Corrigendum

Corrigendum to 'Prospect Theory and market quality' Journal of Economic Theory 149 (2014), 276–310

Paolo Pasquariello[1]

Ross School of Business, University of Michigan

This Corrigendum corrects three typos and a potentially confusing description of equilibrium dealership activity in Pasquariello (2014).

1 Typos

The first typo is in the expressions for $E[\pi|S, \pi < 0]$ and $var[\pi|S, \pi < 0]$ in Eqs. (6) and (7), respectively [p. 283], which should read as

$$E[\pi|S, \pi < 0] = x(\phi S - P) + x sgn(x) \sigma_v \sqrt{1-\phi} \Lambda^- (sgn(x)\chi), \tag{6'}$$

$$var[\pi|S, \pi < 0] = x^2 \sigma_v^2 (1-\phi) \left[1 - \Delta^- (sgn(x)\chi)\right], \tag{7'}$$

instead of

$$E[\pi|S, \pi < 0] = x(\phi S - P)\Phi(sgn(x)\chi) + x sgn(x) \sqrt{\sigma_v^2 (1-\phi)} \Lambda^- (sgn(x)\chi), \tag{6}$$

$$var[\pi|S, \pi < 0] = x^2 \sigma_v^2 (1-\phi) \left[1 - \Delta^- (sgn(x)\chi)\right] \Phi(sgn(x)\chi). \tag{7}$$

This typo does not affect any of the analytical derivations in Pasquariello (2014), as all of them are based on Eqs. (6') and (7').

[1] The author is affiliated with the department of Finance at the Ross School of Business, University of Michigan. Please address comments to the author via email at ppasquar@umich.edu. I am grateful to Uday rajan for comments.

The second typo is in the expression for $E[v|\omega]$ in Eq. (12) and the expressions for $\Pr[S > S_H]$, $\Pr[S < S_L]$, and $\Pr[S_L \leq S \leq S_H|\omega]$ in Eqs. (14), and (16), and (18), respectively [p. 286], which should read as

$$\begin{aligned} E[v|\omega] &= E[v|\omega, S > S_H]\Pr[S > S_H|\omega] + E[v|\omega, S < S_L]\Pr[S < S_L|\omega] \\ &\quad + E[v|\omega, S_L \leq S \leq S_H]\Pr[S_L \leq S \leq S_H|\omega], \qquad (12') \\ \Pr[S > S_H|\omega] &= 1 - \Phi(H), \qquad (14') \\ \Pr[S < S_L|\omega] &= \Phi(L), \qquad (16') \\ \Pr[S_L \leq S \leq S_H|\omega] &= \Phi(H) - \Phi(L), \qquad (18') \end{aligned}$$

instead of

$$\begin{aligned} E[v|\omega] &= E[v|\omega, S > S_H]\Pr[S > S_H] + E[v|\omega, S < S_L]\Pr[S < S_L] \\ &\quad + E[v|\omega, S_L \leq S \leq S_H]\Pr[S_L \leq S \leq S_H], \qquad (12) \\ \Pr[S > S_H] &= 1 - \Phi(H), \qquad (14) \\ \Pr[S < S_L] &= \Phi(L), \qquad (16) \\ \Pr[S_L \leq S \leq S_H] &= \Phi(H) - \Phi(L); \qquad (18) \end{aligned}$$

