( \frac{d\rho}{dp} > \frac{dz}{dp} \).

Part (i) is natural; it implies that over the range \( (\underline{p}_h, E, y) \), a higher price implies a strictly higher intermediate probability that the state is high. Part (ii) ensures that the contracting problem is well-behaved: At each of the three critical thresholds \( \mu^n(p), \mu^w(p), \) and \( \mu_1(p) \), the intermediate belief rises at a faster rate than the threshold.

The next proposition outlines how the optimal contract varies with the price.

**Proposition 4** Suppose that \( p \in (\underline{p}_h, E, y) \) and the credit rating of the risky bond is \( \sigma \). Then, there exist price thresholds \( p_1(\psi, \sigma), p_2(\psi, \sigma) \in (\underline{p}_h, E, y) \) such that the prohibitive contract is optimal if \( p < p_1(\psi, \sigma) \) and the null contract is optimal if \( p > p_2(\psi, \sigma) \). Further,

(a) If the maximal private benefit \( M \) is sufficiently large (in particular, \( M > \hat{M}(p, \delta) \)), then \( p_1(\psi, \sigma) = p_2(\psi, \sigma) \), and the wage contract is never optimal.

(b) If the maximal private benefit \( M \) is sufficiently small (in particular, \( M < \hat{M}(p, \delta) \)), then \( p_1(\psi, \sigma) < p_2(\psi, \sigma) \), and the wage contract is optimal when \( p \in (p_1(\psi, \sigma), p_2(\psi, \sigma)) \).

The key intuition is that higher prices signal that, all else equal, the state is more likely to be high. Further, the posterior belief \( \mu(\sigma, p) \) is both continuous and takes on all values between 0 and 1 as \( p \) varies from a low value of \( \underline{p}_h \) to a high value of \( E, y \). Thus, at low prices (below \( p_1(\psi, \sigma) \)), the prohibitive contract is optimal. Conversely, at high prices (above \( p_2(\psi, \sigma) \)), the null contract is optimal. As is familiar by now, when the maximal private benefit \( M \) is high, the wage contract is not optimal for any values of the parameters. However, when this maximal private benefit is low, there is an intermediate region of posterior belief.
(which translates into a region of credit rating precision) at which the wage contract is optimal.

Observe that the exact thresholds \( p_1(\cdot) \) and \( p_2(\cdot) \) depend both on the rating precision and the realized credit rating, \( \sigma \). In particular, if the intermediate belief \( \rho(p \mid \sigma) \) is independent of the rating \( \sigma \), it follows that the posterior belief given a good rating, \( \mu(g, p) \), exceeds the belief given a bad rating, \( \mu(b, p) \). In this situation, when \( M \) is low, we expect that \( p_1(b, p) > p_1(g, p) \) and \( p_2(b, p) < p_2(g, p) \).

### 4.3 Example

To illustrate our results, consider the following example. Suppose the probability the risky bond fully repays in state \( h \) (\( g_h \)) is 0.9 and the corresponding probability in state \( \ell \) (\( g_\ell \)) is 0.7. If the risky bond defaults, the expected recovery rate is 50\%, so that its expected cash flow is 0.5. Hence, \( E_h y = 0.95 \) and \( E_\ell y = 0.85 \). Set \( \bar{\epsilon} = 0.16, \delta = 0.24, \) and \( M = 0.08 \). Observe that, given these parameters, \( \hat{M}(p, \delta) > 0.19 \) for all \( p \in (p_h, E_\ell y) \), so that throughout we have \( M < \hat{M}(p, \delta) \). Finally, let \( \rho(p \mid \sigma) = \left( \frac{p - p_h}{E_\ell y - p_h} \right)^{0.5} \) for \( p \in [p_h, E_\ell y] \) (note that \( \rho(\cdot) \) is independent of \( \sigma \) in this example).

Figure 4 shows the optimal contracts when the rating is \( g \) (left figure) and \( b \) (right figure) for different values of the price \( p \) and the rating precision \( \psi \). All else equal, a high price signals that the state is more likely to be high. Thus, in Figure 4 (a), which is conditional on a good credit rating for the risky bond, at high prices the optimal contract is the null contract. Further, the range of prices at which the null contract is optimal increases with the precision of the credit rating, as a more precise rating increases the likelihood that a good rating signals the high state. Below the dashed blue line, if either the rating precision \( \psi \) or the price \( p \) is low, the optimal contract is a wage contract. Finally, if both rating precision and the price are very low (in the region to the left of the solid red line), despite the good rating the principal finds it optimal to offer the prohibitive contract, and ensure that the manager invests in the riskless bond.

Next, consider Figure 4 (b), which assumes that the rating on the risky bond is bad.
This figure shows the optimal contract for each price and signal precision when the rating is \( g \) (figure (a)) and when the rating is \( b \) (figure (b)). In each figure, the prohibitive contract is optimal in the region to the left of the solid red line, the wage contract is optimal in the region between the solid red and dashed blue lines, and the null contract is optimal to the right of the dashed blue line. The parameters are \( g_h = 0.9, g_e = 0.7, E(y \mid \text{default}) = 0.5, \bar{\epsilon} = 0.16, \delta = 0.24, \) and \( M = 0.08 \). For \( p \in [p_h, E_{\ell}y] \), we take \( \rho(p \mid \sigma) = \left( \frac{p - p_h}{E_{\ell}y - \bar{p}_h} \right)^{0.5} \).

Figure 4: Optimal Contracts as Price and Rating Precision Vary

Here, at low prices and high rating precisions, the likelihood the state is low is large, so it is optimal to offer the prohibitive contract. Over a large range of intermediate prices and rating precisions, it is optimal to offer a wage contract. Finally, in the bottom right of the figure, if the price is high and the rating precision is low, there is a large likelihood the state is high, so the null contract is offered.

