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Prospect Theory and market quality

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Abstract

We study equilibrium trading strategies and market quality in an economy in which speculators display preferences consistent with Prospect Theory (Kahneman and Tversky, [39]; Tversky and Kahneman, [63]), i.e., loss aversion and mild risk seeking in losses. Loss aversion (risk seeking in losses) induces speculators to trade less (more), and less cautiously (more aggressively), with their private information – but also makes them less (more) inclined to purchase private information when it is costly – in order to mitigate (enhance) their perceived risk of a trading loss. We demonstrate that these forces have novel, nontrivial, state-dependent effects on equilibrium market liquidity, price volatility, trading volume, market efficiency, and information production.

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Over the past two decades, a large and long-standing body of experimental evidence on human behavior has provided support to the notion, first formulated by Kahneman and Tversky [39] as *Prospect Theory*, that the decision-making process of *any* economic agent may depart from the predictions of standard expected utility theory. In Tversky and Kahneman's [63] version, Prospect Theory postulates that economic agents assess gambles with a value function defined over gains and losses relative to a reference point (instead of the absolute level of financial wealth or consumption), concave over gains (*risk aversion*), but convex (*risk seeking*) and steeper (*loss aversion*) over losses. Recent work employs modified versions of this theory to interpret the behavior of financial investors and study the pricing of financial securities. Prospect Theory arguments have been proposed to explain such known asset pricing puzzles as the magnitude of the equity premium, excess stock return volatility, momentum and the disposition effect, the value premium, or stock return predictability and its implications for portfolio selection.²

The past two decades have also been characterized by an increasing interest in the study of the process of price formation in financial markets. Market microstructure research has studied (both theoretically and empirically) such issues as the mechanisms through which private information is acquired, utilized, and impounded into prices, agents' reasons for trade and optimal trading strategies, and the implications for liquidity and volatility.³ Yet, to our knowledge, this literature has not examined any of these issues when investors make decisions according to Prospect Theory.

The main objective of this paper is to investigate the effects of Prospect Theory on market quality. Our theoretical analysis makes two related contributions to the literature. First, its predictions are novel and indicate that these effects are nontrivial and may play an important role in explaining financial market quality. Second, its predictions are testable, thus possibly refutable rather than aimed at matching extant features of the data. As such, they provide an unbiased, albeit more challenging opportunity to assess the empirical relevance of unconventional utility models.

Our theory is based on a one-period model of sequential trading in the spirit of Grossman and Stiglitz [29], Kyle [45], and Vives [67]. The model is populated by a continuum of informed traders (competitive, price-taking speculators endowed with a noisy signal of the asset payoff) submitting demand schedules (i.e., generalized limit orders), noise traders submitting market orders, and competitive, risk-neutral market makers (MM). If a speculator's preferences are described by an exponential utility function (MV speculation), the model's implications for trading strategies, market depth (the inverse of Kyle's [45] "lambda," or price impact of noise trading), price volatility, informed trading volume, and price informativeness are well-known in the literature (e.g., Vives [68]). We depart from this standard setting by assuming that (PT) speculators display preferences capturing parsimoniously *all* of the aforementioned main features of Kahneman and Tversky's [39] Prospect Theory – as well as their *relative* importance (as assessed by Tversky and Kahneman [63]).⁴

² E.g., Benartzi and Thaler [10], Aït-Sahalia and Brandt [2], Barberis, Huang, and Santos [6], Barberis and Huang [4], Berkelaar, Kouwenberg, and Post [11], Gomes [25], Grinblatt and Han [28], Barberis, Huang, and Thaler [7], Kyle, Ou-Yang, and Xiong [46], Barberis and Huang [5], Barberis and Xiong [9], and Li and Yang [52].

³ E.g., see the surveys in O'Hara [57], Hasbrouck [30], and Vives [68].

⁴ More precisely, we assume that a PT speculator makes trading decisions maximizing a tractable piecewise *value function* nesting the mean-variance value function of a risk-averse (MV) speculator. This assumption allows us to char-

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The model's ensuing noisy rational expectations equilibrium yields several novel, microfounded predictions for financial market quality. First, we show that PT speculation *improves* a risky asset's market liquidity (i.e., *reduces* the average price impact of noise trading) but *worsens* its price efficiency and *lowers* its aggregate trading volume relative to standard MV speculation. Intuitively, loss aversion (LA) not only induces a speculator to trade less (or not at all) with her noisy signal – especially when her perceived marginal probability of a trading loss is high – but also increases her trading intensity (i.e., the sensitivity of her demand function to private information shocks) – especially when such probability is low – in order to decrease her conditional expected trading loss. In turn, speculators' trading intensity has two effects of opposite sign on market quality. Ceteris paribus, less cautious speculation increases both the likelihood of informed trading in the aggregate order flow and its information content. The former effect (labeled *selection*) worsens the MM's adverse selection problem, while the latter effect (labeled *efficiency*) alleviates it. In equilibrium, the efficiency effect of LA speculation on MM's perceived adverse selection risk, relative to MV speculation, dominates the corresponding selection effect, leading to greater market depth but lower price volatility, informativeness, and trading volume.

Risk seeking in losses (RSL) instead induces a speculator to trade more, and more aggressively, with her noisy signal – especially when her perceived cumulative probability of a trading loss is high – in order to increase her conditional trading loss variance. As for LA speculation, the equilibrium efficiency effect of RSL speculation on market quality dominates its selection effect, yielding greater market depth and trading volume, but lower price volatility and informativeness. According to Tversky and Kahneman [63], agents' risk seeking in losses is *mild* relative to loss aversion.⁵ Thus, for preference parameters consistent with their assessment, the effects of loss aversion on market quality prevail over the effects of risk seeking in losses in equilibrium, leading to greater market liquidity but worse efficiency and lesser trading than with MV speculation.

As importantly, we also show that PT speculation makes equilibrium market quality *state-dependent*. In the presence of standard MV speculation, market quality instead varies exclusively with exogenous preference and technology parameters. An intuitive explanation for this result is that ceteris paribus, the effects of loss aversion and risk seeking in losses on a speculator's trading intensity depend on the absolute magnitude of the noisy signal she observes relative to its mean, since so does her perceived probability of a trading loss. In particular, RSL (LA) speculation's trading intensity is (first increasing then) decreasing in absolute private signals. In equilibrium, the prevalence of efficiency over selection (for the MM's perceived adverse selection risk from trading) and of loss aversion over risk seeking in losses (in PT speculators' preferences) makes

acterize analytically the demand function of informed PT speculators, hence to clearly identify the effects of each of the main features of Prospect Theory on their optimal trading strategy relative to MV speculation. Further discussion is in Section 2.2. Previous research (e.g., Barberis, Huang, and Santos [6]; Barberis and Huang [4]) concentrates on agents' greater sensitivity to reductions of their financial wealth. Kyle, Ou-Yang, and Xiong [46] develop an exponential version of Kahneman and Tversky's [39] Prospect Theory to examine an agent's decision whether to liquidate an asset before its natural payoff. Tversky and Kahneman [63] also suggest that, in assessing gambles, economic agents employ subjective nonlinear transformations of the objective cumulative probability distribution of payoffs overweighting its tails. Barberis and Huang [5] study the asset pricing implications of the resulting utility model, known as Cumulative Prospect Theory.

⁵ E.g., see Tversky and Kahneman's [63] descriptive utility function U_{TK} in Section 2.2 (Eq. (1)) and Fig. 1a. Tversky and Kahneman [63] base this observation on experimental evidence in which agents select among gambles that can lead to both gains and losses. Consistently, Barberis, Huang, and Santos [6, p. 17] observe that "[for those gambles] – such as the one-year investment in stocks [...] – loss aversion at the kink is far more important than the degree of curvature away from the kink."

price impact, volatility, informativeness, and relative trading volume first decreasing then increasing in those signals.

In most financial markets private information about asset payoffs is costly. This motivates us to extend our model to consider whether Prospect Theory preferences affect speculators' endogenous information acquisition. The amended model generates a rich set of additional, novel implications. In particular, we show that when the noisy signal is sufficiently expensive, the presence of PT speculation diminishes information production and amplifies its effects on market quality but attenuates its state-dependence. Intuitively, the availability of private information mitigates a speculator's perceived risk of a trading loss, yet at a certain cost. When private information is sufficiently expensive, risk aversion induces a fraction of MV speculators faced with this trade-off not to purchase it. In those circumstances, the MM's vulnerability to adverse selection may either decline (if a selection effect prevails) or increase (if an efficiency effect prevails). Loss aversion makes a speculator even less inclined to purchase the noisy signal to mitigate that risk; yet, her lesser trading with it increases returns to private information. Risk seeking in losses makes a speculator more inclined to purchase the noisy signal to magnify that risk; yet, her greater trading with it decreases returns to private information. As above, prevalence of the former set of effects on the latter yields lower information production by PT speculators in equilibrium. In turn, more limited market participation of informed PT speculators not only lowers price impact and market efficiency but also attenuates the state-dependence induced by their trading activity. These insights are important for they suggest that the extent to which Prospect Theory preferences affect financial market quality is sensitive to the market's information environment.

Our work is related to Subrahmanyam [61] and Foster and Viswanathan [24]. Subrahmanyam [61] shows that allowing for imperfectly competitive, risk-averse speculators submitting market orders in the one-period noisy rational expectations model of Kyle [45] has ambiguous effects on market liquidity (but unambiguously lowers price efficiency) relative to risk-neutral speculation. Yet, equilibrium market quality remains non-state-dependent (as in Kyle [45]). In this paper we show that Prospect Theory preferences have unambiguous effects on equilibrium market quality (relative to competitive, risk-averse speculation) and make it state-dependent. Foster and Viswanathan [24] demonstrate that representing strategic speculators' beliefs with nonnormal, elliptically contoured distributions makes price volatility and trading volume (but not price impact) state-dependent. However, there is little or no evidence guiding such modeling choice for those unobservable beliefs. In our model state-dependent market quality ensues from speculators' microfounded (i.e., Prospect Theory-inspired) preferences even when all random variables are normally distributed.⁶ Another related literature explores asset pricing implications of investors exhibiting either irrationality or bounded rationality.⁷ All agents in our model, including the speculators displaying nonconventional preferences, are instead fully rational.⁸

⁶ Fos and Collin-Dufresne [23] show that equilibrium price impact and price volatility depend on stochastically timevarying noise trading volatility in a continuous-time formulation of Kyle [45]. In our model, state-dependent market quality stems from speculators' microfounded preferences, ceteris paribus for the technology parameters of the economy.

⁷ E.g., Barberis, Schleifer, and Vishny [8], Hong and Stein [35], Daniel, Hirshleifer, and Subrahmanyam [19], and Kogan, Ross, Wang, and Westerfield [42].

⁸ In recent work, Mele and Sangiorgi [54] and Ozsoylev and Werner [58] show that Knightian uncertainty (i.e., uncertainty about the quality of some information signals) *and* ambiguity aversion (i.e., aversion to such uncertainty) may lead to a no-trading region for *uninformed* (risk-averse) speculators or (risk-neutral) arbitrageurs in rational expectations

We proceed as follows. In Section 2, we construct a model of trading with PT speculation and discuss its implications for market quality. In Section 3, we enrich the model by endogenizing PT speculators' decision to become informed. We conclude in Section 4.

2. A model of trading with Prospect Theory

In their seminal work, Kahneman and Tversky [39] and Tversky and Kahneman [63] introduce Prospect Theory as a model of decision-making under uncertainty based on experimental evidence of violations of the standard Morgenstern–von Neumann utility theory. The main features of Prospect Theory are a value function i) defined on changes in financial wealth; ii) displaying concavity in the domain of gains (risk aversion) and *mild* convexity in the domain of losses (*risk seeking*); and iii) steeper for losses than for gains (*loss aversion*). This theory is supported by numerous experimental studies of human behavior in the psychology literature.⁹

In this section we describe a noisy rational expectations equilibrium model of sequential trading in the presence of better-informed speculators with Prospect Theory-inspired preferences. The model's structure is similar to Kyle [45] and Subrahmanyam [61]; yet, we assume that the speculators are competitive (instead of strategic) and submit demand schedules (i.e., limit orders instead of market orders), in the spirit of Grossman and Stiglitz [29], Diamond and Verrecchia [20], Verrecchia [65], and Vives [67]. These assumptions are made solely for simplicity. Allowing for imperfect competition and informed market orders complicates the analysis that follows considerably without significantly affecting its main intuition.¹⁰ All proofs are in Appendix A.

2.1. The basic economy

The model is a two-date, one-period economy in which a single risky asset is exchanged. Trading occurs only at the end of the period (t = 1), after which the asset payoff (v) is realized. The economy is populated by three types of traders: A continuum of informed traders with measure one (labeled *speculators*) representing a competitive "speculative sector;" liquidity traders; and competitive, risk-neutral market makers (MM). All traders know the structure of the economy and the decision process leading to order flow and prices.

At time t = 0 there is neither information asymmetry about v nor trading. Sometime between t = 0 and t = 1, each speculator receives private information about v in the form of a noisy signal S = v + u (e.g., Grossman and Stiglitz [29]; Yuan [69]).¹¹ The random variables v and u

equilibrium models, hence to equilibrium market illiquidity, market crashes, and excess price volatility. Our analysis concentrates on the implications of *informed* speculation with Prospect Theory preferences for equilibrium market quality. Condie and Ganguli [18] find that ambiguity averse speculators endowed with initial *nonzero* stock positions and *ambiguous* private information trade away their stock holdings (but their optimal demand is *not* a no-trade position) if prices do not incorporate an ambiguity premium, yielding price volatility and illiquidity jumps.

⁹ E.g., see the surveys in Camerer [15], Barberis, Huang, and Santos [6], and Nofsinger [56].

¹⁰ For instance, in the presence of imperfectly competitive, risk-averse informed traders submitting market orders, a closed-form expression for equilibrium price impact cannot be obtained (e.g., see Subrahmanyam [61]). Further, Vives [68] and Kovalenkov and Vives [43] show that in the presence of risk-averse informed traders submitting limit orders, a competitive rational expectations equilibrium provides a reasonably close approximation to the corresponding strategic equilibrium.

¹¹ As the discussion in Vives [67,68] suggests, allowing for heterogeneous private information would complicate the model without altering its main insights. We examine the implications of Prospect Theory preferences for speculators' endogenous information acquisition in Section 3.



Fig. 1. Prospect Theory preferences. In this figure we plot realizations of a speculator's value functions described in Section 2.2. Specifically, we first assume that the trading profits π are normally distributed with mean zero and unit variance. We then plot, over the domain of π , realizations of an ad-hoc function of π , $R(V_{PT})$ of Eq. (5) such that $E[R(V_{PT})|I] = V_{PT}$, the Prospect Theory-inspired (PT) piecewise value function of Eq. (2): $R(V_{PT}) = \pi - \frac{1}{2}\alpha\pi^2 + \begin{cases} 0 & \text{if } \pi \ge 0, \\ \gamma\pi + \frac{1}{2}\beta(\pi^2 - \frac{2}{pi}) & \text{if } \pi < 0, \end{cases}$ where $pi \equiv \arccos(-1)$, for $\alpha = 1$, $\gamma = 1$, and $\beta = 1.05$ (Fig. 1a, solid line), for $\alpha = 1$, $\gamma = 0$, and $\beta = 0$ (mean-variance [MV], Fig. 1b, dashed line, labeled $R(V_{MV})$), for $\alpha = 1$, $\gamma = 2$, and $\beta = 0$ (loss aversion [LA], Fig. 1b, dotted line, labeled $R(V_{LA}^{\gamma=2})$), and for $\alpha = 1$, $\gamma = 0$, and $\beta = 2$ (risk seeking in losses [RSL], Fig. 1b, thin line, labeled $R(V_{RSL}^{\beta=2})$), as well as realizations of the power utility function of Tversky and Kahneman [63], i.e., U_{TK} of Eq. (1) (Fig. 1a, crossed line).

are assumed to be mutually independent and normally distributed with mean zero and variance σ_v^2 and σ_u^2 , respectively. It then ensues that $var[S] \equiv \sigma_s^2 = \sigma_v^2 + \sigma_u^2$ and $cov[v, S] = \sigma_v^2$. At time t = 1, moving first, liquidity traders submit *market orders* and speculators submit *demand* schedules (i.e., generalized limit orders) to the MM, before the equilibrium price *P* has been set. Liquidity traders generate a random, normally distributed demand *z*, with mean zero and variance σ_z^2 . For simplicity, we assume that *z* is independent of all other random variables. As in Vives [67], we denote a speculator's demand schedule by $x(S, \cdot)$; thus, when the price is *P*, her desired trade is x = x(S, P) and her profits from trading are given by $\pi = x(v - P)$.