in the sentence immediately above Result 1 [p. 286], which should read as "their accompanying probability $\Pr[S > S_H|\omega]$ and $\Pr[S < S_L|\omega]$ as [...]" instead of "their accompanying probability $\Pr[S > S_H]$ and $\Pr[S < S_L]$ as [...];" as well as in the last sentence of the Proof of Result 1 [p. 305] which should read as "$\Pr[S > S_H|\omega] \approx \Pr\left[S > \frac{d_H + f_H P}{1-e_H}\right] = 1 - \Phi\left[\frac{d_H + f_H P}{\sigma_s(1-e_H)}\right]$ and $\Pr[S < S_L|\omega] \approx \Pr\left[S < \frac{d_L + f_L P}{1-e_L}\right] = \Phi\left[\frac{d_L + f_L P}{\sigma_s(1-e_L)}\right]$, respectively; if $\gamma = 0$, then $\Pr[S > S_H|\omega] \approx \Pr\left[S > \frac{P}{\phi}\right] = 1 - \Phi\left[\frac{P}{\phi\sigma_s}\right]$ and $\Pr[S < S_L|\omega] \approx \Pr\left[S < \frac{P}{\phi}\right] = \Phi\left[\frac{P}{\phi\sigma_s}\right]$" instead of "$\Pr[S > S_H] = \Pr\left[S > \frac{d_H + f_H P}{1-e_H}\right] = 1 - \Phi\left[\frac{d_H + f_H P}{\sigma_s(1-e_H)}\right]$ and $\Pr[S < S_L] = \Pr\left[S < \frac{d_L + f_L P}{1-e_L}\right] = \Phi\left[\frac{d_L + f_L P}{\sigma_s(1-e_L)}\right]$, respectively; if $\gamma = 0$, then $\Pr[S > S_H] = \Pr\left[S > \frac{P}{\phi}\right] = 1 - \Phi\left[\frac{P}{\phi\sigma_s}\right]$ and $\Pr[S < S_L] = \Pr\left[S < \frac{P}{\phi}\right] = \Phi\left[\frac{P}{\phi\sigma_s}\right]$."

The third typo is in the numerical analysis of the model's extension with endogenous information acquisition, as described in Appendix A, yielding Table 2 [p. 298] and Figs. 4 [p. 297] and 5 [pp. 300-301]. Specifically, the expression for $V_{PT}\left(S\left(m\right),P_{PT}\left(l,m\right)\right)$ in Eq. (A-28) of Appendix A [p. 306] should read and be computed as

$$\begin{aligned}V_{PT}\left(S\left(m\right),P_{PT}\left(l,m\right)\right) &= x_{PT}\left(l,m\right)\left[\phi S\left(m\right)-P_{PT}\left(l,m\right)\right]-\frac{1}{2}\alpha x_{PT}^{2}\left(l,m\right)\sigma_{v}^{2}\left(1-\phi\right)\\ &+\gamma x_{PT}\left(l,m\right)\left[\phi S\left(m\right)-P_{PT}\left(l,m\right)\right]\Phi\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right)\\ &+\gamma x_{PT}\left(l,m\right)sgn\left[x_{PT}\left(l,m\right)\right]\sigma_{v}\sqrt{1-\phi}\Lambda^{-}\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right)\Phi\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right)\\ &+\frac{1}{2}\beta x_{PT}^{2}\left(l,m\right)\sigma_{v}^{2}\left(1-\phi\right)\left[1-\Delta^{-}\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right)\right]\Phi\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right),\end{aligned} \tag{A-28'}$$

instead of

$$\begin{aligned}V_{PT}\left(S\left(m\right),P_{PT}\left(l,m\right)\right) &= x_{PT}\left(l,m\right)\left[\phi S\left(m\right)-P_{PT}\left(l,m\right)\right]-\frac{1}{2}\alpha x_{PT}^{2}\left(l,m\right)\sigma_{v}^{2}\left(1-\phi\right)\\ &+\gamma x_{PT}\left(l,m\right)\left[\phi S\left(m\right)-P_{PT}\left(l,m\right)\right]\Phi^{2}\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right)\\ &+\gamma x_{PT}\left(l,m\right)sgn\left[x_{PT}\left(l,m\right)\right]\sigma_{v}\sqrt{1-\phi}\Lambda^{-}\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right)\Phi\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right)\\ &+\frac{1}{2}\beta x_{PT}^{2}\left(l,m\right)\sigma_{v}^{2}\left(1-\phi\right)\left[1-\Delta^{-}\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right)\right]\Phi^{2}\left(sgn\left(x_{PT}\left(l,m\right)\right)\chi\left(l,m\right)\right).\end{aligned} \tag{A-28}$$

Below are the corrected Table 2 and Figs. 4 and 5, i.e., those based on the correct expression for $V_{PT}\left(S\left(m\right),P_{PT}\left(l,m\right)\right)$ in Eq. (A-28'). Both the corrected Table 2 and Figs. 4 and 5 are qualitatively similar to the ones reported in Pasquariello (2014). Thus, the correction does not qualitatively affect the model's main insights, as discussed in Section 3.2.2 of Pasquariello (2014).

