Risk Adjustment and Trading Strategies

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We assess the profitability of momentum strategies using a stochastic discount factor approach. In unconditional tests, approximately half of the strategies' profitability is explained. In conditional tests we see a further slight decline in profits. We argue that the risk of these strategies should be increasing in the market risk premium. Empirically, while their risk measures estimated relative to the stochastic discount factor behave as predicted, market betas do not; thus capital asset pricing model (CAPM)-like benchmarks may lead to incorrect inferences. Given that our nonparametric risk adjustment explains roughly half of momentum strategy profits, we cannot rule out the possibility of residual mispricing.

A large number of empirical articles document the ability of investors to achieve abnormal returns through the use of simple trading strategies based on historical returns. Recently Jegadeesh and Titman (1993) show that, over intermediate horizons of 3–12 months, a portfolio that purchases past winners and sells past losers has a positive abnormal return. The evidence that such a "momentum" strategy exhibits abnormal performance has received a great deal of attention. Some authors have proposed risk-based explanations of these apparent profits [see, e.g., Fama and French (1996)], but to date, the evidence that the returns of these strategies are related to identifiable risk measures is mixed. In response, several articles have proposed new theories to explain, among other stylized facts, momentum strategy profits. These articles rely on psychological factors; for example, Barberis, Shleifer, and Vishny (1998) build a model which assumes factors such as representativeness and conservatism, while Daniel, Hirshleifer, and Subramanyam (1998) rely on overconfidence and self-attribution.

In this article we reassess the risk-based explanations for the profitability of momentum trading strategies by measuring these profits against a nonparametric benchmark. The motivation for this analysis is straightforward.

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First, existing articles which examine the risk-adjusted performance of trading strategies rely, by necessity, on a particular pricing model or models to measure risk, and therefore abnormal returns. For example, Jegadeesh and Titman (1993) use a capital asset pricing model (CAPM) benchmark, while Grundy and Martin (2001) use a conditional factor model with two factors (the market portfolio and a size factor). However, if these pricing models are misspecified, then the abnormal returns which follow from their use are misspecified as well. That is, as Fama (1998) points out, these studies are plagued by a "bad model" problem.

The method that we employ follows Chen and Knez (1996) in using a stochastic discount factor retrieved from a set of basis assets to measure risk-adjusted performance. The existence of a stochastic discount factor is a necessary and sufficient condition for equilibrium in securities markets; thus evidence that a stochastic discount factor prices the momentum strategies is consistent with the ability of risk to explain these profits. However, unlike parametric performance measures, which adopt candidate stochastic discount factors implied by particular asset pricing models, nonparametric performance measures attempt to recover a set of admissible stochastic discount factors based on minimal conditions such as the *law of one price* or *no arbitrage* conditions. Thus, relative to parametric approaches, this study asks whether momentum profits can be explained with the minimal restriction of equilibrium in securities markets. Our approach provides insight into whether the profitability of such strategies represents the failure of particular parametric models in describing the cross section of returns or the failure of the rational pricing paradigm.

This approach has several advantages. First, as mentioned above, estimating a stochastic discount factor from a set of basis assets imposes equilibrium pricing conditions without the need to specify a parametric benchmark. Second, the estimation of a discount factor leads to natural measures of risk-adjusted abnormal performance. If the trading strategies considered outperform when measured in this manner, then it is more likely that their performance is due to investor irrationality. However, if these strategies cannot outperform the benchmark, their success may be consistent with rational asset pricing. Third, the nonparametric measures we use can be easily extended to conditional measures which incorporate the possibility that risk premiums are time varying. This may be particularly important given the recent work by Chordia and Shivakumar (2000), who present evidence that momentum profits are related to business cycle conditions. Their model for expected returns, however, imposes no cross-sectional constraints. In our framework, we can test whether time variation in expected returns might contribute to momentum "profits" while simultaneously requiring that those returns satisfy equilibrium constraints such as the law of one price.

In unconditional tests, our results suggest that approximately half of the momentum profits can be achieved by holding a fixed-weight combination of a set of basis assets, which are industry-sorted portfolios. Specifically, the average nonparametric risk-adjusted performance of the strategies is 51% of the profit level in the raw returns. Eight (nine) of the 16 strategies have residual profits which are significant at the 5% (10%) level when we require that the law of one price hold. We cannot rule out the possibility that this remaining half of momentum profits is due to mispricing. A joint test of all 16 strategies fails to reject the null hypothesis of no abnormal performance. This contrasts with the results obtained when using a CAPM benchmark, in which 9 of the strategies have risk-adjusted performance higher than the raw profits and 13 of the strategies examined remain significant at the 5% level after adjusting for market risk. These results also contrast sharply with those obtained using the Fama–French benchmark, in which all 16 of the strategies retain significant positive profits and, indeed, all 16 risk-adjusted profits are magnified relative to their raw levels.

If we require that a stronger no-arbitrage condition hold, that is, we require that the pricing kernel be nonnegative, the results are similar. Average abnormal profits decline substantially, but 8 of the 16 strategies' profits remain unexplained at the 5% level. We briefly examine conditional tests, in which investors' expectations are allowed to vary based on a limited set of publicly known conditioning variables. Momentum profits decline slightly from those observed in unconditional tests, particularly for shorter holding periods.

Our results suggest that a passive weighting of basis assets can explain a significant fraction of the level of momentum profits observed in the data. That is, an unconditional risk adjustment that relies on the risk/return relations observed in benchmark assets reduces momentum profits by approximately 50%. These results are surprising in light of the evidence in the literature that parametric benchmarks explain little of, or even magnify, momentum strategy profits. However, the nonparametric results that we present in this article suggest that momentum strategies so dramatically outperform previously considered benchmarks not because of information contained in past prices, but rather because of the nature of the risk adjustment. We discuss a simple model of the risk measures of securities in the extreme winner and loser portfolios, measured relative to the "true" stochastic discount factor, and show that the risk measure of a momentum portfolio should be increasing in the true risk premium. In contrast, our empirical results, as well as previous work in this area, suggest that momentum portfolios are associated with negative risk measures when estimated *relative to the market portfolio*. Thus CAPM (or CAPM-like) risk measures appear to be misspecified. However, the betas of momentum portfolios estimated relative to the alternative pricing kernels employed in our article conform more closely to the predicted behavior.

These results are subject to an important caveat. We are implicitly assuming that the industry portfolios themselves are correctly priced. This assumption may be incorrect. That is, while some of the momentum strategies' profits can be attributed to variation in the cross section of the basis assets' returns, the basis assets themselves may not be in equilibrium. In that case, we may have simply shifted the focus of the pricing puzzle to industry portfolios. However, subject to this point, this article shows that a portion of momentum returns is attributable to the risk inherent in the strategies.

The remainder of the article is outlined as follows. In Section 1 we discuss the theoretical basis for the existence of the measure of excess performance used in the article, largely following Chen and Knez (1996), and outline the methods used for estimating the risk-adjusted performance of the strategies. In Section 2 we describe the data. Estimation results are presented in Section 3. In Section 4 we formalize the theoretical link between past return information and risk, perform an empirical investigation of these implications, and examine the properties of the test statistics using Monte Carlo techniques. We discuss the interpretation of the results and briefly summarize in Section 5.

1. Nonparametric Performance Measures of Trading Strategy Portfolios

In this section we discuss the implications of the existence of a stochastic discount factor for the analysis of the profitability of trading strategies.

1.1 Stochastic discount factors and performance measures

Under suitable regularity conditions, Harrison and Kreps (1979) prove that the absence of arbitrage opportunities in a securities market implies the existence of a stochastic discount factor. This stochastic discount factor satisfies the pricing relationship

$$E[m_{t+\tau} x_{i,\,t+\tau} \mathscr{F}_t] = 1 \quad \forall\ i, t, \tau, \tag{1}$$

where $x_{i,\,t+\tau} = 1 + R_{i,\,t+\tau}$ denotes the gross return on traded asset i in the economy, $\mathscr{F}_t$ is a filtration, representing information available to the investing public at time t, and $m_{t+\tau}$ denotes the economy-wide stochastic discount factor or pricing kernel. If we assume the existence of a risk-free asset, then for any excess return Equation (1) implies

$$E[m_{t+\tau} r_{i,\,t+\tau} \mathscr{F}_t] = 0, \tag{2}$$

where $r_{i,\,t+\tau} = R_{i,\,t+\tau} - R_{f,\,t}$ denotes the excess return on asset i in the economy. The returns to the trading strategies discussed in the introduction, which buy winning securities and sell losing securities, are of the form of Equation (2). Under the law of one price, an $m_{t+\tau}$ should exist that prices the gross (and excess) returns on these securities as well. Under the stronger condition of no arbitrage, this $m_{t+\tau}$ must also be positive.[1]

[1] Following Hansen and Jagannathan (1991), we use a nonnegativity condition instead of the positivity condition in actual estimation.

Intuitively this formulation states that the "fair" or risk-adjusted return of any asset (or trading strategy) will be determined by the pricing kernel or stochastic discount factor in the economy. Note that this formulation also expressly accounts for risk. That is, Equation (2) can be rewritten as

$$E[r_{i,t+\tau}] = -\frac{1}{E[m_{t+\tau}]}\text{cov}[r_{i,t+\tau}, m_{t+\tau}]. \tag{3}$$

The covariance of the return on the asset with the pricing kernel varies for each asset and reflects its risk: if an asset's payoffs are greater in good states (when the value of m is lower), its expected risk premium is higher.

