NOTES

1. Thanks to Neil Paragiri, Joseph Slunt, Michelle McCarthy, Ken Penningtion, and others at Bankers Trust for their comments and assistance.

2. See *Risk Standards for Institutional Investment Managers and Institutional Investors.*

3. Pensionmetrics is one of the few software products that is targeted specifically to pension funds and evaluates the risks highlighted in this chapter.

4. Winkelman (2000), in a discussion on budgeting active risk at the total fund level, ignores the importance of linking active risk to liabilities.

5. See Muralidhar and U (1997).

6. The author thanks Samir Varma and Sanjay Santhanam for discussions on this topic.

8

DECOMPOSING AND UNDERSTANDING RISK

Arun S. Muralidhar, Kemal Asad-Syed, and Paolo Pasquariello

Portfolio managers may take many bets to outperform a benchmark. This chapter provides two simple methodologies to calculate the contribution of a specific bet to total risk, whether the risk measure is an absolute or a relative one, and demonstrates how investors can develop simple in-house models to measure such risk. In addition, it demonstrates how other measures that are not derived from finance theory, but are used as first approximations, are incorrect. These simple tools allow investors to measure and monitor the risks in their portfolios and thereby manage them more effectively.

OVERVIEW

The issue of risk management and risk budgeting is becoming more important for institutional investors, especially pension funds, and a number of working groups have been formed to evaluate the risk standards that should be adopted by oversight committees for the management of such plans. However, a current shortcoming in the industry is that no uniform model has been adopted to measure risks, which would allow pension funds to manage them. In addition, many software providers have focused only on the absolute risk of

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a portfolio in measuring the value-at-risk (VAR) of a portfolio. Most risk systems have often not captured the largest risk that most pension plans are exposed to, namely, asset-liability risk. This was highlighted in Chapter 7.

Further, when the performance of an institutional investor is measured relative to a passive benchmark, it is imperative to measure not only the absolute risk of the benchmark and the actual portfolio, but also the risk relative to a benchmark (Ambarish and Seigel 1996). Chapter 7 provided high-level measures of risk relative to different benchmarks such as (1) liabilities, (2) strategic asset allocation, (3) tactical asset allocation, and (4) specific market indices. Even those software packages that have focused on relative risk have not adequately captured the contribution to total risk of any specific bet taken by portfolio managers, which is termed marginal risk. This measure is important because it provides the plan sponsor with an indication of the concentration of risk and relatedness of bets in the portfolio.

Litterman (1996) highlights the usefulness of this measure and gives an indication of how it may be computed; however, the article does not provide the methodology for the calculation. In addition, Gibson (1997) suggests that pension funds should monitor the contribution to risk of positions, that is, correlated risks attributable to a position vis-à-vis either liabilities or some asset benchmark. When such adjustments are made, the contribution can be termed the marginal contribution, and the sum of all marginals should equal total risk. The appendices to this chapter provide two simple methodologies to calculate the contribution of a specific bet to total risk, whether the risk measure is an absolute or a relative one, and demonstrates how plan sponsors can develop simple in-house models to measure plan risk.

The first approach develops the mathematical technique suggested by Litterman (1996); the second provides a more intuitive approach, which is derived from the basic theory of asset pricing. In addition, this chapter demonstrates how other measures that are not derived from finance theory, but used as first approximations, are incorrect.[1] This approach also provides valuable insight into the correlation of bets with the entire portfolio of bets, thereby enhancing the evaluation of risk-taking activities. These simple tools can allow sponsors to measure and monitor the risks in their portfolios and thereby manage them more effectively. In addition, the measurement of risk can be done at any level—across asset classes, across managers within an asset

class, and across securities in a specific portfolio. In this chapter the analysis is conducted at the highest level of a pension fund—across asset classes—but the extensions are trivial.

This chapter provides the math for the calculations in the chapter appendices, but the nontechnical reader can skip the technical sections to gain key insights without loss of continuity. The chapter also considers the feasibility of using such statistics for the allocation of risk capital. The discussion is developed in the context of a pension plan, but the concepts and conclusions apply more generally to any investor, whether a portfolio manager or an institutional investor with investment advisers.

