Does the Simple Investment-based Model Explain Equity Returns? Evidence from Euler Equations*

Stefanos Delikouras† **Robert F. Dittmar**‡

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Abstract

We investigate the empirical implications of the investment-based model for stochastic discount factors that price equity returns, investment returns, and both equity and investment returns simultaneously. Our methodology is based on the equivalence between investment and equity returns implied by the investment-based model, and the existence of pricing kernels in the linear span of returns. We find that the pricing kernels satisfying the Euler equation for equity returns cannot satisfy the Euler equation for investment returns, and vice versa. Our results suggest that joint restrictions on the optimality of investment and consumption pose stringent conditions for candidate stochastic discount factors.

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†Department of Finance, Miami Business School, University of Miami, Coral Gables, FL 33124, email: sdelikouras@bus.miami.edu

‡Department of Finance, Stephen Ross School of Business, University of Michigan, Ann Arbor, MI 48109, email: rdittmar@umich.edu

1 Introduction

The structural investment-based model of asset pricing introduced in Cochrane (1991) has been instrumental in shaping our understanding of the relation between firms' investment decisions and the returns on their equity. Zhang (2005) shows that firms with high book-to-market ratios should earn higher expected returns because reducing capital stock in economic downturns is costly. More recently, Hou, Xue, and Zhang (2015) propose a four-factor return model inspired by the investment-based model. Their evidence suggests that the model fares quite well in explaining expected returns for a large set of portfolios.

While the structural investment-based framework has offered considerable insight into why firm characteristics might be related to equity returns, empirical tests of the mechanisms and cross-sectional implications behind the model have been more limited. Cochrane (1991) emphasizes that when firms with linear homogeneous production technologies invest optimally relative to the stochastic discount factor that satisfies the Euler equation for equity returns, investment returns are equal to equity returns in every state of nature. A testable hypothesis of this condition is that regressing investment returns and equity returns on the same variables should produce the same coefficients. In an expanded framework, Liu, Whited, and Zhang (2009) show that in a linear homogeneous investment model, the return on levered investment should be equal to the return on equity, and suggest that a testable implication of this restriction is the equality of the means of investment and equity returns. The authors find statistical, but more limited economic support of this condition.

Even though the above papers have examined several implications of the investment-based model, they tend to ignore its central empirical restriction: the Euler equations for investment and equity returns should hold simultaneously. Our goal in this paper is to help fill this gap by directly testing the Euler equation implied by a specific investment-based model. In particular, we examine the version of the model suggested in Cochrane (1991), Liu, Whited, and Zhang (2009), and Hou, Xue, and Zhang (2015) in which managers determine investment policy for physical capital when investment and production technologies are linear homogeneous. In this model, the optimal investment policy maximizes the present value of dividends, given the stochastic discount factor that satisfies equity holders' Euler equation.

The main premise of our empirical tests lies in the core prediction of the investment-based models of Cochrane (1991) and Liu, Whited, and Zhang (2009) that investment and equity returns are equal in every state of nature. An immediate consequence of this equivalence is that projecting the stochastic discount factor onto the span of equity or investment returns should produce the same coefficients. Put differently, a projected, and therefore minimum variance, stochastic discount factor (e.g., Hansen and Jagannathan (1991)) that is a linear combination of equity returns should satisfy the Euler condition for investment returns. As noted in Cochrane (1991), satisfaction of this condition suggests that firm investment is optimal given equilibrium in the financial markets. Our empirical analysis represents a direct test of this hypothesis. Specifically, using the generalized method of moments (GMM) of Hansen and Singleton (1982), we examine whether a minimum variance stochastic discount factor based on equity returns can price investment returns, and vice versa.

We run these tests in the quarterly cross-section of nine portfolios of firms double-sorted on investment and profitability. We focus on these portfolios for two reasons. First, they are the basis of the investment and profitability factors of Hou, Xue, and Zhang (2015). Second, these authors draw on the investment-based model to motivate the use of these factors in asset pricing tests. Thus, following their reasoning, the investment model should be particularly successful in matching investment returns to equity returns for the investment and profitability portfolios.

Our tests are of particular economic interest because the investment model does not inform us as to the object that drives pricing, the stochastic discount factor. Rather, it states that, conditional on a stochastic discount factor, investment should be optimal, and that investment will produce a return that is a function of firm characteristics. Firms with higher ratios of output to capital, for example, have higher returns not just because their profitability happens to be higher, but rather because these firms' output to capital covaries more negatively with the stochastic discount factor than firms with low ratios of output to capital. Our empirical investigations are designed to explicitly examine whether this is true, and therefore, whether the factors that are designed to maximize variation along these characteristics are consistent with the implications of investment-based asset pricing.

Our baseline empirical results show that, within the workhorse linear homogeneous model of Liu, Whited, and Zhang (2009), it is virtually impossible for a minimum variance stochastic discount factor to simultaneously satisfy Euler equations for both equity and investment returns for the investment and profitability portfolios. Further, in most of our tests, investment returns tend to covary positively with the stochastic discount factor in the linear span of equity returns, suggesting that investment pays off in bad states of the world, that is when the stochastic discount factor is high. As a result, it is difficult to generate a risk premium on investment similar to the risk premium on equity, which tends to pay off in good states of the world, that is when the stochastic discount factor is low.

In addition to minimum variance stochastic discount factors, we test the equivalence between investment and equity returns using the most recent factor-based pricing kernels. Specifically, we consider the four-factor model of Hou, Xue, and Zhang (2015) that is inspired by the investment model, and the Fama and French (2015) specification, which is motivated by clean surplus accounting. According to our results, the four-factor model of Hou, Xue, and Zhang (2015) performs much better than the Fama and French (2015) discount factor in fitting the cross-section of equity returns for the investment and profitability portfolios.

However, our results also suggest that simply because a model fares well in pricing equity returns does not mean it will also satisfy the Euler equation for investment, the key implication of the investment-based model. Both the Hou, Xue, and Zhang (2015) and the Fama and French (2015) models are rejected by the χ^2-test of over-identifying restrictions in the joint cross-section of equity and investment returns. Further, both models tend to generate a positive covariation with investment returns. This finding implies that investment pays off in bad states of nature. For instance, the four-factor model in Hou, Xue, and Zhang (2015) generates positive return covariances for six of the nine investment returns that we examine, and the five-factor of Fama and French (2015) generates positive return covariances for five of the nine investment returns.

In general, our tests strongly reject the equivalence of stochastic discount factors that price investment and equity returns, and suggest that the restriction of equality of investment and equity returns, as implied by the structural investment model, does not hold. This rejection may indicate failure of the economic mechanism of the investment-based model, rejection of the assumption that the capital share and adjustment costs parameters are the same across the nine investment and profitability portfolios, or mis-specification of the production function. We investigate these scenarios in further tests.