2.2. Prospect Theory speculators

The speculators make trading decisions under Prospect Theory (PT). To represent PT preferences, Tversky and Kahneman [63] propose a specific power utility function over trading gains and losses π based on experimental evidence:

$$U_{TK} = \begin{cases} \pi^{0.88} & \text{if } \pi \ge 0, \\ -2.25(-\pi)^{0.88} & \text{if } \pi < 0. \end{cases}$$
(1)

This functional form, plotted in Fig. 1a (crossed line) over the domain of π , captures the main features of Prospect Theory for it is concave over gains, and *mildly* convex and steeper over losses. Yet, it makes PT speculators' problem analytically intractable in our setting. Therefore, in the spirit of Barberis, Huang, and Santos [6], Barberis and Huang [4], and Kyle, Ou-Yang, and Xiong [46], we assume that each PT speculator chooses the optimal trading strategy *x* that maximizes the following tractable piecewise *value function* (conditional on her information set *I*):

$$V_{PT} = V_{MV} + V_{TK},\tag{2}$$

where

$$V_{MV} = E[\pi | I] - \frac{1}{2} \alpha \, var[\pi | I], \tag{3}$$

$$V_{TK} = \left\{ \gamma E[\pi | \pi < 0, I] + \frac{1}{2} \beta var[\pi | \pi < 0, I] \right\} \Pr[\pi < 0 | I],$$
(4)

 $Pr[\cdot]$ is the probability operator, $\alpha > 0$, $\gamma \ge 0$, and $\beta \ge 0$.

This model of asset choice is based on the *mean-variance* approach to rational investment, commonly used in financial theory for its analytical convenience (e.g., Yuan [69]).¹² The expression for V_{PT} is defined over first and second *conditional moments* of a speculator's profits π . The power utility function U_{TK} of Tversky and Kahneman [63] is instead defined over *realiza-tions* of π . Thus, to illustrate the intuition for V_{PT} relative to U_{TK} and calibrate its parameters accordingly, we define an ad-hoc function $R(V_{PT})$ of π such that $E[R(V_{PT})|I] = V_{PT}$ as follows:

$$R(V_{PT}) \equiv \pi - \frac{1}{2}\alpha \left\{ \pi^2 - E[\pi|I]^2 \right\} + \begin{cases} 0 & \text{if } \pi \ge 0, \\ \gamma \pi + \frac{1}{2}\beta \{\pi^2 - E[\pi|\pi < 0, I]^2\} & \text{if } \pi < 0. \end{cases}$$
(5)

Figs. 1a and 1b plot $R(V_{PT})$ of Eq. (5) over the domain of π under the assumption that π is normally distributed with mean zero and unit variance.¹³

The value function V_{PT} of Eq. (2) has several desirable properties. In particular, as we show next (in Section 2.3), this specification allows us to characterize analytically the demand function of PT speculators, thus to clearly and explicitly describe the separate implications of *loss aversion* and *risk seeking in losses* for their optimal trading activity relative to standard risk-averse speculation. For $\gamma = 0$ and $\beta = 0$, V_{PT} reduces to V_{MV} of Eq. (3), the mean-variance value function of a risk-averse (MV) speculator; in Fig. 1b, $R(V_{MV})$ is strictly concave over both gains and losses (e.g., dashed line, for $\alpha = 1$). For $\gamma > 0$ and $\beta = 0$, V_{PT} makes speculators loss averse (LA); e.g., in Fig. 1b, $R(V_{PT})$ is kinked at the origin (where trading gains are zero) and steeper over losses (dotted line, for $\gamma = 2$; $R(V_{LA}^{\gamma=2})$). For $\gamma = 0$ and $\beta > \alpha$, V_{PT} makes speculators risk seeking in losses (RSL); e.g., in Fig. 1b, $R(V_{PT})$ is concave over gains but convex over losses (thin line, for $\beta = 2$; $R(V_{RSL}^{\beta=2})$). Thus, for $\gamma > 0$ and *small* $\beta > \alpha$, V_{PT} captures parsimoniously and tractably both the main features of Kahneman and Tversky's [39] Prospect Theory and Tversky and Kahneman's [63] assessment of their *relative* strength in Eq. (1), as discussed in the

 $-\sqrt{\frac{2}{pi}}$, where $pi \equiv \arccos(-1)$.

¹² There is a large literature comparing the mean-variance and expected utility approaches to rational portfolio choice (e.g., see Johnstone and Lindley [38]). In general, the former is not equivalent to the latter. It is well-known that the mean-variance model can be motivated by assuming quadratic utility or normality (e.g., Huang and Litzenberger [36, pp. 61–62]); however, quadratic utility displays "the undesirable properties of satiation and increasing absolute risk aversion" while normality is "inconsistent with limited liability" for preferences defined over wealth or consumption. The value function V_{PT} of Eq. (2) is defined over trading gains and losses. Properties of normal distributions imply that maximizing the value function V_{MV} of Eq. (3) is equivalent to maximizing the expected utility of a speculator with CARA preferences (e.g., Huang and Litzenberger [36, pp. 265–266]). Levy and Markowitz [50] and Kroll, Levy, and Markowitz [44] find the portfolio ordering by the mean-variance rule to be almost identical to the ordering obtained by using expected utility for various probability distributions (i.e., beyond normal) and utility functions (e.g., quadratic, logarithmic, power, and negative exponential).

¹³ As such, both the expression for $R(V_{PT})$ and its plot in Fig. 1a are meant to facilitate the interpretation of V_{PT} of Eq. (2) relative to U_{TK} of Eq. (1), rather than to represent a subjective utility function for decision-making under risk. Well-known properties of truncated normal distributions (e.g., Greene [26, pp. 951–952]) imply that $E[\pi | \pi < 0] = \sqrt{2}$

Introduction: Loss aversion and *relatively mild* risk seeking in losses. For instance, in Fig. 1a, the graphs for $R(V_{PT})$ (solid line, for $\gamma = 1$ and $\beta = 1.05$) and U_{TK} (crossed line) nearly overlap in the domain of losses.¹⁴

2.3. Prospect Theory trading

At time t = 1, each speculator submits her demand schedule (i.e., limit order) $x(S, \cdot)$ maximizing V_{PT} of Eq. (2) conditional upon her noisy signal S – the best information available about the risky asset's payoff v (i.e., $I = \{S\}$). As such, speculators neither learn from market prices nor internalize the impact of their trades on market prices (e.g., Vives [67,68]). Standard formulas for the moments of a truncated normal distribution (e.g., Greene [26, pp. 950–952]) imply that

$$E[\pi|S,\pi<0] = x(\phi S - P)\Phi(sgn(x)\chi) + x\,sgn(x)\sqrt{\sigma_v^2(1-\phi)\Lambda^-(sgn(x)\chi)},\tag{6}$$

$$var[\pi | S, \pi < 0] = x^2 \sigma_v^2 (1 - \phi) \left[1 - \Delta^- \left(sgn(x)\chi \right) \right] \Phi \left(sgn(x)\chi \right), \tag{7}$$

where $\phi \equiv \frac{\sigma_v^2}{\sigma_s^2}$ is the relative precision of the signal *S*, $sgn(\cdot)$ is the sign function, $\Phi(\cdot)$ and $\psi(\cdot)$ are the standard normal cdf and pdf, $\chi = \frac{P-\phi S}{\sigma_v \sqrt{1-\phi}}$, $\Lambda^-(\cdot) = -\frac{\psi(\cdot)}{\Phi(\cdot)}$, and $\Delta^-(\cdot) = \Lambda^-(\cdot)[\Lambda^-(\cdot) - (\cdot)]$. In light of the aforementioned properties of V_{PT} , both $\Phi(sgn(x)\chi)$ and $\psi(sgn(x)\chi)$ – the conditional *cumulative* and *marginal* probability of a trading loss, respectively – play an important role in each speculator's trading strategy.

Eq. (2) then becomes

$$V_{PT}(S) = E[\pi|S] - \frac{1}{2}\alpha \operatorname{var}[\pi|S] + \gamma E[\pi|S, \pi < 0]\Phi(\operatorname{sgn}(x)\chi) + \frac{1}{2}\beta \operatorname{var}[\pi|S, \pi < 0]\Phi(\operatorname{sgn}(x)\chi).$$
(8)

Substituting Eqs. (6) and (7) into Eq. (8) and differentiating with respect to x yields

$$x_{PT} = \begin{cases} \frac{[1+\gamma\Phi(\chi)]}{\alpha^{*}(\chi)\sigma_{v}^{2}(1-\phi)}(\phi S - P) - \frac{\gamma\psi(\chi)}{\alpha^{*}(\chi)\sigma_{v}\sqrt{1-\phi}} > 0 & \text{if } S > S_{H}, \\ \frac{[1+\gamma\Phi(-\chi)]}{\alpha^{*}(-\chi)\sigma_{v}^{2}(1-\phi)}(\phi S - P) + \frac{\gamma\psi(-\chi)}{\alpha^{*}(-\chi)\sigma_{v}\sqrt{1-\phi}} < 0 & \text{if } S < S_{L}, \\ 0 & \text{if } S_{L} \leqslant S \leqslant S_{H}, \end{cases}$$
(9)

where $S_H = \frac{P}{\phi} + \frac{\gamma \psi(\chi) \sigma_v \sqrt{1-\phi}}{\phi [1+\gamma \Phi(\chi)]}$, $S_L = \frac{P}{\phi} - \frac{\gamma \psi(-\chi) \sigma_v \sqrt{1-\phi}}{\phi [1+\gamma \Phi(-\chi)]}$, and $\alpha^*(\cdot) = \alpha - \beta [1-\Delta^-(\cdot)] \Phi(\cdot) > 0.^{15}$

The optimal demand schedule of Eq. (9) has standard and many novel features. For $\gamma = 0$ and $\beta = 0$, x_{PT} reduces to the optimal generalized limit order of a MV speculator,

$$x_{MV} = \frac{1}{\alpha \sigma_v^2 (1 - \phi)} (\phi S - P) \tag{10}$$

(e.g., Vives [67]). Fig. 2a plots x_{MV} (dashed line) for $\alpha = 1$, $\sigma_v^2 = 1$, and $\sigma_u^2 = 1$ over the domain of the noisy signal S for a price P = 0. In Eq. (10), risk aversion α induces MV speculators,

¹⁵ The s.o.c. is satisfied if and only if risk seeking is not "too high," i.e., iff $\beta < \frac{\alpha}{[1 - \Delta^{-}(\cdot)]\phi(\cdot)}$ such that $\alpha^{*}(\cdot) > 0$.

¹⁴ Importantly, we choose the above parameters only for illustration. Similar insights ensue from alternative parametrizations of V_{PT} of Eq. (2) in the spirit of Tversky and Kahneman's [63] Prospect Theory. See also the discussion in Section 2.4.2.



Fig. 2. Prospect Theory trading. In this figure we plot realizations of the optimal demand schedule *x* and trading intensity $\frac{\partial x}{\partial S}$ (described in Section 2.3) of a MV speculator (i.e., for $\alpha = 1$, $\gamma = 0$, and $\beta = 0$: x_{MV} of Eq. (10) in Fig. 2a; $\frac{\partial x_{MV}}{\partial S} = \frac{1}{\alpha \sigma_u^2}$ in Fig. 2c; dashed lines), as well as of a Prospect Theory (PT) speculator (i.e., for $\alpha = 1$, $\gamma = 1$, and $\beta = 1.05$: x_{PT} of Eq. (9) in Fig. 2a; $\frac{\partial x_{PT}}{\partial S}$ in Fig. 2c, by numerical differentiation; solid lines), of a loss averse (LA) speculator (i.e., for $\alpha = 1$, $\gamma = 2$, and $\beta = 0$: $x_{LA}^{\gamma=2}$ in Fig. 2b; $\frac{\partial x_{LA}^{\gamma=2}}{\partial S}$ in Fig. 2d; dotted lines), and of a risk seeking in losses (RSL) speculator (i.e., for $\alpha = 1$, $\gamma = 0$, and $\beta = 2$: $x_{RSL}^{\beta=2}$ in Fig. 2b; $\frac{\partial x_{RSL}^{\beta=2}}{\partial S}$ in Fig. 2d; thin lines) over the domain of the noisy signal *S* for a price P = 0 when $\sigma_v^2 = 1$ and $\sigma_u^2 = 1$.

even if better informed, to submit cautious limit orders $(|x_{MV}| < \infty)$ to the MM. The stylized Prospect Theory preferences of Eq. (2) – i.e., $\gamma > 0$ and $\beta > \alpha$ in Eq. (9) – have additional effects on speculators' optimal trading activity. These effects are crucial to the analysis. To facilitate their interpretation, we consider a measure of (informed) speculation *aggressiveness* (or trading *intensity*), namely the sensitivity of speculators' demand function to information shocks: $\frac{\partial x_{PT}}{\partial S}$ (e.g., Vives [67]). Ceteris paribus, this variable captures a speculator's *effective* attitude toward risk when trading with her private signal S.

LA speculation. For any given signal *S* and price *P*, loss aversion induces speculators to either *lesser* trading or *no* trading at all (e.g., $|x_{LA}^{\gamma=2}| < |x_{MV}|$ in Fig. 2b, dotted line) to decrease their conditional expected trading loss (Eq. (6)) in the value function $V_{PT}(S)$ of Eq. (8). Consistently, ceteris paribus, the extent of lesser trading and the width of the no-trade interval $[S_L, S_H]$ in x_{PT} of Eq. (9) are increasing in the conditional marginal probability of a trading loss $\psi(\pm \chi)$. However, the relationship between loss averse (LA) and MV speculation's trading intensity is *nonmonotonic*. This is illustrated by plots of $\frac{\partial x_{MV}}{\partial S} = \frac{1}{\alpha \sigma_u^2}$ (Fig. 2c, dashed line) and $\frac{\partial x_{LA}^{\gamma=2}}{\partial S}$ (by numerical differentiation; Fig. 2d, dotted line) with respect to *S* (at *P* = 0). For "small" private signals (i.e., *S* close to its zero mean), marginal loss probability $\psi(\pm \chi)$ is perceived to be high

and induces LA speculators not to trade $\left(\frac{\partial x_{LA}^{\gamma=2}}{\partial S} = 0 < \frac{\partial x_{MV}}{\partial S}\right)$. When *S* is farther from its mean (i.e., "medium" |*S*|), marginal loss probability is lower (but nontrivial), and LA speculators decrease their conditional expected trading loss by revising their trading activity *more* significantly than MV speculation in response to the same information shock $\left(\frac{\partial x_{LA}^{\gamma=2}}{\partial S} > \frac{\partial x_{MV}}{\partial S}\right)$. For instance, in correspondence with negative private news at medium S > 0 in Fig. 2d, LA speculation reduces its purchases *more* than MV speculation. Lastly, when *S* is farthest from its mean (i.e., "large" |*S*|), marginal loss probability is low and LA speculators trade as if driven by risk aversion alone $\left(\frac{\partial x_{LA}^{\gamma=2}}{\partial S} \approx \frac{\partial x_{MV}}{\partial S}\right)$.

RSL speculation. Risk seeking in losses (RSL) induces speculators to trade *more* than MV speculation to increase the conditional variance of their trading losses (Eq. (7)) in the value function $V_{PT}(S)$. In x_{PT} of Eq. (9), this effect is captured by the effective risk aversion coefficient $\alpha^*(\pm \chi)$ being lower than α , (ceteris paribus) the more so the greater is $\Phi(\pm \chi)$, the conditional cumulative probability of a trading loss. E.g., see Fig. 2b (thin line) for $\gamma = 0$ and $\beta = 2$ ($|x_{RSL}^{\beta=2}| > |x_{MV}|$). Once again, the relationship between the trading intensity of RSL and MV speculators is *nonmonotonic*. When private signals *S* are small, the former perceive the cumulative loss probability $\Phi(\pm \chi)$ to be high and revise their trading activity *more* significantly than the latter in response to the same information shock ($\frac{\partial x_{RSL}^{\beta=2}}{\partial S} > \frac{\partial x_{MV}}{\partial S}$; Fig. 2d, thin line). At medium |S|, cumulative loss probability is lower (but nontrivial), and the response of RSL speculators to private news is more *muted* (lower $\frac{\partial x_{RSL}^{\beta=2}}{\partial S}$). For example, Fig. 2d shows that in correspondence with positive private news at some medium S < 0, RSL speculators increase their conditional trading loss variance by reducing their sales *less* than MV speculation ($\frac{\partial x_{RSL}^{\beta=2}}{\partial S} < \frac{\partial x_{MV}}{\partial S}$). Lastly, for large |S|, cumulative loss probability is low, leading both RSL and MV speculation to *similar* trading aggressiveness ($\frac{\partial x_{RSL}^{\beta=2}}{\partial S} \approx \frac{\partial x_{MV}}{\partial S}$).

PT speculators' optimal demand schedule x_{PT} (e.g., Fig. 2a, solid line, for $\gamma = 1$ and $\beta = 1.05$) and underlying effective attitude toward risk $\frac{\partial x_{PT}}{\partial S}$ (Fig. 2c, solid line) reflect the tension between these forces in accordance with the (state-dependent) predominance of loss aversion over (relatively mild) risk seeking in losses, as advocated in Tversky and Kahneman's [63] utility function of Eq. (1).