Looking at the contracts \((w, A)\), it is clear that the rating is relevant to the contract for most parameter values, except in the southwest (where the prohibitive contract is offered for both ratings) and southeast corners (where the null contract is offered for both ratings). The common feature of these regions is that the price is highly informative and the rating precision is low; the combination implies that the rating adds no value to the contract. In all other regions, the contract depends on the rating. Note that when wage contracts are offered,
the wage given a good rating is different from the wage given a bad rating. In particular, all else equal, the posterior belief \( \mu \) is lower when the rating is bad, and so higher wages are offered in this case. Further, over the region that the wage contract is optimal, the wages offered vary with price and rating precision.

5 Discussion

As mentioned in the introduction, the view that credit ratings provide market participants with new information about the security or the issuer is pervasive in the academic literature. Some (but not all) implications for market observables are similar across the contracting and information views. However, the policy implications are very different, making it important to differentiate between the two views.

5.1 Implications for Market Observables

We begin by discussing market reactions that are similar under both the contracting and information views. First, if the delegated portfolio management sector is large, the mere fact that bond prices react to rating changes cannot distinguish between the two views. In our model, when the price is not fully informative, an increase in the credit rating from \( b \) to \( g \) leads to more managers buying the risky bond in state \( \ell \) regardless of the contract offered, and possibly in state \( h \) as well. If the delegated portfolio management sector is large, this will have a feedback effect on price as long as there are arbitrage frictions that lead to an upward-sloping supply curve, with the increased demand leading to a price increase. Thus, our model is consistent with the results of Tang (2009), who finds that when Moody’s refined its rating system to include + and – levels, there was a response in bond prices, and Cornaggia, Cornaggia, and Israelsen (2018), who show that when Moody’s revised its rating scale for municipal bonds, prices reacted accordingly. Therefore, such results cannot establish by themselves that ratings contain new information.

Observation 1  An increase (decrease) in a bond price after a credit rating upgrade (down-
grade) is consistent with both the contracting and the information views.

In applying our model, we expect both the rating precision $\psi$ and the private benefits $m$ to vary across bond types. For example, rating precision is likely to be high for corporate and municipal bonds, and low for new kinds of securities such as non-agency mortgage-backed securities. The private benefit $m$, as mentioned in Section 2, is likely to be high for bonds with high mark-ups.

Consider the effect of rating precision. In the information view, the rating releases new information to the market. Thus, the scope for the manager to benefit from their own private information is higher when the rating precision $\psi$ is lower. In the contracting view, when $\psi$ is high, the manager is often either prohibited from holding a bad-rated bond or is incentivized to do so through the wage. In our model, the bond’s cash flow distribution is independent of its rating, so bad-rated bonds, which have lower prices, have higher returns. Thus, preventing a manager from investing in them leads to a lower alpha relative to a benchmark that has both kinds of bonds. Conversely, when $\psi$ is low, wage and null contracts are more prevalent, allowing the manager to continue to invest in bad-rated bonds, and thus generate a higher gross alpha.

**Observation 2** Under both views, managers have a higher gross alpha when trading in assets for which the rating precision $\psi$ is lower.

Differences between the contracting and information views emerge when we consider either the release of a rating on a new bond or a change in rating on an existing one. To the extent that a credit rating contains new information, some informed traders are likely to possess the information beforehand as well. When the credit rating is issued, the information is released publicly to the entire market. Therefore, measures of adverse selection in the stock (such as, e.g., microstructure or spread-based measures) should decrease. In the contracting view, the release of a credit rating should have no effect on adverse selection in the market for a stock (recall that, in our model, the state is known to both parties).
Observation 3 *The release of a credit rating should decrease adverse selection in the market for the stock of a firm under the information view, and leave it unchanged under the contracting view.*

Consider a change in the credit rating on a bond. To the extent such a change releases information to the market, it reduces the manager’s informational advantage, lowering their alpha. In the contracting view, the effect on alpha is asymmetric. All else equal, a switch from a good to a bad rating will lead to a reduction in alpha under a prohibitive or wage contract, whereas a switch from a bad to a good rating leads to an increase in alpha.

Observation 4 *In the information view, a change in the credit rating of a bond reduces the manager’s alpha. In the contracting view, gross alpha falls if the released rating is bad and increases if the released rating is good.*

Rating changes and rating precision have implications for bond turnover. Suppose a bond rating changes from good to bad. In the contracting view, when \( \psi \) is high: (i) prohibitive contracts are more likely to be offered, and (ii) the wage in a wage contract is higher than when \( \psi \) is low. Thus, if the rating changes from good to bad, a manager is either outright forced to sell the bond, or is provided a greater incentive to hold the riskless bond, leading to high turnover in both cases. Conversely, when \( \psi \) is low, it is less likely that a manager sells the bond. The reverse effect occurs when the rating changes from bad to good. The information view, however, does not make a clear prediction on the relationship between turnover and rating precision.

Observation 5 *Under the contracting view, a change in the credit rating of a bond has a larger effect on turnover for assets with a high rating precision.*

Finally, consider the cross-sectional implications of the private benefit varying across types of bonds. All else equal, an increase in the maximal private benefit \( M \) reduces the wages in wage contract. As the zero wage in our model is just a normalization to a base salary level, a positive wage corresponds to the slope of the incentive contract. Thus, our model
predicts that incentives are less high-powered for managers who trade in bonds with high mark-ups. The information view again does not offer a clear prediction on this point. Two caveats are in order. First, note that this prediction holds assuming all else is held equal, including the rating precision. Second, in measuring the steepness of the incentive contract, it is important to take into account that, for a fund manager, current performance connects to future compensation through fund flows.

**Observation 6** Holding rating precision constant, under the contracting view, incentive contracts should be less steep when bonds have higher mark-ups.

### 5.2 Policy Implications

We focus on three policy implications of our model: (1) Should contracts and regulations be contingent on credit ratings? (2) What is the optimal precision of credit ratings? (3) Who should pay for ratings?