PENSION PLAN RISKS

Prior to discussing the contribution to risk of a specific bet, the different risks that a plan is exposed to are briefly recapitulated. Risk is generated in pension plans at different levels. At the highest level, selecting a benchmark for the asset portfolio creates the possibility for risks from asset-liability mismatches (or asset-liability risk). Alternatively, selecting an asset benchmark for purely asset reasons implies targeting an absolute risk point or a target variability of returns. At the next level, once target asset-class allocations and benchmarks have been determined, a plan sponsor may create additional risk by investing tactically in the actual portfolio away from these target levels (or tactical risk) (Mashayekhi-Beschloss and Muralidhar 1996). At the simplest level, tactical risk is created by underweighting or overweighting individual asset classes.

In this chapter, the focus is only on (1) the absolute risk of the benchmark portfolio; (2) the absolute risk of the actual portfolio on any given day (which, if tactical bets are permitted, could be quite different from that of the benchmark); and (3) the relative risk implied by the actual portfolio vis-à-vis the benchmark or tactical risk. Thereafter, it is possible to demonstrate the contribution to the total risk or variability of returns of each asset class in which the plan has made either a target allocation or a tactical deviation.[2] The concept of the "marginal" is very well developed in economics in determining optimal consumption, pricing, and so on, and in an analogous fashion this chapter attempts to demonstrate whether the marginal risk measure can be used in the optimal utilization of a risk budget.

DEFINITION OF TERMS

For convenience, two portfolios are defined, namely the benchmark and the actual portfolio, and three risk measures identified: the absolute risk of the benchmark, the absolute risk of the actual portfolio, and the relative risk of the actual portfolio.

Benchmark portfolio: This is the strategic long-term asset allocation of the plan that is described by listing the various asset classes in which the plan is invested and the long-term target allocations. A hypothetical benchmark portfolio is provided in Table 8.1.

Actual portfolio: This is the investor's portfolio on any measurement day. As a consequence of portfolio managers overweighting or underweighting asset classes, the live portfolio can and will differ from the benchmark. For illustrative purposes, a hypothetical actual portfolio is provided in Table 8.1, which is relative to the benchmark. The last column gives the percentage deviation of each asset class from its benchmark; the sum of these deviations is zero.[3]

Absolute risk of benchmark: In asset space, the variance or standard deviation of the expected returns of this portfolio describes the risk of the benchmark portfolio.[4] Mathematically, the absolute risk is

Table 8.1
Benchmark, Actual, and Relative Portfolios: Weighting of Assets

Asset Classes	*Benchmark Portfolio (%) (v)*	*Actual Portfolio (%) (w)*	*Relative Portfolio (%) (z)*
U.S. equities	30.0	32.0	2.0
Non-U.S. equities	35.0	29.0	−6.0
Emerging equities	5.0	8.0	3.0
U.S. fixed income	7.0	9.0	2.0
Non-U.S. fixed income	10.0	8.0	−2.0
High-yield bonds	2.0	4.0	2.0
Private equities	10.0	8.0	−2.0
Cash	1.0	2.0	1.0
Total	100.0	100.0	0.0
Standard deviation	11.69%	11.37%	1.24%

estimated by taking the benchmark or target weights and multiplying them through a variance-covariance matrix:

$$\sigma^2(\text{benchmark}) = (v^T \Gamma v) \tag{8.1}$$

where v = matrix of benchmark asset class weights (v^T is the transpose of v), Γ is the assumed variance-covariance matrix, and v_i is the target weight of the ith asset class. The square root, or the standard deviation, is also a risk measure, as it captures the dispersion of the portfolio return around its mean. Using the hypothetical benchmark portfolio in Table 8.1 and the assumed variance-covariance matrix in the appendix (Table A8.3.1), the standard deviation (i.e., risk) of this portfolio is provided in Table 8.1.[5]

Absolute risk of the actual portfolio: The variance of the actual portfolio is calculated in a fashion identical to that of the benchmark:

$$\sigma^2(\text{actual}) = (w^T \Gamma w) \tag{8.2}$$

where w = matrix of actual asset class weights, and w_i is the actual weight of the ith asset class. The square root or standard deviation is an alternative expression of this risk measure and is provided in Table 8.1.

Relative risk of the actual portfolio: This is the risk engendered by off-benchmark positions. Ambarish and Seigel (1996) demonstrate why this measure should be used when a portfolio is measured relative to a benchmark. The relative risk or variance of the active portfolio is calculated in a fashion identical to those above:

$$\sigma^2(\text{relative}) = (z^T \Gamma z) \tag{8.3}$$

where z = matrix of the differences between the actual and target asset class weights, and z_i is the deviation from benchmark in the ith asset class. Any component of the z matrix can be positive or negative, as the investment team could have chosen to underweight or overweight a particular asset class. The square root of σ^2(relative) per unit of time is referred to as the tracking error of a portfolio. Mathematically,

$$\text{Tracking error} = \sqrt{z^T \Gamma z} = \frac{z^T \Gamma z}{\sqrt{z^T \Gamma z}} \tag{8.4}$$

The tracking error measures the amount by which the performance of the actual portfolio can deviate from the benchmark and is provided in Table 8.1.