Hansen and Jagannathan (1991) investigate how to retrieve the stochastic discount factors $m_{t+\tau}$ from a given set of tradable, or basis, assets. The key underlying assumption therein is that there is no pricing inconsistency among the basis assets: that is, the stochastic discount factors are *admissible*. Hansen and Jagannathan (1991) suggest two particular solutions for $m_{t+\tau}$ which are the minimum-norm discount factors defined in different metrics. The first is defined as the $m_{t+\tau}$ that satisfies Equation (1) [or equivalently Equation (2)] and is in the linear span. That is,

$$m_{t+\tau}^{LOP} = \mathbf{x}_{t+\tau}'\boldsymbol{\delta}. \tag{4}$$

This stochastic discount factor has been studied extensively in the literature [e.g., Chamberlain and Rothschild (1983), Hansen and Jagannathan (1991, 1997)]. We follow Chen and Knez (1996) and term this solution for the stochastic discount factor the law of one price (LOP) discount factor, since its existence necessitates only that the law of one price hold.

The second stochastic discount factor satisfies Equation (1) and the further requirement that $m_{t+\tau}$ is nonnegative, or

$$m_{t+\tau}^{NA} = \left(\mathbf{x}_{t+\tau}'\boldsymbol{\delta}\right)^{+}, \tag{5}$$

where $(\mathbf{x}_{t+\tau}'\boldsymbol{\delta})^{+} \equiv \max(\mathbf{x}_{t+\tau}'\boldsymbol{\delta}, 0)$. This stochastic discount factor satisfies the stronger condition of no arbitrage, thereby ruling out investment opportunities with positive payoffs and nonpositive prices. We refer to this discount factor as the no-arbitrage stochastic discount factor.

The implications of Equations (1) and (2) for trading strategies are straightforward. Consider the set of basis assets from which the strategies are to be implemented, $\mathbf{x}_{t+\tau}$. Under the law of one price (or alternatively, no arbitrage), there must be a stochastic discount factor that correctly prices $\mathbf{x}_{t+\tau}$ and all linear combinations thereof. Furthermore, this must hold even in the case in which the combinations represent zero-cost portfolios. Thus the testable implication of Equations (1) and (2) for trading strategies is that

$$\begin{aligned} &\exists\ m^*_{t+\tau}\ s.t. \\ &E\left[m^*_{t+\tau}\mathbf{x}_{B,\,t+\tau}\middle|\mathcal{F}_t\right] = \mathbf{1}_N \\ &E\left[m^*_{t+\tau}r_{TS,\,t+\tau}\middle|\mathcal{F}_t\right] = 0, \end{aligned} \tag{6}$$

where $\mathbf{x}_{B,\,t+\tau}$ represents the gross returns on the basis assets over τ periods and $r_{TS,\,t+\tau}$ represents the return to the trading strategy over τ periods.

Chen and Knez (1996) show that Equation (6) provides the basis for a test of the risk-adjusted performance of a portfolio. According to Equation (6),

$$\alpha_\tau = E[\alpha_{t,\,\tau}] = E\left[m^*_{t+\tau}r_{TS,\,t+\tau}\right] = 0. \tag{7}$$

Therefore a natural hypothesis to test is whether α_τ is equal to zero. This measure is similar to other abnormal performance measures such as Jensen's (1968) alpha, which obtains as a special case of Equation (7). However, unlike the Jensen measure, Equation (7) does not rely on the existence of a particular model of equilibrium asset prices.

It is important to note the differences between this method and other approaches in assessing the abnormal performance of momentum strategies. Other researchers assume a particular parametric pricing model, which is assumed to price all assets, then test whether the momentum strategy outperforms relative to this model. For example, Jegadeesh and Titman (1993) employ a CAPM benchmark, thereby using the pricing kernel

$$m_{t+1} = \phi_0 + \phi_1 R_{M,\,t+1},$$

where $R_{M,\,t+1}$ is the return on a particular market-proxy portfolio. Grundy and Martin (2001) employ a conditional factor model, with the market portfolio and a size factor representing the set of priced factors. In their results, these researchers find evidence that risk-adjusted returns of momentum strategies are significant, and frequently larger than the raw returns themselves. Thus, *conditional on a particular parametric pricing model being true*, their results suggest that momentum strategies generate statistically and economically significant profits and in fact may in some cases have negative risk measures. Given the lack of consensus in the literature about which pricing model is more "correct" (and, in fact, the large body of literature that suggests that the CAPM is not empirically supported in the data), it seems worthwhile to examine the abnormal performance of such strategies without the need to specify a particular parametric model. Rather, the model we employ assumes, first, that the set of basis assets chosen form a basis of comparison for the remaining assets in the economy and, second, that the pricing kernel can be accurately retrieved from this set of assets. Given such a pricing kernel, we test whether the momentum strategy enhances the investor's opportunity set, or whether the momentum profits can be achieved through a combination of the basis assets. We provide some additional comparisons between the pricing kernel we extract and other parametric benchmarks in Section 4.

1.2 Methods

Estimating α and testing Equation (7) is straightforward and can be accomplished easily via Hansen's (1982) generalized method of moments (GMM). In order to implement this procedure, we substitute $\mathbf{x}'_{t+\tau}\boldsymbol{\delta}((\mathbf{x}'_{t+\tau}\boldsymbol{\delta})^{+})$ for $m^{LOP}_{t+\tau}(m^{NA}_{t+\tau})$ and collect the vector of errors

$$\boldsymbol{\epsilon}_{t+\tau} = \left(\mathbf{u}'_{t+\tau}, v_{t+\tau}\right)', \tag{8}$$

where

$$\begin{aligned} \mathbf{u}_{t+\tau} &= \mathbf{x}_{B,\,t+\tau}\mathbf{x}'_{B,\,t+\tau}\boldsymbol{\delta} - \mathbf{1}_n \\ v_{t+\tau} &= r_{TS,\,t+\tau}\mathbf{x}'_{B,\,t+\tau}\boldsymbol{\delta}. \end{aligned}$$

We then form the vector of sample moment conditions

$$\mathbf{g}_T(\delta) = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\epsilon}_{t+\tau}. \tag{9}$$

Under the null hypothesis of no abnormal performance, Equation (7), $E[\mathbf{g}_T(\boldsymbol{\delta})] = 0$. We test this hypothesis by minimizing the quadratic form

$$J_T = T\mathbf{g}'_T(\boldsymbol{\delta})\mathbf{W}_T\mathbf{g}'_T(\boldsymbol{\delta}), \tag{10}$$

where the weighting matrix is

$$\mathbf{W}_T = E\left[\mathbf{g}_T(\boldsymbol{\delta})\mathbf{g}'_T(\boldsymbol{\delta})\right]^{-1}.$$

As shown by Hansen (1982), under the null $J_T \sim \chi^2_{n-k}$, where n is the number of moment conditions and k is the number of parameters. Each of the n assets is associated with one moment condition. We incorporate the return on a risk-free asset into the estimation as well; Dahlquist and Soderlind (1999) emphasize the importance of this technique in order to fix the mean of the stochastic discount factor at a reasonable level. The trading strategy return adds an additional moment condition so that $J_T \sim \chi^2_1$.

We have limited our discussion thus far to agents employing static trading strategies in the formulation of the basis portfolios, that is, buy-and-hold strategies. This approach is equivalent to asking whether the momentum strategy returns are spanned by a constant-weight, linear combination of the basis assets or a constant risk premium. This static approach ignores a large body of evidence that suggests that expectations vary over time.[2] In fact, Chordia and Shivakumar (2000) argue that such time variation in expected returns can fully explain momentum strategy profits (although as we mention above, they examine expected returns in a setting that does not impose equilibrium constraints). Similarly, using size and book-to-market portfolios, Lewellen (2001) argues that macroeconomic factors, rather than firm-specific returns, are responsible for momentum profits in these assets. Consequently we briefly consider and estimate an alternative pricing kernel, allowing for the use of a limited set of conditioning information. The conditioning information that we use does not rely explicitly (or only) on information contained in past prices; rather it takes the form of public information known to economic agents at the time of portfolio formation. As noted by Fama (1991), this type of conditioning does not violate market efficiency if risk premia are time varying.[3]

In order to incorporate the idea of time-varying expectations, we follow Hansen and Singleton (1982) in assuming that conditional expectations are linear in time t information variables, Z_t. This case yields the following representation for the pricing relation of Equation (1):

$$E[(\mathbf{x}_{t+\tau}\otimes\mathbf{Z}_t)(\mathbf{x}_{t+\tau}\otimes\mathbf{Z}_t)'\boldsymbol{\delta}]=\mathbf{1}_N. \tag{11}$$

We can then collect the vector of errors,

$$\boldsymbol{\epsilon}_{z,\,t+\tau}=\left(\mathbf{u}'_{z,\,t+\tau},\,v_{z,\,t+\tau}\right)', \tag{12}$$

where

$$\begin{aligned}\mathbf{u}_{z,\,t+\tau}&=(\mathbf{x}_{t+\tau}\otimes\mathbf{Z}_t)(\mathbf{x}_{t+\tau}\otimes\mathbf{Z}_t)'\boldsymbol{\delta}-\mathbf{1}\otimes\mathbf{Z}_t\\ \mathbf{v}_{z,\,t+\tau}&=r_{TS,\,t+\tau}(\mathbf{x}_{t+\tau}\otimes\mathbf{Z}_t)'\boldsymbol{\delta}.\end{aligned}$$

The no-arbitrage discount factor can also be employed in the methods discussed above. It requires that we replace $\mathbf{x}'\boldsymbol{\delta}$ with $(\mathbf{x}'\boldsymbol{\delta})^+\equiv\max(\mathbf{x}'\boldsymbol{\delta},0)$ in the unconditional setting and $(\mathbf{x}\otimes\mathbf{Z})'\boldsymbol{\delta}$ with $((\mathbf{x}\otimes\mathbf{Z})'\boldsymbol{\delta})^+$ in the conditional setting. The estimation of this quantity is significantly more complicated, but

[2] The literature documenting time variation in expected returns is voluminous. Some articles include Campbell (1987), Fama and French (1988), and Shanken (1990). The implications of this time variation for asset pricing models has been investigated theoretically in Hansen and Richard (1987) and empirically in Gibbons and Ferson (1985), Harvey (1989), and Ferson and Harvey (1991), among others.