2.4. Equilibrium

The MM do not receive any information, but observe the aggregate order flow (i.e., the noisy limit-order book schedule) $\omega = x_{PT} + z$ before setting the market clearing price $P = P(\omega)$. Dealership competition and risk neutrality then imply semi-strong market efficiency (e.g., Kyle [45]; Hirshleifer, Subrahmanyam, and Titman [34]; Vives [67]):

$$P(\omega) = E[v|\omega]. \tag{11}$$

The expression for x_{PT} of Eq. (9) makes clear that the order flow's informativeness about the asset payoff v depends on the net effect of risk aversion, loss aversion, and risk seeking on PT speculators' trading activity. That effect also depends, in complex fashion, on the market clearing price. For instance if, at a given price P, speculators' noisy signal S falls within the no-trade interval $[S_L, S_H]$, the resulting aggregate order flow is uninformative about v (i.e., $\omega = z$). Thus,

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the MM must also conjecture speculators' trading status. In the spirit of Yuan [69], the MM's inference problem can be expressed as

$$E[v|\omega] = E[v|\omega, S > S_H] \Pr[S > S_H] + E[v|\omega, S < S_L] \Pr[S < S_L] + E[v|\omega, S_L \leqslant S \leqslant S_H] \Pr[S_L \leqslant S \leqslant S_H],$$
(12)

where $\Pr[S > S_H | \omega]$, $\Pr[S < S_L | \omega]$, and $\Pr[S_L \leq S \leq S_H | \omega]$ are the probability of the order flow being informative ($S > S_H$ or $S < S_L$) or uninformative ($S_L \leq S \leq S_H$) about v, respectively.

Unfortunately, x_{PT} of Eq. (9) makes ω , S_H , and S_L nonlinear functions of both the price P and the normally distributed noisy signal S, thus the MM's inference problem in Eq. (12) in the trade region outside of $[S_L, S_H]$ analytically intractable. There are several approaches in the literature for approximating nonlinear rational expectations equilibrium models.¹⁶ In this paper we employ a *numerical* approach to express both conditional first moments $E[v|\omega, S > S_H]$ and $E[v|\omega, S < S_L]$ and their accompanying probability $Pr[S > S_H]$ and $Pr[S < S_L]$ as explicit, linear functions of ω and P estimated via ordinary least squares (OLS).¹⁷ This approach, described in Appendix A, yields the following result.

Result 1. *The MM approximate the conditional means of* v *given the order flow* ω *and accompanying probability as*

$$E[v|\omega, S > S_H] = a_H + b_H\omega + c_H P, \tag{13}$$

$$\Pr[S > S_H] = 1 - \Phi(H), \tag{14}$$

$$E[v|\omega, S < S_L] = a_L + b_L \omega + c_L P, \tag{15}$$

$$\Pr[S < S_L] = \Phi(L), \tag{16}$$

$$E[v|\omega, S_L \leqslant S \leqslant S_H] = \phi \sigma_s \Lambda(L, H), \tag{17}$$

$$\Pr[S_L \leqslant S \leqslant S_H] = \Phi(H) - \Phi(L), \tag{18}$$

where $H = \frac{d_H + f_H P}{\sigma_s(1 - e_H)}$, $L = \frac{d_L + f_L P}{\sigma_s(1 - e_L)}$, $\Lambda(L, H) = \frac{\psi(L) - \psi(H)}{\Phi(H) - \Phi(L)}$, and the exogenous parameters a_H , a_L , b_H , b_L , c_H , c_L , d_H , d_L , e_H , e_L , f_H , and f_L are OLS coefficients.

Given Result 1, Proposition 1 accomplishes the task of solving for the equilibrium of this economy.

Proposition 1. *The rational expectations equilibrium price function of the model described by Eqs.* (9) *and* (11) *is the unique fixed point of the implicit function*

$$P_{PT} = P_{PT}^{0} + L_{PT}\omega = \begin{cases} P_{PT}^{0,H} + G_{PT}^{H}S + L_{PT}^{H}z & \text{if } S > S_{H}, \\ P_{PT}^{0,L} + G_{PT}^{L}S + L_{PT}^{L}z & \text{if } S < S_{L}, \\ P_{PT}^{0} + L_{PT}z & \text{if } S_{L} \leqslant S \leqslant S_{H}, \end{cases}$$
(19)

where the variables P_{PT}^0 and L_{PT} are functions of P, and $P_{PT}^{0,H}$, $P_{PT}^{0,L}$, G_{PT}^H , G_{PT}^L , L_{PT}^H , and L_{PT}^L are functions of P and S, defined in Appendix A.

¹⁶ E.g., see Blanchard and Kahn [13], Bernardo and Judd [12], Klein [41], and Sims [60], as well as the discussion in Lombardo and Sutherland [53].

¹⁷ Intuitively, a rational MM may be constrained by computational ability to estimate linear functions of ω and *P* via OLS to best predict the asset payoff *v* (e.g., see Hayashi [32, pp. 138–140]). We thank a referee for this insight. As we discuss in Appendix A, this approach yields very accurate estimates.

Within our stylized economy, the market microstructure literature suggests several (potentially estimable) variables capturing equilibrium market quality (e.g., see Vives [68]). The first one is market liquidity (depth), the inverse of the *price impact* λ_{PT} of noise trading z (as in Kyle [45]): $\lambda_{PT} \equiv \frac{\partial P_{PT}}{\partial z}$. The second one is ex-ante *price* (price change, or return) *volatility*: $\sigma_{PT}^2 \equiv var[P_{PT}]$ (since E[v] = 0; e.g., Vives [67,68]; Ozsoylev and Werner [58]). As such, σ_{PT}^2 is consistent with measures of realized return volatility conventionally computed in the empirical literature from observed asset price changes (e.g., see Hasbrouck [30]). The third one is expected informed trading volume (as in Vives [67]): $Vol_{PT} \equiv E[|x_{PT}|]$. The fourth one is price informativeness (efficiency), the posterior precision of the asset payoff v conditional on the price (as in Subrahmanyam [61]; Vives [67]): $Q_{PT} \equiv E\{var[v|P_{PT}]\}^{-1}$.¹⁸

We characterize the properties of the equilibrium of Proposition 1 relative to these variables in two steps. We begin by describing equilibrium market quality with MV speculation. We then explore the implications of PT speculation for market quality by virtue of numerical analysis.

2.4.1. MV speculation

If speculators have MV preferences (i.e., for $\gamma = 0$ and $\beta = 0$), there exists a closed-form solution to Eq. (19).¹⁹

Remark 1. In the presence of MV speculators, the unique rational expectations equilibrium price P_{MV} of the model described by Eqs. (10) and (11) is

$$P_{MV} = \frac{\sigma_v^2}{\alpha \sigma_u^2 \sigma_z^2} \omega = G_{MV}S + L_{MVZ},$$
(20)

where $G_{MV} \equiv \frac{1}{\alpha \sigma_u^2} L_{MV}$ and

$$L_{MV} \equiv \frac{\alpha \sigma_u^2 \sigma_v^2}{\sigma_u^2 (1 + \alpha^2 \sigma_u^2 \sigma_z^2) + \sigma_v^2} > 0.$$
⁽²¹⁾

As in Kyle [45] and Vives [67], equilibrium price impact is positive ($\lambda_{MV} \equiv \frac{\partial P_{MV}}{\partial z} = L_{MV} > 0$ of Eq. (21)), reflecting the MM's attempt to offset losses due to the adverse selection of the speculators with profits from noise trading. As such, market liquidity deteriorates (λ_{MV} increases) the more uncertain is the asset payoff v (i.e., the greater is σ_v^2 , as in Kyle [45]) since the more valuable is speculators' private information, the greater is their expected trading volume (i.e., the greater is $Vol_{MV} \equiv E[|x_{MV}|] = \sqrt{\frac{2}{pi}} \frac{\sigma_s \sigma_z}{\sqrt{\sigma_u^2 (1 + \alpha^2 \sigma_u^2 \sigma_z^2) + \sigma_v^2}}$, despite a riskier

asset payoff), and the more vulnerable are the MM to adverse selection.²⁰ Accordingly, mar-

²⁰ If a random variable x is normally distributed with mean zero and variance σ_x^2 , well-known properties of half-normal distributions imply that $E[|x|] = \sigma_x \sqrt{\frac{2}{pi}}$ (e.g., Vives [68, p. 149]). It can then be shown that $\frac{\partial Vol_{MV}}{\partial \sigma_v^2} > 0$. However, the two aforementioned effects exactly offset each other in MV speculators' trading intensity $\frac{\partial x_{MV}}{\partial S} = \frac{1}{\alpha \sigma^2}$.

¹⁸ This definition is based on the observation that in any (i.e., not necessarily normal) bivariate distribution of v and P_{PT} , payoff volatility $\sigma_v^2 = E\{var[v|P_{PT}]\} + var\{E[v|P_{PT}]\}$ (e.g., Greene [26, p. 83]; Vives [68, p. 131]). ¹⁹ In particular, Remark 1 is a special case of the linear equilibrium in Vives [67, Proposition 1.1], [68, Proposition 4.2]

when a continuum of risk-averse speculators receives identical noisy signals of the asset payoff.

ket liquidity improves (λ_{MV} decreases) – while both ex-ante price volatility ($\sigma_{MV}^2 \equiv var[P_{MV}] = \frac{\sigma_v^4}{\sigma_u^2(1+\alpha^2\sigma_u^2\sigma_z^2)+\sigma_v^2}$) and price informativeness ($Q_{MV} \equiv E\{var[v|P_{MV}]\}^{-1} = \frac{\sigma_u^2(1+\alpha^2\sigma_u^2\sigma_z^2)+\sigma_v^2}{\sigma_u^2\sigma_v^2(1+\alpha^2\sigma_u^2\sigma_z^2)}$) decline – in the presence of more intense noise trading (i.e., the greater is σ_z^2 ; e.g., as in Subrahmanyam [61]).²¹

The relationship between equilibrium market liquidity and either MV speculators' risk aversion (α) or the quality of their noisy signal $S(\sigma_u^2)$ – i.e., between λ_{MV} and speculative trading intensity $(\frac{\partial x_{MV}}{\partial S} = \frac{1}{\alpha \sigma_u^2})$ – is instead *nonmonotonic*, consistent with Vives [67]. Less risk-averse (or better informed) MV speculators (lower α or σ_u^2) trade more, and more aggressively with their private information (greater Vol_{MV} and $\frac{\partial x_{MV}}{\partial S}$), thus also making prices more efficient (higher Q_{MV}) and volatile (higher σ_{MV}^2). The former effect (labeled *selection*) worsens the MM's perceived adverse selection risk, while the latter effect (labeled *efficiency*) alleviates it. The efficiency effect dominates the selection effect, and market liquidity improves (λ_{MV} decreases), if risk aversion (or signal noise) is sufficiently low.²² These effects are crucial to understand equilibrium market quality in the presence of PT speculation.

2.4.2. PT speculation

When speculators display Prospect Theory-inspired preferences (e.g., $\gamma = 1$ and $\beta = 1.05$), additional forces affect equilibrium market quality. Unfortunately, none of the ensuing equilibrium variables of interest (price impact, λ_{PT} ; price volatility, σ_{PT}^2 ; expected informed trading volume, V_{PT} ; price efficiency, Q_{PT}) can be expressed in closed-form. Therefore, we illustrate those forces and the intuition behind them via numerical analysis of an economy in which $\alpha = 1$, $\sigma_v^2 = 1$, $\sigma_u^2 = 1$ (as in Fig. 2), and $\sigma_z^2 = 1$. Alternative calibrations of the basic economy's technology and preference parameters (e.g., as in Leland [47]; Easley, O'Hara, and Yang [22]) yield qualitatively similar implications.²³

We begin by observing that Proposition 1 and the definition of price impact $\lambda_{PT} \equiv \frac{\partial P_{PT}}{\partial z}$ imply that equilibrium market liquidity (depth) is the inverse of

²¹ Variance decomposition and joint normality of v and P_{MV} imply that the nonrandom $var[v|P_{MV}] = \sigma_v^2 - var[P_{MV}]$ in Q_{MV} (e.g., Greene [26, pp. 83, 90]; Vives [67, p. 133]). Thus, as in Kyle [45], ceteris paribus price volatility is increasing in price informativeness. Accordingly, it can further be shown that because of semi-strong market efficiency (Eq. (11)), the unconditional variance of the asset payoff less the equilibrium price $var[v - P_{MV}] = \sigma_v^2 - var[P_{MV}] =$ Q_{MV}^{-1} (e.g., Vives [68, p. 131]). An increase in noise trading also leads to greater informed trading volume Vol_{MV} . As observed by Vives [67], in this class of models there is trade both because speculators are endowed with better information than the MM and because of the presence of noise trading. Yet, MV speculators' trading intensity $\frac{\partial x_{MV}}{\partial S} = \frac{1}{\alpha \sigma_u^2}$ is independent of σ_z^2 since so are their conditional expected trading profit and risk when endowed with identical private signals (see Section 2.3).

²² It is straightforward to show that $\frac{\partial \lambda_{MV}}{\partial \alpha} > 0$ iff $\alpha < \frac{\sigma_s}{\sigma_u^2 \sigma_z}$, while $\frac{\partial \lambda_{MV}}{\partial \sigma_u^2} > 0$ iff $\sigma_u^2 < \frac{\sigma_v}{\alpha \sigma_z}$. ²³ For instance, this literature proposes market-specific calibrations of technology parameters (e.g., consistent with

²⁵ For instance, this literature proposes market-specific calibrations of technology parameters (e.g., consistent with annual S&P500 data, such as $\sigma_v^2 = 0.04$, $\sigma_u^2 = 0.20$, and $\sigma_z^2 = 0.10$), as well as risk aversion $\alpha = 2$, when attempting to match such equilibrium outcomes as the expected risk premium (or cost of capital) to available estimates in those markets (e.g., the U.S. stock market). We choose $\alpha = 1$ since Tversky and Kahneman [63] model economic agents as *nearly risk neutral* over trading gains in U_{TK} of Eq. (1), e.g., such that $U_{TK} \approx R(V_{PT})$ of Eq. (5) over realizations of $\pi > 0$ for $\alpha = 0.25$ (and $\beta = 0.40$). Our model's insights are meant to apply to a broad range of financial markets, and are robust to all of these parametrizations.

$$\lambda_{PT} = \begin{cases} \frac{L_{PT}^{H}}{1 - \frac{\partial P_{PT}^{0,H}}{\partial P} - \frac{\partial G_{PT}^{H}}{\partial P} S - \frac{\partial L_{PT}^{H}}{\partial P} z} & \text{if } S > S_{H}, \\ \frac{L_{PT}^{L}}{1 - \frac{\partial P_{PT}^{0,L}}{\partial P} - \frac{\partial G_{PT}^{L}}{\partial P} S - \frac{\partial L_{PT}^{L}}{\partial P} z} & \text{if } S < S_{L}, \\ \frac{L_{PT}}{1 - \frac{\partial P_{PT}^{0}}{\partial P} - \frac{\partial L_{PT}}{\partial P} z} & \text{if } S_{L} \leqslant S \leqslant S_{H} \end{cases}$$

$$(22)$$

(by the Implicit Function Theorem, given that neither $P_{PT}(\cdot)$ nor $G_{PT}(\cdot)$ in the expression for P_{PT} of Eq. (19) are functions of z). We then plot (in Fig. 3a) the average price impact λ_{PT} of Eq. (22) over the domain of S (by virtue of numerical integration, with respect to all possible noise trading shocks z) for $\gamma = 1$ and $\beta = 1.05$ ($E[\lambda_{PT}|S]$, solid line), as well as $\lambda_{MV} = L_{MV}$ of Eq. (21) (dashed line). Integration over z allows us to focus on the relation between λ_{PT} and PT speculators' optimal trading strategy given their private signal S (i.e., x_{PT} of Eq. (9)).

Fig. 3a motivates a noteworthy conclusion, novel to the literature, about a risky asset's equilibrium market liquidity in the presence of PT speculation.

Conclusion 1. The presence of better-informed speculators with Prospect Theory preferences improves equilibrium market liquidity and makes it state-dependent – lower for large and small |S|, higher otherwise.

The intuition for this conclusion is that market liquidity depends on (and mirrors) PT speculators' trading intensity in equilibrium. This is best explained in three steps. First, as discussed in Section 2.3, loss aversion and risk seeking in losses have *state-dependent*, often *conflicting* effects on PT speculators' effective attitude toward risk when trading (i.e., on their trading intensity $\frac{\partial x}{\partial S}$ in Figs. 2c and 2d), relative to MV speculation.²⁴ Ceteris paribus, RSL speculators ($\beta > \alpha$) trade more with their noisy signal *S*, yet most (least) aggressively when |S| is "small" ("medium") and the conditional cumulative probability of a loss ($\Phi(\pm \chi)$) is high (nontrivial). LA speculators ($\gamma > 0$) trade less or not at all with *S*, yet most (least) cautiously when |S| is small (medium) and the conditional marginal probability of a loss ($\psi(\pm \chi)$) is high (nontrivial). However, when |S| is "large" and both conditional cumulative and marginal loss probability are low, risk aversion explains exhaustively the trading activity and intensity of both LA and RSL speculators.