2 Equilibrium Dealership

Lastly, the discussion about the nature of dealership and of the equilibrium of the ensuing one-shot auction throughout the text is potentially confusing. In particular, if market makers (MM) could observe the aggregate limit-order book schedule $\omega(\cdot) \equiv x(\cdot) + z$ (like in the model of sequential trading of Vives, 1995a [as inadvertently stated on p. 285]), they would be able to perfectly infer the amount of liquidity trading z in the order book and set fully revealing equilibrium prices in the presence of PT speculation, since the demand schedule of PT speculators ($x_{PT}(\cdot)$ of Eq. (9)) is equal to zero within a range of prices (given their private signal S).[2]

The noisy rational expectations equilibrium in Pasquariello (2014) is instead based on the notion that MM set the market clearing price from their expectation of the asset payoff v conditional on a realization ω of the aggregate order flow ($E[v|\omega]$ of Eq. (12), as in Kyle, 1985). Vives (1995b [pp. 182-183]) suggests an equivalent, yet more intuitive way to describe the market clearing process through which equilibrium prices are set in Pasquariello (2014) by recurring to the notion of simultaneous placement of orders to a centralized auctioneer (CA) (see also Yuan, 2005; Ozsoylev and Werner, 2011). Liquidity (noise) traders submit market orders z to the CA; competitive, price-taking, privately informed speculators submit optimal demand schedules (generalized limit orders) $x = \arg\max V(S, P)$ (as in Vives, 1995a) to the CA; competitive, risk-neutral MM submit demand schedules based on prices to the CA according to semi-strong market efficiency $P = E[v|\omega]$ of Eq. (11) (as in Kyle, 1985; Vives, 1995a, b);[3] the CA then sets the equilibrium price P that clears the market. Thus, for any pair (S, z) of private signal and liquidity trading, the noisy rational expectations equilibrium of the economy (e.g., as in Proposition 1 and Remark 1) is a price function $P = P(\cdot)$, speculators' trades $x = x(\cdot)$, and

[2] In Vives (1995a), both sequential and simultaneous order placement yield the same equilibrium outcome because of competitive, risk-neutral dealership and informed speculators' linear trading strategies (e.g., as in the economy of Remark 1; see also Vives, 1995b, 2008). We thank Pierre Chaigneau and Nicolas Sahuguet at HEC Montréal for bringing this issue to our attention.

[3] As in Kyle (1985), the efficient pricing condition can be justified as the outcome of Bertrand competition among equally-informed risk-neutral MM yielding zero expected profits from dealership. Equivalently, Vives (1995b [p. 183]) notes that market clearing has to satisfy $P = E[v|P]$ because the MM would otherwise assume unbounded positions; given the aggregate order flow ω, it then follows that $P = E[v|\omega]$ since, in equilibrium, P and ω are equivalent sufficient statistics for v in the Blackwell sense (see also Vives, 2008 [pp. 131, 372-373]).

order flow $\omega = x(\cdot) + z$ such that:

- $x = \arg\max V(S, P)$;
- $P = E[v|\omega]$.

Accordingly, one may interpret the MM as competitive, uninformed liquidity suppliers, i.e., providing liquidity to informed speculators and noise traders (liquidity demanders) according to semi-strong market efficiency.

References

A. Kyle, Continuous auctions and insider trading, Econometrica 53 (1985), 1315-1335.

H. Ozsoylev, J. Werner, Liquidity and asset prices in rational expectations equilibrium with ambiguous information, Econ. Theory 48 (2011), 269-291.

P. Pasquariello, Prospect Theory and market quality, J. Econ. Theory 149 (2014), 276-310.

X. Vives, Short-term investment and the informational efficiency of the market, Rev. Financ. Stud. 8 (1995a), 125-160.

X. Vives, The speed of information revelation in a financial market mechanism, J. Econ. Theory 67 (1995b), 178-204.

X. Vives, Information and Learning in Markets, Princeton Univ. Press, Princeton, NJ, 2008.

K. Yuan, Asymmetric price movements and borrowing constraints: A REE model of crisis, contagion, and confusion, J. Financ. 60 (2005), 379-411.