[3] If risk premia are not time varying, of course, then any predictability would be the result of (marketwide) lags in response to public information. Our explicit assumption that the law of one price (or no-arbitrage) condition holds rules out this type of lagged price adjustment (as do most, if not all, parametric pricing models). If this assumption is counterfactual, then our measure of abnormal performance is a measure of the *incremental* profit that individual security momentum strategies add to the marketwide lag in price adjustment to our three public information variables. Intuitively, allowing for the use of conditioning information expands the set of basis assets to include managed portfolios, which take into account changes in the moments of returns that are related (only) to changes in the conditioning variables. [See also Cochrane (1997) for an excellent discussion of the nature of conditional tests.]

can be achieved through the procedure in Hansen and Jagannathan (1991). This approach also imposes the restriction that the estimated stochastic discount factor has the minimum second moment in the set of positive pricing kernels.

2. Data

As a starting point, we consider the universe of firms listed by the Center for Research in Security Prices (CRSP) over the period December 31, 1962, through December 31, 1997, yielding 420 monthly observations. We construct the momentum payoffs and the basis portfolios as follows.

2.1 Momentum portfolios

We follow the method outlined in Jegadeesh and Titman (1993) for the construction of the momentum strategy payoffs. New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) firms are classified into 10 deciles on the basis of the past $J = \{3, 6, 9, 12\}$-month cumulative holding period returns at month t. The firms offering the best performance over these horizons are termed "winner" firms and the firms offering the worst performance over these horizons are termed "loser" firms. The winner and loser firms are then held in portfolios for the next $K = \{3, 6, 9, 12\}$ months. That is, at time t for horizon J, we purchase an equally weighted portfolio of firms that were winners from $t - J$ through t and sell an equally weighted portfolio of firms that were losers over period $t - J$ through t. Since the $J = 12, K = 12$ strategy requires 24 months to implement, this reduces the number of time series observations to 396, covering the period January 31, 1965, through December 31, 1997.

2.2 The basis assets

An important consideration in estimating the measure of abnormal performance discussed in the article is the choice of reference assets. In complete markets, the stochastic discount factor will be unique. However, when markets are incomplete, there exists a multiplicity of stochastic discount factors that will correctly price the assets in the economy [Harrison and Kreps (1979)]. If the reference set from which the stochastic discount factor is formed spans the payoff opportunity set which is available to investors, then measuring abnormal performance relative to this reference set will provide a correct (and unique) inference. However, if the reference set does not span the payoffs, it is possible to incorrectly reject the null hypothesis of zero abnormal performance.[4]

[4] Ahn and Shivdasani (1999) discuss this issue more fully and present methods for addressing the spanning problem.

Ideally, to prevent an incorrect rejection of the null, the reference assets should mimic the entire opportunity set from which the trading strategies are chosen. However, since the momentum strategies we investigate are formed from the set of all NYSE and AMEX securities, this approach cannot be implemented in practice. Therefore we must choose a more parsimonious set of reference assets that capture as much of the investment opportunity set as possible; that is, we wish to group securities in a manner that maximizes intragroup correlation and minimizes intergroup correlation. King (1966) demonstrates that industry groupings do precisely that. In an exhaustive analysis of factors important in the determination of stock returns, he concludes that market and industry factors capture most, if not all, of the common variation in stock returns. For example, he demonstrates that "large" positive covariances in returns cluster strongly within industry groupings, and negative covariances are observed exclusively across industry groupings. Therefore we form the reference set by forming portfolios on the basis of industry. Specifically, each year we form 20 equally weighted portfolios of NYSE and AMEX firms on the basis of two-digit SIC code groupings. Definitions of the groups can be found in Ahn, Conrad, and Dittmar (2000).[5]

Moskowitz and Grinblatt (1999) implement a variant of the momentum strategy that relies on the use of industry portfolios. Specifically, they examine a strategy that buys the top three industry winners and sells the bottom three industry losers each period; they control for specific sources of risk (in their case given by the three Fama–French portfolios) to measure abnormal performance. That is, as in Jegadeesh and Titman (1993) and Grundy and Martin (2001), Moskowitz and Grinblatt choose a *particular* parametric risk adjustment. In contrast, we assume that industry portfolios are rationally priced by some pricing kernel $m_{t+\tau}$; the purpose of the industry portfolios in this article is merely to retrieve this kernel. In their article, Moskowitz and Grinblatt vary the weights on industry portfolios based on relative past performance of the portfolios. In contrast, in the pricing kernel, the weights assigned to our industry portfolios are fixed (in the unconditional analysis) or are allowed to vary only with the three public information variables in the conditional tests. That is, in our unconditional (conditional) tests, we are asking whether momentum strategy profits can be priced by *passive* combinations of industry (industry plus a particular set of managed) portfolios. These two weighting schemes will coincide only if the industry momentum strategy is a reflection of the publicly available information we use in our tests.[6]

[5] We have also used industry portfolios reformed each month. The explanatory power of the basis assets is improved in that case. Note that Chen and Knez (1996) also use industry portfolios in their analysis of the performance of mutual funds.

[6] As mentioned, another difference between our analysis and that of Moskowitz and Grinblatt is that we employ an (annually rebalanced) equal weighting in our industry portfolios, while Moskowitz and Grinblatt (1999) use a value weighting. We examined the possibility of using value weights in our industry portfolios.

Table 1
Summary statistics: momentum portfolios

		Mean returns			
		$K=3$	$K=6$	$K=9$	$K=12$
$J=3$:	Win	0.0132	0.0131	0.0148	0.0154
	Lose	0.0121	0.0125	0.0099	0.0093
	Win-Lose	0.0011	0.0006	0.0049	0.0061
$J=6$:	Win	0.0158	0.0167	0.0172	0.0164
	Lose	0.0102	0.0090	0.0081	0.0090
	Win-Lose	0.0056	0.0077	0.0091	0.0074
$J=9$:	Win	0.0180	0.0185	0.0175	0.0163
	Lose	0.0090	0.0075	0.0081	0.0094
	Win-Lose	0.0090	0.0090	0.0094	0.0079
$J=12$:	Win	0.0188	0.0178	0.0168	0.0156
	Lose	0.0072	0.0075	0.0087	0.0100
	Win-Lose	0.0116	0.0103	0.0081	0.0056
		Standard deviations			
		$K=3$	$K=6$	$K=9$	$K=12$
$J=3$:	Win	0.0629	0.0628	0.0635	0.0645
	Lose	0.0860	0.0850	0.0850	0.0802
	Win-Lose	0.0520	0.0484	0.0452	0.0368
$J=6$:	Win	0.0637	0.0644	0.0647	0.0644
	Lose	0.0910	0.0883	0.0857	0.0846
	Win-Lose	0.0625	0.0567	0.0507	0.0484
$J=9$:	Win	0.0651	0.0651	0.0651	0.0649
	Lose	0.0924	0.0905	0.0890	0.0881
	Win-Lose	0.0648	0.0612	0.0577	0.0552
$J=12$:	Win	0.0662	0.0657	0.0655	0.0653
	Lose	0.0914	0.0911	0.0906	0.0899
	Win-Lose	0.0641	0.0627	0.0605	0.0583

This table presents monthly means and standard deviations of the returns to 16 relative strength strategies formed as in Jegadeesh and Titman (1993). The portfolios are formed by purchasing the portfolio formed of firms in the tenth decile of returns and selling the portfolio formed of firms in the first decile of returns over the past J months. Portfolios are held for the subsequent K months. The data cover the period February 28, 1965, through December 31, 1997.

3. Estimation Results

3.1 Returns to relative strength portfolios

Summary statistics for the returns to the momentum trading strategies are presented in Table 1. Consistent with Jegadeesh and Titman (1993), the mean monthly returns for the strategies vary widely, from 6 basis points per month for the 3-month/6-month strategy to 116 basis points per month for the 12-month/3-month strategy. As in Jegadeesh and Titman, the returns to the strategies tend to increase in the length of the portfolio formation period. However, the average returns to the strategies in Table 1 are almost always

However, these portfolios performed poorly as basis assets, perhaps due to a lack of diversification in some portfolios. In our sample we observed individual security weights as high as 60% in some months; this led to a large standard deviation of returns in these portfolios and a Hansen–Jagannathan bound much lower than that observed for equally weighted portfolios. That is, the equally weighted portfolios capture wider cross-sectional variation in asset returns and thus impose a more informative restriction on the admissible set of stochastic discount factors. Consequently we use equal-weighted portfolios in our analysis, but rebalance only annually to ensure that the transaction costs of holding the basis assets is not prohibitive.

lower than those reported by Jegadeesh and Titman. At the extreme, the 12-month/3-month strategy reported in this article has a mean return of 116 basis points per month compared to a return of 131 basis points per month reported by Jegadeesh and Titman. Much of this discrepancy is attributable to the sample period; the data in this article cover an additional eight years of returns. Over a common sample period, the difference in mean returns is negligible.