Second, as discussed in Section 2.4.1, the effect of higher (lower) speculative trading intensity on market liquidity is *nonmonotonic* because it both worsens (alleviates) the MM's adverse selection problem – *selection*, leading to higher (lower) price impact – and increases (decreases) the potential informativeness of the order flow – *efficiency*, leading to lower (higher) price impact. Fig. 3b plots average equilibrium price impact in the presence of either LA speculators $(E[\lambda_{LA}^{\gamma=2}|S], \text{ dotted line})$ or RSL speculators $(E[\lambda_{RSL}^{\beta=2}|S], \text{ thin line})$. In both cases, the *efficiency* effect dominates the *selection* effect in equilibrium such that risk seeking in losses (loss aversion) makes market liquidity (first decreasing then) increasing in |S|, i.e., mimicking the corresponding trading intensity in Fig. 2d as follows.

LA speculation. When the noisy signal *S* is close to its (zero) mean ("small" |S|), LA speculators (e.g., $\gamma = 2$ and $\beta = 0$) display the lowest trading intensity, hence are most likely not to

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²⁴ Plots of conditional *equilibrium* trading intensity $E[\frac{\partial x_{LA}^{\gamma=2}}{\partial S}|S]$, $E[\frac{\partial x_{LA}^{\beta=2}}{\partial S}|S]$, and $E[\frac{\partial x_{PT}}{\partial S}|S]$ display similar state-dependence, hence are omitted for economy of space.



Fig. 3. Prospect Theory and market quality. In this figure we plot, over the domain of speculators' noisy signal *S* (via numerical integration), conditional equilibrium outcomes of each of the measures of market quality defined in Section 2.4 (price impact λ_{PT} , Figs. 3a and 3b; price volatility σ_{PT}^2 , Figs. 3c and 3d; expected informed trading volume Vol_{PT} , Figs. 3e and 3f; price informativeness Q_{PT} , Figs. 3g and 3h) for the equilibrium of Proposition 1 in the presence of a continuum of Prospect Theory (PT) speculators ($\alpha = 1$, $\gamma = 1$, and $\beta = 1.05$: $E[\lambda_{PT}|S]$, $\sigma_{PT}^2|S$, $Vol_{PT}|S$, $Q_{PT}|S$; solid lines), loss averse (LA) speculators ($\alpha = 1$, $\gamma = 2$, and $\beta = 0$: $E[\lambda_{LA}^{\gamma=2}|S]$, $\sigma_{LA}^{2,\gamma=2}|S$, $Vol_{LA}^{\gamma=2}|S$, dotted lines), speculators risk seeking in losses (RSL) ($\alpha = 1$, $\gamma = 0$, and $\beta = 2$: $E[\lambda_{RSL}^{\beta=2}|S]$, $\sigma_{RSL}^{2,\beta=2}|S$, $Vol_{RSL}^{\beta=2}|S$, $Q_{RSL}^{\beta=2}|S$; thin lines), or MV speculators ($\alpha = 1$, $\gamma = 0$, and $\beta = 0$: λ_{MV} of Eq. (21), $\sigma_{MV}^2|S$, and $Q_{MV}|S$; dashed lines) when $\sigma_v^2 = 1$, $\sigma_u^2 = 1$, and $\sigma_z^2 = 1$.

trade with *S* (and the order flow to be uninformative about the asset payoff *v* relative to MV speculation). In those circumstances, the efficiency effect of low $\frac{\partial x_{LA}^{\gamma=2}}{\partial S}$ prevails over its selection effect in equilibrium, i.e., yielding high $E[\lambda_{LA}^{\gamma=2}|S]$ in Fig. 3b. When *S* is farther from its mean ("medium" |*S*|), LA speculators display high sensitivity to private information shocks, i.e., the order flow is more likely to be informative about *v* relative to MV speculation. Once again, the efficiency effect of high $\frac{\partial x_{LA}^{\gamma=2}}{\partial S}$ prevails over its selection effect in equilibrium, i.e., yielding lower $E[\lambda_{LA}^{\gamma=2}|S]$. Lastly, when *S* is farthest from its mean ("large" |*S*|), LA speculators display nearly the same trading intensity as MV speculators, leading to $E[\lambda_{LA}^{\gamma=2}|S] \approx \lambda_{MV}$.

RSL speculation. At small |S|, RSL speculators (e.g., $\gamma = 0$ and $\beta = 2$) display the highest trading intensity, hence the order flow is most likely to be informative about v relative to MV speculation. In those circumstances, the efficiency effect of high $\frac{\partial x_{RSL}^{\beta=2}}{\partial S}$ prevails over its selection effect in equilibrium, i.e., yielding low $E[\lambda_{RSL}^{\beta=2}|S]$ in Fig. 3b. At medium |S|, RSL speculators display low sensitivity to private information shocks, i.e., the order flow is less likely to be informative about v relative to MV speculation. Once again, the efficiency effect of low $\frac{\partial x_{RSL}^{\beta=2}}{\partial S}$ prevails over its selection effect in equilibrium, i.e., yielding higher $E[\lambda_{RSL}^{\beta=2}|S]$. Lastly, large realizations of |S| lead both RSL and MV speculation to similar trading aggressiveness, such that in equilibrium $E[\lambda_{RSL}^{\beta=2}|S] \approx \lambda_{MV}$.

Third, equilibrium market liquidity in the presence of PT speculation ($\gamma > 0$ and $\beta > \alpha$), as captured by the inverse of λ_{PT} of Eq. (22), stems from the interaction of these possibly conflicting forces. As discussed in the Introduction and Section 2.2, the literature provides guidance about the relative importance of loss aversion and risk seeking in losses in economic agents' preferences. According to Tversky and Kahneman [63], a large body of experimental evidence indicates that agents display *mild* risk seeking in losses relative to loss aversion when faced with gambles yielding potential gains and losses. In light of this evidence, Tversky and Kahneman [63] propose the *specific* power utility function U_{TK} of Eq. (1) (in Fig. 1a, crossed line). Fig. 3a suggests that for preference parameters consistent with their assessment (e.g., $\gamma = 1$ and $\beta = 1.05$), PT speculation improves equilibrium market liquidity ($E[\lambda_{PT}|S] < \lambda_{MV}$) in a fashion reflecting the relative, state-dependent prevalence of both loss aversion over risk seeking in losses *and* the accompanying efficiency effect over the selection effect. Identical insights ensue from plotting $E[\lambda_{PT}|S]$ with respect to either the asset payoff (v) or such observable equilibrium outcomes as average prices (i.e., returns: $E[P_{PT}|S]$) or aggregate order flow ($E[\omega|S]$).

PT speculation has similar, nontrivial effects on additional dimensions of equilibrium market quality. Fig. 3 plots (via numerical integration, conditional on the noisy signal *S*) equilibrium price volatility $\sigma_{PT}^2 | S$ (Fig. 3c, solid line), expected informed trading volume $Vol_{PT} | S$ (Fig. 3e, solid line), and price informativeness $Q_{PT} | S$ (Fig. 3g, solid line) – as well as their conditional MV counterparts $\sigma_{MV}^2 | S$, $Vol_{MV} | S$, and $Q_{MV} | S$ of Section 2.4.1 in closed-form (dashed lines).²⁵ Table 1 reports unconditional market quality outcomes (i.e., over all possible signals *S* and noise trading shocks *z*), as well as speculators' average equilibrium trading intensity $E[\frac{\partial x}{\partial S}]$.

²⁵ Since $\sigma_{PT}^2|S$, $Vol_{PT}|S$, and $Q_{PT}|S$ are conditional on *S* (i.e., are computed over all possible noise trading shocks *z* for a given *S*), we use conditional asset payoff uncertainty $var[v|S] = \sigma_v^2(1-\phi)$ and price volatility $var[P_{MV}|S] = L_{MV}^2 \sigma_z^2$

Equinorium market quanty.									
Speculators	$E[\lambda]$	σ^2	Vol	Q	$E[\frac{\partial x}{\partial S}]$				
$MV: \alpha = 1, \gamma = 0, \beta = 0$	0.33	0.33	0.65	1.50	0.33				
$_{PT}$: $\alpha = 1, \gamma = 1, \beta = 1.05$	0.27	0.16	0.61	1.19	0.44				
$\stackrel{\gamma=2}{PT}: \alpha = 1, \gamma = 2, \beta = 0$	0.26	0.12	0.54	1.14	0.45				
$\frac{\beta=2}{PT}: \alpha = 1, \gamma = 0, \beta = 2$	0.28	0.22	0.73	1.29	0.42				

Table 1

In this table we compute *unconditional* equilibrium market quality outcomes (average price impact $E[\lambda]$; price volatility σ^2 ; expected informed trading volume *Vol*; price informativeness *Q*; as well as speculators' average trading intensity $E[\frac{\partial x}{\partial S}]$; see Section 2.4), i.e., over all possible signals *S* and noise trading shocks *z*, in the presence of a continuum of mean-variance (MV) speculators ($\alpha = 1$, $\gamma = 0$, and $\beta = 0$), loss averse (LA) speculators ($\alpha = 1$, $\gamma = 2$, and $\beta = 0$), risk seeking in losses (RSL) ($\alpha = 1$, $\gamma = 0$, and $\beta = 2$), and Prospect Theory (PT) speculators ($\alpha = 1$, $\gamma = 1$, and $\beta = 1.05$), when $\sigma_v^2 = 1$, $\sigma_u^2 = 1$, and $\sigma_z^2 = 1$.

Consistent with the above intuition, loss aversion lowers speculators' average trading volume relative to MV speculation $(Vol_{LA}^{\gamma=2} < Vol_{MV})$, while risk seeking in losses increases it $(Vol_{RSL}^{\beta=2} > Vol_{MV})$; yet, both increase speculators' sensitivity to private information shocks $(E[\frac{\partial x_{LA}^{\gamma=2}}{\partial S}] > \frac{\partial x_{MV}}{\partial S}$ and $E[\frac{\partial x_{RSL}^{\beta=2}}{\partial S}] > \frac{\partial x_{MV}}{\partial S}$). In equilibrium, the latter makes the market for the risky asset more liquid in both circumstances $(E[\lambda_{LA}^{\gamma=2}] < \lambda_{MV}$ and $E[\lambda_{RSL}^{\beta=2}] < \lambda_{MV})$ via a prevailing efficiency effect, such that on average $E[\frac{\partial x_{PT}}{\partial S}] > \frac{\partial x_{MV}}{\partial S}$ and $E[\lambda_{PT}] < \lambda_{MV}$ (Conclusion 1). Expected informed trading volume is always increasing in |S|; see $Vol_{MV}|S$ (Fig. 3e), $Vol_{LA}^{\gamma=2}|S$, and $Vol_{RSL}^{\beta=2}|S$ (Fig. 3f). Intuitively, ceteris paribus for market efficiency, speculators

Expected informed trading volume is always increasing in |S|; see $Vol_{MV}|S$ (Fig. 3e), $Vol_{LA}^{\gamma=2}|S$, and $Vol_{RSL}^{\beta=2}|S$ (Fig. 3f). Intuitively, ceteris paribus for market efficiency, speculators trade more the farther is the noisy signal they observe from its mean. However, the *relative* magnitude of expected absolute speculation depends on speculators' relative trading intensity $\frac{\partial x}{\partial S}$ (in Figs. 2c and 2d). For instance, at small |S|, LA speculators are least sensitive to private news relative to MV and RSL speculators (see Fig. 2d); hence, their average trading volume is lowest. Because of semi-strong market efficiency (Eq. (11)), equilibrium price volatility and informativeness also depend on speculators' effective risk attitude, via its effect on equilibrium market depth. In particular, in the presence of RSL (LA) speculation, both variables are also (first decreasing then) increasing in |S|; see $\sigma_{RSL}^{2,\beta=2}|S$ and $Q_{RSL}^{\beta=2}|S$ ($\sigma_{LA}^{2,\gamma=2}|S$ and $Q_{LA}^{\gamma=2}|S$) in Figs. 3d and 3h, respectively (thin [dotted] lines). For example, at medium |S|, the prevailing efficiency effect of RSL speculation's low sensitivity to private news (see Fig. 2d) on MM's adverse selection risk yields high price impact $E[\lambda_{RSL}^{\beta=2}|S]$, hence high price volatility and informativeness. Accordingly, Table 1 reports that in aggregate not only $\sigma_{LA}^{2,\gamma=2} < \sigma_{MV}^2$ and $Q_{LA}^{\gamma=2} < Q_{MV}$ but also $\sigma_{RSL}^{2,\beta=2} < \sigma_{MV}^2$ and $Q_{RSL}^{\beta=2} < Q_{MV}$.

Once again, for parameters capturing parsimoniously the greater importance of loss aversion relative to risk seeking in losses in Tversky and Kahneman's [63] Prospect Theory (e.g., $\gamma = 1$

to compute $\sigma_{MV}^2|S = L_{MV}^2\sigma_z^2$ and $var[v|P_{MV}, S] = \sigma_v^2(1-\phi) - L_{MV}^2\sigma_z^2$ in $Q_{MV}|S$. Accordingly, we use well-known properties of folded normal distributions (e.g., Nelson [55]) to compute $Vol_{MV}|S = \sqrt{\frac{2}{p_i}}\sqrt{var[x_{MV}|S]}e^{-\frac{E[x_{MV}|S]^2}{2var[x_{MV}|S]}} + E[x_{MV}|S][1 - 2\phi(-\frac{E[x_{MV}|S]}{\sqrt{var[x_{MV}|S]}})]$, where $E[x_{MV}|S] = [\frac{\phi - G_{MV}}{\alpha\sigma_v^2(1-\phi)}]S$ and $var[x_{MV}|S] = \frac{L_{MV}^2\sigma_z^2}{\alpha^2\sigma_v^4(1-\phi)^2}$, while Eq. (30) in Section 3.2 implies that $E\{var[v|P_{PT}]|S\} = \sigma_v^2(1-\phi) - var[P_{PT}|S]$ in $Q_{PT}|S$ (see Vives [68, p. 131]).

and $\beta = 1.05$ in Eq. (4)), Fig. 3 and Table 1 suggest the following additional conclusions about a risky asset's equilibrium market quality in the presence of PT speculation.

Conclusion 2. The presence of better-informed speculators with Prospect Theory preferences lowers equilibrium price volatility and makes it state-dependent – higher for large and small |S|, lower otherwise.

Conclusion 3. The presence of better-informed speculators with Prospect Theory preferences lowers expected informed trading volume and makes it state-dependent – larger for large |S|, smaller for small |S|.

Conclusion 4. The presence of better-informed speculators with Prospect Theory preferences lowers equilibrium price informativeness and makes it state-dependent – higher for large and small |S|, lower otherwise.

3. Endogenous information acquisition

An important characteristic of most financial markets is that information about the fundamentals of traded securities is available only at a cost. There is a vast literature showing that endogenous information acquisition affects both the dynamics of asset prices and the quality of their trading venues.²⁶ In this section, we investigate whether Prospect Theory preferences affect speculators' production of information and accompanying trading decisions by studying an extension of the economy of Section 2 in which the noisy signal is costly.

3.1. The amended economy

We consider a market that is identical to that of Section 2. We further assume that at time t = 1, before trading, each speculator decides whether to pay a fixed cost c > 0 to observe the noisy signal S of the asset payoff v.