The mathematical details for the calculation of marginal risk and correlation of bets are provided in the chapter appendices for the more technical reader. In the next few sections the specific benchmark and actual portfolio are examined to decompose the risks of the portfolio.

A CASE STUDY IN MARGINAL RISK ANALYSIS

Table 8.1 gives a benchmark portfolio and a tactical portfolio that is maintained relative to the benchmark. This hypothetical "actual" portfolio is overweight U.S. equities, U.S. fixed income, and high yield by 2 percent, underweight non-U.S. equities by 6 percent, overweight emerging equities by 3 percent, underweight non-U.S. fixed income and private equities by 2 percent, and overweight cash by 1 percent. This causes a tracking error versus the benchmark of 1.24 percent. Tables 8.2 and 8.3 provide the marginal contribution to total risk (in percentage points) and percentage contribution to total risk, respectively, for the portfolios in Table 8.1. These diagnostics provide a number of useful insights. It is apparent from the tables that the risk is

Table 8.2
Contribution to the Standard Deviation of the Three Portfolios

	Absolute Risk		
Asset Classes	*Benchmark (%)*	*Actual (%)*	*Relative Portfolio (%)*
U.S. equities	3.8	4.1	−0.045
Non-U.S. equities	5.8	4.9	0.783
Emerging equities	0.5	0.8	0.241
U.S. fixed income	0.1	0.2	0.019
Non-U.S. fixed income	0.2	0.2	0.001
High-yield bonds	0.1	0.1	0.040
Private equities	1.2	1.0	0.203
Cash	0.0	0.0	0.000
Total standard deviation	11.69%	11.37%	1.24%

Table 8.3
Percentage Contribution to the Standard Deviation of the Three Portfolios

	Absolute Risk		
Asset Classes	*Benchmark (% of Total)*	*Actual (% of Total)*	*Relative Portfolio (% of Total)*
U.S. equities	32.2	36.2	−3.7
Non-U.S. equities	49.7	43.4	63.1
Emerging equities	4.4	7.4	19.4
U.S. fixed income	1.1	1.5	1.5
Non-U.S. fixed income	1.6	1.3	0.1
High-yield bonds	0.6	1.3	3.3
Private equities	10.5	8.8	16.4
Cash	0.0	0.0	0.0
Total standard deviation	100.0	100.0	100.0

Table 8.4
Implied Correlation of Asset Class Bet to Portfolio of Bets

Asset Classes	*Relative Portfolio*
U.S. equities	−0.152
Non-U.S. equities	−0.670
Emerging equities	0.343
U.S. fixed income	0.179
Non-U.S. fixed income	−0.009
High-yield bonds	0.206
Private equities	−0.376
Cash	0.000

additive, thereby validating the "marginal" label. Hence this measure allows fo a decomposition of risk (standard deviation) of all asset classes/bets whil capturing the correlation with other asset classes/bets.

First, it is relevant to observe that although the actual portfolio in Table 8.1 overweight U.S. equities, overweighting this asset class reduces the tracking erro as this bet is negatively correlated with other bets in the portfolio (Table 8.4 thereby lowering the total relative risk (Tables 8.2 and 8.3). This is evident by t negative coefficient on U.S. equities in the Relative Portfolio column.

Second, the absolute or relative size of a bet may mask the actual contribution to the total risk. For example, although the 2 percent overweight in U.S. equities actually lowers the tracking error, the same absolute bet in high-yield bonds (+2 percent) contributes positively to the relative risk (3.3 percent in Table 8.3).[6] In addition, the 2 percent underweight in non-U.S. fixed income has a negligible impact on the tracking error, whereas the same absolute and relative deviation in private equities contributes 16 percent of the total tracking error. Although private equities are more volatile, there is a more complex relationship at work, which includes the relationship with other bets in the portfolio.[7]

Third, in evaluating the correlation of bets with the overall portfolio of bets as in Table 8.4, it is relevant to notice that the bets in U.S. equities, non-U.S. equities, non-U.S. fixed income, and private equities are all negatively correlated with the portfolio of bets. One could ask if all these are therefore risk reducing by offsetting other bets in the portfolio. However, where the portfolio is long in respect of the benchmark and is negatively correlated, the contribution to the tracking error is negative (as in U.S. equities). On the other hand, where the portfolio is short in respect of the benchmark (non-U.S. equities, non-U.S. fixed income, and private equities), the negative correlation, in conjunction with the short position, contributes positively to the tracking error.