3.2 Measuring abnormal performance

As noted in the previous section, the returns to the strategies are fairly large and vary across both formation and holding periods. In this section, we assess the degree to which risk can explain this variation.

3.2.1 CAPM-based performance measure. Table 2 presents an analysis of the abnormal performance of the trading strategies using a CAPM benchmark for risk. This analysis follows that presented in Jegadeesh and Titman (1993) and consists of regressing the returns to the trading strategies on the excess market return and a constant,

$$r_{TS,\,t+\tau} = \alpha_{TS} + \beta_{TS}(R_{M,\,t+\tau} - R_{f,\,t+\tau}) + \epsilon_{TS,\,t+\tau}. \tag{13}$$

The trading strategy earns an abnormal return if $\alpha_{TS} \neq 0$.

The results of Table 2 suggest that 13 of the 16 trading strategies earn positive abnormal returns. In fact, in 9 of the 16 cases, the abnormal profits after CAPM risk adjustment are larger than the raw profits observed in

Table 2
CAPM performance measure

	$J=3,\ K=3$	$J=3,\ K=6$	$J=3,\ K=9$	$J=3,\ K=12$
α_{W-L}	0.0021	0.0015	0.0035	0.0050
p	(0.4306)	(0.5229)	(0.1087)	(0.0073)
	$J=6,\ K=3$	$J=6,\ K=6$	$J=6,\ K=9$	$J=6,\ K=12$
α_{W-L}	0.0065	0.0085	0.0082	0.0068
p	(0.0377)	(0.0033)	(0.0015)	(0.0058)
	$J=9,\ K=3$	$J=9,\ K=6$	$J=9,\ K=9$	$J=9,\ K=12$
α_{W-L}	0.0099	0.0094	0.0085	0.0063
p	(0.0028)	(0.0026)	(0.0038)	(0.0245)
	$J=12,\ K=3$	$J=12,\ K=6$	$J=12,\ K=9$	$J=12,\ K=12$
α_{W-L}	0.0123	0.0103	0.0082	0.0057
p	(0.0002)	(0.0013)	(0.0076)	(0.0528)

This table presents results from estimating the excess returns on the relative strength strategies based on Jensen's (1968) performance measure. Specifically, the table presents estimates of the intercepts from the least squares regression,

$$r_{TS,\,t+\tau} = \alpha_{TS} + \beta_{TS}(R_{M,\,t+\tau} - R_{f,\,t+\tau}) + \epsilon_{TS,\,t+\tau},$$

where $r_{TS,\,t+\tau}$ denotes the excess return on relative strength portfolio TS and $R_{M,\,t+\tau}$ represents the return on the value-weighted CRSP index over the one-month Treasury bill return.

Table 1. The "average" profit across all 16 strategies is 70 basis points per month, and is virtually unchanged from the average raw or unadjusted profits of 71 basis points per month. At the extremes, the 3-month/6-month strategy earns the smallest abnormal return, 15 basis points per month, whereas the 12-month/3-month strategy earns the largest, 123 basis points per month. Jegadeesh and Titman (1993) present results only for the 6-month/6-month strategy and find that the strategy earns an abnormal return of 100 basis points per month, which is slightly higher than the abnormal return of 85 basis points per month shown in Table 2. However, this differential is proportional to the difference in raw returns discussed in the previous section.

Under the assumption that the CAPM is the relevant pricing model and that risk measures are well estimated, the conclusion drawn from Table 2 is that a momentum trading strategy provides significant excess returns on a risk-adjusted basis. This evidence is consistent with that in Jegadeesh and Titman (1993), who report that momentum profits persist even after CAPM risk adjustments. However, given the large body of evidence which suggests that the CAPM does not hold [e.g., MacKinlay (1987), Fama and French (1992)], we explore alternative benchmarks.

3.2.2 Fama–French performance measure. Table 3 presents abnormal performance measures of momentum trading strategies using the Fama and French (1992) three-factor model to adjust for risk. Thus the following model

Table 3
Fama–French performance measure

	$J=3, K=3$	$J=3, K=6$	$J=3, K=9$	$J=3, K=12$
α_{W-L}	0.0056	0.0048	0.0082	0.0090
SE	(0.0022)	(0.0021)	(0.0018)	(0.0016)
	$J=6, K=3$	$J=6, K=6$	$J=6, K=9$	$J=6, K=12$
α_{W-L}	0.0108	0.0123	0.0130	0.0114
SE	(0.0027)	(0.0025)	(0.0022)	(0.0021)
	$J=9, K=3$	$J=9, K=6$	$J=9, K=9$	$J=9, K=12$
α_{W-L}	0.0144	0.0156	0.0142	0.0119
SE	(0.0028)	(0.0026)	(0.0024)	(0.0022)
	$J=12, K=3$	$J=12, K=6$	$J=12, K=9$	$J=12, K=12$
α_{W-L}	0.0169	0.0157	0.0137	0.0112
SE	(0.0028)	(0.0027)	(0.0025)	(0.0024)

This table presents results from estimating the excess returns on the relative strength strategies based on a Jensen's (1968) performance measure. Specifically, the table presents estimates of the intercepts from the least squares regression,

$$r_{TS,t+\tau} = \alpha_{TS} + \beta_{TS,MRP}(R_{MRP,t+\tau}) + \beta_{TS,SMB}(R_{SMB,t+\tau}) + \beta_{TS,MRP}(R_{HML,t+\tau}) + \epsilon_{TS,t+\tau},$$

where $r_{TS,t+\tau}$ denotes the excess return on relative strength portfolio TS, R_{MRP} represents the return on the value-weighted CRSP index over the one-month Treasury bill return, R_{SMB} represents the return on a portfolio that is long in large-capitalization stocks and short in small-capitalization stocks, and R_{HML} represents the return on a portfolio that is long in high book-to-market stocks and short in low book-to-market stocks. Heteroscedasticity-consistent standard errors are presented in parentheses.

is estimated:

$$r_{TS,\,t+\tau} = \alpha_{TS} + \beta_{TS,\,MRP}(R_{MRP,\,t+\tau}) + \beta_{TS,\,SMB}(R_{SMB,\,t+\tau}) + \beta_{TS,\,HML}(R_{HML,\,t+\tau}) + \epsilon_{TS,\,t+\tau}, \tag{14}$$

where R_{MRP} is the return on a market proxy, R_{SMB} is the return on a zero-cost portfolio that buys large-capitalization firms and sells small-capitalization firms, and R_{HML} is the return on another zero-cost portfolio that buys high book-to-market firms and sells low book-to-market firms. As before, we test whether the intercept is significantly different from zero. The results in Table 3 show that, using this model to adjust for risk, 16 of 16 trading strategies earn positive abnormal returns, with very small associated p-values. In addition, abnormal profits increase relative to raw profits in all 16 cases, averaging 118 basis points per month. As with the CAPM risk adjustment, the largest abnormal return is in the 12-month/3-month strategy with an abnormal return of 1.69% per month. This evidence is consistent with the results presented in Fama and French (1996) and Carhart (1997), who show that the three-factor model does not explain momentum profits.

3.2.3 Unconditional performance measures. Next we examine the implications of the unconditional performance measure based on the law of one price. As discussed previously, the basis used for the tests consists of the 20 industry-sorted portfolios augmented by the return on the riskless asset. Results of this estimation are shown in Table 4.

The results in Table 4 suggest that the abnormal performance of momentum strategies declines when the unconditional performance measure is used.

Table 4
LOP-based performance measures—unconditional

	$J=3,\ K=3$	$J=3,\ K=6$	$J=3,\ K=9$	$J=3,\ K=12$
α_{W-L}	−0.0002	−0.0006	0.0028	0.0035
p	(0.9265)	(0.7476)	(0.1021)	(0.0163)
	$J=6,\ K=3$	$J=6,\ K=6$	$J=6,\ K=9$	$J=6,\ K=12$
α_{W-L}	0.0032	0.0049	0.0055	0.0037
p	(0.2587)	(0.0632)	(0.0141)	(0.0471)
	$J=9,\ K=3$	$J=9,\ K=6$	$J=9,\ K=9$	$J=9,\ K=12$
α_{W-L}	0.0056	0.0065	0.0048	0.0026
p	(0.0444)	(0.0105)	(0.0215)	(0.1390)
	$J=12,\ K=3$	$J=12,\ K=6$	$J=12,\ K=9$	$J=12,\ K=12$
α_{W-L}	0.0068	0.0052	0.0031	0.0009
p	(0.0144)	(0.0326)	(0.1272)	(0.5881)

This table presents results from the unconditional law of one price estimation of performance measures. The table presents results for 16 strategies representing different combinations of sorting and holding periods. Portfolios are ranked on the basis of J-month lagged returns and held for K months. α represents average monthly excess performance over the portfolio of basis assets for the strategy. The numbers in parentheses represent the p-values from a chi-squared test of the hypothesis $H_0: \alpha = 0$. The basis assets in this sample consist of 20 industry sorted portfolios.