Corrected Table II. Equilibrium Market Quality with Endogenous Information Acquisition

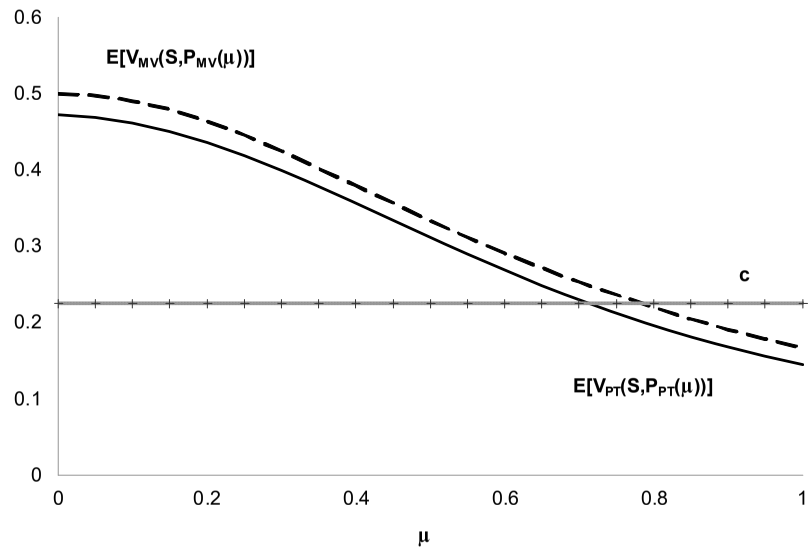
In this table we compute *unconditional* equilibrium market quality outcomes (average price impact $E[\lambda]$; price volatility σ^2; expected informed trading volume Vol; price informativeness Q; as well as average informed trading intensity $E[\frac{\partial x}{\partial S}]$; see Section 2.4), i.e., over all possible signals S and noise trading shocks z given a private information cost $c = 0.225$, in the presence of the ensuing equilibrium fraction $\mu(c)$ of informed mean-variance (MV) speculators ($\alpha = 1$, $\gamma = 0$, and $\beta = 0$), informed loss averse (LA) speculators ($\alpha = 1$, $\gamma = 2$, and $\beta = 0$), informed risk-seeking in losses (RSL) speculators ($\alpha = 1$, $\gamma = 0$, and $\beta = 2$), and informed Prospect Theory (PT) speculators ($\alpha = 1$, $\gamma = 1$, and $\beta = 1.05$), when $\sigma_v^2 = 1$, $\sigma_u^2 = 1$, and $\sigma_z^2 = 1$.

Speculators	$\mu(c)$	$E[\lambda(\mu)]$	$\sigma^2(\mu)$	$Vol(\mu)$	$Q(\mu)$	$E[\frac{\partial x(\mu)}{\partial S}]$
$_{MV}: \alpha = 1, \gamma = 0, \beta = 0$	0.78	0.35	0.28	0.59	1.38	0.45
$_{PT}: \alpha = 1, \gamma = 1, \beta = 1.05$	0.71	0.25	0.09	0.54	1.10	0.58
$_{PT}^{\gamma=2}: \alpha = 1, \gamma = 2, \beta = 0$	0.68	0.23	0.06	0.45	1.07	0.61
$_{PT}^{\beta=2}: \alpha = 1, \gamma = 0, \beta = 2$	0.85	0.27	0.16	0.68	1.19	0.54

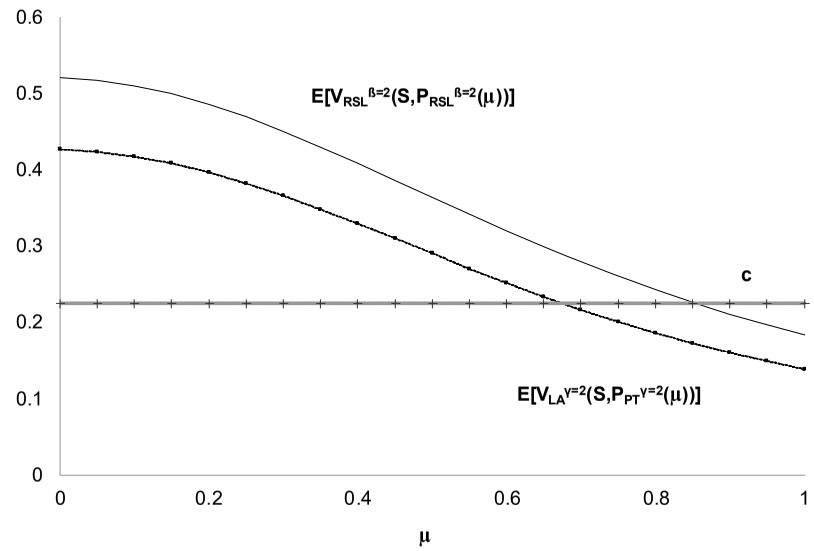
Corrected Figure 4. Prospect Theory and Endogenous Information Acquisition