The ensuing information acquisition model resembles those in Grossman and Stiglitz [29], Admati and Pfleiderer [1], and Foster and Viswanathan [24]. When informed, speculators learn about v exclusively from S (i.e., their information set $I = \{S\}$); when uninformed, speculators learn about v from the equilibrium price P (i.e., $I^u = \{P\}$). Speculators simultaneously decide whether to become informed at a given c, and the equilibrium fraction $\mu(c)$ of informed speculation is determined. As in Grossman and Stiglitz [29], each speculator's ex-ante returns to information depend on the mass of informed speculation. Speculators purchase the signal Suntil their expected value function $(V_{PT}(S))$ over all possible signals net of the information cost is equal to their expected value function when remaining uninformed $(V_{PT}^u(P))$, as follows:

$$E[V_{PT}(S)] - c = E[V_{PT}^{u}(P)], \qquad (23)$$

where $V_{PT}(S)$ is described in Eq. (8), $V_{PT}^{u}(P)$ is given by

²⁶ An incomplete list includes Grossman and Stiglitz [29], Verrecchia [65], Li, McKelvey, and Page [51], Admati and Pfleiderer [1], Vives [66], Foster and Viswanathan [24], Burguet and Vives [14], and Hellwig and Veldkamp [33]. See also Vives [68] and Veldkamp [64] for a review.

$$V_{PT}^{u}(P) = E[\pi|P] - \frac{1}{2}\alpha \operatorname{var}[\pi|P] + \gamma E[\pi|P, \pi < 0]\Phi(\operatorname{sgn}(x)\chi^{u}) + \frac{1}{2}\beta \operatorname{var}[\pi|P, \pi < 0]\Phi(\operatorname{sgn}(x)\chi^{u}),$$
(24)

and $\chi^{u} = \frac{P - E[v|P]}{\sqrt{var[v|P]}}$. Informed and uninformed speculators then submit their optimal demand schedules x_{PT} and x_{PT}^{u} , respectively, contingent upon their information sets I and I^{u} . As discussed in Section 2.3, informed speculators trade x_{PT} of Eq. (9); uninformed speculators choose the optimal demand schedule that maximizes $V_{PT}^{u}(P)$ of Eq. (24):

$$x_{PT}^{u} = \begin{cases} \frac{[1+\gamma\Phi(\chi^{u})]}{\alpha^{*}(\chi^{u}) \operatorname{var}[v|P]} \{E[v|P] - P\} - \frac{\gamma\psi(\chi^{u})}{\alpha^{*}(\chi^{u})\sqrt{\operatorname{var}[v|P]}} > 0 & \text{if } P < P_{H}^{u}, \\ \frac{[1+\gamma\Phi(-\chi^{u})]}{\alpha^{*}(-\chi^{u}) \operatorname{var}[v|P]} \{E[v|P] - P\} + \frac{\gamma\psi(-\chi^{u})}{\alpha^{*}(-\chi^{u})\sqrt{\operatorname{var}[v|P]}} < 0 & \text{if } P > P_{L}^{u}, \\ 0 & \text{if } P_{L}^{u} \leqslant P \leqslant P_{H}^{u}, \end{cases}$$
(25)

where $P_H^u = E[v|P] - \frac{\gamma \psi(\chi^u) \sqrt{var[v|P]}}{[1+\gamma \Phi(\chi^u)]}$ and $P_L^u = E[v|P] + \frac{\gamma \psi(-\chi^u) \sqrt{var[v|P]}}{[1+\gamma \Phi(-\chi^u)]}$. For $\gamma = 0$ and $\beta = 0$, Eq. (25) reduces to the optimal generalized limit order of an uninformed MV speculator,

$$x_{MV}^{u} = \frac{1}{\alpha \, var[v|P]} \{ E[v|P] - P \}.$$
(26)

Lastly, the MM set P according to semi-strong market efficiency (Eq. (11)) for the observed aggregate order flow $\omega = \mu(c)x_{PT} + (1 - \mu(c))x_{PT}^{u} + z$. Thus, the decision rules of both uninformed speculation and the MM depend on the fraction $\mu(c)$ of informed speculation in the noisy limit-order book schedule ω .

3.2. Equilibrium

The equilibrium of such an economy is typically found in several steps (e.g., see Grossman and Stiglitz [29]; Yuan [69]). First, we use Result 1 and Proposition 1 to solve for the equilibrium of a *fictitious* economy populated *exclusively* by a given fraction μ of informed speculators, i.e., where $\omega^{fic} = \mu x_{PT} + z$. This is accomplished in Result 2 and Proposition 2.

Result 2. If $\omega^{fic} = \mu x_{PT} + z$, the MM approximate the conditional means of v given ω^{fic} as

$$E[v|\omega^{fic}, S > S_H] = a_H(\mu) + b_H(\mu)\omega^{fic} + c_H(\mu)P,$$
(27)

$$E[v|\omega^{fic}, S < S_L] = a_L(\mu) + b_L(\mu)\omega^{fic} + c_L(\mu)P, \qquad (28)$$

where the exogenous parameters $a_H(\mu)$, $a_L(\mu)$, $b_H(\mu)$, $b_L(\mu)$, $c_H(\mu)$, and $c_L(\mu)$ are OLS coefficients. The conditional mean $E[v|\omega^{fic}, S_L \leq S \leq S_H]$ is given by Eq. (17) of Result 1.

Proposition 2. If $\omega^{fic} = \mu x_{PT} + z$, the rational expectations equilibrium price function of the model described by Eqs. (9) and (11) is the unique fixed point of the implicit function

$$P_{PT}^{fic}(\mu) = P_{PT}^{0}(\mu) + L_{PT}(\mu)\omega^{fic} \\ = \begin{cases} P_{PT}^{0,H}(\mu) + G_{PT}^{H}(\mu)S + L_{PT}^{H}(\mu)z & \text{if } S > S_{H}, \\ P_{PT}^{0,L}(\mu) + G_{PT}^{L}(\mu)S + L_{PT}^{L}(\mu)z & \text{if } S < S_{L}, \\ P_{PT}^{0}(\mu) + L_{PT}(\mu)z & \text{if } S_{L} \leqslant S \leqslant S_{H}, \end{cases}$$

$$(29)$$

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where the variables $P_{PT}^{0}(\mu)$ and $L_{PT}(\mu)$ are functions of P and μ , and $P_{PT}^{0,H}(\mu)$, $P_{PT}^{0,L}(\mu)$, $G_{PT}^{H}(\mu)$, $G_{PT}^{L}(\mu)$, $L_{PT}^{H}(\mu)$, and $L_{PT}^{L}(\mu)$ are functions of P, S, and μ , defined in Appendix A.

Importantly, because uninformed speculators learn about v only from P, the equilibrium price $P_{PT}^{fic}(\mu)$ (and order flow ω^{fic}) of this fictitious economy has the *same* information content of the equilibrium price $P_{PT}(\mu)$ (and order flow $\omega = \omega^{fic} + (1 - \mu)x_{PT}^{u}$) of the economy populated by both informed and uninformed speculators. Namely, the equilibrium prices $P_{PT}^{fic}(\mu)$ and $P_{PT}(\mu)$ (and order flows ω^{fic} and ω) are *equivalent sufficient statistics* for v in the Blackwell sense (e.g., see Yuan [69, pp. 386–387]; Vives [68, pp. 372–373]). Hence, semi-strong market efficiency implies (via $E[v|\omega^{fic}] = E[v|\omega]$) that $P_{PT}^{fic}(\mu) = P_{PT}(\mu)$.

Second, uninformed speculators learn about the asset payoff v from $P_{PT}(\mu)$. Their inference problem is similar to the inference problem of the MM, as described in Section 2.4. Being rational, the uninformed speculators use Proposition 2 (i.e., Eq. (29)) to infer the realized order flow ω from $P_{PT}(\mu)$ and the expression for x_{PT} of Eq. (9) to assess the information content of ω for v. Accordingly, semi-strong market efficiency (Eq. (11)) directly implies that

$$E[v|P_{PT}(\mu)] = P_{PT}(\mu)$$
(30)

(e.g., Vives [67,68]).²⁷

Third, we use Eq. (30) to specify the optimal demand schedule of the uninformed speculators. It is straightforward to show that Eq. (25) implies that it is optimal for uninformed speculators not to trade, i.e., $x_{PT}^u(P_{PT}(\mu)) = 0.^{28}$ Intuitively, in our setting uninformed speculative trading is insufficiently rewarding given the risk if equilibrium prices are semi-strong efficient (i.e., because of dealership competition and risk neutrality).²⁹

Fourth, we use $x_{PT}(P_{PT}(\mu))$ of Eq. (9) to solve for the equilibrium in the information market, $\mu(c)$, from Eq. (23), and Proposition 2 to solve for the ensuing market equilibrium given $\mu(c)$. Since $V_{PT}^{u}(P_{PT}(\mu)) = 0$ for $x_{PT}^{u}(P_{PT}(\mu)) = 0$, at any given μ we can interpret a speculator's expected value function over all possible signals, $E[V_{PT}(S, P_{PT}(\mu))]$, as the maximum price she is willing to pay to purchase the noisy signal S. Hence, $\mu(c)$ is the fraction of informed speculation at which a speculator's reservation price for private information is equal to its cost:

$$E\left[V_{PT}\left(S, P_{PT}(\mu(c))\right)\right] = c.$$
(31)

Given the distributional assumptions in Section 2, the expression for $E[V_{PT}(S, P_{PT}(\mu))]$ is analytically intractable except in the presence of MV speculators. Therefore, once again we begin by examining endogenous information acquisition with MV speculation. We then characterize equilibrium information production and market quality with PT speculation when private information is costly via numerical analysis.

²⁸ To that purpose, note that $x_{PT}^{u}(P_{PT}(\mu)) = \arg \max V_{PT}^{u}(P_{PT}(\mu))$. Because $\chi^{u} = 0$, $\psi(0) = \frac{1}{\sqrt{2pi}} > 0$, and $\Phi(0) = \frac{1}{2}$, the s.o.c. of this problem is satisfied iff risk seeking is not "too high," i.e., iff $\beta < \frac{2\alpha}{(1-\frac{2}{pi})}$ such that

²⁷ I.e., as above, $P_{PT}(\mu)$ and ω are equivalent sufficient statistics for v in the Blackwell sense $(E[v|P_{PT}(\mu)] = E[v|\omega])$. For instance, Vives [68, p. 131] observes that if $E[v|P_{PT}(\mu)] \neq P_{PT}(\mu)$ the competitive risk-neutral MM "would like to take unbounded positions" in the risky asset.

 $[\]alpha^*(0) = \alpha - \frac{1}{2}\beta(1 - \frac{2}{pi}) > 0$. Hence, x_{PT}^u of Eq. (25) can be neither positive for $P < P_H^u$ nor negative for $P > P_L^u$.

²⁹ Accordingly, while uninformed risk-averse speculators trade in absence of competitive, risk-neutral market making in the market clearing condition (e.g., see Grossman and Stiglitz [29]), Eqs. (26) and (30) imply that $x_{MV}^u = 0$.

3.2.1. MV speculation

When speculators have MV preferences ($\gamma = 0$ and $\beta = 0$), the equilibrium of Proposition 2 is in closed-form and implies the following remark.

Remark 2. In the presence of a fraction μ of informed MV speculators, their reservation price for private information is

$$E[V_{MV}(S, P_{MV}(\mu))] = \frac{\alpha \sigma_v^2 \sigma_u^2 \sigma_z^2}{2[\sigma_u^2(\mu^2 + \alpha^2 \sigma_u^2 \sigma_z^2) + \mu^2 \sigma_v^2]},$$
(32)

such that if private information costs c, there is a unique rational expectations equilibrium in which

$$\mu_{MV}(c) = \begin{cases} 1 & \text{if } c \leqslant c_L, \\ \frac{\sigma_u \sigma_z}{\sigma_s \sqrt{2c}} \sqrt{\alpha(\sigma_v^2 - 2\alpha c \sigma_u^2)} & \text{if } c_L < c < c_H, \\ 0 & \text{if } c \geqslant c_H, \end{cases}$$
(33)

where $c_L = \frac{\alpha \sigma_u^2 \sigma_v^2 \sigma_z^2}{2[\sigma_u^2(1+\alpha^2 \sigma_u^2 \sigma_z^2)+\sigma_v^2]}$ and $c_H = \frac{\sigma_v^2}{2\alpha \sigma_u^2}$, and

$$P_{MV}(\mu_{MV}(c)) = \frac{\mu_{MV}(c)\sigma_v^2}{\alpha\sigma_u^2\sigma_z^2}\omega = G_{MV}(\mu_{MV}(c))S + L_{MV}(\mu_{MV}(c))z,$$
(34)

where $G_{MV}(\mu_{MV}(c)) = \frac{\mu_{MV}(c)}{\alpha \sigma_u^2} L_{MV}(\mu_{MV}(c))$ and

$$L_{MV}(\mu_{MV}(c)) = \begin{cases} \frac{\alpha \sigma_u^2 \sigma_v^2}{\sigma_u^2 (1 + \alpha^2 \sigma_u^2 \sigma_z^2) + \sigma_v^2} = L_{MV} > 0 & \text{if } c \leq c_L, \\ \frac{\sigma_u \sqrt{2\alpha c}}{\sigma_z \sigma_s} \sqrt{\sigma_v^2 - 2\alpha c \sigma_u^2} > 0 & \text{if } c_L < c < c_H, \\ 0 & \text{if } c \geq c_H. \end{cases}$$
(35)

As in Grossman and Stiglitz [29], the maximum price a MV speculator is willing to pay to observe the noisy signal *S* is strictly monotonically decreasing in the fraction of informed MV speculation in the economy. E.g., see the plot of $E[V_{MV}(S, P_{MV}(\mu))]$ of Eq. (32) in Fig. 4a (dashed line) over the domain of $\mu \in [0, 1]$ in an economy in which $\alpha = 1$, $\sigma_v^2 = 1$, $\sigma_u^2 = 1$, and $\sigma_z^2 = 1$ (as in Figs. 2 and 3). When private information is "too cheap" ($c \leq c_L$), all MV speculators purchase *S* and the equilibrium reduces to the one described in Remark 1. When private information is "too expensive" ($c \geq c_H$), no MV speculator either purchases *S* or trades and the equilibrium reduces to $P_{MV}(\mu_{MV}(c)) = 0$. Otherwise, the equilibrium fraction of informed MV speculators, $\mu_{MV}(c)$ of Eq. (33), is decreasing in the cost of information (c) and increasing in fundamental uncertainty (σ_v^2) or the intensity of noise trading (σ_z^2).³⁰ Intuitively, a more uncertain asset payoff v makes *S* more valuable, while more noise trading lowers price informativeness (see Section 2.4.1); hence, the greater are returns to private information, and more MV speculators purchase *S*.

However, the relationship between information production and either MV speculators' risk aversion (α) or the quality of private information (σ_u^2) – i.e., between $\mu_{MV}(c)$ and speculative trading intensity ($\frac{\partial x_{MV}}{\partial S} = \frac{1}{\alpha \sigma_u^2}$) – is *nonmonotonic*. Intuitively, for any given μ , MV speculators

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³⁰ It is straightforward to show that for any $c \in (c_L, c_H)$, $\frac{\partial \mu_{MV}(c)}{\partial c} < 0$, $\frac{\partial \mu_{MV}(c)}{\partial \sigma_v^2} > 0$, and $\frac{\partial \mu_{MV}(c)}{\partial \sigma_z^2} > 0$.



Fig. 4. Prospect Theory and endogenous information acquisition. In this figure we plot the maximum price that a speculator is willing to pay to purchase the noisy signal *S* in the amended economy of Section 3 over the domain of $\mu \in [0, 1]$, the fraction of informed speculators in the market. Specifically, we plot the expected value function of a MV speculator $(\alpha = 1, \gamma = 0, \text{ and } \beta = 0: E[V_{MV}(S, P_{MV}(\mu))]$ of Eq. (32); Fig. 4a, dashed line), a Prospect Theory (PT) speculator $(\alpha = 1, \gamma = 1, \text{ and } \beta = 1.05: E[V_{PT}(S, P_{PT}(\mu))]$ of Eq. (A.29); Fig. 4a, solid line), a loss averse (LA) speculator $(\alpha = 1, \gamma = 2, \text{ and } \beta = 0: E[V_{LA}^{\gamma=2}(S, P_{LA}^{\gamma=2}(\mu))]$ of Eq. (A.29); Fig. 4b, dotted line), and of a speculator risk seeking in losses $(\alpha = 1, \gamma = 0, \text{ and } \beta = 2: E[V_{RSL}^{\beta=2}(S, P_{RSL}^{\beta=2}(\mu))]$ of Eq. (A.29); Fig. 4b, thin line), as well as the information cost c = 0.225 (crossed line), when $\sigma_v^2 = 1, \sigma_u^2 = 1$, and $\sigma_z^2 = 1$.

are less willing to pay a certain cost *c* for private information yielding uncertain returns the more risk-averse they are (higher α) or the less precise is that information (higher σ_u^2). Yet, more riskaverse (or worse-informed) MV speculators trade less, and less aggressively with their signal *S* (lower $|x_{MV}|$ and $\frac{\partial x_{MV}}{\partial S}$; see Section 2.4.1), thus making prices less efficient and increasing returns to private information. Ceteris paribus, if information is sufficiently expensive (i.e., if *c* is "large"), the former effect dominates the latter and the equilibrium fraction of informed MV speculation declines.³¹

Importantly, costly private information has two effects of opposite sign on equilibrium market liquidity (the inverse of $\lambda_{MV}(\mu_{MV}(c)) = L_{MV}(\mu_{MV}(c))$ of Eq. (35)), relative to the equilibrium of Remark 1 ($\mu = 1$). The first one is a *selection* effect: For any cost $c \in (c_L, c_H)$, only a fraction $\mu_{MV}(c) < 1$ of MV speculation is privately informed (such that the expected informed trading volume $Vol_{MV}(\mu_{MV}(c)) \equiv E[|\mu_{MV}(c)x_{MV}|] = \sqrt{\frac{2}{p_i}} \frac{\sigma_z}{\sigma_v} \sqrt{\sigma_v^2 - 2\alpha c \sigma_u^2} < Vol_{MV})$, lowering the MM's adverse selection risk. The second one is an *efficiency* effect: For any $\mu_{MV}(c) < 1$, equilibrium price informativeness deteriorates $(Q_{MV}(\mu_{MV}(c))) = \frac{\sigma_v^2 + \sigma_u^2}{\sigma_u^2 \sigma_v^2 (2\alpha c + 1)} < Q_{MV})$, making the MM more vulnerable to adverse selection. The efficiency effect dominates the selection effect, and market liquidity deteriorates $(\lambda_{MV}(\mu_{MV}(c)) > \lambda_{MV})$, if private information is sufficiently cheap (and MV speculators' risk aversion is sufficiently low).³² E.g., see the equilibrium market quality outcomes in Table 2 for c = 0.225, relative to those in Table 1.