USEFULNESS OF THIS MEASURE

Any ability to drill down into a total risk measure and attribute the value to its components is useful for portfolio managers. As highlighted in the results, when the marginal contribution is negative, all else being equal, a marginal unit increase in the direction of the current bet lowers the total tracking error.[8] Only the U.S. equity bet changes the risk posture by effectively being risk reducing. Therefore, this breakdown can be used to size bets more effectively and capture the maximum alpha for a given risk tolerance.[9] In addition, the portfolio manager determines whether the bets are all correlated and is able to disaggregate how diversified their bets may be. For example, in Table 8.1 there are eight asset class bets; however, the three bets in non-U.S. equity, emerging markets, and private equity contribute 99 percent of the risk exposure. If the marginal contribution is concentrated in a few bets, even the

implementation of a large number of bets suggests that risks are not diversified. Similarly, a negative correlation in isolation is insufficient information for ascertaining whether bets are risk increasing or risk reducing, as demonstrated above.

The marginal contribution is dependent on current allocations; hence a slight change to a position implies very different results. For example, by shifting 2 percent more to U.S. equities from non-U.S. fixed income (i.e., extending the previous bet), the contribution from U.S. equities to the tracking error turns positive, and the correlation of U.S. equities and non-U.S. fixed income to other bets in the portfolio is now positive. However, now the contribution to the tracking error from non-U.S. fixed income turns mildly negative as the correlation has shifted sign. Therefore, the sensitivity of the marginal risk analysis to the portfolio changes would make it very difficult to allocate risk capital on this basis, for it would require a constant fine-tuning, and each asset class manager would need to be cognizant not only of his or her view on that specific market, but also its impact on other views.

This technique has also been adopted by investment managers to manage the risk of their portfolios. For example, this can easily be used for equity, bond, or currency management, where instead of having asset classes one can list the securities or currencies and thereby decompose the risk relative to the passive benchmark. In addition, Muralidhar and Pasquariello (2001) extend this analysis to imply the currency views of various positions implemented by currency overlay managers for a client. They are able to demonstrate some seeming inconsistencies in positions and views in a fashion similar to some seeming inconsistencies highlighted above. This method effectively allows for a clearer understanding of the concentration of bets and which bets can lower tracking error for optimal risk utilization.

COMPARISON WITH OTHER METHODOLOGIES

This section covers other methodologies that are applied in standard risk management software and explains the deficiencies of each. The content i based on the experience in evaluating a number of risk-management softwar programs for implementation in the World Bank's pension fund portfoli

Table 8.5
Comparing Methods of Estimating Contribution to Benchmark Risk Percentage

Asset Classes	*Assuming Independence (% Contribution)*	*Marginal Not Rebalanced (% Contribution)*	*Marginal Rebalanced (% Contribution)*	*Proposed Method (% Contribution)*
U.S. equities	71.5	40.67	−61.2	32.2
Non-U.S. equities	21.5	37.7	−70.5	49.7
Emerging equities	1.6	5.9	4.2	4.4
U.S. fixed income	1.8	2.7	92.7	1.1
Non-U.S. fixed income	0.1	2.1	70.8	1.6
High-yield bonds	0.5	2.4	27.8	0.6
Private equities	3.0	8.5	8.8	10.5
Cash	0.0	0.0	27.4	0.0
Total	100.0	100.0	100.0	100.0
Total variance of portfolio	0.60%	1.66%	−0.26%	1.37%
Standard deviation	7.73%	12.90%	N/A	11.69%

Table 8.6
Comparing Methods of Estimating Contribution to Relative Risk Percentage

Asset Classes	*Marginal Not Rebalanced (% Contribution)*	*Proposed Method (% Contribution)*
U.S. equities	45.3	−3.7
Non-U.S. equities	−34.6	63.1
Emerging equities	35.3	19.4
U.S. fixed income	−3.3	1.5
Non-U.S. fixed income	−1.0	0.1
High-yield bonds	6.4	3.3
Private equities	51.9	16.4
Cash	0.0	0.0
Total	100.0	100.0
Total tracking error	1.06%	1.24%

In Tables 8.5 and 8.6, the contribution to the total risk is compared using the method proposed above and the following three methods, which appear to be most commonly used by vendors to provide an estimate of the magnitude of the error of not capturing the diversification benefits of an asset class.