Specifically, our tests continue to reject the equivalence of investment and equity returns when we consider alternative assumptions for the investment-based model. These assumptions are designed to improve the ability of the model to fit the cross-section of mean equity returns for the investment and profitability portfolios. In one of these tests, we allow for different production parameters across test assets. In this case, the increased flexibility of the investment model significantly improves its performance in explaining the cross-section of mean equity returns. Despite the improvement in fitting first moments, our specification tests continue to reject the hypothesis that the generated investment returns can span equity returns. To verify the robustness of our results, we also consider annual returns, alternative timing assumptions for the release of financial information and the expectations of new investment projects, and an investment-based model that accounts for leverage. Overall, our findings suggest that the joint restrictions on the optimality of investment and consumption pose stringent conditions for candidate stochastic discount factors.

Several earlier papers empirically investigate the implications of investment-Euler equations for cross-sectional variation in returns. Our approach is closely related to Cochrane (1996) and Gomes, Yaron, and Zhang (2006), who investigate investment-Euler equations' implications for expected equity returns. Cochrane (1996) uses investment returns as factors, and investigates the ability of a stochastic discount factor that is a linear function of investment returns to price a set of size-sorted portfolios. He finds that the model performs about as well as the capital asset pricing model (CAPM) or the Chen, Roll, and Ross (1986) factor model in explaining cross-sectional variation in returns on these portfolios.

Gomes, Yaron, and Zhang (2006) pursue a similar exercise in investigating the role financial frictions play in explaining cross-sectional variation in returns. Our approach differs significantly from theirs in that we construct our investment returns from firm characteristics, following Liu, Whited, and Zhang (2009), rather than aggregate macroeconomic data. Our focus is also explicitly on the role that optimal investment plays in understanding the investment and profitability premia across firms.

Our results also add to the recent findings in Golubov and Konstantinidi (2018) that neither the investment-based model of Kogan and Papanikolaou (2014) with investment specific technology shocks nor the operating leverage framework of Zhang (2005) are likely to explain the value premium. The authors reach this conclusion by decomposing the market-to-book ratio, the key characteristic behind the value premium, into a market-to-value term, which carries large and significant risk premia, and a value-to-book term that does not yield any cross-sectional variation in expected returns. Even though our empirical approach and test assets are different to those used in Golubov and Konstantinidi (2018), both sets of results cast doubt on the ability of simple investment models to explain empirical regularities in the cross-section of returns.

Our empirical work focuses on the linear homogeneous investment-based model with physical capital because it has been the workhorse in the recent asset pricing literature and the inspiration for a number of investment-based explanations for cross-sectional variation in equity returns. Liu, Whited, and Zhang (2009) show that the condition implied by the model that expected equity returns equal expected investment returns holds reasonably well in the data. The same condition is examined in Belo, Xue, and Zhang (2013), together with the implication that average Tobin's q is equal to the average Tobin's q implied by the model.

Lin and Zhang (2013) argue forcefully for interpreting these results as indicating that the physical investment-based pricing model is a new paradigm for asset pricing. As a result, Hou, Xue, and Zhang (2015) propose a four-factor model, dubbed the q-factor model, that the authors suggest is a theoretically motivated alternative to the Fama and French (2015) five-factor model. Our results show that these conclusions are perhaps premature; the models fare poorly when confronted with Euler equations in the joint cross-section of equity and investment returns for the investment and profitability portfolios, even though the former model performs better in the set of equity returns.

It is critically important to note that we are not saying that an investment-based model *cannot* be consistent with equity Euler equations, and thus expected equity returns. Rather, we are simply saying that a simple, physical capital-based investment model is not. Instead, additional features may need to be incorporated into the model in order to reconcile its implications with the data. For example, Kuehn, Simutin, and Wang (2015) propose a model in which labor is the relevant input to production and search frictions generate cross-sectional asset pricing implications. Belo, Gala, Salamao, and Vitorino (2018) decompose firm value into components related to physical, labor, brand, and knowledge capital, and find that physical capital accounts for at most about half of firm value.

Rather than suggesting that the investment-based model is invalid, our results should be understood to suggest two conclusions. The first is that a simple linear homogeneous investment-based model with investment returns that are a function of investment intensity and operating efficiency is unlikely to satisfy Euler equations, and thus provide a complete explanation for the determinants of cross-sectional variation in equity returns. Therefore, it is unclear to what extent that factors motivated by models such as Hou, Xue, and Zhang (2015) and Hou, Mo, Xue, and Zhang (2018) command risk premia because of firms' optimal investment decisions. The second conclusion is that in testing investment-based asset pricing models, one should explicitly consider the Euler equation implied by optimal investment. Failing to do so may result in tests that are insufficiently powerful to reject these models.

2 Investment-Based Pricing and the Cross-Section of Returns

2.1 Firm Investment Decisions and Expected Equity Returns

Cochrane (1991) provides one of the first explicit links between firms' investment policies and equity returns. He notes that, following Hansen and Richard (1987) and Ingersoll (1988), prices of contingent claims on real assets will satisfy an Euler equation,

$$E_t\left[M_{t+1}R^S_{t+1}\right] = 1. \tag{1}$$

R^S_{t+1} represents the gross return to a contingent claim, which we superscript S for stock, as our main interest in this paper is the firm's equity return. The variable M_{t+1} is a stochastic discount factor, which in this framework captures the information in the returns to a set of Arrow-Debreu securities that return one dollar in a particular state of the economy and zero in all others. Equation (1) is common to equilibrium models of asset pricing in which a consumer optimizes her consumption and portfolio choice given a distribution of payoffs to the contingent claims on the real assets in the economy.

Producers in the economy will have access to the same set of Arrow-Debreu securities, but a different objective function. The producer's objective is to choose an investment policy to maximize the value of its firm's contingent claim price. Cochrane (1991) shows that this optimal investment decision will satisfy another Euler equation,

$$E_t\left[M_{t+1}R^I_{t+1}\right] = 1, \tag{2}$$

where R^I_{t+1} is the gross return to the firm's investment. A large body of literature has investigated the implications of equation (2) for asset pricing under different assumptions about the stochastic discount factor, the production function that determines the return on investment, and the constraints that affect the return on the firm's investment.

There are at least three ways to interpret the Euler equation as discussed by Cochrane (1991). First, equation (1) represents a hyperplane in which all contingent claim prices must lie to prevent arbitrage. Since the producer has access to the contingent claims market, he will adjust investment until there is no arbitrage between the investment return and portfolios of the contingent claims. Second, if markets are complete, there will always be a unique contingent claim return that is proportional to the investment return. Again, to prevent arbitrage, the producer will adjust investment to make these returns equal. Finally, Cochrane (1991) notes that equation (2) implies that firms will adjust investment until the M_{t+1} that satisfies the Euler equation for contingent claims also satisfies the Euler equation for investment returns.