³¹ Eq. (33) implies that $\frac{\partial \mu_{MV}(c)}{\partial \alpha} < 0$ iff $c > \frac{\sigma_v^2}{4\alpha \sigma_u^2}$, while $\frac{\partial \mu_{MV}(c)}{\partial \sigma_u^2} < 0$ iff $c > \frac{\sigma_v^4}{2\alpha \sigma_u^2 (2\sigma_v^2 + \sigma_u^2)}$.

³² It ensues from Eqs. (21) and (35) that $\lambda_{MV}(\mu_{MV}(c)) > \lambda_{MV}$ iff $c_L < c < \frac{\sigma_v^2 \sigma_s^2}{2\alpha \sigma_u^2 [\sigma_u^2 (1 + \alpha^2 \sigma_u^2 \sigma_z^2) + \sigma_v^2]}$ (and $\alpha < \frac{\sigma_s}{\sigma_u^2 \sigma_z}$). In those circumstances, informed MV speculators' equilibrium trading intensity is higher than for c = 0 (i.e.,

Equilibrium market quality with endogenous information acquisition.									
Speculators	$\mu(c)$	$E[\lambda(\mu)]$	$\sigma^2(\mu)$	$Vol(\mu)$	$Q(\mu)$	$E[\frac{\partial x(\mu)}{\partial S}]$			
$MV: \alpha = 1, \gamma = 0, \beta = 0$	0.78	0.35	0.28	0.59	1.38	0.45			
$PT: \alpha = 1, \gamma = 1, \beta = 1.05$	0.52	0.20	0.05	0.43	1.06	0.68			
$PT^{\gamma=2}_{PT}: \alpha = 1, \gamma = 2, \beta = 0$	0.29	0.11	0.01	0.23	1.01	0.75			
$\stackrel{\beta=2}{PT}: \alpha = 1, \gamma = 0, \beta = 2$	0.81	0.25	0.16	0.68	1.20	0.54			

Table 2

In this table we compute *unconditional* equilibrium market quality outcomes (average price impact $E[\lambda]$; price volatility σ^2 ; expected informed trading volume Vol; price informativeness Q; as well as average informed trading intensity $E[\frac{\partial x}{\partial S}]$; see Section 2.4), i.e., over all possible signals S and noise trading shocks z given a private information cost c = 0.225, in the presence of the ensuing equilibrium fraction $\mu(c)$ of informed mean-variance (MV) speculators ($\alpha = 1$, $\gamma = 0$, and $\beta = 0$), informed loss averse (LA) speculators ($\alpha = 1$, $\gamma = 2$, and $\beta = 0$), informed risk seeking in losses (RSL) speculators ($\alpha = 1$, $\gamma = 0$, and $\beta = 2$), and informed Prospect Theory (PT) speculators ($\alpha = 1$, $\gamma = 1$, and $\beta = 1.05$), when $\sigma_v^2 = 1$, $\sigma_u^2 = 1$, and $\sigma_z^2 = 1$.

Accordingly, equilibrium price volatility $(\sigma_{MV}^2(\mu_{MV}(c))) = \frac{\sigma_v^2(\sigma_v^2 - 2\alpha c \sigma_u^2)}{\sigma_v^2 + \sigma_u^2} < \sigma_{MV}^2)$, informed trading volume, and price efficiency are monotonically decreasing in private information costs; equilibrium market liquidity is not. As discussed above, costly information has both a selection effect and an efficiency effect on the MM's perceived adverse selection risk. Only if MV speculators' risk aversion (α) is sufficiently low does market liquidity deteriorate ($\lambda_{MV}(\mu_{MV}(c))$) increase) when the noisy signal S becomes more expensive.³³ In those circumstances, a higher cmitigates MV speculators' incentive to purchase S less than it would if α were high. Since less risk-averse MV speculators trade *more* aggressively with their signal S (see Section 2.4.1), ceteris paribus a higher c reinforces the selection effect of nonzero c (lower adverse selection risk) less than it reinforces the corresponding efficiency effect (greater adverse selection risk), worsening equilibrium market liquidity. Once again we use these basic insights to interpret equilibrium market quality in the presence of PT speculation facing costly private information.

3.2.2. PT speculation

When Prospect Theory-inspired speculation (e.g., $\gamma = 1$ and $\beta = 1.05$) faces nonzero information costs, the equilibrium of Proposition 2 cannot be expressed in closed-form. Thus, we describe its properties via numerical analysis of the economy of Sections 2.4.1 and 2.4.2 (in which $\alpha = 1$, $\sigma_v^2 = 1$, $\sigma_u^2 = 1$, and $\sigma_z^2 = 1$, as in Figs. 2 and 3) with information cost c = 0.225(as for MV speculation in Section 3.2.1).

As discussed above, we proceed in two main steps. In the first step, we solve for the equilibrium of the information market, i.e., the fraction $\mu(c)$ of PT speculators paying c to purchase the noisy signal S such that Eq. (31) is satisfied, by virtue of numerical integration (see Appendix A, Eq. (A.29)). Fig. 4a plots $E[V_{PT}(S, P_{PT}(\mu))]$ (solid line) over the domain of $\mu \in [0, 1]$. Fig. 4a implies a novel conclusion about information production in the presence of PT speculation.

 $\frac{\partial x_{MV}(\mu_{MV}(c))}{\partial S} = \frac{2c}{\sigma_v^2} > \frac{\partial x_{MV}}{\partial S} [= \lambda_{MV}], \text{ but not high enough to offset the efficiency effect of } \mu_{MV}(c) < 1 \text{ on equilibrium market depth, iff } \frac{\alpha \sigma_v^4 \sigma_u^2}{2[\sigma_u^2(1+\alpha^2 \sigma_u^2 \sigma_z^2)+\sigma_v^2]} < c < \frac{\sigma_v^2 \sigma_s^2}{2\alpha \sigma_u^2[\sigma_u^2(1+\alpha^2 \sigma_u^2 \sigma_z^2)+\sigma_v^2]}.$ ³³ In particular, it can be shown from Eq. (35) that $\frac{\partial \lambda_{MV}(\mu_{MV}(c))}{\partial c} > 0$ iff $\alpha < \frac{\sigma_v^2}{4c\sigma_u^2}.$

Conclusion 5. The presence of speculators with Prospect Theory preferences diminishes equilibrium information production.

The intuition for this conclusion is as follows. Relative to MV speculation, loss aversion and risk seeking in losses have additional, *opposite* effects on a speculator' information decision.

LA speculation. Loss aversion ($\gamma > 0$) makes a speculator *less* willing to purchase *S* at a certain cost *c*, especially when the conditional marginal probability of a loss ($\psi(\pm \chi)$) is high. Yet, LA speculators also trade *less* (albeit less cautiously) or *not at all* with their costly signal *S* (see $|x_{LA}^{\gamma=2}| < |x_{MV}|$ in Fig. 2b) to decrease their conditional expected trading loss, thus making prices *less* efficient and increasing returns to private information. Ceteris paribus, if information is sufficiently expensive, the former effect dominates the latter, yielding *lower* information production. E.g., see $E[V_{LA}^{\gamma=2}(S, P_{LA}^{\gamma=2}(\mu))]$ in Fig. 4b (dotted line), yielding $\mu_{LA}^{\gamma=2}(c) < \mu_{MV}(c)$ in Table 2.

RSL speculation. Risk seeking in losses ($\beta > \alpha$) makes a speculator *more* willing to pay a certain cost *c* for private information yielding uncertain returns, especially when the conditional cumulative probability of a loss ($\Phi(\pm \chi)$) is high (and her effective risk aversion coefficient $\alpha^*(\pm \chi)$ is low). However, RSL speculators also trade *more* (and often more aggressively) with their signal *S* (see $|x_{RSL}^{\beta=2}| > |x_{MV}|$ in Fig. 2b) to increase their conditional trading loss variance, thus making prices *more* efficient and decreasing returns to private information. Ceteris paribus, if information is sufficiently expensive, the former effect dominates the latter, yielding *higher* information production than MV speculation. E.g., see $E[V_{RSL}^{\beta=2}(S, P_{RSL}^{\beta=2}(\mu))]$ in Fig. 4b (thin line), yielding $\mu_{RSL}^{\beta=2}(c) > \mu_{MV}(c)$ in Table 2.

Fig. 4a then suggests that for preference parameters capturing *mild* risk seeking in losses relative to loss aversion (e.g., see $E[V_{PT}(S, P_{PT}(\mu))]$, solid line, for $\gamma = 1$ and $\beta = 1.05$), as advocated by Tversky and Kahneman [63], PT speculation engages in lower information production than MV speculation: $\mu_{PT}(c) < \mu_{MV}(c)$ in Table 2 (Conclusion 2).

This conclusion has important implications for equilibrium market quality when information is costly. In the second step, we describe these implications by solving for the equilibrium price given $\mu_{PT}(c)$ (i.e., $P_{PT}(\mu_{PT}(c))$ of Eq. (29)) in the market for the risky asset, and computing conditional and unconditional measures of market quality (as in Section 2.4.2 and Fig. 3). We begin by applying the Implicit Function Theorem to Eq. (29) to define equilibrium price impact in the amended economy with endogenous information acquisition as

$$\lambda_{PT}(\mu_{PT}(c)) = \begin{cases} \frac{L_{PT}^{H}(\mu_{PT}(c))}{1 - \frac{\partial P_{PT}^{0,H}(\mu_{PT}(c))}{\partial P} - \frac{\partial G_{PT}^{H}(\mu_{PT}(c))}{\partial P} S - \frac{\partial L_{PT}^{H}(\mu_{PT}(c))}{\partial P} z} & \text{if } S > S_{H}, \\ \frac{L_{PT}^{L}(\mu_{PT}(c))}{1 - \frac{\partial P_{PT}^{0,L}(\mu_{PT}(c))}{\partial P} - \frac{\partial G_{PT}^{L}(\mu_{PT}(c))}{\partial P} S - \frac{\partial L_{PT}^{L}(\mu_{PT}(c))}{\partial P} z} & \text{if } S < S_{L}, \\ \frac{L_{PT}(\mu_{PT}(c))}{1 - \frac{\partial P_{PT}^{0}(\mu_{PT}(c))}{\partial P} - \frac{\partial L_{PT}(\mu_{PT}(c))}{\partial P} z} & \text{if } S_{L} \leqslant S \leqslant S_{H}, \end{cases}$$
(36)

consistent with Eq. (22). We then plot the average price impact $\lambda_{PT}(\mu_{PT}(c))$ of Eq. (36) over the domain of *S* (via numerical integration) for $\gamma = 1$ and $\beta = 1.05$ in Fig. 5a ($E[\lambda_{PT}(\mu_{PT}(c))|S]$,



Fig. 5. Endogenous information acquisition and market quality. In this figure we plot, over the domain of speculators' noisy signal *S* (via numerical integration), conditional equilibrium outcomes of each of the measures of market quality defined in Section 3.2 (price impact λ_{PT} , Figs. 5a and 5b; price volatility σ_{PT}^2 , Figs. 5c and 5d; expected informed trading volume *Vol*_{PT}, Figs. 5e and 5f; price informativeness Q_{PT} , Figs. 5g and 5h) for the equilibrium of Proposition 2 when information is costly (c = 0.225) in the presence of the equilibrium fraction $\mu_{PT}(c)$ of informed Prospect Theory (PT) speculators ($\alpha = 1, \gamma = 1, \text{ and } \beta = 1.05$: $E[\lambda_{PT}(\mu_{PT}(c))|S]$, $\sigma_{PT}^2(\mu_{PT}(c))|S$, $Vol_{PT}(\mu_{PT}(c))|S$, $Q_{PT}(\mu_{PT}(c))|S]$, solid lines), $\mu_{PT}^{\gamma=2}(c)$ of informed loss averse (LA) speculators ($\alpha = 1, \gamma = 2, \text{ and } \beta = 0$: $E[\lambda_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c))|S]$, $\sigma_{LA}^{2,\gamma=2}(\mu_{LA}^{\gamma=2}(c))|S$, $Vol_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c))|S$, $Q_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c))|S$, dotted lines), $\mu_{RSL}^{\beta=2}(c)$ of informed speculators risk seeking in losses (RSL) ($\alpha = 1, \gamma = 0, \text{ and } \beta = 2$: $E[\lambda_{RSL}^{\beta=2}(\mu_{RSL}^{\beta=2}(c))|S]$, $\sigma_{RSL}^{2,\beta=2}(\mu_{RSL}^{\beta=2}(c))|S$; thin lines), or $\mu_{MV}(c)$ of MV speculators ($\alpha = 1, \gamma = 0, \text{ and } \beta = 0$: $\lambda_{MV}(\mu_{MV}(c))$ of Eq. (35), $\sigma_{MV}^{\beta=2}(\mu_{MV}(c))|S$, $Vol_{MV}(\mu_{MV}(c))|S$, and $Q_{MV}(\mu_{MV}(c))|S$; dashed lines) when $\sigma_v^2 = 1, \sigma_u^2 = 1, \text{ and } \sigma_z^2 = 1$.

solid line). The corresponding plots of conditional equilibrium price volatility $(\sigma_{PT}^2(\mu_{PT}(c))|S)$, expected informed trading volume $(Vol_{PT}(\mu_{PT}(c))|S)$, and price efficiency $(Q_{PT}(\mu_{PT}(c))|S)$ are in Figs. 5c, 5e, and 5g, respectively, together with their conditional MV counterparts $\sigma_{MV}^2(\mu_{PT}(c))|S$, $Vol_{MV}(\mu_{PT}(c))|S$, and $Q_{MV}(\mu_{PT}(c))|S$ of Section 3.2.1 in closed-form (dashed lines).³⁴ Table 2 reports their unconditional outcomes, as well as informed speculators' average equilibrium trading intensity $E[\frac{\partial x(\mu)}{\partial S}]$.

In aggregate, Table 2 and Fig. 5 suggest the following novel, noteworthy conclusion.

³⁴ As in Section 2.4.2, we use conditional asset payoff uncertainty $var[v|S] = \sigma_v^2(1-\phi)$ and price volatility $var[P_{MV}(\mu_{MV}(c))|S] = L_{MV}^2(\mu_{MV}(c))\sigma_z^2$ to compute $\sigma_{MV}^2(\mu_{MV}(c)) = L_{MV}^2(\mu_{MV}(c))\sigma_z^2$ and $var[v|P_{MV}(\mu_{MV}(c)), S] = \sigma_v^2(1-\phi) - L_{MV}^2(\mu_{MV}(c))\sigma_z^2$ in $Q_{MV}(\mu_{MV}(c))$. Accordingly, conditional expected informed trading volume is given by $Vol_{MV}(\mu_{MV}(c))|S = \sqrt{\frac{2}{pi}}\sqrt{var[x_{MV}(\mu_{MV}(c))|S]}e^{-\frac{E[x_{MV}(\mu_{MV}(c))|S]^2}{2var[x_{MV}(\mu_{MV}(c))|S]}} + E[x_{MV}(\mu_{MV}(c))|S] \times [1 - 2\Phi(-\frac{E[x_{MV}(\mu_{MV}(c))|S]}{\sqrt{var[x_{MV}(\mu_{MV}(c))|S]}})]$, where $E[x_{MV}(\mu_{MV}(c))|S] = [\frac{\phi - G_{MV}(\mu_{MV}(c))}{\alpha\sigma_v^2(1-\phi)}]S$ and $var[x_{MV}(\mu_{MV}(c))|S] = \frac{L_{MV}^2(\mu_{MV}(c))\sigma_z^2}{\alpha^2\sigma_v^4(1-\phi)^2}$, while $E\{var[v|P_{PT}(\mu_{PT}(c))]|S\} = \sigma_v^2(1-\phi) - var[P_{PT}(\mu_{MV}(c))|S]$ in $Q_{PT}(\mu_{MV}(c))|S$.



Fig. 5. (continued).

Conclusion 6. Costly private information amplifies the implications of the presence of speculators with Prospect Theory preferences for equilibrium market quality but mitigates its state-dependence.