In this figure we plot the maximum price that a speculator is willing to pay to purchase the noisy signal S in the amended economy of Section 3 over the domain of $\mu \in [0,1]$, the fraction of informed speculators in the market. Specifically, we plot the expected value function of a MV speculator ($\alpha = 1$, $\gamma = 0$, and $\beta = 0$: $E\left[V_{MV}\left(S, P_{MV}\left(\mu\right)\right)\right]$ of Eq. (32); Figure 4a, dashed line), a Prospect Theory (PT) speculator ($\alpha = 1$, $\gamma = 1$, and $\beta = 1.05$: $E\left[V_{PT}\left(S, P_{PT}\left(\mu\right)\right)\right]$ of Eq. (A-29); Figure 4a, solid line), a loss averse (LA) speculator ($\alpha = 1$, $\gamma = 2$, and $\beta = 0$: $E[V_{LA}^{\gamma=2}(S, P_{LA}^{\gamma=2}\left(\mu\right))]$ of Eq. (A-29); Figure 4b, dotted line), and of a speculator risk-seeking in losses ($\alpha = 1$, $\gamma = 0$, and $\beta = 2$: $E[V_{RSL}^{\beta=2}(S, P_{RSL}^{\beta=2}\left(\mu\right))]$ of Eq. (A-29); Figure 4b, thin line), as well as the information cost $c = 0.225$ (crossed line), when $\sigma_v^2 = 1$, $\sigma_u^2 = 1$, and $\sigma_z^2 = 1$.

a) $E\left[V_{MV}\left(S, P_{MV}\left(\mu\right)\right)\right]$ & $E\left[V_{PT}\left(S, P_{PT}\left(\mu\right)\right)\right]$