The average profit level is 51% of the raw profits observed in Table 1. The residual profit is frequently statistically significant: of the 16 individual strategies analyzed, 8 continue to exhibit abnormal performance at the 5% level. In one additional case, the abnormal performance is significant at the 10% level. A joint test across all 16 momentum strategies fails to reject the null hypothesis of no abnormal performance, with a p-value of 0.463. This evidence suggests that a stochastic discount factor can be constructed from the industry portfolios, which encompasses approximately half of the profits of the momentum portfolios we form.

In Table 5 we require that the stronger no-arbitrage condition hold, and again investigate whether a stochastic discount factor formed from the basis assets can explain the momentum profits observed in the data. The results are similar: 8 of the 16 strategies retain profits which are significant at the 5% level and an additional one has profits significant at the 10% level. On average, abnormal profits decline to average 48% of the levels observed in Table 1. In addition, a joint test across all 16 momentum strategies fails to reject the null hypothesis of no abnormal performance with a p-value of 0.469.

Overall the evidence in Tables 4 and 5 suggest that, while a specific factor model (i.e., the CAPM) cannot explain momentum profits (and in fact appears to magnify these profits), a pricing kernel estimated from industry-sorted portfolios, which satisfies the much weaker condition of the law of one price and which is stationary over time, can explain roughly half of these profits. That is, these results suggest that up to one-half of the trading strategy profits may be attributable to the risk inherent in the strategy. The remaining half of momentum profits fall outside of the risk/return relations represented in the set of basis assets; thus, using industry portfolios as our benchmark, we cannot rule out the existence of residual mispricing in the momentum portfolios.

Table 5
No-arbitrage-based performance measures—unconditional

	$J=3,\ K=3$	$J=3,\ K=6$	$J=3,\ K=9$	$J=3,\ K=12$
α_{W-L}	−0.0003	−0.0006	0.0026	0.0032
p	(0.9317)	(0.7474)	(0.1031)	(0.0161)
	$J=6,\ K=3$	$J=6,\ K=6$	$J=6,\ K=9$	$J=6,\ K=12$
α_{W-L}	0.0031	0.0047	0.0051	0.0034
p	(0.2570)	(0.0641)	(0.0143)	(0.0448)
	$J=9,\ K=3$	$J=9,\ K=6$	$J=9,\ K=9$	$J=9,\ K=12$
α_{W-L}	0.0054	0.0061	0.0044	0.0023
p	(0.0475)	(0.0112)	(0.0209)	(0.1311)
	$J=12,\ K=3$	$J=12,\ K=6$	$J=12,\ K=9$	$J=12,\ K=12$
α_{W-L}	0.0063	0.0047	0.0028	0.0007
p	(0.0153)	(0.0315)	(0.1197)	(0.5658)

This table presents results from the unconditional no arbitrage estimation of performance measures. The table presents results for 16 strategies representing different combinations of sorting and holding periods. Portfolios are ranked on the basis of J-month lagged returns and held for K months. α represents average monthly excess performance over the portfolio of basis assets for the strategy. The numbers in parentheses represent the p-values from a chi-squared test of the hypothesis $H_0: \alpha = 0$. The basis assets in this sample consist of 20 industry sorted portfolios.

Table 6
LOP- and no-arbitrage-based performance measures—conditional

Panel A: LOP estimation

	$J=3, K=3$	$J=3, K=6$	$J=3, K=9$	$J=3, K=12$
α_{W-L}	−0.0017	−0.0020	0.0015	0.0026
p	(0.0914)	(0.0553)	(0.0634)	(0.0858)
	$J=6, K=3$	$J=6, K=6$	$J=6, K=9$	$J=6, K=12$
α_{W-L}	0.0015	0.0027	0.0047	0.0031
p	(0.5378)	(0.3986)	(0.1421)	(0.1550)
	$J=9, K=3$	$J=9, K=6$	$J=9, K=9$	$J=9, K=12$
α_{W-L}	0.0052	0.0044	0.0034	0.0024
p	(0.3246)	(0.1634)	(0.1899)	(0.1944)
	$J=12, K=3$	$J=12, K=6$	$J=12, K=9$	$J=12, K=12$
α_{W-L}	0.0054	0.0040	0.0032	0.0016
p	(0.1446)	(0.1075)	(0.1352)	(0.1904)

Panel B: No-arbitrage estimation

	$J=3, K=3$	$J=3, K=6$	$J=3, K=9$	$J=3, K=12$
α_{W-L}	−0.0035	−0.0040	0.0007	0.0023
p	(0.2993)	(0.2046)	(0.7257)	(0.2155)
	$J=6, K=3$	$J=6, K=6$	$J=6, K=9$	$J=6, K=12$
α_{W-L}	0.0006	0.0026	0.0035	0.0023
p	(0.8466)	(0.3687)	(0.1256)	(0.3308)
	$J=9, K=3$	$J=9, K=6$	$J=9, K=9$	$J=9, K=12$
α_{W-L}	0.0029	0.0043	0.0031	0.0015
p	(0.3422)	(0.0882)	(0.1856)	(0.5448)
	$J=12, K=3$	$J=12, K=6$	$J=12, K=9$	$J=12, K=12$
α_{W-L}	0.0051	0.0040	0.0021	0.0000
p	(0.0812)	(0.1028)	(0.4070)	(0.9854)

This table presents results from the conditional law of one price and no-arbitrage estimation of performance measures. Each panel presents results for 16 strategies representing different combinations of sorting and holding periods. Portfolios are ranked on the basis of J-month lagged returns and held for K months. α represents average monthly excess performance over the portfolio of basis assets for the strategy. The numbers in parentheses represent the p-values from a chi-squared test of the hypothesis $H_0 : \alpha = 0$. The basis assets in this sample consist of 20 industry-sorted portfolios, augmented by managed portfolios, which consist of the product of the equity portfolios with the instrumental variables $z_t = \{tb, ts, dy\}$, for a total of 80 basis assets.

3.2.4 Conditional performance measures. We examine whether our results are affected if we allow investors' expectations to vary conditional on three publicly known information variables: the dividend yield on the S&P 500 index, the return on the Treasury bill with maturity closest to one month, and the term spread, measured as the difference in yields on 10-year maturity Treasury bonds over one-year maturity Treasury bills.[7] These results are presented in Table 6.

Measured relative to a time-varying benchmark, LOP momentum profits decline slightly from the unconditional measures of abnormal performance. Averaging across all 16 strategies, abnormal profits are 72% of those

[7] These variables have been shown to have predictive power for returns in numerous studies, including Fama and Schwert (1977), Ferson (1989), and Fama and French (1988). In addition, these variables are similar to three (out of four) variables included in Chordia and Shivakumar (2000); they add a default spread variable as well.

observed in the unconditional tests and 37% of the raw profits. The use of this particular set of conditioning variables appears to affect profits of strategies with shorter holding periods to a greater extent. Despite the relatively small difference in the level of abnormal profits, none of the profits are statistically significant at the 5% level, and only five are significant at the 10% level; however, the larger standard errors may not be surprising given the larger number of moment conditions estimated.[8] The no-arbitrage results in panel B give qualitatively similar results.[9]

4. An Analysis of Risk Adjustment

The empirical results in the previous section suggest that a portion of momentum profits can be explained by a fixed-weight combination of a set of basis assets. Why, then, have existing studies of momentum profits, which use specific parametric benchmarks, failed to detect any evidence that momentum profits may be compensation for risk bearing? In this section, we explore the link between past return information and risk in order to address this question. This study also provides us with an additional diagnostic tool for evaluating whether the benchmark stochastic discount factor proposed is *admissible* in the context of measuring the (risk-adjusted) profitability of the trading strategies.[10]

Given the existence of a stochastic discount factor, m, it is well known [see, e.g., Cochrane (1997)] that one can construct a hedge portfolio or constant consumption portfolio which is perfectly correlated with the stochastic discount factor. Then, if we denote the return on such a hedge portfolio as R_d, the equilibrium expected return on asset i can be written as

$$E[R_i]-R_f=\beta_{di}(E[R_d]-R_f). \tag{15}$$

The above expression is similar to the CAPM, but the hedge portfolio does not need to be a market portfolio; in addition, this expression is still consistent with a multifactor model.[11] In this characterization, β_{di} is similar to the market beta and measures the sensitivity of the asset's payoff to that of

[8] We discuss the power characteristics of our tests in Section 4.2.

[9] The results in Table 6 are weak evidence that a portion of momentum profits are related to common macroeconomic factors. Chordia and Shivakumar (2000) find that momentum profits vary strongly with the business cycle and a four-factor time-varying model of expected returns appears to subsume much of the profits associated with momentum portfolios. In our tests, however, any time-varying expected returns in the basis assets must still satisfy the equilibrium requirement of the law of one price (or no arbitrage).

[10] For a more detailed analysis, see Ahn, Conrad, and Dittmar (2000).

[11] The existence of multiple factors in any form can be nested since Equation (3) is a mathematical identity to Equation (15). The difference between the two equations is similar to the difference in the representation of expected returns between the consumption CAPM of Breeden (1979) and the intertemporal CAPM of Merton (1973). The multiple factors are reduced to a single stochastic variable which represents the intertemporal marginal rate of substitution. In our context, the pricing kernel m and the return on its corresponding hedge portfolio, R_d, can be a function of multiple factors.

the constant consumption portfolio, R_d. In addition, given Equation (8), the *realized* return on asset i can be expressed as a function of its beta and the excess return on the hedge portfolio.