**Contracts in ratings**

In our model, we focus on contracts between an investor and a fund manager, but the intuition applies just as straightforwardly when the principal is a regulator and the agent is a relevant participant in the financial market. That is, when states are unverifiable (and therefore regulation cannot be contingent on them), the use of credit ratings in regulation should reduce the cost to a regulator of inducing the right behavior from market participants. As Kisgen and Strahan (2010) point out, credit ratings have been used in regulation in the US since 1931, to regulate institutions including banks, mutual funds, pension funds, and insurance companies. Our model offers a justification for this use of credit ratings.

However, there has been much criticism of the use of credit ratings in regulation. In the models of Opp, Opp, and Harris (2013) and Kartasheva and Yilmaz (2014), the use of ratings in regulation has an adverse effect on the quality of the rating, and leads to rating inflation. In both these models, welfare is enhanced by reducing the use of ratings in regulation. Consistent with this, the Dodd-Frank Act of 2010, in Section 939A, requires
each US federal agency to substitute an appropriate “standard of credit-worthiness” instead of credit ratings in regulations.

We disagree with this view. Our results on the benefits of ratings are qualitatively robust to frictions (such as rating inflation) in the rating process. Such frictions effectively lead to a lower precision of the credit rating, but imprecise ratings too are valuable in our setting. We argue that ratings should be banned only if other signals about the state (such as bond or CDS prices) are relatively precise. Of course, in this case, rational investors would anyway reject the use of ratings, so a ban is immaterial. Therefore, moves such as those in the European Union in 2012 to ban the use of credit ratings are short-sighted at best. In designing regulation on ratings, it is critical to remember the positive role ratings play in contracting.

Optimal precision of ratings

Many of the ill-effects of ratings-based regulation are tied to rating inflation, which, under the information view can lead to over-investment in bad projects. If there is no technological cost to increasing the precision of ratings, greater precision increases surplus by preventing inefficient investment.

In our model, the total surplus in the transaction between the investor and the manager includes the private benefit of the manager, $m$, whenever they buy the risky bond in state $\ell$. Suppose the precision of the rating improves, and the optimal contract at some price changes from a wage contract to a prohibitive contract. In a prohibitive contract, the manager loses their entire private benefit. Thus, while the investor clearly benefits from the increased precision, at the threshold at which the investor switches from a wage to a prohibitive contract, the manager’s payoff will go down, as will the total surplus in the transaction. This feature arises from the fact that the wage contract leaves the manager with rent, earned partly through the wages themselves and partly through the private benefit, whereas the prohibitive contract removes both sources of rent.

A second potential pitfall to increasing the precision of ratings is that the widespread use of credit ratings in the optimal contract induces a correlation in the actions of portfolio managers. In turn, fixing the state (and therefore the fundamentals on the bond), this induces
a difference in the returns of a bond with a good rating and one with a low rating. That is, the bond return is volatile even when its fundamentals are held fixed, simply because of a noisy credit rating.

**Paying for ratings**

The issuer-pay model for credit ratings has been widely criticized, with some suggesting that the investor- or subscriber-pay model provides better incentives for a credit rating agency.\(^{10}\) Kashyap and Kovrijnykh (2016) point out that, under the investor-pay model, investors may seek ratings too often relative to the social optimum. In our model, as mentioned above, the payoff of the manager decreases once the prohibitive contract is used. Thus, to the extent that ratings transfer surplus from manager to investors, the investor and manager will disagree on the desired precision in credit ratings, with managers preferring a lower precision. Therefore, if investors delegate the authority to request and pay for credit ratings to a fund manager, even in an investor-pay model, low quality ratings may persist.

### 6 Conclusion

We show that an investor should condition on both the rating and the price of a risky bond to set the terms of the contract with an investment manager. More broadly, the price in our model can be thought of as a proxy for all non-rating information available to the investor. Our results then imply that ratings should be used in contracts if their marginal informativeness, given all other information the investor has, is sufficiently high.

What properties should a contractible signal have? This is an intriguing question, and one that we defer to future work. For now, a few observations are in order. First, a contractible signal should be forward-looking. Thus, macro-economic indices at the national level or reports by auditors at the firm level, which are inherently backward-looking, would be less useful. Second, the contracting signal cannot be too volatile—contracts have to be enforce-

\(^{10}\)See, for example the discussion on the respective models in “Report to Congress on Assigned Credit Ratings,” prepared by the Staff of the Division of Trading and Markets of the U.S. Securities and Exchange Commission, December 2012.
able, and if signals change at too high a frequency relative to the actions of the manager, it is difficult to determine if he behaved appropriately given the contract. Given the way they are currently structured, credit ratings have a few characteristics that make them extremely useful in contracts—they are stable, change relatively infrequently, and are forward-looking.

In conclusion, we note that for close to a century credit ratings have been produced and used in financial markets. In order to ensure that they provide the largest social benefit, it is important to understand how they add value. Fleshing out the implications of various possible uses is the first step to such an understanding.
Appendix: Proofs

Proof of Lemma 1

(i) Suppose that $p \in (E_{\ell}, E_{h})$. Then, the state must be $h$. Consider the contract with $w(V, y \mid \sigma, p) = 0$ for all $V, y, \sigma, p$, and $A(\sigma, p) = \{a_y, a_f\}$ for all $\sigma, p$. As the state is $h$, faced with this contract, the manager purchases the risky bond; i.e., takes action $a_y$. Further, this action is optimal for the investor. Thus, the specified contract achieves the desired action at zero cost, and so must be optimal.