Contribution assuming an identity correlation matrix: Under this method, it is assumed that diagonal elements are unity, and off-diagonal elements in the correlation matrix are zero. This is done to simplify the calculation. Therefore, the assumption of independence between assets would provide a variance estimate whereby adding the weighted variance of each asset class equals the portfolio variance. The problem in assuming that off-diagonal elements are zero is that the true benefits of diversification are never captured in such analyses. In addition, as Table 8.5 demonstrates, the total risk of the benchmark portfolio is wrongly estimated, thereby rendering this approach incorrect (7.73 percent versus 11.69 percent).

"Marginal contribution" (with and without rebalancing): Under this methodology, which is embedded in the most commonly available software, the user calculates the variance using all the assets and then extracts one asset class at a time from the portfolio and recomputes the variance or standard deviation. In fact, Rahl (2000) describes marginal VAR as "the difference between overall portfolio VAR and VAR excluding certain accounts, risk factors or positions." This new standard deviation (excluding a particular asset class) is compared to the full portfolio risk to give an estimate of the "contribution" of that particular asset class. The most critical problem is that the contribution to risk for a total portfolio is to be computed when portfolios are complete (i.e., with all asset classes including the one whose contribution is being estimated) and not by using subsets of portfolios. Therefore, even if correct, the sum of all "marginal estimates" should equal the true variance of the portfolio (in the case of the benchmark portfolio, 11.69 percent). As is evident from Table 8.5, this is not the case, and the marginal method overestimates th total risk of the portfolio (12.9 percent). There are actually two ways to perform this calculation: not to rebalance the remaining asset class weights (i.e., so that the sum of the assets need not total 100 percent) ar to rebalance the remaining assets.[10] The rebalancing method is clearly incorrect as it excludes the possibility of the asset class ever being in the portfolio. As indicated in Table 8.5, it gives a negative variance, which i an infeasible result for an asset portfolio.

- *Tracking error of each bet in isolation:* Under this method the bet being evaluated is assumed to be the only bet in the portfolio and its risk is considered in isolation. This assumes that the bets are independent and is identical to the first method (identity correlation matrix).

Clearly, the marginal method assuming rebalancing and the method assuming independence of assets are incorrect in estimating either the total variance or the percentage contribution of an asset class or an asset class bet. Similarly, it can be shown numerically that these alternative methods are inadequate for estimating the percentage contribution to relative risk. For completeness, Table 8.6 includes the results of the marginal, no rebalancing calculation vis-à-vis the proposed method for the tracking error calculation. Not only are the resultant totals wrong, but also the magnitude, and often the signs too, are incorrect for the portfolio bets, providing the user with incorrect statistics on the contribution of bets to the risk of the overall portfolio.

CAVEATS

In the case of the two absolute measures of benchmark risk and actual risk, the implied correlations are meaningful. However, the implied correlations of the asset class bet to the portfolio of bets are based on the assumption that the variance-covariance matrix for asset classes is applicable also for asset class bets (which need not always be true). However, this is an acceptable first approximation and assumes no bias in the bets away from the respective benchmarks. This can be easily corrected by using a revised variance-covariance matrix should the plan sponsor so decide.

SUMMARY

This chapter set out to demonstrate simple methods by which the contribution of an asset class allocation or an asset class bet to the total absolute or relative risk of the portfolio could be determined. Such an evaluation is useful for a plan sponsor not only to measure risk but also to effectively manage risk by decomposing it into its constituent parts. Four key investment truths, highlighted in the accompanying box, emerge from this discussion.

INVESTMENT TRUTHS

- *The size of bet need not be a good indicator of the contribution to total risk.*
- *The direction of a bet need not be a good indicator of the contribution to total risk.*
- *It is difficult to allocate risk capital on the basis of marginal risk measures.*
- *One can determine if risk is concentrated in a few bets and correct for this through better portfolio construction.*

In addition, this methodology is superior to other methodologies that are available to institutional investors for implementation by risk management software companies. However, a plan sponsor can easily implement these tools using spreadsheet-type computer programs. Finally, this chapter has demonstrated only how contributions of asset class allocations to total risk are determined; the extensions to estimate the contribution of a selection of a benchmark of a manager or an individual manager's security selection (in either equities, bonds, or currencies) to an entire portfolio is a simple extension of this methodology.