In this setting, consider the problem of selecting firms as "winners" and "losers." When the constant-consumption portfolio return is positive, the highest beta asset will be the winner. In contrast, when the market reverses, the lowest beta asset will be the winner. Assuming that the market price of risk is, on average, positive, then on average winners should be high beta securities and losers should be low beta securities. Consequently a momentum portfolio, which consists of buying past winners and selling past losers, should on average have a positive risk measure since a portion of that portfolio's high relative return is due to high expected return.

The intuition behind this argument is an extension of the analysis of Lo and MacKinlay (1990), who show that momentum (contrarian) strategies have, even in the absence of time-series effects, an average return that is positive (negative). It is also similar to the point made by Ball, Kothari, and Shanken (1995), who point out that "if the realized premium in the ranking period is positive, the loser portfolio is more likely to consist of low-beta stocks." Note that this result does not imply that abnormal returns cannot be earned with a momentum strategy—merely that the abnormal returns must be earned after allowing for the (on average) positive risk such a strategy entails.

We use this posited relationship between betas and the likelihood of being chosen as winner or loser as a diagnostic for the "admissibility" of the pricing kernels we consider. That is, we estimate the risk measures of the winner and loser portfolios using both the CAPM and our constructed pricing kernel as the benchmark. The results of this estimation are summarized in Table 7. As we move from loser to winner portfolios down the table, we see that the portfolios' kernel betas increase for all four estimation techniques employed

Table 7
Portfolio betas

		Unconditional		Conditional		
Decile	Mean	LOP	No arbitrage	LOP	No arbitrage	Value weighted
1	0.0090	1.4411	1.3673	0.5863	1.2504	1.3797
2	0.0013	1.5027	1.4555	0.6634	1.3512	1.1784
3	0.0125	1.4987	1.4615	0.7702	1.4298	1.1095
4	0.0126	1.4532	1.4250	0.8610	1.4011	1.0726
5	0.0127	1.4328	1.4107	0.9399	1.4611	1.0474
6	0.0131	1.4689	1.4547	0.9615	1.4881	1.0417
7	0.0134	1.5184	1.5069	1.0351	1.6054	1.0475
8	0.0137	1.5955	1.5872	1.1032	1.7612	1.0779
9	0.0146	1.7415	1.7316	1.1915	1.9192	1.1369
10	0.0167	1.9379	1.9203	1.3230	2.2963	1.2588

This table presents betas and mean returns for the decile portfolios of the six-month/six-month momentum strategy. Betas are estimated with respect to the six-month/six-month unconditional LOP, unconditional no-arbitrage, conditional LOP, and unconditional no-arbitrage pricing kernels and the value-weighted portfolio.

(LOP or NA, unconditional or conditional). The differences between the loser and the winner portfolio betas are striking: for the conditional LOP measure, the winner portfolio beta is more than twice the loser portfolio beta. The difference using the other estimation techniques is also substantial. These results imply that at least a portion of the returns to the momentum strategies examined represent compensation for bearing increased risk.

In contrast, firms' market betas are high for both extreme winner and loser portfolios. Thus, using this diagnostic, the CAPM kernel appears to be misspecified—the market betas of loser firms are "too high." In fact, note that the market betas of losers are slightly larger than the market betas of winners in this sample, consistent with the results of Jegadeesh and Titman (1993). This results in the market beta of the combined (winner minus loser) portfolio being slightly negative, which would correspond to a negative market price of risk, $\bar{\lambda}$, and magnify the risk-adjusted profit of the strategy. Since the market price of risk implied by the CAPM is positive, this result seems to contradict the theory. In addition, this result contributes to the inference that the risk-adjusted return of the momentum strategy is positive. The difference in results across the two types of pricing kernels in this context is another illustration of the importance of benchmark specification in assessing abnormal profits; as Roll (1978) shows, even small differences in benchmarks can lead to large differences in inferences about performance.

The difference in our nonparametric benchmark and the CAPM and the resulting difference in inferences can be illustrated graphically since DeSantis (1995) and Bekaert and Urias (1996) show that there is a direct relation between the LOP and the more familiar mean-variance analysis. In the CAPM, the market is mean-variance efficient, that is, it is the tangency portfolio with the highest possible Sharpe ratio. In contrast, in the LOP estimation procedure that we use, the 20 industry portfolios are used to form an "efficient frontier" relative to which the market portfolio may or may not compare favorably. The test of abnormal performance in this context is a test of whether the momentum portfolio's returns represent a significant improvement over the opportunity set formed by the industry portfolios. Figure 1 presents the mean-variance opportunity set created by the industry portfolios, the six-month/six-month momentum strategy and (for comparison) the value-weighted market.

It is clear from this figure that the market portfolio is dominated by combinations of the industry portfolios that we use. For example, the Sharpe ratio of the market portfolio is 0.1096; in contrast, the maximum Sharpe ratio estimated from the set of industry portfolios is substantially higher, at 0.3255. Corrected for the bias caused by searching for the maximum [as outlined in MacKinlay (1995)], it is 0.1961. In comparison, the six-month/six-month momentum portfolio has a Sharpe measure of 0.2084. Thus the fact that the abnormal performance of momentum portfolios is significant with respect to the CAPM pricing kernel, but declines substantially when industry-sorted

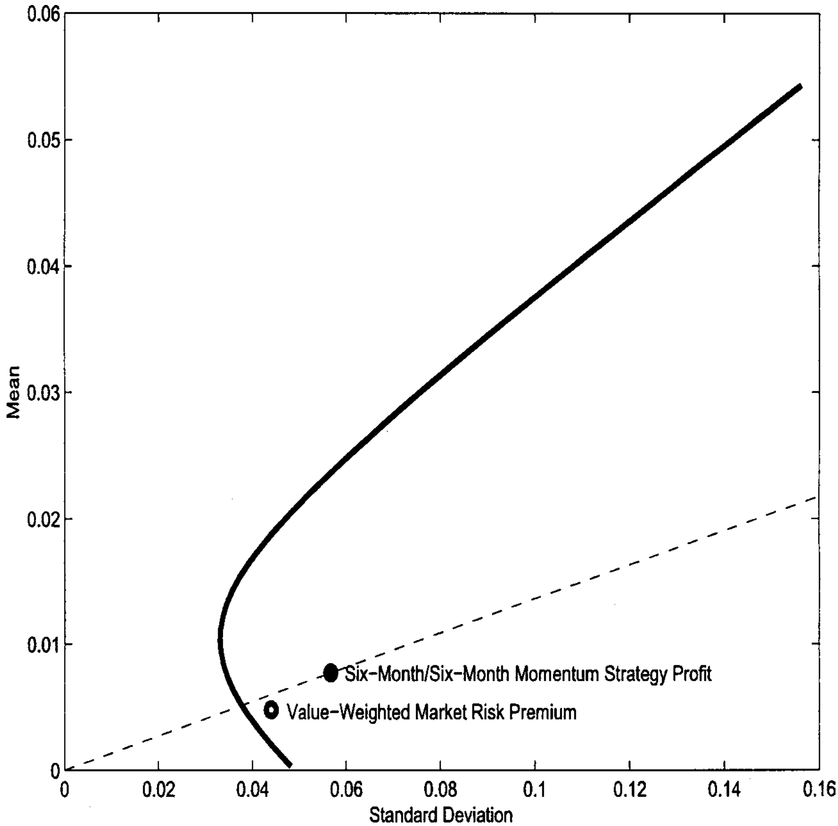

Figure 1
The ex post mean standard deviation frontier for the industry portfolios utilized as basis assets in this article. In addition to this frontier, the figure depicts the plot of the market portfolio, the industry momentum strategy, and the six-month/six-month momentum strategy. The dashed line represents a line with slope equal to the Sharpe ratio of the six-month/six-month momentum strategy and intercept equal to the risk-free rate.

portfolios are used to form the benchmark, is a consequence of the fact that the opportunity set created by industry portfolios dominates the market portfolio.

The finding that the Sharpe ratio obtained from the sample of industry portfolios is larger than that observed using the market proxy is consistent with the evidence in Cohen and Polk (1998). MacKinlay (1995) argues that ratios larger than 0.175 are implausibly high; the ratio observed from the industry-sorted portfolios in our sample exceeds that threshold. Thus, while our evidence suggests that momentum portfolio strategies are not mispriced relative to industry portfolios, it may be that industry portfolios are themselves mispriced. In that case, we have succeeded only in shifting the puzzle to another venue. Since the use of industry portfolios is widespread both among academic and practitioners, however, this puzzle is an important one.

If we accept the industry-sorted portfolios as a valid set of basis assets, note that the higher Sharpe measure they imply suggests that it may be feasible to construct a linear multifactor model which prices momentum strategies. For example (and independent of the literature on momentum trading strategies), Dittmar (1999) develops a pricing model based on investors' preferences in which kurtosis measures are associated with a positive risk premium. Using this *particular* parametric pricing model as the benchmark and industry portfolios as the set of benchmark assets, we cannot reject the hypothesis that momentum portfolios earn zero abnormal return ($\alpha = 0.0023$, p-value $= 0.23$).