(ii) Suppose that $p \in [p_{\ell}, p_h)$. Then, the state is $\ell$. Consider the contract with $w(V, y \mid \sigma, p) = 0$ for all $V, y, \sigma, p$, and $A(\sigma, p) = \{a_f\}$ for all $\sigma, p$. Then, the manager must buy the risk-free bond; as the state is $\ell$, this action is optimal for the investor. Again, the specified contract achieves the desired action at zero cost, and so must be optimal. ■

Proof of Lemma 2

The prohibitive contract has zero wages and forces the manager to invest in the risk-free bond, and so has an expected payoff to the investor equal to 1. The expected payoff to the investor from the null contract, $\Pi^n$ is exhibited in equation (6). The inequality $\Pi^n > 1$ is equivalent to

$$\mu(\sigma, p) > \frac{\delta - (E_{\ell}y - 1)}{\delta + E_{h}y - E_{\ell}y} = \frac{\delta - R_{\ell}(p)}{\delta + R_{h}(p) - R_{\ell}(p)} = \mu^n(p), \quad (9)$$

where for each $s$, $R_s = \frac{E_s y}{p} - 1$ is the net return of the risky bond.

Similarly, the inequality $\Pi^n < 1$ is equivalent to $\mu(\sigma, p) < \mu^n(p)$. ■

Proof of Lemma 3

(i) There are two steps to the proof of part (i). Fix $\sigma$ and $p$. In what follows, in the notation for convenience we suppress the dependence of $w^y_r, w^f_r, w^y_d$ and $w^f_d$ on $\sigma$ and $p$.

Step 1: In an optimal wage contract, at least one out of $w^f_r$ and $w^y_d$ must equal zero, and similarly at least one out of $w^f_d$ and $w^y_r$ must equal zero.
First, suppose that $w^f_d > 0$ and $w^y_d > 0$. Reduce each by some small amount $\gamma > 0$. Then, $\hat{w}^f_\ell$ and $\hat{w}^y_\ell$ each fall by the amount $(1 - g_\ell)\gamma$, and $\hat{w}^f_h$ and $\hat{w}^y_h$ both fall by the amount $(1 - g_h)\gamma$. Therefore, the agent takes the same decision as before in state $h$, and, fixing $m$, takes the same decision as before in state $\ell$. However, payments to the agent have fallen, so the conjectured contract could not be optimal. Hence, at least one of $w^f_d$ and $w^y_d$ must be zero.

Next, suppose that $w^f_r > 0$ and $w^y_r > 0$. Again, reduce each by a small amount $\gamma > 0$. Then, both $\hat{w}^f_\ell$ and $\hat{w}^y_\ell$ fall by $g_\ell\gamma$, and both $\hat{w}^f_h$ and $\hat{w}^y_h$ fall by $g_h\gamma$. Therefore, the agent’s decision in state $h$ is unaffected, and, fixing $m$, their decision in state $\ell$ is also unaffected. However, payments to the agent have fallen, so again the conjectured contract could not be optimal. Hence, at least one of $w^f_d$ and $w^y_d$ must be zero.

Step 2: In an optimal wage contract, $w^y_d = w^f_r = 0$.

Given Step 1, there are now four cases to consider:

(a) $w^f_d = w^f_r = 0$. Observe that the investor’s payoff is now strictly decreasing in $\hat{w}^y_h$ and $\hat{w}^y_\ell$.

Therefore, it is optimal to set $w^y_d = w^y_\ell = 0$.

(b) $w^f_d = w^y_r = 0$, but $w^f_r > 0$ (otherwise we are back in case (a)). Here, $w^y_d$ may or may not equal zero.

First, suppose that $w^y_d = 0$. Consider the following adjustment to the contract: decrease $w^f_r$ by a small amount $\gamma > 0$, and increase $w^f_d$ by $\frac{g_\ell g_t}{1 - g_\ell}\gamma$. This adjustment leaves $\hat{w}^f_\ell = g_\ell w^f_\ell + (1 - g_\ell)w^f_d$ the same. Therefore, the payoff in state $\ell$ is unchanged. Consider the effect on $\hat{w}^f_h$. We have

\[
\hat{w}^f_h = g_h(w^f_r - \gamma) + (1 - g_h)\frac{g_\ell}{1 - g_\ell}\gamma = g_h w^f_r + \left(\frac{(1 - g_h)g_\ell}{1 - g_\ell} - g_h\right)\gamma.
\] (10)

Now, $g_h > g_\ell$ implies that $g_h > \frac{(1 - g_h)g_\ell}{1 - g_\ell}$. Therefore, $\hat{w}^f_h$ decreases. As $\hat{w}^y_h = 0$, the agent’s decision in state $h$ is unaffected, but is achieved at a smaller expected wage. Therefore, the original contract could not have been optimal. Observe that the same argument goes
through whenever \( w^y_d > 0 \) but \( \hat{w}^y_h = (1 - g_h)w^y_d < \hat{w}^f = g_h w^f \).

Now, suppose that \( w^f > 0 \) and \( w^y > 0 \), with \( \hat{w}^y_h \geq \hat{w}^f \). Decrease \( w^f \) by a small amount \( \gamma > 0 \), and decrease \( w^y \) by an amount \( \frac{g_h}{1-g_h} \gamma \). Then, the difference \( \hat{w}^f - \hat{w}^y \) is unchanged, so the decision in state \( \ell \) is unchanged. However, the expected wage in this state is reduced. In this case, \( \hat{w}^y_h \) falls by \( \frac{1-g_h}{1-g_h} \gamma \), and \( \hat{w}^f_h \) falls by \( g_h \gamma \). As argued, \( g_h > \frac{1-g_h}{1-g_h} \gamma \), so that, after the change, \( \hat{w}^y_h - \hat{w}^f_h > 0 \). Therefore, the decision in state \( h \) is also unaffected, but is obtained at a lower wage. Hence, the original contract could not have been optimal.

Therefore, it cannot be that \( w^f = w^y = 0 \).

(c) \( w^y_d = w^y_r = 0 \).