APPENDIX

8.1

THE MATHEMATICAL SOLUTION FOR MARGINAL CONTRIBUTION

The marginal contribution to total risk from an individual bet is nothing but a function of the first derivative of the risk measure vis-à-vis the bet under consideration. Litterman (1996) defines it loosely as "the marginal rate of change in risk per unit change in the position (at the current position size) times the position size itself, can be thought of as the rate of change in risk with respect to a small percentage change in the size of the position." For simplicity, the tracking error is used for this estimation.

Marginal contribution of the bet in the ith asset class (zi) to tracking error

$$= z_i \frac{\partial(\text{tracking error})}{\partial z_i} \tag{A8.1.1}$$

(such that $\sum_i z_i \frac{\partial(\text{tracking error})}{\partial z_i} = \text{total tracking error}$)[11]

$$= z_i \frac{\partial\sqrt{z^T\Gamma z}}{\partial z_i}$$

$$= z_i \frac{z^T\Gamma}{\sqrt{z^T\Gamma z}} \tag{A8.1.1'}$$

where $\left(\frac{z^T\Gamma}{\sqrt{z^T\Gamma z}}\right)$ is a $1 \times N$ matrix measuring the marginal risk per unit of deviation.

Notice that the denominator in the second term is nothing but the tracking error, thereby normalizing the calculation.

The same approach can be followed to measure the marginal contribution of each individual position to the total absolute risk of the portfolio. In this case, the marginal contribution of the position in the *i*th asset class to the portfolio's risk is given by

$$w_i \times \frac{\partial(\text{std dev})}{\partial w_i} \tag{A8.1.2}$$

which equals

$$w_i \frac{w^T\Gamma}{\sqrt{w^T\Gamma w}} \tag{A8.1.2'}$$

where $(w^T\Gamma)/\sqrt{w^T\Gamma w}$ is a $1 \times N$ matrix measuring the marginal risk per unit of the positions.

Finally, the marginal contribution of the position in the *i*th asset class to the benchmark's risk is given by

$$= v_i \times \frac{\partial(\text{std dev})}{\partial v_i} \tag{A8.1.3}$$

$$= v_i \frac{v^T\Gamma}{\sqrt{v^T\Gamma v}} \tag{A8.1.3'}$$

where $(v^T\Gamma)/\sqrt{v^T\Gamma v}$ is a $1 \times N$ matrix measuring the marginal risk per unit of the positions.

APPENDIX

8.2

THE INTUITIVE APPROACH

There is another approach to estimating the contribution of an allocation to total risk that is derived from asset pricing theory. For simplicity, this is called the intuitive approach. Define the contribution of a stock I to the total risk of a portfolio of N stocks (P) as r_i. Define the contribution of an asset class I to the total risk of a portfolio of N asset classes (P) as c_i. From the basics in finance, the contribution to total risk of a stock I to a total portfolio of N stocks (P) or r_i is equal to

$$r_i = s_i \times \text{covariance}(I, P) \tag{A8.2.1}$$

where s_i is the weight of stock i in portfolio P. Mathematically, this is equivalent to

$$r_i = s_i \times \sigma(I, P) = s_i \times \rho_{I,P} \times \sigma(I) \times \sigma(P) \tag{A8.2.2}$$

where $\rho_{I,P}$ is the correlation between I and P, $\sigma(I, P)$ is the covariance between I and P, and $\sigma(P)$ and $\sigma(I)$ represent the standard deviations of P and I respectively. Usually, the correlation among stocks is known and stable, whereas that of an individual stock to a specific portfolio is uncertain. Where the correlation factor is unknown ex-ante, the contribution to risk can be calculated by the following:

$$r_i = s_i \times \sum s_j \times \sigma(i, j) \tag{A8.2.3}$$

where $\sum$ is the summation operator for $j = 1$ through N stocks and $\sigma(i, j)$ is the covariance of stocks i and j. The sum of all r_i in the portfolio must equal $\sigma^2(P)$. Hence in percentage terms, the contribution of stock I to the variance of portfolio P would be $r_i/\sigma^2(P)$.