Conditions (1) and (2) do not automatically establish equivalence between investment and equity returns. However, as shown in Cochrane (1991) and Liu, Whited, and Zhang (2009), if the firms' production and investment technologies are linear homogeneous functions of their arguments, then the principal empirical implication of equations (1) and (2) is that the return on investment and the return on equity are identical,

$$R^{I}_{t+1} = R^{S}_{t+1}, \quad \forall t. \tag{3}$$

The implicit assumption underlying this statement is that equity represents the entire claim to the firm's assets. In the presence of debt and taxes, Liu, Whited, and Zhang (2009) note that

$$R^{IL}_{t+1} = R^{S}_{t+1}, \quad \forall t, \tag{4}$$

where R^{IL}_{t+1} is the return on a levered claim to investment. In this context, the return on investment will be identical to the weighted average cost of capital. Both sets of authors note that it is not realistic to take the equality condition literally due to specification and measurement errors. Therefore, the authors instead exploit expectational conditions of the restrictions.

Liu, Whited, and Zhang (2009) exploit condition (4), and note that it implies

$$\begin{aligned} E\left[R^{IL}_{t+1}\right] &= E\left[R^{S}_{t+1}\right] \\ E\left[\left(R^{IL}_{t+1}\right)^2\right] &= E\left[\left(R^{S}_{t+1}\right)^2\right], \end{aligned}$$

that is, the expected levered investment return should equal the expected equity return and the variances of each return should be the same. The authors find that, in their sample, neither condition can be statistically rejected in the data. Further, when confronted only with the restrictions on means, absolute pricing errors on portfolios are reasonably small. However, when both the mean and volatility restrictions are imposed, pricing errors are larger. The authors conclude that their results suggests that firms align their investment policies with costs of capital and that this alignment can explain many of the perceived anomalies detected in empirical studies of equity returns.

An alternative implication, exploited in Cochrane (1991), is that condition (3) implies that regressing stock and investment returns on any variables should produce the same coefficients. That is, in regressions of (de-meaned) variables X_{t+j},

$$\begin{aligned} R^I_{t+1} &= b^I X_{t+j} + e^I_{t+1} \\ R^S_{t+1} &= b^S X_{t+j} + e^S_{t+1}, \end{aligned}$$

we obtain

$$\widehat{b}^I = \frac{Cov\left(X_{t+j}, R^I_{t+1}\right)}{Var\left(X_{t+j}\right)} = \frac{Cov\left(X_{t+j}, R^S_{t+1}\right)}{Var\left(X_{t+j}\right)} = \widehat{b}^S, \quad \text{for any } j,$$

assuming that $Cov\left(X_{t+j}, R^I_{t+1}\right) = Cov\left(X_{t+j}, R^S_{t+1}\right)$ as implied by condition (3).

Cochrane (1991) also investigates whether regressing a candidate variable on investment and equity returns produces the same coefficients. That is,

$$\begin{aligned} X_{t+j} &= d^I R^I_{t+1} + u^I_{t+1} \\ X_{t+j} &= d^S R^S_{t+1} + u^S_{t+1}, \end{aligned}$$

yield point estimates

$$\widehat{d}^I = \frac{Cov\left(X_{t+j}, R^I_{t+1}\right)}{Var\left(R^I_{t+1}\right)} = \frac{Cov\left(X_{t+j}, R^S_{t+1}\right)}{Var\left(R^S_{t+1}\right)} = \widehat{d}^S,$$

assuming that $Cov\left(X_{t+j}, R^I_{t+1}\right) = Cov\left(X_{t+j}, R^S_{t+1}\right)$ and $Var\left(R^S_{t+1}\right) = Var\left(R^I_{t+1}\right)$ from equation (3). The above relations imply that in using returns on investment and equity to forecast various aggregate variables, the forecasts from both sets of returns should be the same. Indeed, the empirical results in Cochrane (1991) suggest that one cannot reject equality in coefficients of forecasting regressions where aggregate on equity and investment are used to forecast future GNP growth. However, Cochrane (1991) focuses on aggregate returns and does not offer any insights on the cross-sectional relation between returns on equity and returns on investment.

Our empirical work exploits a version of this last condition, where we utilize a vector of equity and investment returns ($\mathbf{R}^{S,I}$). According to equation (3), an implication of the investment model is that for some random variable X_{t+1}

$$X_{t+1}\mathbf{R}^{I}_{t+1} = X_{t+1}\mathbf{R}^{S}_{t+1},\ \forall t.$$

It is unrealistic to ask that either this condition, or the condition stated in equation (3), hold exactly state-by-state in the data. However, as in Liu, Whited, and Zhang (2009), it is more reasonable to ask whether the condition holds in expectation and

$$E\left[X_{t+1}\mathbf{R}^{I}_{t+1}\right] = E\left[X_{t+1}\mathbf{R}^{S}_{t+1}\right]. \tag{5}$$

This condition is similar to the regressions explored in Cochrane (1991), as it implies

$$E\left[X_{t+1}\right]E\left[\mathbf{R}^{I}_{t+1}\right] + Cov\left(X_{t+1}, \mathbf{R}^{I}_{t+1}\right) = E\left[X_{t+1}\right]E\left[\mathbf{R}^{S}_{t+1}\right] + Cov\left(X_{t+1}, \mathbf{R}^{S}_{t+1}\right).$$

From Cochrane (1991) and Liu, Whited, and Zhang (2009), we know that the expectation of the return on investment should be equal to the expectation of the return on equity ($E[\mathbf{R}^{I}_{t+1}] = E[\mathbf{R}^{S}_{t+1}]$), and that the covariances of each return with a given variable X_{t+1} should be the same ($Cov\left(X_{t+1}, \mathbf{R}^{I}_{t+1}\right) = Cov\left(X_{t+1}, \mathbf{R}^{S}_{t+1}\right)$). In addition to being a testable restriction implied by the investment-based model, the restriction has interesting economic implications that we discuss below.

2.2 Implications of the Investment Model for the Stochastic Discount Factor

One possible choice of a random variable to examine in equation (5) is the stochastic discount factor, i.e., $X_{t+1} = M_{t+1}$. This choice is of particular economic interest, as it yields equality of the Euler equations for equity returns and investment returns, which is implied by equilibrium in the investment-based model.