Suppose that \( w^f > 0 \). Then, we can achieve a superior outcome by increasing \( w^f \) by a small amount and reducing \( w^f \) while keeping \( \hat{w}^f \) the same. Such a change reduces \( \hat{w}^f_h \), and results in the agent taking the same actions in each state with the expected wage remaining constant in state \( \ell \) and falling in state \( h \). Therefore, it must be that \( w^f = 0 \).

(d) \( w^f = w^y = 0 \).

This outcome obtains in cases (a) and (c) above as well, and case (b) has been ruled out. Therefore, it must be that \( w^f = w^y = 0 \).

(ii) From step (i), we have that \( w^f = w^y_d = 0 \). Suppose that \( w^y > 0 \) (else, as shown in case (i) (c) above, it also follows that \( w^f = 0 \)).

Observe that it must be that \( w^f_d > 0 \). Otherwise, we can reduce \( w^y \) to zero, which leaves the action in both states unchanged while lowering the expected wage in each state.

Suppose that \( \hat{w}^y_h > \hat{w}^f_h \). Consider some small \( \gamma > 0 \). Increase \( w^f_d \) by an amount \( \frac{\gamma}{1-g_h} \), and reduce \( w^y \) by \( \frac{\gamma}{g_d} \). This changes leaves \( \hat{w}^f - \hat{w}^y \) unchanged, so cannot affect the agent’s actions in state \( \ell \). After the change, if \( \gamma \) is small, it must be that \( \hat{w}^y_h \) continues to exceed \( \hat{w}^f_h \). Further, \( \hat{w}^y_h \) is clearly lower than before. Hence, the same action in each state is induced at
with strictly lower expected wages in each state, so that the original contract could not have been optimal.

Next, suppose that \( \hat{w}_h^y < \hat{w}_h^f \). In this case, for some small \( \gamma > 0 \), we can decrease \( w_d^f \) by an amount \( \frac{\gamma}{y} \) and increase \( w_d^y \) by an amount \( \frac{\gamma}{y} \). This leaves \( \hat{w}_\ell^y - \hat{w}_\ell^y \) unchanged, but reduces each of \( \hat{w}_\ell^f, \hat{w}_\ell^y, \) and \( \hat{w}_h^f \). For \( \gamma \) small enough, it must be that \( \hat{w}_h^f \) continues to exceed \( \hat{w}_h^y \). Therefore, actions in either state are unchanged, but the expected wage in each state falls. Thus, the original contract could not have been optimal.

Hence, it must be that \( \hat{w}_h^y = \hat{w}_h^f \), or \( g_h w_d^y = (1 - g_h) w_d^f \), so that \( w_d^y = \frac{1 - g_h}{g_h} w_d^f \) and \( \hat{w}_h^y = (1 - g_h) w_d^f \). Further, \( \hat{w}_\ell^f = (1 - g_\ell) w_d^f \), and \( \hat{w}_\ell^y = \frac{g_\ell (1 - g_h)}{g_h} w_d^f \). Therefore, \( \hat{w}_\ell^y - \hat{w}_\ell^y = \left(1 - g_\ell - \frac{g_\ell (1 - g_h)}{g_h}\right) w_d^f = \frac{g_h - g_\ell}{g_h} w_d^f \geq 0 \).

(iii) Suppose that \( \hat{w}_\ell^y - \hat{w}_\ell^y \leq M \). Then, keeping in mind that \( \hat{w}_h^y = \hat{w}_h^f \), we can write the payoff of the investor as:

\[
\Pi(\sigma, p) = \mu(\sigma, p) \left( \frac{E_h y}{p} - \hat{w}_h^y \right) + (1 - \mu(\sigma, p)) \left( \frac{\hat{w}_\ell^y - \hat{w}_\ell^y}{M} \right) (1 - \hat{w}_\ell^f) \\
+ \left(1 - \hat{w}_\ell^y - \hat{w}_\ell^y\right) \left( \frac{E_\ell y}{p} - \delta - \hat{w}_\ell^y \right) \\
= \mu(\sigma, p) \frac{E_h y}{p} + (1 - \mu(\sigma, p)) \left( \frac{E_\ell y}{p} - \delta \right) - \mu(\sigma, p) \hat{w}_h^y + \\
(1 - \mu(\sigma, p)) \left( - \hat{w}_\ell^y + \frac{\hat{w}_\ell^y - \hat{w}_\ell^y}{M} \left(1 - \frac{E_\ell y}{p} + \delta - (\hat{w}_\ell^f - \hat{w}_\ell^f)\right) \right) \\
\] (11)

Making the appropriate substitutions for \( \hat{w}_h^y, \hat{w}_h^f, \) and \( \hat{w}_\ell^f \), we have

\[
\Pi(\sigma, p) = \mu(\sigma, p) \frac{E_h y}{p} + (1 - \mu(\sigma, p)) \left( \frac{E_\ell y}{p} - \delta \right) - \mu(\sigma, p) (1 - g_h) w_d^f + \\
(1 - \mu(\sigma, p)) \left\{ - \frac{g_\ell (1 - g_h)}{g_h} w_d^f + \frac{1}{M} \frac{g_h - g_\ell}{g_h} \left(1 - \frac{E_\ell y}{p} + \delta - g_h - g_\ell w_d^f\right) \right\} \\
= \mu(\sigma, p) \frac{E_h y}{p} + (1 - \mu(\sigma, p)) \left( \frac{E_\ell y}{p} - \delta \right) - \mu(\sigma, p) (1 - g_h) w_d^f + \\
(1 - \mu(\sigma, p)) w_d^f \left\{ - \frac{g_\ell (1 - g_h)}{g_h} + \frac{1}{M} \frac{g_h - g_\ell}{g_h} \left(1 - \frac{E_\ell y}{p} + \delta - g_h - g_\ell w_d^f\right) \right\} \\
\] (13)
The first-order condition is \( \frac{\partial \Pi}{\partial w_d} = 0 \), or
\[-\mu(\sigma, p)(1 - g_h) + (1 - \mu(\sigma, p)) \left\{ - \frac{g_\ell (1 - g_h)}{g_h} + \frac{1}{M} \frac{g_h - g_\ell}{g_h} \left( 1 - \frac{E_\ell y_p}{p} + \delta \right) \right\} - 2 \frac{1}{M} \left( \frac{g_h - g_\ell}{g_h} \right)^2 w^f_d \right\} = 0 \] (15)
\[ w^f_d = \frac{g_h}{2(g_h - g_\ell)} \left[ 1 - \frac{E_\ell y_p}{p} + \delta - \frac{g_h (1 - g_h)}{g_h - g_\ell} \left( \frac{\mu(\sigma, p)}{1 - \mu(\sigma, p)} + \frac{g_\ell}{g_h} \right) M \right]. \] (16)