In an analogous fashion to equations (A8.2.1–3), the contribution to the total risk of an asset class for either absolute or relative risk can be defined as above. However, in the case of asset class structuring, the correlation between that of a specific asset class and the total portfolio (or those of asset class bets with the portfolio of bets) is difficult to determine ex-ante and probably changes as the portfolio composition changes. The correlation between two asset classes is easier to estimate. Hence an adaptation of equation (A8.2.3) is applied to estimate the

ntribution of an asset class to portfolio risk. Thus, in the case of the actual risk f the portfolio:

$$c_i(\text{actual}) = w_i \times \sum w_j \times \sigma(i, j) \tag{A8.2.4}$$

where $\sigma(i, j)$ is the covariance between the ith and the jth asset class and $\sum$ is the summation operator for $j = 1$ through N asset classes. For the absolute risk of the benchmark portfolio, define

$$c_i(\text{benchmark}) = v_i \times \sum v_j \times \sigma(i, j) \tag{A8.2.5}$$

In the case of the relative risk calculations, the correlation of a tactical bet in an asset class with the portfolio of tactical bets can be determined as

$$c_i(\text{tactical deviation}) = z_i \times \sum z_j \times \sigma(i, j) \tag{A8.2.6}$$

It is clear that the c_i are calculated using variance as a measure of risk. To normalize for the standard deviation being the measure of risk and using equation (8.4), define

$$c_i'(\text{actual}) = \frac{c_i(\text{absolute})}{\sqrt{w^T \Gamma w}} = \frac{w_i \times \sum w_j \times \sigma(i, j)}{\sqrt{w^T \Gamma w}} \tag{A8.2.7}$$

$$c_i'(\text{benchmark}) = \frac{c_i(\text{benchmark})}{\sqrt{v^T \Gamma v}} = \frac{v_i \times \sum v_j \times \sigma(i, j)}{\sqrt{v^T \Gamma v}} \tag{A8.2.8}$$

$$c_i'(\text{tactical}) = \frac{c_i(\text{tactical})}{\sqrt{z^T \Gamma z}} = \frac{z_i \times \sum z_j \times \sigma(i, j)}{\sqrt{z^T \Gamma z}} \tag{A8.2.9}$$

Note that the last equation describes the marginal risk of a single bet to the total racking error. For the portfolios in Table 8.1 (benchmark, actual, and deviation), Tables 8.2 and 8.3 provide the marginal contribution to the total risk (in percentage points) and the *percentage contribution* of each asset class or asset class bet to the total risk, respectively.

APPENDIX

8.3

CORRELATIONS OF ASSET ALLOCATIONS TO PORTFOLIO ALLOCATIONS

An interesting statistic that can be derived from the above is the correlation of an asset class allocation to the overall allocation (as differentiated from the asset class correlations in Table A8.3.1) or a specific asset class bet to a portfolio of bets. This section develops the analytical solutions for estimating these correlations. This statistic is useful as it allows the portfolio managers to determine whether bets are positively correlated, negatively correlated or uncorrelated with other bets—something that is not obvious at the time of constructing portfolios.

Measures of correlation of a single position with the total portfolio and of a single bet with the total portfolio of bets can be explicitly obtained from the following definitions. First, for the absolute portfolio:

$$\operatorname{cov}(w_i y_i, y_P) = \operatorname{cov}\left(w_i y_i, \sum_{j=1}^{N} w_j y_j\right) = w_i^2 \operatorname{var}(y_i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} w_i w_j \operatorname{cov}(y_i, y_j)$$

$$= \sum_{j=1}^{N} w_i w_j \operatorname{cov}(y_i, y_j) \qquad (A8.3.1)$$

$$\rho_{iP} = \frac{\operatorname{cov}(w_i y_i, y_P)}{\sigma_{w_i y_i} \sigma_P} = \frac{\sum_{j=1}^{N} w_i w_j \operatorname{cov}(y_i, y_j)}{w_i \sigma_i \sqrt{w^T \Gamma w}} = \frac{c_i(\text{actual})}{w_i \sigma_i \sqrt{w^T \Gamma w}}$$

where y_i is the return from the ith asset class.