Unfortunately, there is not universal agreement as to the identification of the stochastic discount factor. However, Hansen and Jagannathan (1991) suggest an approach for identifying a set of stochastic discount factors that satisfy Euler equations and are consistent with the framework explored in Cochrane (1991). The authors suggest projecting the stochastic discount factor onto the linear span of returns,

$$M_{t+1} = c + \mathbf{d}' \left(\mathbf{R}_{t+1} - \overline{\mathbf{R}}\right) + \xi_{t+1}, \tag{6}$$

where c is a pre-specified mean of the stochastic discount factor, $\overline{\mathbf{R}}$ is the vector of average returns, and ξ_{t+1} is an orthogonal shock. The authors note that for a choice of parameters

$$\widehat{\mathbf{d}} = E\left[\left(\mathbf{R}_{t+1} - \overline{\mathbf{R}}\right)\left(\mathbf{R}_{t+1} - \overline{\mathbf{R}}\right)'\right]^{-1} E\left[\left(\mathbf{R}_{t+1} - \overline{\mathbf{R}}\right)(M_{t+1} - c)\right].$$

Further, when sample moments converge to population moments, the Euler equations for asset returns imply that the parameters of the stochastic discount factor in the linear span of returns are given by

$$\widehat{\mathbf{d}} = \widehat{\mathbf{\Sigma}}^{-1}\left(\mathbf{1} - c\overline{\mathbf{R}}\right), \tag{7}$$

where $\widehat{\mathbf{\Sigma}}$ is the sample covariance matrix of returns. Since the projection is orthogonal to the error, the resulting stochastic discount factor is minimum variance given its mean.

As stated in the previous section, an implication of the investment-based model is that if one regresses a random variable on the returns on investment, one should obtain the same regression coefficients that one gets regressing that random variable on the returns on equity. Thus, if we regress the stochastic discount factor on both sets of returns,

$$\begin{aligned} M_{t+1}^{I} &= c + \left(\mathbf{d}^{I}\right)'\left(\mathbf{R}_{t+1}^{I} - \overline{\mathbf{R}}^{I}\right) + \xi_{t+1}^{I} \\ M_{t+1}^{S} &= c + \left(\mathbf{d}^{S}\right)'\left(\mathbf{R}_{t+1}^{S} - \overline{\mathbf{R}}^{S}\right) + \xi_{t+1}^{S}, \end{aligned}$$

the resulting coefficients, $\mathbf{d}^I$ and $\mathbf{d}^S$ should be equal. A different way of stating this is that if one takes the coefficients on the equity returns and derives the stochastic discount factor for stock returns, then the Euler equation for investment returns should hold

$$E\left[\left(c + \left(\mathbf{d}^{S}\right)'\left(\mathbf{R}_{t+1}^{S} - \overline{\mathbf{R}}^{S}\right)\right)\mathbf{R}_{t+1}^{I}\right] = \mathbf{1}. \tag{8}$$

Similarly, the Euler equation for equity returns should hold using the stochastic discount factor in the linear span of investment returns

$$E\left[\left(c + \left(\mathbf{d}^{I}\right)' \left(\mathbf{R}_{t+1}^{I} - \overline{\mathbf{R}}^{I}\right)\right) \mathbf{R}_{t+1}^{S}\right] = \mathbf{1}. \tag{9}$$

An appealing economic feature of this restriction is that it is fully in accordance of the interpretation in Cochrane (1991) of the Euler equation for investment. That is, firms will adjust investment until the stochastic discount factor that satisfies the Euler equation for contingent claims also satisfies the Euler equation for investment. Thus, equation (8) represents a null hypothesis for the investment-based pricing model. Next, we discuss a framework for testing this null hypothesis and its complement in equation (9).

3 Empirical Methodology

3.1 Stochastic Discount Factors for Investment and Equity Returns

Our empirical approach is based in extensions of the insights of Hansen and Jagannathan (1991) to generate testable restrictions on stochastic discount factors that price a set of assets. Specifically, we follow Chen and Knez (1996) and Bekaert and Urias (1996) in implementing a testing methodology.

Chen and Knez (1996) investigate the performance of mutual funds by asking whether the Euler equation for the mutual fund return can be satisfied by a stochastic discount factor in the linear span of a set of basis asset returns. The authors note that according to Hansen and Jagannathan (1991), when the set of assets includes a riskless payoff, the minimum variance stochastic discount factor in equation (6) and the solution for its parameters in equation (7) can be simplified. Specifically, there are some coefficients, $\mathbf{d}$, such that for the returns $\mathbf{R}_{t+1}^{J}$ of an M-dimensional set of basis assets, the following holds

$$E\left[M_{t+1}\mathbf{R}_{t+1}^{J}\right] = E\left[\left(\mathbf{d}'\mathbf{R}_{t+1}^{J}\right)\mathbf{R}_{t+1}^{J}\right] = \mathbf{1}_{M}, \quad \text{with solution } \; \widehat{\mathbf{d}} = E\left[\mathbf{R}_{t+1}^{J}\mathbf{R}_{t+1}^{J\prime}\right]^{-1}\mathbf{1}_{M}.$$

The authors suggest two testing approaches in this framework. The first is to use the explicit solution $\widehat{\mathbf{d}}$ above and calculate a vector of pricing errors for the returns $\mathbf{R}_{t+1}^{K}$ of a set of N alternative assets,

$$\mathbf{e}_T = \frac{1}{T}\sum_t \left[\left(\widehat{\mathbf{d}}'\mathbf{R}_{t+1}^{J}\right)\mathbf{R}_{t+1}^{K} - \mathbf{1}_N\right].$$

Then, one can conduct a Wald test of the hypothesis that the errors, $\mathbf{e}_T$, are zero. Alternatively, one can specify moment conditions

$$\mathbf{h}_T\left(\mathbf{d}\right) = \frac{1}{T}\sum_t \left[\begin{array}{c} \left(\mathbf{d}'\mathbf{R}_{t+1}^{J}\right)\mathbf{R}_{t+1}^{J} - \mathbf{1}_M \\ \left(\mathbf{d}'\mathbf{R}_{t+1}^{J}\right)\mathbf{R}_{t+1}^{K} - \mathbf{1}_N \end{array}\right], \tag{10}$$

and estimate parameters $\mathbf{d}$ via GMM. The test of overidentifying restrictions with N degrees of freedom represents a test that the pricing errors are jointly zero. In this paper, we follow the GMM approach.

A limitation of the Chen and Knez (1996) approach, noted in Dahlquist and Söderlind (1999) and Farnsworth, Ferson, Jackson, and Todd (2002), is that in general, this approach does not constrain the mean of the stochastic discount factor. Consequently, one can obtain a stochastic discount factor estimate with a mean greater than one, implying a negative risk-free rate. One solution to this problem is to include the risk-free asset in the set of basis assets, $\mathbf{R}_{t+1}^{J}$. Alternatively, one can pursue the approach in Bekaert and Urias (1996), which uses the more general stochastic discount factor specification from equation (6) that does not require the inclusion of a risk-free payoff in the set of basis assets.