It is immediate to see that the second-order condition is satisfied. Recognizing that \( \frac{g_h}{g_h - g_\ell} = \frac{1}{1 - \frac{g_\ell}{g_h}} \), this is the same expression as in the first part of the “min” term in equation (8).

Now, observe that it cannot be optimal to set \( \hat{w}^f_\ell - \hat{w}^y_\ell > M \). Recall from part (ii) that at the optimal wages \( \hat{w}^f_\ell - \hat{w}^y_\ell = \frac{q_h - q_\ell}{q_h} w^f_d \). When this quantity equals \( M \), we already have the manager taking the correct action in state \( \ell \) (i.e., investing in the riskless bond) with probability 1. Increasing the difference \( w^f_d \) at this point merely adds to the cost of the contract without changing the action of the manager in either the high or the low state, so must be sub-optimal. Therefore, the wage exhibited in equation (16) is optimal only if 
\[ (1 - \frac{w^f_d}{g_h})w^f_d \leq M; \] or, equivalently, \( w^f_d \leq M \frac{1}{1 - \frac{g_\ell}{g_h}} \). The latter is the second expression in the “min” term in equation (8).

Finally, note from equation (8) that \( w^f_d \geq 0 \) is equivalent to 
\[ \frac{\mu(\sigma, p)}{1 - \mu(\sigma, p)} \leq \frac{1 - \frac{q_\ell}{q_h}}{1 - g_h} \frac{\delta - R_\ell(p)}{M} - \frac{q_\ell}{g_h}. \]
Define \( \mu^w \) as the value of \( \mu(\sigma, p) \) that satisfies this weak inequality as a strict equality. That is, denoting \( \lambda(\delta, p, M) = \frac{1 - \frac{q_\ell}{q_h}}{1 - g_h} \frac{\delta - R_\ell(p)}{M} - \frac{q_\ell}{g_h} \),
\[ \mu^w = \frac{\lambda(\delta, p, M)}{1 + \lambda(\delta, p, M)}. \] (17)

The statement of part (iii) now follows. \( \blacksquare \)

**Proof of Proposition 1**

(a) For convenience, denote \( w = w^f_d \). Then, the payoffs to the investor from the three
contracts, prohibitive, null, and wage, are as follows:

\[
\Pi^{\text{PRO}} = 1
\]

\[
\Pi^{n}(\sigma, p) = \mu(\sigma, p) \frac{E_y y}{p} + (1 - \mu(\sigma, p)) \left( \frac{E_y y}{p} - \delta \right)
\]

\[
\Pi^{w}(\sigma, p) = \mu(\sigma, p) \left\{ E_y y - (1 - g)w \right\} + (1 - \mu(\sigma, p)) \left\{ (1 - g)w \frac{M}{M} + \left( 1 - (1 - g) \frac{w}{M} \right) \left( \frac{E_y y}{p} - \delta - (1 - g)w \right) \right\}
\]

Define \( \hat{M}(\delta, p) \) as the value of \( M \) at which \( \mu^{w} = \mu^{n} \). Observe that \( \lambda(\delta, p, M) \) (as defined at the end of the proof of Lemma 3) is strictly decreasing in \( M \); it follows from equation (17) that \( \mu^{w} \) is strictly decreasing in \( M \). From the definition of \( \mu^{n} \) (in the text immediately before Lemma 2), it is immediate that \( \mu^{n} \) is invariant to \( M \). Therefore, if \( M < \hat{M}(\delta, p) \), we have \( \mu^{w} > \mu^{n} \), and if \( M < \hat{M}(\delta, p) \), we have \( \mu^{w} < \mu^{n} \).

(a) Suppose that \( M \geq \hat{M}(\delta, p) \). Then, it follows that \( \mu^{w} \leq \mu^{n} \). If \( \mu(p, \sigma) > \mu^{n} \), then by Lemma 2 we have \( \Pi^{n} > \Pi^{\text{PRO}} \), and \( \mu^{n} \geq \mu^{w} \) implies that \( \Pi^{n} \geq \Pi^{w} \). Thus, in this region, it is optimal to offer the null contract. Consider \( \mu(p, \sigma) \in [\mu^{w}, \mu^{n}] \). Here, it follows that \( \Pi^{\text{PRO}} > \Pi^{n} \) and \( \Pi^{n} \geq \Pi^{w} \), so that \( \Pi^{\text{PRO}} > \Pi^{w} \), and the prohibitive contract is optimal. If \( \mu(p, \sigma) < \mu^{w} \), as \( \Pi^{w} \) is decreasing in \( \mu \) and \( \Pi^{\text{PRO}} \) is constant, it must continue to be that \( \Pi^{\text{PRO}} > \Pi^{w} \), and we still have \( \Pi^{\text{PRO}} > \Pi^{n} \). Thus, the prohibitive contract remains optimal in this region. Hence, for \( M \geq \hat{M}(\delta, p) \), it is not optimal to offer a wage contract.

Denote \( \mu_{1} = \mu^{n} \) in this case. The statement of part (a) follows.