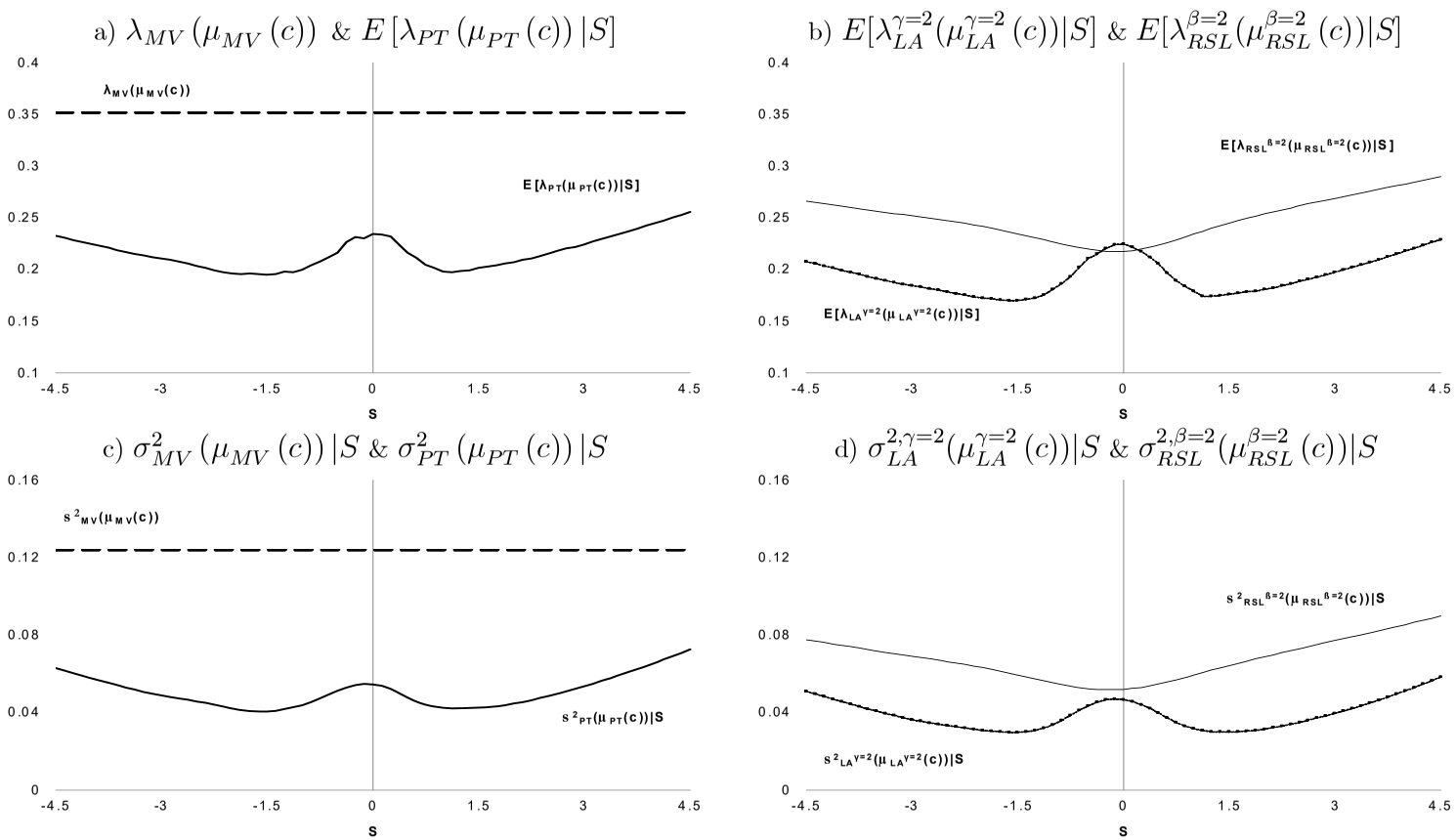


b) $E[V_{LA}^{\gamma=2}(S, P_{LA}^{\gamma=2}\left(\mu\right))]$ & $E[V_{RSL}^{\beta=2}(S, P_{RSL}^{\beta=2}\left(\mu\right))]$



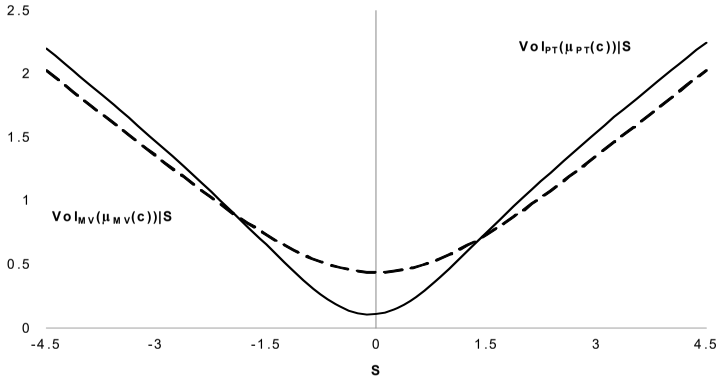
Corrected Figure 5. Endogenous Information Acquisition and Market Quality