Then, for the correlation of an individual bet with the portfolio of bets:

$$\operatorname{cov}(y_{Zi}, y_{Zp}) = \operatorname{cov}\left(z_i y_i, \sum_{j=1}^{N} z_j y_j\right) = z_i^2 \operatorname{var}(y_i) + \sum_{\substack{j=1 \\ j \neq i}}^{N} z_i z_j \operatorname{cov}(y_i, y_j)$$

$$= \sum_{j=1}^{N} z_i z_j \operatorname{cov}(y_i, y_j) \qquad (A8.3.2)$$

$$\rho_{ZiZP} = \frac{\operatorname{cov}(y_{Zi}, y_{ZP})}{\sigma_{Zi} \sigma_{ZP}} = \frac{\sum_{j=1}^{N} z_i z_j \operatorname{cov}(y_i, y_j)}{z_i \sigma_i \sqrt{z^T \Gamma z}} = \frac{c_i'(\text{tactical})}{z_i \sigma_i}$$

where y_{zi} is the spread expected return from the ith asset class.

Alternatively, using the intuitive approach, since the correlation between an asset class and the portfolio is unknown ex-ante, from equations (A8.3.1–2) and (A8.2.2), the following can also imply the correlation coefficient of each asset class to the benchmark portfolio:

$$\rho_{i,B} = \frac{c_i(\text{benchmark})}{\sigma(P) \times \sigma(I) \times v_i} \qquad (A8.3.3)$$

he correlation coefficient of each asset class to the actual portfolio (or $\rho_{i,P}$)

$$= \frac{c_i(\text{actual})}{\sigma(\text{actual}) \times \sigma(I) \times w_i} \qquad (A8.3.4)$$

r the correlation of the bet in asset class I to the portfolio of bets ($\rho_{Zi,ZP}$)

$$= \frac{c_i(\text{tactical})}{\sigma(TE) \times \sigma(I) \times z_i} \qquad (A8.3.5)$$

where TE is the tracking error.

Table 8.4 provides the implied correlation coefficient of each asset class bet to portfolio of bets based on their respective allocations.[12] This table shows that the bets in four asset classes are negatively correlated with the portfolio of bets, at the current position.

Table A8.3.1
Data on Asset Classes

Asset Classes	*Standard Deviation (%)*	*Correlations*							
		USEQ	*NUSEQ*	*EMEQ*	*USFI*	*NUSFI*	*HY*	*PE*	*Cash*
U.S. equities	15.0%	1.0	0.5	0.3	0.4	0.4	0.5	0.4	−0.08
Non-U.S. equities	19.5%	0.5	1.0	0.3	0.2	0.3	0.2	0.1	−0.13
Emerging equities	23.3%	0.3	0.3	1.0	0.3	0.3	0.2	0.1	−0.10
U.S. fixed income	5.2%	0.4	0.2	0.3	1.0	0.4	0.3	0.0	−0.02
Non-U.S. fixed income	4.5%	0.4	0.3	0.3	0.4	1.0	0.3	0.0	−0.05
High-yield bonds	9.8%	0.5	0.2	0.2	0.3	0.3	1.0	0.0	−0.07
Private equities	27.0%	0.4	0.1	0.1	0.0	0.0	0.0	1.0	0.00
Cash	1.0%	−0.08	−0.13	−0.10	−0.02	−0.05	−0.07	0.00	1.00

NOTES

1. Litterman (1996) makes a similar point in a footnote for one of these methods.

2. For the purpose of this analysis it is demonstrated how asset class allocations at a target or tactical level can be used to determine contribution to risk. The extension of determining the contribution of any deviation from a benchmark (e.g., security, country, or currency selection) is trivial.

3. We are assuming unleveraged deviations from the benchmark, but the results would be unaffected if leverage is appropriately captured.

4. See, for example, Markowitz (1959).

5. Since numerical simulations are provided to illuminate the key points in this chapter, we provide a variance-covariance matrix of the various asset classes in Table A.8.3.1. Every institutional investor can select his or her own matrix; these values were based on estimates from historical data.

6. This point has been made elsewhere, more specifically with respect to managing the risks of currency overlays. See Mashayekhi-Beschloss and Muralidhar (1996).

7. Litterman (1996), who makes a similar observation, terms the point where risk contribution is zero as a candidate for a "best hedge."

8. Up to a point. If the bet size increases, this becomes the dominant bet in the portfolio and will contribute positively to the tracking error.

9. A cautionary note: any risk analysis depends on the correlations and variances remaining stable over time, and a violation of this assumption would put any risk analysis in doubt. Also, once the positions are changed, the statistics need to be recalculated for the new portfolio.

10. I would like to thank Mr. P.S. Srinivas for pointing this out.

11. See also Litterman (1996).

12. As the allocation weights change, the total risk of a portfolio and hence the implied correlation will also change.

IV

IMPLEMENTATION OF ASSET ALLOCATION