(b) Suppose that \( M < \hat{M}(\delta, p) \). It then follows that \( \mu^{w} > \mu^{n} \). Recall that \( \mu^{w} \) is the threshold value of \( \mu \) at which the optimal wage \( w_{d}^{f} \) is exactly equal to zero. By definition, \( \Pi^{w} = \Pi^{n} \) at this threshold.

There are now two cases to consider. First, suppose that \( \mu(\sigma, p) > \mu^{w} \). Then, \( \Pi^{n} > \Pi^{w} \). Further, \( \mu^{w} > \mu^{n} \) implies that in this region \( \Pi^{n} > \Pi^{\text{PRO}} \); that is, the optimal contract is a null contract.

Second, suppose that \( \mu(\sigma, p) < \mu^{w} \), in which case \( \Pi^{w} > \Pi^{n} \). Observe that, as \( \mu^{n} > 0 \),
\( \mu^w > \mu^n \) implies that \( \mu^w > 0 \). If \( \mu(\sigma, p) \in (\mu^n, \mu^w) \), it follows that \( \Pi^w > \Pi^{\text{pro}} \) as well, so that the wage contract is optimal.

Now, observe that (after taking into account that \( w(\mu) \) changes with \( \mu \)) \( \Pi^w \) is strictly increasing in \( \mu \). This follows from the fact that at any fixed \( \mu \), \( \frac{E_{t}\gamma - (1 - g_{t})w}{p} - (1 - g_{t})w \). If \( \mu \) increases by a small amount, it is always feasible for the investor to keep \( w \) fixed, in which case \( \Pi^w \) strictly increases. Therefore, at the optimal \( w \), this property must continue to hold.

At \( \mu = \mu^n \), we have \( \Pi^{\text{pro}} = \Pi^n \) and \( \Pi^w > \Pi^n \), so it follows that \( \Pi^w > \Pi^{\text{pro}} \). Conversely, if \( \mu = 0 \), then the state is low for sure. Here, \( \mu < \mu^w \) implies that \( w_{d}^f > 0 \). If the manager buys the risky bond, the investor obtains \( \frac{E_{t}\gamma - \delta}{p} - (1 - g_{t})w_{d}^f \). If the manager buys the riskless bond, the investor obtains \( 1 - w_{d}^f \). In either case, the payoff to the investor is strictly less than 1, so \( \Pi^w < 1 = \Pi^{\text{pro}} \). Thus, there must exist some \( \mu_1 \in (\mu^n, \mu_w) \) such that \( \Pi^w > \Pi^{\text{pro}} \) when \( \mu(p, \sigma) > \mu_1 \) and \( \Pi^w < \Pi^{\text{pro}} \) when \( \mu(p, \sigma) < \mu_1 \).

Denote \( \mu_2 = \mu^w \) in this case. The statement of part (b) follows.

**Proof of Proposition 2**

Suppose that the rating is \( b \), and fix a price \( p \in (p_h, E_i y) \). From equation (5), we have \( \mu(b, p) = \frac{\rho(p|b)(1-\psi)}{\rho(p|b)(1-\psi)+\rho(\{p\|b\} b)\psi} \), which is decreasing in \( \psi \). Further, when \( \psi = \frac{1}{2} \), \( \mu(b, p) = \rho(p|b) \), and as \( \psi \to 1 \), \( \mu(b, p) \to 0 \). From Proposition 1, regardless of the value of \( M \), there exists some \( \mu_1(p) \) such that the prohibitive contract is optimal when \( \mu(b, p) < \mu_1 \). Define \( \psi_1(p) \) to be the value of \( \psi \) at which \( \mu(b, p) = \mu_1(p) \). As \( \mu(b, p) \) is strictly decreasing in \( \psi \), it follows that for \( \psi > \psi_1(p) \), the prohibitive contract is optimal.

(a) Suppose that \( M \geq \hat{M}(p, \delta) \). Then, from Proposition 1 (a), the wage contract is never optimal. The null contract is optimal if \( \mu(b, p) > \mu^n \), and the prohibitive contract is optimal if \( \mu(b, p) < \mu^n \). As argued above, the condition \( \mu(b, p) > \mu^n \) translates into a threshold rating precision \( \psi_1(p) \) such that \( \mu(b, p) > \mu^n \) if and only if \( \psi < \psi_1(p) \).

(b) Consider \( M < \hat{M}(p, \delta) \).
(i) Suppose that $\rho(p | b) > \mu^w$. From Proposition 1 (b), we know that the null contract is optimal when $\mu(b, p) > \mu^w$. As $\mu(b, p)$ is strictly decreasing in $\psi$ and $\mu(b, p) = \rho(p | b)$ when $\psi = \frac{1}{2}$, it follows that there exists some $\psi_2(p)$ such that when $\psi < \psi_2(p)$, we have $\mu(b, p) > \mu^w$ and the null contract is optimal. Further, when $\psi \in (\psi_2(p), \psi_1(p))$, we have $\mu(b, p) \in (\mu_1, \mu^w)$, where $\mu_1$ is the threshold identified in Proposition 1 (b). In this range of $\mu$ (equivalently, of $\psi$), the wage contract is optimal.

(ii) Suppose next that $\rho(p | b) < \mu^w$. Then, $\mu(b, p) < \mu^w$ for all $\psi \in [\frac{1}{2}, 1]$, so from Proposition 1, the null contract is never optimal. Instead, if $\mu(b, p) > \mu_1$, the wage contract is optimal and if $\mu(b, p) < \mu_1$, the prohibitive contract is optimal, where $\mu_1$ is the threshold identified in Proposition 1 (b). If $\rho(p | b) < \mu_1$, then there is no region of rating precision for which the wage contract is optimal. Conversely, if $\rho(p | b) > \mu_1$, the wage contract is optimal for $\psi \in [\frac{1}{2}, \psi_1(p))$.