4.1 Industry momentum and individual security momentum

Moskowitz and Grinblatt (1999) document that *industry momentum* strategies are profitable. The authors find profits of 43 basis points per month in a strategy that buys an equally weighted portfolio of the top three performing industries and shorts an equally weighted portfolio of the bottom three performing industries, holding this position for six months. The authors attribute these profits to industry components in the data-generating process that are unrelated to priced factors and present evidence that industry momentum is responsible for much of the momentum in individual securities. The authors suggest that their results are potentially attributable to either behavioral or rational explanations for the profitability of momentum strategies.

In this section we investigate whether our pricing kernel approach can explain the profits to these industry momentum portfolios. The industry portfolios employed in our study are the same as those investigated in Moskowitz and Grinblatt (1999), with the exception that our industry portfolios are equally weighted rather than value weighted. We find that using equally weighted portfolios and our sample period results in slightly lower returns, although these returns are still statistically significant. For example, the return to the six-month/six-month industry momentum strategy is 27 basis points per month (t-statistic $= 2.60$).

Table 8 displays the results of the LOP and no-arbitrage performance evaluation for the six-month/six-month industry momentum strategy. When profits are measured relative to the reference assets, the LOP point estimate of profits remains roughly the same at 26 basis points (unconditional) and 28 basis points (conditional), although they are not statistically significant at the 5% or 10% level. The no-arbitrage profits are similar, at 26 and 21 basis points, respectively, for the unconditional and conditional tests. These results suggest that while our basis assets can explain approximately half of individual security momentum profits, they explain little or none of the profits of the industry momentum strategy. This appears to confirm the contention of Grundy and Martin (2001), who argue that individual and industry momentum are distinct phenomena. Moreover, the evidence that passive industry (or industry plus managed) portfolios cannot explain "industry momentum"

Table 8
Performance assessment of industry momentum strategies

Panel A: Unconditional

Measure:	LOP	No arbitrage
α_{W-L}	0.0026	0.0026
p	(0.2500)	(0.2520)

Panel B: Conditional

Measure:	LOP	No arbitrage
α_{W-L}	0.0028	0.0021
p	(0.1581)	(0.3627)

This table presents results from the LOP and no-arbitrage performance measures in unconditional (panel A) and conditional (panel B) settings for the six-month/six-month *industry* momentum strategy. α represents average monthly excess performance over the portfolio of basis assets for the strategy. The numbers in parentheses represent the p-values from a chi-squared test of the hypothesis $H_0 : \alpha = 0$. The basis assets in this sample consist of 20 industry-sorted portfolios (for unconditional measures), augmented by managed portfolios (for conditional measures), which consist of the product of the equity portfolios with the instrumental variables $z_t = \{tb, ts, dy\}$, for a total of 80 basis assets.

suggests that it is the *dynamic* nature of the "industry momentum" strategy, rather than the "industry" component, that is important.

The dynamic nature of the Moskowitz and Grinblatt (1999) strategy compared to ours can be seen in the contrast between the weights on the industry portfolios in our application and theirs. In our unconditional setting, the pricing kernel is represented by a fixed-weight combination of the industry portfolios; the weights exhibit no variation through time. In contrast, Figure 2 depicts the time-series weight on each industry in the industry momentum strategy. As is shown in the figure, one substantial, and apparently important, difference in the industry momentum strategy and our approach is that in the industry momentum strategy the weights vary considerably from month to month. In fact, this variation is sufficiently high that its returns cannot be mimicked by a fixed-weight portfolio. Consequently the (dynamic) industry momentum strategy earns abnormal profits relative to the fixed-weight industry benchmark.[12]

In conjunction with the results of Moskowitz and Grinblatt (1999), our results suggest the possibility that two components of industry returns are important for explaining the profits to momentum strategies. First, as we show, since a fixed-weight portfolio of industries explains roughly half of the momentum profits, static risk accounts for about half of the general momentum strategy performance. Since Moskowitz and Grinblatt show that the industry momentum strategy, with its dynamic reweighting over time, subsumes virtually all of the profits to the general momentum strategy, we conjecture that the remaining component of momentum returns is related to dynamic rebalancing. It is not clear, however, whether this dynamic component arises from dynamic sources of risk or investors' biases.

[12] When industry portfolios are reformed every month, instead of annually, we find that industry momentum profits can be explained by basis assets. Again, this suggests that it is the dynamic nature of the strategy, rather than the industry component, that is important.

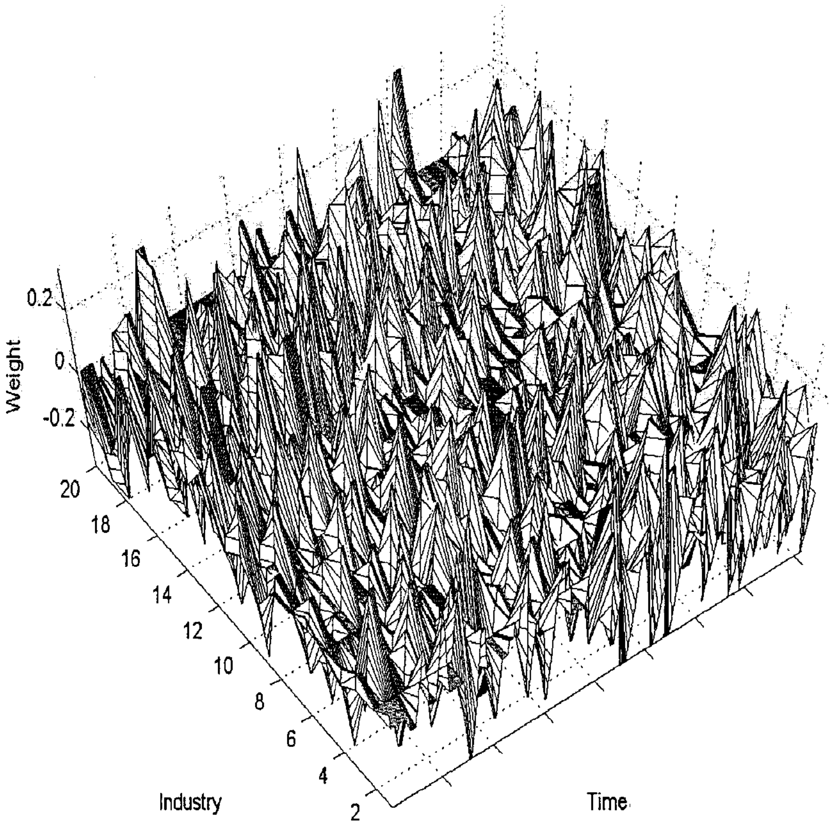

Figure 2
The time series of the weights on each industry portfolio in the industry momentum strategy. A weight of 1.0 indicates that the industry comprises 100% of the winner portfolio at the time of portfolio formation, whereas a weight of −1.0 indicates that the industry comprises 100% of the loser portfolio at the time of portfolio formation. Portfolios are formed as in Moskowitz and Grinblatt (1999).

4.2 Robustness checks

One issue that arises when utilizing the risk-free rate as a basis asset is its near unit-root behavior. We examine the robustness of the performance of our test assets as a basis by conducting alternate tests of the ability of the excess industry returns to span the strategy returns. Specifically we utilize the methodology of Huberman and Kandel (1987) as explained in Kan and Zhou (2001). That is, we perform a Wald test of the null hypothesis that the basis assets do not span the strategies. The test fails to reject the null hypothesis (p-value 0.999), suggesting that the industry portfolios serve well as a basis for the momentum strategy portfolios.[13] Thus the result suggests

[13] This result is untabulated, but is available from the authors upon request.

Table 9
Performance assessment: winners and losers removed from basis

	Unconditional		Conditional	
	LOP	No arbitrage	LOP	No arbitrage
p	(0.453)	(0.440)	(0.530)	(0.371)

This table presents results of joint tests for the significance of 16 momentum strategies using both the LOP and no-arbitrage measures in unconditional and conditional settings when winners and losers are removed from basis assets. The p-values shown are the result of testing the hypothesis $H_0 : \alpha = 0$. The basis assets in this sample consist of 20 industry-sorted portfolios (for unconditional measures), augmented by managed portfolios (for conditional measures), which consist of the product of the equity portfolios with the instrumental variables $z_t = \{tb, ts, dy\}$, for a total of 80 basis assets.

that our conclusions are not driven by the statistical properties of the risk-free asset.

In a further robustness check, we investigate whether the inclusion of winner/loser securities in the industry portfolios contribute to our finding that momentum portfolio profits are explained by a set of industry portfolios. Specifically, we identify individual winner/loser securities and remove them from their respective industry portfolios. We repeat the analysis of momentum strategy abnormal performance; the results of joint tests are shown in Table 9. Clearly, removing individual securities with extreme price movements from the basis does not change our results.

Finally, we investigate the power and size characteristics of our tests of abnormal performance. Our results (available on request) suggest that for levels of abnormal performance similar to those observed in the literature, the power characteristics of these tests are reasonable for both the LOP and no-arbitrage measures. For example, for abnormal performance levels of 100 basis points per month and significance levels of 5%, the rejection rates never fall below 80% (75%) for the unconditional (conditional) tests.