The empirical methodology in Bekaert and Urias (1996) investigates whether the basis assets span the joint space of the basis assets and the returns of interest, $\mathbf{R}_{t+1}^{K}$. This approach also implicitly asks whether any discount factor in the linear span of $\mathbf{R}_{t+1}^{J}$ prices the assets $\mathbf{R}_{t+1}^{K}$. Specifically, the Bekaert and Urias (1996) procedure suggests forming sample moment conditions for the Euler equations for the joint set of returns $\mathbf{R}_{t+1} = \{\mathbf{R}_{t+1}^{J}; \mathbf{R}_{t+1}^{K}\}$,

$$\mathbf{h}_T\left(\mathbf{d}_1, \mathbf{d}_2\right) = \frac{1}{T}\sum_t \left(\begin{array}{c} \left(c_1 + \mathbf{d}_1'\left(\mathbf{R}_{t+1} - \overline{\mathbf{R}}\right)\right)\mathbf{R}_{t+1} - \mathbf{1}_{M+N} \\ \left(c_2 + \mathbf{d}_2'\left(\mathbf{R}_{t+1} - \overline{\mathbf{R}}\right)\right)\mathbf{R}_{t+1} - \mathbf{1}_{M+N} \end{array}\right), \tag{11}$$

where $\mathbf{d}_k = \{\mathbf{d}_k^J; \mathbf{d}_k^K\}$ for $k = 1, 2$. The constants c_1 and c_2 are two arbitrary choices for the mean of the stochastic discount factor. Similar to the insights in Black (1972) that any efficient portfolio can be represented as the combination of two arbitrary efficient portfolios, any minimum variance stochastic discount factor can be represented as the combination of two arbitrary minimum variance stochastic discount factors with means c_1 and c_2. The parameters $\mathbf{d}_1$ and $\mathbf{d}_2$ can be estimated via GMM; without restrictions, the system is exactly identified.

Bekaert and Urias (1996) suggest two test statistics associated with estimation of parameters in equation (11). The first, a Wald statistic, leaves all parameters unrestricted, with parameters estimated using the exactly identified GMM system or equivalently, the explicit solution from equation (7). This statistic tests the hypothesis that the parameters $\{\mathbf{d}_1^K; \mathbf{d}_2^K\}$, which correspond to the vector of returns $\mathbf{R}_{t+1}^K$, are jointly equal to zero. Failure to reject this hypothesis implies that information in the returns $\mathbf{R}_{t+1}^J$ alone are sufficient to satisfy the Euler equations given means c_1 and c_2. Put differently, the Euler equation for returns $\mathbf{R}_{t+1}^K$ is satisfied by stochastic discount factors in the linear span of $\mathbf{R}_{t+1}^J$ for means c_1 and c_2, and therefore for any arbitrary c_k. Replacing J with equity returns and K with investment returns, this test represents a test that the Euler equation for investment returns is satisfied by a stochastic discount factor in the linear span of equity returns, our null hypothesis in equation (8).

The second test statistic explicitly restricts the parameters that correspond to the returns $\mathbf{R}_{t+1}^K$ to be zero, i.e., $\mathbf{d}_1^K = \mathbf{d}_2^K = 0$. Because the test imposes this restriction, the authors refer to it as a likelihood ratio test. In practice, imposing these restrictions results in overidentifying restrictions in the GMM estimation. Consequently, the GMM test of overidentifying restrictions represents a test of whether stochastic discount factors in the linear span of $\mathbf{R}_{t+1}^J$ with means c_1 and c_2 satisfy the Euler equation for asset $\mathbf{R}_{t+1}^K$. Again, replacing J and K with equity and investment returns allows us to test equation (8), our null hypothesis for the investment model.

3.2 Measuring the Return on Investment

Thus far, we have discussed the return on investment as if it were an observable quantity. In reality, this return is the outcome of the firm's choice of investment given an unobserved production technology and law of motion for capital accumulation. As a result, one must posit a production technology and costs of adjusting capital. Hence, any results that we obtain are conditioned on our choice of the parametric form of this technology and costs. A failure of the implications of the investment model may be a result of our misspecification of these functions.

We proceed, however, to use a production technology and adjustment costs that has been used widely in the investment-based asset pricing literature. Specifically, we utilize a simplified version of the model explored in Liu, Whited, and Zhang (2009), where we ignore debt and taxes as in Hou, Xue, and Zhang (2015). In this model, a firm generates consumption goods by deploying capital, K_t, to generate output, Y_t, via a constant return to scale production technology, $\Pi(K_t, Z_t)$, where Z_t is a productivity shock.

Investing in new capital, I_t, results in convex adjustment costs,

$$\Phi(I_t, K_t) = \frac{a}{2}\left(\frac{I_t}{K_t}\right)^2 K_t,$$

where $a > 0$ is the adjustment cost parameter of investment. Capital accumulates according to the law of motion

$$K_{t+1} = K_t(1 - \delta_t) + I_t,$$

where δ_t is the depreciation rate. Finally, firms distribute profits as dividends D_t,

$$D_t = \Pi(K_t, Z_t) - \Phi(I_t, K_t) - I_t.$$

The firm chooses the level of investment in order to maximize the value of cum-dividend equity, defined as the expected discounted value of current and future dividends. Discounting is achieved by a stochastic discount factor, M_{t+1}, determined by consumers' optimal consumption and portfolio decisions.

In this framework, optimal investment satisfies the Euler condition in equation (2). Further, the assumed linear homogeneous production and investment technologies establish equivalence between returns on equity, R^S_{t+1}, and investment returns, R^I_{t+1}, which, in the absence of debt and taxes, are given by the following expression:

$$R_{t+1}^S = R_{t+1}^I = \frac{\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{a}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + (1-\delta_{t+1})\left(1 + a\frac{I_{t+1}}{K_{t+1}}\right)}{1 + a\frac{I_t}{K_t}}, \ \forall t. \tag{12}$$

The parameter $\alpha \in (0, 1)$ above represents the share of capital in the firm's production function. Given parameter estimates for α and a, and measurement of output, capital, and investment, one can use equation (12) to compute investment returns.