Intuitively, endogenous information acquisition has both a selection effect and an efficiency effect on equilibrium market quality, relative to the equilibrium in which PT speculators are endowed with a noisy signal *S* (Proposition 1). As discussed above, when information is sufficiently expensive only a fraction $\mu_{PT}(c) < 1$ of PT speculators becomes informed. This attenuates the MM's perceived adverse selection risk when faced with the aggregate limit-order book $\omega = \mu_{PT}(c)x_{PT}(P_{PT}(\mu(c))) + z$ (selection). However, limited information production also lowers the informativeness of ω , worsening the MM's perceived adverse selection risk (*efficiency*). The implications of a sufficiently high information cost *c* for equilibrium market quality depend once again on the relative strength of these conflicting effects.

LA speculation. When speculators are loss averse ($\gamma > 0$), the same cost *c* induces them not only to *lower* information production (Fig. 4b) and trading than MV speculation, but also to even *higher* trading intensity. E.g., see $Vol_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c))|S$ in Fig. 5f, as well as $Vol_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c)) < Vol_{MV}(\mu_{MV}(c))$ and $E[\frac{\partial x_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c))}{\partial S}] > E[\frac{\partial x_{LA}^{\gamma=2}}{\partial S}] > \frac{\partial x_{MV}(\mu_{MV}(c))}{\partial S}$ in Table 2 for c = 0.225. Intuitively, informed LA speculators bearing a certain information cost revise their trading activity *more* significantly, in response to the same information shock, to decrease their conditional expected trading loss. Ceteris paribus, this trading behavior *amplifies* both the selection and efficiency effects of $\mu_{LA}^{\gamma=2}(c) < 1$ on equilibrium market quality, thus not only further improving market liquidity but also further worsening price informativeness (with respect to the equilibrium of Proposition 1). E.g., see both $E[\lambda_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c))|S]$ in Fig. 5f (dotted line; or

 $E[\lambda_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c))] < E[\lambda_{LA}^{\gamma=2}]$ in Table 2) and $Q_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}(c))|S$ in Fig. 5h (dotted line; or $Q_{LA}^{\gamma=2}(\mu_{PT}^{\gamma=2}(c)) < Q_{LA}^{\gamma=2}$ in Table 2).

RSL speculation. When speculators are risk seeking in losses $(\beta > \alpha)$, the same cost *c* induces them not only to *higher* information production (Fig. 4b) and trading than MV speculation, but also to even *higher* trading intensity. E.g., see $Vol_{RSL}^{\beta=2}(\mu_{RSL}^{\beta=2}(c))|S$ in Fig. 5f, as well as $Vol_{RSL}^{\beta=2}(\mu_{RSL}^{\beta=2}(c)) > Vol_{MV}(\mu_{MV}(c))$ and $E[\frac{\partial x_{RSL}^{\beta=2}(\mu_{RSL}^{\beta=2}(c))}{\partial S}] > E[\frac{\partial x_{RSL}^{\beta=2}}{\partial S}] > \frac{\partial x_{MV}(\mu_{MV}(c))}{\partial S}$ in Table 2. Intuitively, RSL speculators paying *c* adjust their trading activity *more* significantly to the same private news, to increase their conditional trading loss variance. Ceteris paribus, this trading behavior *attenuates* (but does not fully extinguish) both the selection and efficiency effects of $\mu_{RSL}^{\beta=2}(c) < 1$ on equilibrium market quality, thus yielding not only a less dramatic improvement in market liquidity but also a less pronounced decline in price volatility and informativeness (with respect to the equilibrium of Proposition 1). E.g., see both $E[\lambda_{RSL}^{\beta=2}(\mu_{RSL}^{\beta=2}(c))|S|$ in Fig. 5f (thin line; or $E[\lambda_{RSL}^{\beta=2}(\mu_{RSL}^{\beta=2}(c))] \approx E[\lambda_{RSL}^{\beta=2}]$ in Table 2) and $\sigma_{RSL}^{2,\beta=2}(\mu_{RSL}^{\beta=2}(c))|S|$ in Fig. 5h (or $\sigma_{RSL}^{2,\beta=2}(\mu_{RSL}^{\beta=2}(c)) < \sigma_{RSL}^{2,\beta=2}$ in Table 2).

Lastly, as observed in Section 2.4.2, both informed LA and RSL speculation make equilibrium market quality state-dependent. Accordingly, Fig. 5 indicates that if speculators display *mild* risk seeking in losses relative to loss aversion (e.g., $\gamma = 1$ and $\beta = 1.05$, consistent with Tversky and Kahneman [63]), a sufficiently high *c* limits their market participation (e.g., $\mu_{PT}(c) < 1$ in Table 2), hence not only further improving both market liquidity (while possibly worsening it in the presence of MV speculation: E.g., $E[\lambda_{PT}(\mu_{PT}(c))] < E[\lambda_{PT}]$ while $E[\lambda_{MV}(\mu_{MV}(c))] > \lambda_{MV}$ in Table 2) and price efficiency (e.g., $Q_{PT}(\mu_{PT}(c)) < Q_{PT}$), but also mitigating their state-dependence.

Conclusions 5 and 6 provide an important qualification to the insights in Conclusions 1–4 (based on PT speculation endowed with private information). In particular, they suggest that the extent to which PT preferences affect equilibrium market quality is crucially related to speculation's endogenous information acquisition and the traded asset's information environment.

4. Conclusions

Prospect Theory is a popular non-standard, descriptive model of decision-making under uncertainty originally proposed by Kahneman and Tversky [39] and Tversky and Kahneman [63] on the basis of a large body of experimental research. This paper studies the implications of the main features of Prospect Theory – loss aversion and mild risk seeking in losses – for market quality.

Our theoretical analysis demonstrates that introducing informed speculators with these preferences in an otherwise standard economy (with or without information costs) yields important predictions for the properties of equilibrium market depth, price volatility, trading volume, price informativeness, and information production. Intuitively, risk seeking in losses induces speculators to trade more (and more aggressively) with private information (and a larger fraction of them to pay for it when expensive), while loss aversion induces them to trade less (but less cautiously) or not at all with private information (and fewer of them to purchase it). These forces affect market makers' perceived adverse selection risk, their liquidity provision, and the ensuing market efficiency in a complex, state-dependent fashion. These microfounded predictions are novel and warrant future empirical investigation, for they suggest that Prospect Theory preferences may be a meaningful determinant of observed measures of financial market quality. For instance, recent empirical studies indicate that country and firm-level proxies for stock market liquidity and price informativeness may be state-dependent and display significant cross-sectional heterogeneity. E.g., among others, see Chordia, Roll, and Subrahmanyam [16], Amihud [3], Pastor and Stambaugh [59], Chordia, Shivakumar, and Subrahmanyam [17], Griffin, Nardari, and Stulz [27], Kamara, Lou, and Sadka [40], and Hasbrouck [31]. According to our model, these properties may stem from the extent of (and asset-level dispersion in) better-informed speculation by agents displaying Prospect Theory preferences, as well as from its interaction with marketwide (and asset-level) information costs.

Our study also raises several unexplored questions. In particular, we note that for simplicity's sake, our model allows for a single round of trading over a single risky asset. Loss aversion and risk seeking in losses are likely to provide further insights for speculators' multiperiod, multiasset trading strategies and equilibrium price dynamics, especially if combined with that particular form of *mental accounting* [62], known as *narrow framing*, often displayed by individuals in experimental settings.³⁵ Accordingly, the portfolio decisions of so-modeled investors and their effects on asset prices have received increasing attention in the literature (e.g., Levy, De Giorgi, and Hens [48]; Levy and Levy [49]; Barberis and Huang [5]).³⁶ Our analysis also suggests that Prospect Theory preferences may affect the role of private information risk (and its interaction with the availability of public information) for a firm's cost of capital in rational expectations equilibrium models (e.g., Easley and O'Hara [21]). Lastly, as is customary, our model assumes risk-neutral market making. It may be of interest to analyze inventory management, liquidity provision, and market quality in the presence of market makers with Prospect Theory preferences, e.g., in the spirit of Subrahmanyam [61] and Ozsoylev and Werner [58]. We look forward to future research on these and other implications of Prospect Theory for the process of price formation in financial markets.

Appendix A

Numerical approach. Semi-strong market efficiency (Eq. (11)) requires the computation of the MM's estimates of the conditional first moments of the asset payoff v when PT speculators are conjectured to trade (i.e., if either their noisy signal $S > S_H$ or $S < S_L$) and their accompanying probability in Eq. (12). Because each PT speculator's optimal demand schedule x_{PT} (Eq. (9)) is nonlinear, these moments and probability cannot be computed analytically, except when speculators have MV preferences ($\gamma = 0$ and $\beta = 0$; see Remark 1). We compute these moments and probability numerically, as explicit functions of ω and P. Conceptually, the rational MM would use knowledge of the economy's structure and each PT speculator's trading rule in Eq. (9) – based on S and P – to learn about v from the order flow ω . Consistent with Hayashi [32, pp. 138–140], we capture the spirit of this inference problem by allowing the MM to form conditional expectations about v from simulating a large number of realizations of the economy and estimating (via

³⁵ E.g., in those circumstances, economic agents derive direct utility from gains and losses of individual assets rather than from portfolio fluctuations, as in Barberis and Huang [4].

³⁶ Relatedly, Prospect Theory has often been informally linked to the disposition effect (and the latter to the momentum effect; e.g., see Grinblatt and Han [28]). In a recent study, Li and Yang [52] formally show that Prospect Theory preferences can generate both price momentum (via a disposition effect) and price reversal in an overlapping generation model.

OLS) a linear relation between v and both ω and P – as the MM would do analytically in the presence of MV speculation (see the proof of Remark 1) – as follows.

Step 1: We specify a price grid P(n) made of N points between P_L and P_H . Our procedure is robust to sensible grid choices. We set $P_L(P_H)$ as the equilibrium price in the basic economy of Section 2 under the assumptions that speculators have MV preferences (i.e., $\gamma = 0$, $\beta = 0$), noise trading z and signal error u are at their unconditional means of zero, and $v = -3\sigma_v$ ($v = 3\sigma_v$). As shown in Remark 1, closed-form solutions for these economies exist.

Step 2: We draw *M* triplets $\{v(m), u(m), z(m)\}$ from their independent normal distributions, and compute the corresponding noisy signal, S(m) = v(m) + u(m).

Step 3: For each triplet $\{v(m), u(m), z(m)\}$, at each price P(n) we compute

$$\chi(m,n) = \frac{P(n) - \phi S(m)}{\sqrt{\sigma_v^2 (1 - \phi)}},$$
(A.1)

$$S_H(m,n) = \frac{P(n)}{\phi} + \frac{\gamma \psi(\chi(m,n))\sigma_v \sqrt{1-\phi}}{\phi[1+\gamma \Phi(\chi(m,n))]},$$
(A.2)

$$S_L(m,n) = \frac{P(n)}{\phi} - \frac{\gamma \psi(-\chi(m,n))\sigma_v \sqrt{1-\phi}}{\phi[1+\gamma \Phi(-\chi(m,n))]},\tag{A.3}$$

as well as $x_{PT}(m, n)$ of Eq. (9), and $\omega(m, n) = x_{PT}(m, n) + z(m)$, and populate the matrix $PT_H = \{v(m), S(m), P(n), \omega(m, n), S_H(m, n)\}$ when $S(m) > S_H(m, n)$ and the matrix $PT_L = \{v(m), S(m), P(n), S(m), S_L(m, n)\}$ when $S(m) < S_L(m, n)$.

Step 4: Based on the observation that any random variable y can be expressed as

$$y = E[y|x] + (y - E[y|x]) = E[y|x] + \varepsilon$$
(A.4)

(e.g., Greene [26, p. 80]), we use PT_H and PT_L to represent $E[v|\omega, S > S_H]$ and $E[v|\omega, S < S_L]$ as tractable functions of ω and P, and S_H and S_L as tractable functions of S and P. We consider the following linear regressions for v(m), $S_H(m, n)$, and $S_L(m, n)$:

$$v(m) = a_H + b_H \omega(m, n) + c_H P(n) + \varepsilon(m, n), \tag{A.5}$$

$$S_H(m,n) = d_H + e_H S(m) + f_H P(n) + \varepsilon(m,n), \qquad (A.6)$$

$$v(m) = a_L + b_L \omega(m, n) + c_L P(n) + \varepsilon(m, n), \qquad (A.7)$$

$$S_L(m,n) = d_L + e_L S(m) + f_L P(n) + \varepsilon(m,n).$$
(A.8)

Estimation (via OLS) of Eqs. (A.5) and (A.6) over PT_H , and Eqs. (A.7) and (A.8) over PT_L then yields

$$E[v|\omega, S > S_H] \approx \hat{a}_H + \hat{b}_H \omega + \hat{c}_H P, \tag{A.9}$$

$$S_H \approx \widehat{d}_H + \widehat{e}_H S + \widehat{f}_H P, \tag{A.10}$$

$$E[v|\omega, S < S_L] \approx \widehat{a}_L + \widehat{b}_L \omega + \widehat{c}_L P, \qquad (A.11)$$

$$S_L \approx \widehat{d}_L + \widehat{e}_L S + \widehat{f}_L P. \tag{A.12}$$

Further investigation reveals the linear functional forms in Eqs. (A.9) to (A.12) to be accurate, and the ensuing insights to be insensitive to either using higher-order polynomials in Eqs. (A.5) to (A.8) or imposing exact symmetry to their coefficients. For instance, the adjusted R^2 from OLS estimation of Eqs. (A.5) and (A.7) are close to the coefficient of determination ρ^2 from $E[v|\omega]$ of Eq. (A.22) below, the MM's linear projection of v on ω with MV speculation (i.e., $\rho_{MV}^2 \equiv \frac{cov[v,\omega]^2}{\sigma_v^2 var[\omega]} = G_{MV}$ of Remark 1; e.g., Greene [26, p. 85]; Vives [68, pp. 377–378]), and nearly one from OLS estimation of Eqs. (A.6) and (A.8).

Proof of Result 1. The expressions for $E[v|\omega, S > S_H]$ and $E[v|\omega, S < S_L]$ of Eqs. (13) and (15) stem from Eqs. (A.9) and (A.11), respectively (where we omit the OLS estimation superscripts for economy of notation), i.e., from the best linear predictors of v (e.g., Hayashi [32, pp. 138–140]). The conditional mean $E[v|\omega, S_L \leq S \leq S_H]$ can be expressed in closed-form, as Eq. (17), using well-known properties of incidentally truncated bivariate normal variables (see, e.g., Johnson and Kotz [37]; Greene [26, p. 975]), since $x_{PT} = 0$ and $\omega = z$ over the no-trade interval $[S_L, S_H]$. Since S is normally distributed with mean zero and variance σ_s^2 , Eqs. (A.10) and (A.12) imply that $\Pr[S > S_H] \approx \Pr[S > \frac{d_H + f_H P}{1 - e_H}] = 1 - \Phi[\frac{d_H + f_H P}{\sigma_s(1 - e_H)}]$ and $\Pr[S < S_L] \approx \Pr[S < \frac{d_L + f_L P}{1 - e_L}] = \Phi[\frac{d_L + f_L P}{\sigma_s(1 - e_L)}]$, respectively; if $\gamma = 0$, then $\Pr[S > S_H] = \Pr[S > \frac{P}{\phi}] = 1 - \Phi[\frac{P}{\phi\sigma_s}]$ and $\Pr[S < S_L] = \Pr[S < \frac{P}{\phi}] = \Phi[\frac{P}{\phi\sigma_s}]$.