5. Conclusions and Extensions

This article investigates the profitability of momentum trading strategies by using a nonparametric test that asks whether these profits can be explained by an equilibrium pricing model that satisfies some minimal restrictions. We investigate two benchmark cases for such a pricing model: the first requires only that the LOP holds and the second that a no-arbitrage condition holds. We find that a stochastic discount factor can be constructed from a basis set of industry-sorted portfolios, which explains approximately half the level of momentum profits in an unconditional setting. That is, a risk measure which assumes only that the LOP holds can account for an average of 49% of the profitability of the trading strategies when agents cannot use conditioning information in forming their expectations. A joint test on the significance of the remaining profits fails to reject the null hypothesis of no abnormal performance. If we require that the stochastic discount factor be positive, the results are similar; individual momentum strategies retain residual profitability, with the average profits lower, and a joint test of significance fails to reject. In further tests, when agents are allowed to implement dynamic trading strategies based only on a limited set of public information, we find that the abnormal performance of the strategies we consider decline further, to slightly more than one-third (one-fourth) of the level of raw profits using the LOP (no-arbitrage) estimation technique.

These results suggest the possibility that at least a portion of the returns to momentum strategies is due to the risk of the strategies rather than investor underreaction. Based on a simple extension of the equilibrium model of security returns we employ, we argue that the risk measure of momentum strategies should, on average, be positive. We develop a diagnostic measure of the risk adjustment used in momentum strategies and show that the stochastic discount factor we estimate appears to be a better fit than the pricing models previously used to adjust for the risk of such strategies.

We also examine the ability of this nonparametric benchmark to explain the profits of an industry momentum strategy, such as Moskowitz and Grinblatt (1999) devise. Of interest is that the profit levels of the industry momentum strategy are virtually unaffected after risk adjustment. Since the nonparametric benchmark essentially consists of adjusting for the risk and return levels of a fixed-weight industry portfolio, this result suggests that it is the dynamic nature of the industry momentum strategy, rather than the cross-sectional variation in industry returns, that is responsible for its profitability.

It is important to note that, although we avoid assuming that a particular parametric pricing model holds, we are requiring that securities markets be in equilibrium, that is, by assuming that the LOP (or no arbitrage) holds, we are ruling out mispricing in the basis assets. If this assumption does not hold, then measuring profits in relation to these basis assets results in a misspecified measure of abnormal performance.[14] However, this criticism would apply to any risk-adjusted measure of performance, since such a measure would have to assume an (equilibrium) pricing model. Subject to this criticism, our results provide some evidence that a portion of the profits to momentum strategies are the result of rational, rather than irrational, pricing behavior. The nonparametric benchmark leaves a nontrivial fraction of momentum profits unexplained, however; consequently we cannot rule out the existence of mispricing in momentum portfolios relative to our set of basis assets.

References

Ahn, D., and A. Shivdasani, 1999, "Long-Term Returns of Seasoned Equity Issues: Bad Performance or Bad Model?" unpublished manuscript, University of North Carolina–Chapel Hill.

Ahn, D., J. Conrad, and R. Dittmar, 2000, "Risk Adjustment and Trading Strategies," working paper, University of North Carolina.

[14] In this case it may still be useful to know that the abnormal return of the trading strategy reflects mispricing of the (passively managed) basis assets rather than the dynamic nature of the strategy itself.

Ball, R., S. Kothari, and J. Shanken, 1995, "Problems in Measuring Portfolio Performance: An Application to Contrarian Investment Strategies," *Journal of Financial Economics*, 38, 79–107.

Barberis, N., A. Shleifer, and R. Vishny, 1998, "A Model of Investor Sentiment," *Journal of Financial Economics*, 49, 307–343.

Bekaert, G., and M. S. Urias, 1996, "Diversification, Integration and Emerging Market Closed-End Funds," *Journal of Finance*, 51, 835–869.

Breeden, D., 1979, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 7, 265–296.

Campbell, J., 1987, "Stock Returns and the Term Structure," *Journal of Financial Economics*, 14, 359–375.

Carhart, M., 1997, "On Persistence in Mutual Fund Performance," *Journal of Finance*, 52, 57–82.

Chamberlain, G., and M. Rothschild, 1983, "Arbitrage, Factor Structure, and Mean-Variance Analysis on Large Asset Markets," *Econometrica*, 51, 1281–1304.

Chen, Z., and P. Knez, 1996, "Portfolio Performance Measurement: Theory and Applications," *Review of Financial Studies*, 9, 511–555.

Chordia, T., and Shivakumar, 2000, "Momentum, Business Cycle and Time-Varying Expected Returns," forthcoming in *Journal of Finance*.

Cochrane, J., 1997, "Asset Pricing," unpublished manuscript, University of Chicago.

Cohen, R. B., and C. K. Polk, 1998, "The Impact of Industry Factors," unpublished manuscript, Northwestern University.

Dahlquist, M., and P. Soderlind, 1999, "Evaluating Portfolio Performance with Stochastic Discount Factors," *Journal of Business*, 72, 347–383.

Daniel, K., D. Hirshleifer, and A. Subrahmanyam, 1998, "Investor Overconfidence, Covariance Risk, and Predictors of Securities Returns," *Journal of Finance*, 53, 1839–1886.

DeSantis, G., 1995, "Volatility Bounds for Stochastic Discount Factors: Tests and Implications from International Financial Markets," unpublished manuscript, University of Southern California.

Dittmar, R., 2002, "Nonlinear Pricing Kernels, Kurtosis Preference, and the Cross-Section of Equity Returns," forthcoming in *Journal of Finance*.

Fama, E., 1991, "Efficient Capital Markets: II," *Journal of Finance*, 46, 1575–1617.

Fama, E., 1998, "Market Efficiency, Long-Term Returns, and Behavioral Finance," *Journal of Financial Economics*, 33, 283–306.

Fama, E., and K. French, 1988, "Dividend Yields and Expected Stock Prices," *Journal of Political Economy*, 96, 246–273.

Fama, E., and K. French, 1992, "The Cross-Section of Expected Stock Returns," *Journal of Finance*, 47, 427–465.

Fama, E., and K. French, 1996, "Multifactor Explanations of Asset Pricing Anomalies," *Journal of Finance*, 51, 55–84.

Fama, E., and G. W. Schwert, 1977, "Asset Returns and Inflation," *Journal of Financial Economics*, 5, 115–146.

Ferson, W., 1989, "Changes in Expected Security Returns, Risk and the Level of Interest Rates," *Journal of Finance*, 44, 1191–1217.

Ferson, W., and C. Harvey, 1991, "The Variation of Economic Risk Premiums," *Journal of Political Economy*, 99, 385–415.

Gibbons, M., and W. Ferson, 1985, "Testing Asset Pricing Models with Changing Expectations and an Unobservable Market Portfolio," *Journal of Financial Economics*, 14, 217–236.

Grundy, B. D., and J. S. Martin, 2001, "Understanding the Nature of the Risks and the Source of the Rewards to Momentum Investing," *Review of Financial Studies*, 14, 29–78.

Hansen, L. P., 1982, "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50, 1029–1079.

Hansen, L. P., and R. Jagannathan, 1991, "Implications of Security Market Data for Models of Dynamic Economies," *Journal of Political Economy*, 99, 225–262.

Hansen, L. P., and R. Jagannathan, 1997, "Assessing Specification Errors in Stochastic Discount Factor Models," *Journal of Finance*, 52, 557–590.

Hansen, L. P., and S. Richard, 1987, "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models," *Econometrica*, 55, 587–613.

Hansen, L. P., and K. Singleton, 1982, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations," *Econometrica*, 50, 1269–1286.

Harrison, J., and D. Kreps, 1979, "Martingales and Arbitrage in Multiperiod Securities Markets," *Journal of Economic Theory*, 20, 381–408.

Harvey, C., 1989, "Time-Varying Conditional Covariances in Tests of Asset Pricing Models," *Journal of Financial Economics*, 24, 289–317.

Huberman, G., and S. Kandel, 1987, "Mean-Variance Spanning," *Journal of Finance*, 42, 873–888.

Jegadeesh, N., and S. Titman, 1993, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance*, 48, 65–91.

Jensen, M., 1968, "The Performance of Mutual Funds in the Period 1945–1964," *Journal of Finance*, 23, 389–416.

Kan, R., and G. Zhou, 2001, "Tests of Mean-Variance Spanning," working paper, Washington University.

King, B. F., 1966, "Market and Industry Factors in Stock Price Behavior," *Journal of Business*, 39, 139–190.

Lewellen, J., 2001, "Momentum and Autocorrelation in Stock Returns," forthcoming in *Review of Financial Studies*.

Lo, A. W., and A. C. MacKinlay, 1990, "When Are Contrarian Profits Due to Stock Market Overreaction?" *Review of Financial Studies*, 3, 175–205.

MacKinlay, A. C., 1987, "On Multivariate Tests of the CAPM," *Journal of Financial Economics*, 18, 341–372.

MacKinlay, A. C., 1995, "Multifactor Models do not Explain Deviations from the CAPM," *Journal of Financial Economics*, 38, 3–28.

Merton, R., 1973, "An Intertemporal Capital Asset Pricing Model," *Econometrica*, 41, 867–888.

Moskowitz, T. J., and M. Grinblatt, 1999, "Do Industries Explain Momentum?" *Journal of Finance*, 54, 1249–1290.

Roll, R., 1978, "Ambiguity When Performance Is Measured by the Security Line," *Journal of Finance*, 33, 1051–1069.

Shanken, J., 1990, "Intertemporal Asset Pricing: An Empirical Investigation," *Journal of Econometrics*, 45, 99–120.