Liu, Whited, and Zhang (2009) explicitly model the impact of debt financing and taxes on firms' investment decisions. Incorporating these considerations results in an altered return on investment, which takes into account the benefits of tax shields from debt and depreciation. In principle, this framework is more realistic, as most publicly traded firms have some degree of long term debt. In practice, obtaining estimates of the after-tax bond return for firms is quite difficult, as noted in Liu, Whited, and Zhang (2009). Therefore, for our baseline tests, we opt to consider the unlevered investment return of equation (12) for this reason. This decision can also be motivated by the fact that the stochastic discount factor that satisfies the Euler equation for equity returns should also satisfy the Euler equation for both investment and bond returns. Finally, the "no-debt" assumption is also consistent with the motivating framework of Hou, Xue, and Zhang (2015). Nevertheless, for robustness, we investigate results of our tests for samples of firms with zero long-term debt reported in Compustat and firms with imputed debt returns.[1]

Equation (12) provides a specification for investment returns conditional on parameters α and a, which must be estimated. Liu, Whited, and Zhang (2009) suggest estimating these parameters using moment conditions implied by equation (12),

$$\mathbf{e}_T = \frac{1}{T}\sum_t \left(\mathbf{R}_t^S - \mathbf{R}_t^I\right). \tag{13}$$

For a set of two assets ($N = 2$), the above system is exactly identified. When $N > 2$, the system is overidentified, and the authors use these overidentifying restrictions as a test of the

[1]Equation (12) is nested by the expression for levered investment returns in Liu, Whited, and Zhang (2009) if we assume zero leverage and zero tax rates. See also equation (17) in Section 6.4.

implications of the investment-based model. We follow their approach and use single-stage GMM to estimate the parameters α and a. We note, however, that rejection of the test of overidentifying restrictions suggests at least three possibilities. First, the rejection could indicate failure of the economic mechanism of the investment-based model. Second, the rejection could be attributed to the assumption of a cross-sectionally constant share of capital in production, α, and adjustment cost, a. Third, the rejection could indicate that the functional form of the return on investment is misspecified. We consider these possibilities in our empirical tests below.

3.3 Data and Summary Statistics

In our empirical tests, we utilize a set of portfolios proposed in Hou, Xue, and Zhang (2015). The authors derive a four-factor model that is inspired by a simplified version of the investment-based model discussed in the previous section. In that model, the return on investment is driven by the ratios of output to capital and investment to capital. The authors suggest that the functional form in the resulting simplified version of equation (12) implies that expected returns should be increasing in profitability and decreasing in investment intensity.[2] This motivates the construction of portfolios sorted on the dimensions of return on equity, which is the measure of profitability adopted in Hou, Xue, and Zhang (2015), and the ratio of investment to assets, which is the corresponding measure of investment intensity. These portfolios also form the basis of our empirical tests because, according to the reasoning in Hou, Xue, and Zhang (2015), the investment-based model should be particularly successful in explaining equity returns for the investment and profitability cross-section.

To construct our test portfolios, we sort all firms in the annual CRSP/Compustat merged database into terciles on the basis of their ratio of investment to assets. We follow Hou, Xue, and Zhang (2015) and define investment to assets as the change in total assets, Compustat item AT, divided by beginning of period assets. Following the authors' procedure, and consistent with Fama and French (1993), we assume that financial statement information becomes available to investors no sooner than six months after the end of the fiscal year. Therefore, investment to

[2] Their two period model results in investment returns of the form $R^I_{t+1} = \Pi_{t+1} / (1 + a(I_t/K_t))$, where Π_{t+1} is profitability, which is empirically measured by the return on equity.

asset ratios in calendar year t are matched to CRSP returns from July of year $t+1$ through June of year $t+2$.

We also sort all firms in the quarterly CRSP/Compustat merged database into terciles on the basis of their return on equity (ROE), defined as the ratio of income before extraordinary items, Compustat item IBQ, to book value of equity at the beginning of the fiscal quarter. As in Hou, Xue, and Zhang (2015), we define book value of equity following Davis, Fama, and French (2000), shareholder's equity plus balance sheet deferred taxes and investment tax credits, less preferred stock.[3] We treat the return on equity as observed on the quarterly earnings reporting date, Compustat item RDQ. Returns on equity are matched to equity returns from CRSP for the month following the month of the earnings reporting date. Finally, we require that earnings be reported for the fiscal quarter that is no more than six months from portfolio formation, consistent with the authors' approach.

According to equation (12), the return on investment is a function of output, Y_{t+1}, capital, K_{t+1}, investment, I_{t+1}, and depreciation rate, δ_{t+1}. Following Liu, Whited, and Zhang (2009), we measure output as total net sales, Compustat item SALEQ, and depreciation rate as the ratio of depreciation and amortization expense, Compustat item DPQ, to beginning of period capital. Liu, Whited, and Zhang (2009) and Hou, Xue, and Zhang (2015) suggest different measures of capital. The former authors use gross property, plant and equipment, whereas the latter use total assets. Since total assets data are available for more firms, we follow Hou, Xue, and Zhang (2015) and measure capital, K_{t+1}, as beginning of period total assets, Compustat item ATQ. Similarly, as in Hou et al., investment is measured as the change in total assets over the quarter. In the quarterly Compustat data, total asset information is not broadly available until 1975. As a result, our final sample spans the period January, 1975 through December, 2014.[4]

We utilize quarterly data largely due to issues regarding econometric inference and sample size. Data to compute the return on equity is not generally available until 1972, which would

[3] For a detailed description of the construction of these variables, refer to Hou, Xue, and Zhang (2015).

[4] We delete observations with missing total assets, total liabilities (Compustat item LTQ), and sales. We also delete financials (SIC codes 4900-4999 and 6000-6999) and observations with return on equity less than -100%. For the tests on zero debt firms, we also delete observations with positive long term debt (Compustat item DLTTQ), and for the tests on levered investment returns, we delete observations with missing imputed debt returns.

yield only 43 annual observations. With nine portfolios and 43 annual observations, statistical power for estimation of the various GMM systems described above would be very low. However, seasonalities may impact the measurement of the quantities of interest in the return on investment from quarterly data. Therefore, for robustness, we repeat the tests in annual data.

The timing of variables used to measure the return on investment is an issue discussed at length in Liu, Whited, and Zhang (2009). The authors propose matching financial statement information from December of year t through December of year $t+1$ to returns on portfolios held from June of year $t+1$ to June of year $t+2$. Their rationale is that the market may be unaware of financial statement information for year t until June of year $t+1$, similar to the rationale used in constructing the book-to-market ratio as a basis for portfolio formation.

However, the economic link between returns on equity and investment associates the return on equity from year t to year $t+1$ to the investment decisions over year t to year $t+1$ (see equation (12)). Based on this relation, we choose to match quarterly financial information contemporarily with returns on equity portfolios, except as the financial information contributes to identifying terciles for portfolio formation. In order to ensure that this implicit foresight on the part of the equity market does not drive our results, we also repeat our tests matching quarterly data entering the return on investment to equity returns two quarters in the future.