Proof of Proposition 1. To prove this statement, we substitute Eqs. (13) to (17) from Result 1 in Eq. (12) and the resulting expression for $E[v|\omega]$ of Eq. (11) to get

$$P = (a_H + b_H \omega + c_H P) [1 - \Phi(H)] + (a_L + b_L \omega + c_L P) \Phi(L) + \phi \sigma_s \Lambda(L, H) [\Phi(H) - \Phi(L)].$$
(A.12)

Solving Eq. (A.12) for P yields

$$P = P_{PT}^0 + L_{PT}\omega, \tag{A.13}$$

where

$$P_{PT}^{0} = \frac{a_{H}[1 - \Phi(H)] + a_{L}\Phi(L) + \phi\sigma_{s}\Lambda(L,H)[\Phi(H) - \Phi(L)]}{1 - c_{H}[1 - \Phi(H)] - c_{L}\Phi(L)},$$
(A.14)

$$L_{PT} = \frac{b_H [1 - \Phi(H)] + b_L \Phi(L)}{1 - c_H [1 - \Phi(H)] - c_L \Phi(L)}.$$
(A.15)

Since $\omega = x_{PT} + z$, substituting the expressions for x_{PT} of Eq. (9) in Eq. (A.13) and then solving again for *P* leads to the implicit function of Eq. (19), where

$$P_{PT}^{0,H} = \frac{\alpha^*(\chi)\sigma_v^2(1-\phi)P_{PT}^0 - L_{PT}\gamma\psi(\chi)\sigma_v\sqrt{1-\phi}}{\alpha^*(\chi)\sigma_v^2(1-\phi) + L_{PT}[1+\gamma\Phi(\chi)]},$$
(A.16)

$$G_{PT}^{H} = \frac{L_{PT}[1 + \gamma \Phi(\chi)]\phi}{\alpha^{*}(\chi)\sigma_{*}^{2}(1 - \phi) + L_{PT}[1 + \gamma \Phi(\chi)]},$$
(A.17)

$$L_{PT}^{H} = \frac{\alpha^{*}(\chi)\sigma_{v}^{2}(1-\phi)L_{PT}}{\alpha^{*}(\chi)\sigma_{v}^{2}(1-\phi) + L_{PT}[1+\gamma\Phi(\chi)]},$$
(A.18)

$$P_{PT}^{0,L} = \frac{\alpha^*(-\chi)\sigma_v^2(1-\phi)P_{PT}^0 + L_{PT}\gamma\psi(-\chi)\sigma_v\sqrt{1-\phi}}{\alpha^*(-\chi)\sigma_v^2(1-\phi) + L_{PT}[1+\gamma\Phi(-\chi)]},$$
(A.19)

$$G_{PT}^{L} = \frac{L_{PT}[1 + \gamma \Phi(-\chi)]\phi}{\alpha^{*}(-\chi)\sigma_{v}^{2}(1 - \phi) + L_{PT}[1 + \gamma \Phi(-\chi)]},$$
(A.20)

$$L_{PT}^{L} = \frac{\alpha^{*}(-\chi)\sigma_{v}^{2}(1-\phi)L_{PT}}{\alpha^{*}(-\chi)\sigma_{v}^{2}(1-\phi) + L_{PT}[1+\gamma\Phi(-\chi)]}.$$
(A.21)

The equilibrium price P_{PT} is a fixed point of Eq. (19) (or, equivalently, of Eq. (A.13)). Let f(P) be the right side of Eq. (19) and g(P) = f(P) - P. Because of properties of $\Phi(\cdot)$ and $\psi(\cdot)$ (in particular, $\lim_{y\to+\infty} \Phi(y) = 1$, $\lim_{y\to-\infty} \Phi(y) = 0$, and $\lim_{y\to\pm\infty} \psi(y) = 0$), it is immediate that $\lim_{P\to+\infty} g(P) < 0$ and $\lim_{P\to-\infty} g(P) > 0$. Existence of a solution to g(P) = 0 follows from the Intermediate Value Theorem, since g(P) is a continuous function of P. As g(P) is decreasing in P, such solution is therefore unique. \Box

Proof of Remark 1. We prove this statement by using Eq. (10) and properties of conditional normal distributions to solve for P_{MV} from Eq. (11). The distributional assumptions of Section 2.1 imply that the order flow $\omega = x_{MV} + z$ is normally distributed with mean $E[\omega] = -C_{MV}P$ and variance $var[\omega] = C_{MV}^2 \phi^2 \sigma_s^2 + \sigma_z^2$, where $C_{MV} = \frac{1}{\alpha \sigma_v^2 (1-\phi)}$. Since $cov[v, \omega] = C_{MV} \phi \sigma_v^2$, it then follows (e.g., Greene [26, p. 90]) that:

$$P = \frac{cov[v,\omega]}{var[\omega]} \left\{ \omega - E[\omega] \right\} = \frac{C_{MV}\phi\sigma_v^2}{C_{MV}^2\phi^2\sigma_s^2 + \sigma_z^2 - C_{MV}^2\phi\sigma_v^2}\omega.$$
(A.22)

Substituting the expressions for C_{MV} and $\phi = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2}$ into Eq. (A.22) and solving for *P* yields

$$P = \frac{\sigma_v^2}{\alpha \sigma_u^2 \sigma_z^2} \omega.$$
(A.23)

Since $\omega = x_{MV} + z$, substituting the expression for x_{MV} of Eq. (10) into Eq. (A.23) and solving again for *P* leads to P_{MV} of Eq. (20). \Box

Amended numerical approach. In the presence of a given fraction $\mu \in (0, 1)$ of informed speculators (as in Section 3.2), we repeat Steps 1 to 4 over a grid $\mu(l)$ made of *L* points over the interval (0, 1), such that for each $\mu(l)$ we compute $\omega^{fic}(l, m, n) = \mu(l)x_{PT}(m, n) + z(m)$ and estimate (via OLS) the following linear regressions for v(m):

$$v(m) = a_H + b_H \omega^{fic}(l, m, n) + c_H P(n) + \varepsilon(m, n), \qquad (A.24)$$

$$v(m) = a_L + b_L \omega^{fc}(l, m, n) + c_L P(n) + \varepsilon(m, n), \qquad (A.25)$$

over PT_H and PT_L , respectively, to yield

$$E[v|\omega^{fic}, S > S_H] \approx \widehat{a}_H(\mu) + \widehat{b}_H(\mu)\omega^{fic} + \widehat{c}_H(\mu)P, \qquad (A.26)$$

$$E[v|\omega^{fic}, S < S_L] \approx \widehat{a}_L(\mu) + \widehat{b}_L(\mu)\omega^{fic} + \widehat{c}_L(\mu)P.$$
(A.27)

Given Result 2 and Proposition 2, we then compute $E[V_{PT}(S, P_{PT}(\mu))]$ from Eqs. (6) to (8), for any given μ , as the average of

$$V_{PT}(S(m), P_{PT}(l, m)) = x_{PT}(l, m) [\phi S(m) - P_{PT}(l, m)] - \frac{1}{2} \alpha x_{PT}^2(l, m) \sigma_v^2(1 - \phi) + \gamma x_{PT}(l, m) [\phi S(m) - P_{PT}(l, m)] \Phi^2 (sgn(x_{PT}(l, m)) \chi(l, m)) + \gamma x_{PT}(l, m) sgn[x_{PT}(l, m)] \sigma_v \sqrt{1 - \phi} \times \Lambda^- (sgn(x_{PT}(l, m)) \chi(l, m)) \Phi (sgn(x_{PT}(l, m)) \chi(l, m)) + \frac{1}{2} \beta x_{PT}^2(l, m) \sigma_v^2(1 - \phi) [1 - \Delta^- (sgn(x_{PT}(l, m)) \chi(l, m))] \times \Phi^2 (sgn(x_{PT}(l, m)) \chi(l, m)),$$
(A.28)

where $\chi(l,m) = \frac{P_{PT}(l,m) - \phi S(m)}{\sqrt{\sigma_v^2(1-\phi)}}$ and $P_{PT}(l,m)$ is the equilibrium price of the fictitious economy in which a fraction $\mu(l)$ of informed speculators observe a signal S(m) = v(m) + u(m) and the aggregate order flow $\omega^{fic}(l,m) = \mu(l)x_{PT}(m) + z(m)$, over all *M* triplets $\{v(m), u(m), z(m)\}$ for each $\mu(l)$. Lastly, we find $\mu(c)$ (by linear interpolation, if necessary) as the fraction $\mu(l^*)$ such that

$$E\left[V_{PT}\left(S, P_{PT}(\mu(c))\right)\right] \approx \frac{1}{M} \sum_{m=1}^{M} V_{PT}\left(S(m), P_{PT}(l^*, m)\right) = c. \qquad \Box \qquad (A.29)$$

Proof of Result 2. As in the proof of Result 1, the expressions for $E[v|\omega^{fic}, S > S_H]$ and $E[v|\omega^{fic}, S < S_L]$ in Eqs. (27) and (28) stem from those in Eqs. (A.26) and (A.27), respectively (where we omit the OLS estimation superscripts for economy of notation), i.e., from the best linear predictors of v. The conditional mean $E[v|\omega^{fic}, S_L \leq S \leq S_H]$ can be expressed in closed-form, as Eq. (17) of Result 1, since $x_{PT} = 0$ and $\omega^{fic} = z$ over the no-trade interval $[S_L, S_H]$. \Box

Proof of Proposition 2. The proof of this statement mimics the proof of Proposition 1. Specifically, we substitute Eq. (17) (from Result 1) and Eqs. (27) and (28) (from Result 2) in Eq. (12) and the resulting expression for $E[v|\omega^{fic}]$ of Eq. (11) to get

$$P = \left[a_{H}(\mu) + b_{H}(\mu)\omega^{fic} + c_{H}(\mu)P\right] \left[1 - \Phi(H)\right] + \left[a_{L}(\mu) + b_{L}(\mu)\omega^{fic} + c_{L}(\mu)P\right] \Phi(L) + \phi\sigma_{s}\Lambda(L,H) \left[\Phi(H) - \Phi(L)\right].$$
(A.30)

Solving Eq. (A.30) for *P* yields

$$P = P_{PT}^{0}(\mu) + L_{PT}(\mu)\omega^{fic},$$
(A.31)

where

$$P_{PT}^{0}(\mu) = \frac{a_{H}(\mu)[1 - \Phi(H)] + a_{L}(\mu)\Phi(L) + \phi\sigma_{s}\Lambda(L,H)[\Phi(H) - \Phi(L)]}{1 - c_{H}(\mu)[1 - \Phi(H)] - c_{L}(\mu)\Phi(L)},$$
(A.32)

$$L_{PT}(\mu) = \frac{b_H(\mu)[1 - \Phi(H)] + b_L(\mu)\Phi(L)}{1 - c_H(\mu)[1 - \Phi(H)] - c_L(\mu)\Phi(L)}.$$
(A.33)

Since $\omega^{fic} = \mu x_{PT} + z$, substituting the expression for x_{PT} of Eq. (9) in Eq. (A.31) and then solving for *P* leads to the implicit function of Eq. (29), where

$$P_{PT}^{0,H}(\mu) = \frac{\alpha^*(\chi)\sigma_v^2(1-\phi)P_{PT}^0(\mu) - \mu L_{PT}(\mu)\gamma\psi(\chi)\sigma_v\sqrt{1-\phi}}{\alpha^*(\chi)\sigma_v^2(1-\phi) + \mu L_{PT}(\mu)[1+\gamma\Phi(\chi)]},$$
(A.34)

$$G_{PT}^{H}(\mu) = \frac{\mu L_{PT}(\mu) [1 + \gamma \Phi(\chi)]\phi}{\alpha^{*}(\chi) \sigma_{\nu}^{2} (1 - \phi) + \mu L_{PT}(\mu) [1 + \gamma \Phi(\chi)]},$$
(A.35)

$$L_{PT}^{H}(\mu) = \frac{\alpha^{*}(\chi)\sigma_{v}^{2}(1-\phi)L_{PT}(\mu)}{\alpha^{*}(\chi)\sigma_{v}^{2}(1-\phi) + \mu L_{PT}(\mu)[1+\gamma\Phi(\chi)]},$$
(A.36)

$$P_{PT}^{0,L}(\mu) = \frac{\alpha^*(-\chi)\sigma_v^2(1-\phi)P_{PT}^0(\mu) + \mu L_{PT}(\mu)\gamma\psi(-\chi)\sigma_v\sqrt{1-\phi}}{\alpha^*(-\chi)\sigma_v^2(1-\phi) + \mu L_{PT}(\mu)[1+\gamma\Phi(-\chi)]},$$
(A.37)

$$G_{PT}^{L}(\mu) = \frac{\mu L_{PT}(\mu)[1 + \gamma \Phi(-\chi)]\phi}{\alpha^{*}(-\chi)\sigma_{v}^{2}(1 - \phi) + \mu L_{PT}(\mu)[1 + \gamma \Phi(-\chi)]},$$
(A.38)

$$L_{PT}^{L}(\mu) = \frac{\alpha^{*}(-\chi)\sigma_{v}^{2}(1-\phi)L_{PT}(\mu)}{\alpha^{*}(-\chi)\sigma_{v}^{2}(1-\phi) + \mu L_{PT}(\mu)[1+\gamma\Phi(-\chi)]}.$$
(A.39)

The equilibrium price $P_{PT}^{fic}(\mu)$ is the unique fixed point of Eq. (29) (or, equivalently, of Eq. (A.31)). \Box

Proof of Remark 2. We prove this statement in five steps, as discussed in Section 3.2. First, we solve for the equilibrium of the fictitious economy in which $\omega^{fic} = \mu x_{MV} + z$. The distributional assumptions of Section 2.1 and x_{MV} of Eq. (10) imply that ω^{fic} is normally distributed with mean $E[\omega^{fic}] = -\mu C_{MV}P$ and variance $var[\omega^{fic}] = \mu^2 C_{MV}^2 \phi^2 \sigma_s^2 + \sigma_z^2$, where $C_{MV} = \frac{1}{\alpha \sigma_v^2 (1-\phi)}$. Since $cov[v, \omega^{fic}] = \mu C_{MV} \phi \sigma_v^2$, it then follows (e.g., Greene [26, p. 90]) that:

$$P = \frac{cov[v, \omega^{fic}]}{var[\omega^{fic}]} \{ \omega^{fic} - E[\omega^{fic}] \} = \frac{\mu C_{MV} \phi \sigma_v^2}{\mu^2 C_{MV}^2 \phi^2 \sigma_s^2 + \sigma_z^2 - \mu^2 C_{MV}^2 \phi \sigma_v^2} \omega^{fic}.$$
(A.40)

Substituting the expressions for C_{MV} and $\phi = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2}$ into Eq. (A.40) yields

$$P = \frac{\mu \sigma_v^2}{\alpha \sigma_u^2 \sigma_z^2} \omega^{fic}.$$
 (A.41)

Since $\omega^{fic} = \mu x_{MV} + z$, substituting the expression for x_{MV} of Eq. (10) into Eq. (A.41) and solving again for *P* leads to

$$P_{MV}^{fic}(\mu) = G_{MV}(\mu)S + L_{MV}(\mu)z, \tag{A.42}$$

where $G_{MV}(\mu) = \frac{\mu}{\alpha \sigma_u^2} L_{MV}(\mu)$ and $L_{MV}(\mu) = \frac{\mu \alpha \sigma_u^2 \sigma_v^2}{\sigma_u^2 (\mu^2 + \alpha^2 \sigma_u^2 \sigma_v^2) + \mu^2 \sigma_v^2}$. Second, we note that in the economy populated by both informed and uninformed MV speculators ($\omega = \omega^{fic} + (1 - \mu) x_{MV}^{u}$), semi-strong market efficiency implies that $P_{MV}(\mu) = P_{MV}^{fic}(\mu)$ and $E[v|P_{MV}(\mu)] = P_{MV}(\mu)$, such that $x_{MV}^u(P_{MV}(\mu)) = 0$ (see Eq. (26)) and $V_{MV}^u(P_{MV}(\mu)) = 0$ (see Eq. (3)). Third, we compute the unconditional expectation of the value function of an informed MV speculator when a fraction μ of MV speculators is informed, $E[V_{MV}(S, P_{MV}(\mu))]$. Eqs. (3) and (10) imply that for a given noisy signal *S*,

$$V_{MV}(S, P_{MV}(\mu)) = E\left[\pi\left(S, P_{MV}(\mu)\right)|S\right] - \frac{1}{2}\alpha var[\pi\left(S, P_{MV}(\mu)\right)|S]$$

= $x_{MV}(S, P_{MV}(\mu))[\phi S - P_{MV}(\mu)] - \frac{1}{2}\alpha x_{MV}^{2}(S, P_{MV}(\mu))\sigma_{v}^{2}(1-\phi)$
= $\frac{[\phi S - P_{MV}(\mu)]^{2}}{2\alpha\sigma_{v}^{2}(1-\phi)}.$ (A.43)

Once substituting the expression for $P_{MV}(\mu)$ of Eq. (A.42) into Eq. (A.43), the unconditional expectation of the resulting expression yields

$$E[V_{MV}(S, P_{MV}(\mu))] = \frac{1}{2\alpha\sigma_v^2(1-\phi)} \left[\frac{\alpha^4\sigma_u^8\sigma_z^4\phi^2S^2}{(\sigma_s^2\mu^2 + \alpha^2\sigma_u^4\sigma_z^2)^2} + \frac{\mu^2\alpha^2\sigma_v^4\sigma_u^4z^2 - 2\mu\alpha^3\sigma_v^4\sigma_u^6\sigma_z^2\sigma_z^2Sz}{(\sigma_s^2\mu^2 + \alpha^2\sigma_u^4\sigma_z^2)^2} \right].$$
 (A.44)

Since the distributional assumptions of Section 2.1 imply that $E[S^2] = \sigma_s^2$, $E[z^2] = \sigma_z^2$, and E[Sz] = 0, Eq. (A.44) reduces to $E[V_{MV}(S, P_{MV}(\mu))]$ of Eq. (32), a strictly monotone decreasing function of μ . Fourth, we solve for the unique equilibrium in the information market by

computing the unique fraction μ of MV speculation paying the cost *c* to observe the noisy signal *S* such that $E[V_{MV}(S, P_{MV}(\mu))] = c$, yielding $\mu_{MV}(c)$ of Eq. (33). Lastly, we compute the equilibrium price of the economy in which a fraction $\mu_{MV}(c)$ of MV speculators chooses to purchase *S* by paying *c* by substituting the expression of Eq. (33) in Eq. (A.42) to yield Eqs. (34) and (35). \Box

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