Proof of Proposition 3

Suppose that the rating is $g$, and fix a price $p \in (\underline{p}_h, E_y)$. From equation (4), we have $\mu(g, p) = \frac{\rho(p | g) \psi}{\rho(p | g) \psi + (1 - \rho(p | g)) (1 - \psi)}$, which is increasing in $\psi$. Further, when $\psi = \frac{1}{2}$, $\mu(g, p) = \rho(p | g)$, and as $\psi \to 1$, $\mu(g, p) \to 1$. From Proposition 1, regardless of the value of $M$, there exists some threshold ($\mu^n(p)$ when $M \geq \hat{M}$ and $\mu_2(p)$ when $M > \hat{M}$) such that the null contract is optimal when $\mu(g, p)$ exceeds this threshold. Denote this threshold as $v(p, M)$, and let $\psi_3(p)$ be the value of $\psi$ at which $\mu(g, p) = v(p, M)$. As $\mu(g, p)$ is strictly increasing in $\psi$, it follows that for $\psi > \psi_3(p)$, the null contract is optimal.

(a) Suppose that $M \geq \hat{M}(p, \delta)$. Then, from Proposition 1 (a), the wage contract is never optimal. The null contract is optimal if $\mu(g, p) > \mu^n$, and the prohibitive contract is optimal if $\mu(g, p) < \mu^n$. As argued above, the condition $\mu(g, p) > \mu^n$ translates into a threshold rating precision $\psi_3(p)$ such that $\mu(g, p) > \mu^n$ if and only if $\psi > \psi_3(p)$.

(b) Consider $M < \hat{M}(p, \delta)$.

(i) Suppose that $\rho(p | b) < \mu_1$, where $\mu_1$ is the threshold in Proposition 1 (b). From Proposition 1 (b), we know that the prohibitive contract is optimal when $\mu(g, p) < \mu_1$. As
$\mu(g,p)$ is strictly increasing in $\psi$ and $\mu(g,p) = \rho(p \mid g)$ when $\psi = \frac{1}{2}$, it follows that there exists some $\psi_4(p)$ such that when $\psi < \psi_4(p)$, we have $\mu(g,p) < \mu_1$ and the prohibitive contract is optimal. Further, when $\psi \in (\psi_4(p),\psi_3(p))$, we have $\mu(g,p) \in (\mu_1,\mu^w)$. In this range of $\mu$ (equivalently, of $\psi$), the wage contract is optimal.

(ii) Suppose next that $\rho(p \mid g) > \mu_1$. Then, $\mu(g,p) > \mu_1$ for all $\psi \in \left[\frac{1}{2}, 1\right]$, so from Proposition 1, the prohibitive contract is never optimal. In this case, if $\mu(g,p) < \mu^w$, the wage contract is optimal and if $\mu(g,p) > \mu^w$, the null contract is optimal. If $\rho(p \mid g) > \mu^w$, then there is no region of rating precision for which the wage contract is optimal. Conversely, if $\rho(p \mid g) < \mu^w$, the wage contract is optimal for $\psi \in (\frac{1}{2}, \psi_3(p))$.

**Proof of Proposition 4**

Suppose that the price is $p \in (p_h,E_iy)$, and the credit rating is $\sigma$. Observe that, under Assumption 1 part (iii), $\rho(p \mid \sigma)$, the probability of state $h$ given price $p$, is continuous and weakly increasing in $p$. As $\rho(p \mid \sigma) = 0$ for all $p < p_h$, it must be that $\rho(p_h \mid \sigma) = 0$, and similarly it must be that $\rho(E_iy \mid h) = 1$.

Now, $\mu(\sigma,p)$ is continuous in $\rho$ for each $\sigma$. Therefore, it must be that $\mu(\sigma,p)$ is also continuous and increases from 0 at $p = p_h$ to 1 at $p = E_iy$. From Proposition 1, it now follows that there exists some price $p_1(\psi,\sigma)$ such that for $p < p_1(\psi,\sigma)$, we have $\mu(\sigma,p) < \mu_1$, where $\mu_1$ is the threshold in either part (a) or part (b) of Proposition 1 (depending on whether $M$ is greater or less than $\hat{M}(p,\delta)$) such that when $p < p_1(\psi,\sigma)$, we have $\mu(p,\sigma) < \mu_1$, and the prohibitive contract is optimal. Similarly, as $\mu(p,\sigma)$ is continuous, there must exist a threshold $\psi_2(p,\sigma)$ such that, when $p > \psi_2(\sigma)$, we have $\mu(\sigma,p) > \mu_1$ (if $M \geq \hat{M}(p,\delta)$) or $\mu(\sigma,p) > \mu_2$ (if $M < \hat{M}(p,\delta)$), so that the null contract is optimal.

(a) Suppose that $M \geq \hat{M}(p,\delta)$. Then, from Proposition 1 (a), the wage contract is not optimal for any value of $\mu(\sigma,p)$. Instead, if $\mu(\sigma,p) < \mu^n$, the prohibitive contract is optimal, and if $\mu(\sigma,p) > \mu^n$, the null contract is optimal. Define $\psi_1(p,\sigma) = \psi_2(p,\sigma)$ to be the rating precision $\psi$ for which $\mu(\sigma,p) = \mu^n(p)$. From Assumption 2, it follows that this threshold is unique.
(b) Suppose that $M < \hat{M}(p, \delta)$. From Proposition 1 (b), it follows that there exists some range of posterior belief $(\mu_1, \mu^w)$ such that if $\mu(\sigma, p) \in (\mu_1, \mu^w)$, the wage contract is optimal. Defined $\psi_1(p, \sigma)$ to be the rating precision at which $\mu(\sigma, p) = \mu_1$, and $\psi_2(p, \sigma)$ to be the rating precision at which $\mu(\sigma, p) = \mu^w$. Both thresholds depend on $p$ and $M$. Again, from Assumption 2, it follows that these thresholds are unique. The statement of part (b) now follows. ■
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