Summary statistics for the variables are presented in Table 1. As suggested by Fama and French (1995), we compute ratios by separately aggregating the numerator and denominator to the portfolio level, and then dividing. That is, the ratio represents the sum across firms of the numerator variable divided by the sum across firms of the denominator variable. Average returns exhibit large differences across investment and profitability terciles. In each of the three return-on-equity (ROE) terciles, average returns decrease monotonically across investment terciles, and in each of the three investment terciles, returns increase monotonically across ROE terciles. These results are consistent with the hypotheses and empirical findings of Hou, Xue, and Zhang (2015). Averaging across investment-to-asset (IA) terciles, firms in the top profitability tercile earn an average return of 4.25% per quarter, compared to 1.19% for the low profitability tercile, for a profitability premium of 3.06% per quarter. Averaging across profitability terciles, low investment firms earn an average return of 3.61% per quarter, compared to 2.24% per quarter for high investment firms, for an investment premium of 1.37% per quarter.

Average ratios are presented in the remaining panels of the table. The ratio of sales to capital, or return on assets, is reported in Panel B. Return on assets (Y/K) increases monotonically across return on equity terciles, as one might expect. The association of return on assets with investment to assets, however, is modest at best. The pattern is relatively flat across any of the return on equity terciles, although there is a tendency for return on assets for the lowest investment tercile to be higher than that of the highest investment tercile.

We report the ratio of investment to capital (I/K) in Panel C. By construction, the ratio of investment to capital increases across the investment terciles. Interestingly, however, the ratio also increases monotonically across profitability terciles. This pattern suggests that more profitable firms tend to invest relatively intensively, but that it is not the case that all investment-intensive firms are relatively profitable. Another point of interest is that the average ratio of investment to capital for the low profitability/low investment portfolio is negative. This result is surprising on face value, as one would not expect a firm to be divesting on average. However, it is instead indicative of turnover in the portfolio; by holding a portolio of low profitability, low investment firms, one is holding firms that on average are divesting.

4 Estimation Results

4.1 Returns to Investment

We first estimate the parameters of the investment return, α and a, representing the share of capital in the production function and the cost of adjustment of new capital, respectively. We follow Liu, Whited, and Zhang (2009) in estimating the production parameters using GMM and the moment conditions in equation (13). Specifically, we employ the identity matrix in single-stage GMM estimation and calculate standard errors using the Newey and West (1987) correction with five lags. Our results suggest that inference about standard errors is not particularly sensitive to the number of lags in the Newey-West estimate.

Results of the estimation are presented in Table 2. Point estimates and model specification tests are presented in Panel A. The point estimate for the share of capital in the production function, α, is 0.174 (standard error = 0.03). This estimate is similar in magnitude to that estimated in Liu, Whited, and Zhang (2009) for the corporate investment portfolio sort of Titman, Wei, and Xie (2004). The point estimate for the adjustment cost parameter, a, is 0.367 (standard error = 0.77), which is smaller in magnitude than the point estimates in Liu, Whited, and Zhang (2009).

One possible explanation for our lower investment adjustment cost parameter is that Liu, Whited, and Zhang (2009) measure capital by tangible assets and investment by capital expenditures, whereas we follow Hou, Xue, and Zhang (2015) and measure capital by total assets and investment by change in total assets. Additionally, our adjustment cost estimate could be low because we use value-weighted portfolios for our tests. Value-weighted portfolios exhibit lower cross-sectional variation in returns than equal-weighted portfolios. Therefore, the investment-based model attempts to fit the cross-section of value-weighted returns with an adjustment cost function that exhibits moderate convexity.[5]

Another interesting finding in Panel A of Table 2 is that unlike the capital share parameter α, which is statistically significant, the adjustment cost coefficient a cannot be accurately estimated, despite the use of quarterly returns that allow for larger samples than annual returns. Finally, the test of moment restrictions suggests that equality of means can be rejected since the χ^2-statistic is quite large.

In Panel B of Table 2, we present average investment returns, moment condition errors, and associated t-statistics that these errors are statistically insignificant. According to the results for the GMM errors, mean equity returns are matched fairly well for the medium profitability/high investment and high profitability/high investment portfolios, with absolute values of pricing errors of less than 30 basis points each. Overall performance is somewhat poorer, however, with mean absolute pricing errors of 83 basis points per quarter. The poor performance is driven in particular by the low profitability portfolios, which exhibit errors of -80 (t-statistic =-2.61), -103 (t-statistic=-2.24), and -212 (t-statistic=-3.92) basis points per quarter for the low, medium,

[5] Liu, Whited, and Zhang (2009) use equal-weighted returns in their analysis. However, we conduct our tests using value-weighted returns as advocated by the findings in Asparouhova, Bessembinder, and Kalcheva (2012).

and high investment portfolios, respectively.

As discussed above, one can interpret these results in several ways. From a statistical standpoint, we can reject equality of means of the return on equity and the return on investment, an implication of the investment-based pricing model explored in Liu, Whited, and Zhang (2009). Moreover, the economic magnitude of several of the pricing errors seems large. This may indicate failure of the economic mechanism of the investment model, rejection of the assumption that the capital share and adjustment costs parameters are the same across the nine investment and profitability portfolios, or mis-specification of the production function. We investigate the first possibility in detail in Sections 4.2 and 4.3, and the second possibility in our robustness tests.

4.2 SDFs Implied in Equity and Investment Returns

In this section, we utilize the investment returns implied by the parameter estimates in the previous section to test restrictions implied by equations (8) and (9). Specifically, we investigate whether a stochastic discount factor in the linear span of equity returns satisfies the Euler equation for investment, and whether a stochastic discount factor in the linear span of investment returns satisfies the Euler equation for equity returns. As discussed in Section 3.1, we test these hypotheses using three test statistics. The first is the overidentifying restriction test from Chen and Knez (1996) using GMM and the moment conditions in equation (10). The second and third are the Wald and likelihood ratio tests from Bekaert and Urias (1996) using the conditions in equation (11).

The results of these tests are presented in Table 3. The answers appear unequivocal; according to all three test statistics, the null hypotheses are rejected. First, the Chen and Knez (1996) test indicates that the minimum variance stochastic discount factor in the linear span of equity returns cannot satisfy the Euler equation for investment returns, indicated by the GMM χ^2-statistic of 73.535 (p-value=0.000). Similarly, the minimum variance stochastic discount factor in the linear span of investment returns fails to satisfy the Euler equation for equity returns.

Results of the more flexible tests of Bekaert and Urias (1996) are similar. The likelihood ratio tests, which are direct complements to the Chen and Knez (1996) test, reject the restrictions of zero coefficients on either the investment or the equity returns in the stochastic discount factor when pricing both sets of assets at below the 1% critical level. Similarly, the Wald tests of whether the unrestricted coefficients are zero rejects the null hypothesis at below the 1% critical value in both specifications. We interpret these results as a failure for the investment-based pricing model, since they suggest that projecting the stochastic discount factor on the space of investment returns produces different coefficients than projecting on equity returns.