In this figure we plot, over the domain of speculators' noisy signal S (via numerical integration), conditional equilibrium outcomes of each of the measures of market quality defined in Section 3.2 (price impact λ_{PT}, Figures 5a and 5b; price volatility σ^2_{PT}, Figures 5c and 5d; expected informed trading volume Vol_{PT}, Figures 5e and 5f; price informativeness Q_{PT}, Figures 5g and 5h) for the equilibrium of Proposition 2 when information is costly ($c = 0.225$) in the presence of the equilibrium fraction $\mu_{PT}(c)$ of informed Prospect Theory (PT) speculators ($\alpha = 1$, $\gamma = 1$, and $\beta = 1.05$: $E[\lambda_{PT}(\mu_{PT}(c))|S]$, $\sigma^2_{PT}(\mu_{PT}(c))|S$, $Vol_{PT}(\mu_{PT}(c))|S$, $Q_{PT}(\mu_{PT}(c))|S$; solid lines), $\mu^{\gamma=2}_{PT}(c)$ of informed loss averse (LA) speculators ($\alpha = 1$, $\gamma = 2$, and $\beta = 0$: $E[\lambda^{\gamma=2}_{LA}(\mu^{\gamma=2}_{LA}(c))|S]$, $\sigma^{2,\gamma=2}_{LA}(\mu^{\gamma=2}_{LA}(c))|S$, $Vol^{\gamma=2}_{LA}(\mu^{\gamma=2}_{LA}(c))|S$, $Q^{\gamma=2}_{LA}(\mu^{\gamma=2}_{LA}(c))|S$, dotted lines), $\mu^{\beta=2}_{RSL}(c)$ of informed speculators risk-seeking in losses (RSL) ($\alpha = 1$, $\gamma = 0$, and $\beta = 2$: $E[\lambda^{\beta=2}_{RSL}(\mu^{\beta=2}_{RSL}(c))|S]$, $\sigma^{2,\beta=2}_{RSL}(\mu^{\beta=2}_{RSL}(c))|S$, $Vol^{\beta=2}_{RSL}(\mu^{\beta=2}_{RSL}(c))|S$, $Q^{\beta=2}_{RSL}(\mu^{\beta=2}_{RSL}(c))|S$; thin lines), or $\mu_{MV}(c)$ of MV speculators ($\alpha = 1$, $\gamma = 0$, and $\beta = 0$: $\lambda_{MV}(\mu_{MV}(c))$ of Eq. (35), $\sigma^2_{MV}(\mu_{MV}(c))|S$, $Vol_{MV}(\mu_{MV}(c))|S$, and $Q_{MV}(\mu_{MV}(c))|S$; dashed lines) when $\sigma^2_v = 1$, $\sigma^2_u = 1$, and $\sigma^2_z = 1$.

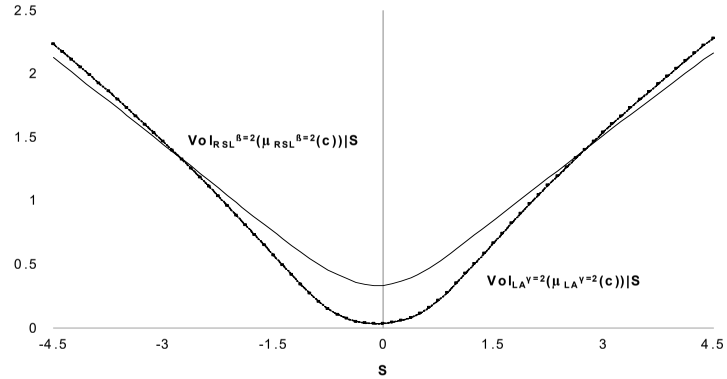


Corrected Figure 5. (*Continued*)

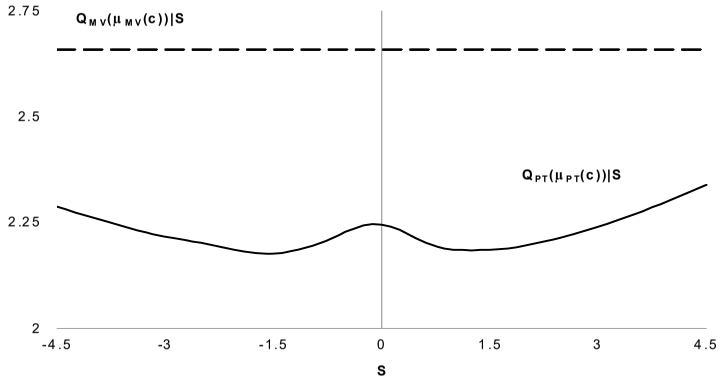
e) $Vol_{MV}\left(\mu_{MV}\left(c\right)\right)|S$ & $Vol_{PT}\left(\mu_{PT}\left(c\right)\right)|S$



f) $Vol_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}\left(c\right))|S$ & $Vol_{RSL}^{\beta=2}(\mu_{RSL}^{\beta=2}\left(c\right))|S$



g) $Q_{MV}\left(\mu_{MV}\left(c\right)\right)|S$ & $Q_{PT}\left(\mu_{PT}\left(c\right)\right)|S$



h) $Q_{LA}^{\gamma=2}(\mu_{LA}^{\gamma=2}\left(c\right))|S$ & $Q_{RSL}^{\beta=2}(\mu_{RSL}^{\beta=2}\left(c\right))|S$