A visual interpretation of the test statistics is presented in Figure 1, where we plot the Hansen and Jagannathan (1991) bounds on the volatility of the stochastic discount factor implied by equity returns, investment returns, and the combined set of investment of equity returns. The figure demonstrates that the lowest bounds are for equity returns. The usual interpretation of this relatively low bound is that it represents a low threshold for a candidate stochastic discount factor; a stochastic discount factor that prices equity returns needs to have less volatility for a given mean than a stochastic discount factor that prices investment returns. However, although the bound for investment returns is higher, it is not as high as the combined bounds for equity and investment returns. Thus, even if a candidate stochastic discount factor generates enough volatility to price investment returns, it will not necessarily be sufficiently volatile to price both equity and investment returns. While such a stochastic discount factor exists, it does not seem to satisfy the condition that equity returns and investment returns are equivalent.

In Panel B of Table 3, we present slope coefficients ($Cov(\mathbf{R}_{t+1}^{S,I}, M_{t+1}^{S,I})/Var(M_{t+1}^{S,I})$) obtained by regressing equity and investment returns on a minimum variance stochastic discount factor based on equity returns, and a complementary set of results by regressing returns on the minimum variance stochastic discount factor implied by the returns on investment. Expected returns for investment and equity returns should be negatively proportional to these slope coefficients because the unconditional Euler equations for equity and investment imply that

$$E\left[\mathbf{R}_{t+1}^{S,I}\right] = \frac{1}{E\left[M_{t+1}\right]} - \frac{Cov\left(\mathbf{R}_{t+1}^{S,I}, M_{t+1}\right)}{E\left[M_{t+1}\right]}. \tag{14}$$

For theses tests, we choose the stochastic discount factor with a mean $E\left[M_{t+1}^{S,I}\right] = 0.988$, since this is the mean of the stochastic discount factor implied by the average of the inverse gross quarterly return on 30-day Treasury Bills from CRSP over our sample period. Hansen and Jagannathan (1991) show that the minimum variance stochastic discount factor with mean equal to the inverse of the risk-free rate corresponds to the tangency portfolio on the efficient frontier with the maximum Sharpe ratio.

The first set of columns in Panel B represents point estimates of regressions of equity and investment returns on the minimum variance stochastic discount factor implied by equity returns. By construction, the stochastic discount factor in the linear span of equity returns satisfies the Euler equation for equity. Consequently, the equity slope coefficients, b_S^S, are perfectly negatively correlated with average equity returns, $E\left[\mathbf{R}_{t+1}^S\right]$. In the final column, we present t-statistics for the equality of the slope coefficients ($b_S^S = b_S^I$). These t-statistics represent a test of the condition suggested in Cochrane (1991) that when regressing returns on investment and equity on other variables, one should obtain the same point estimates. The table shows that these hypotheses are strongly rejected for six out of nine portfolios.

The second set of columns provide the complement to the first; returns on equity and investment are regressed on the minimum variance stochastic discount factor in the linear span of investment returns. Average returns on investment, $E\left[\mathbf{R}_{t+1}^I\right]$, are perfectly negatively correlated with the coefficients of the investment-based stochastic discount factor, b_I^I, and these coefficients are uniformly negative. One issue that arises is that the slope coefficients b_I^S from regressing equity returns on the investment-based stochastic discount factor are positive for seven out of nine portfolios. This implies that equities have relatively high payoffs when the investment-based stochastic discount factor is high, and low when the stochastic discount factor is low. This is counterintuitive, as we generally think of equities as assets that pay off when marginal utility is low and provide poor insurance against bad states of the world. Nevertheless, there is a negative association between the slope coefficients and average equity returns as implied by the Euler equation; the two exhibit a correlation of -56%.

4.3 Relation between the Equity-based SDF and Components of the Return on Investment

In order to get a bit more insight into the results of this section, we conduct time series regressions of the minimum variance stochastic discount factor in the linear span of equity returns, M_{t+1}^S, with mean equal to the average of the inverse gross risk-free return, on the components of investment returns: the output to capital and the investment to capital ratios.[6] According to equation (12), the return on investment is positively associated with the contemporaneous ratios of output to capital and investment to capital and negatively associated with the lagged ratio of investment to capital.

As emphasized in equation (14), an asset earns a higher expected return if its returns covary more negatively (less positively) with the stochastic discount factor. Thus, the investment-based pricing model suggests that higher expected investment and equity returns should be associated with greater negative, or less positive, covariance of the contemporaneous ratios of output to capital and investment to capital with the stochastic discount factor, and less negative, or more positive covariance of the ratio of lagged investment to capital with the discount factor.

Results of these regressions are presented in Table 4. The table shows that, as predicted by the investment-based model, there is a negative relation between contemporaneous output to capital ratios and the equity-based minimum variance stochastic discount factor across all nine portfolios investment and profitability portfolios. We can interpret this result as saying that output to capital tends to be high in states that are good for equity returns and low in states that are bad for returns. Six of these coefficients are statistically significant at the 10% critical level. While the output to capital coefficients are negative, it is not clear that their magnitudes align well with investment returns since the correlation between the output to capital coefficients and average investment returns is zero. For instance, the most negative point estimate is for the low ROE/medium IA portfolio, which has the second lowest average investment return of the nine portfolios (Panel B in Table 2).

Both the contemporaneous and lagged ratios of investment to capital exhibit mostly positive coefficients, although fewer of these coefficients are statistically different than zero. Only the three low investment portfolios have investment to capital coefficients that are at least marginally

[6] In untabulated tests, we also conduct time series regressions of equity returns for each portfolio on the corresponding components of investment returns. The results are qualitative very similar to the ones presented here, i.e., statistically weak estimates and poor fit of the regressions.

statistically distinguishable from zero. The preponderance of positive coefficients suggests that firms in general tend to invest more intensively when the stochastic discount factor is high, or alternatively less intensively when the stochastic discount factor is low. This finding runs against the predictions of the structural investment-based model and the definition of investment returns in equation (12), where the contemporaneous investment to capital ratio should be high when equity returns are high, i.e., during periods when the stochastic discount factor is low.

In general, the results in this section suggest that the implications of the structural investment-based asset pricing model are violated. In particular, since the investment model predicts that equity returns will be equal to investment returns in every state of nature, we expect that a projection of the stochastic discount factor onto the two sets of assets will yield the same coefficients. This restriction appears to be strongly rejected. At least some of the reason for this rejection appears to be the fact that the investment-based model predicts that current investment to capital should be high when the stochastic discount factor is low, and low when the stochastic discount factor is high. The data suggest a tendency to the opposite; the contemporaneous investment to capital ratio tends to covary positively with the stochastic discount factor in seven out nine portfolios.

5 Parametric Stochastic Discount Factors

The results above do not suggest that there is no stochastic discount factor that can price both investment returns and equity returns, but rather that it will be difficult for a minimum variance stochastic discount factor in the span of equity returns to price both equity and investment returns. In order to investigate this idea a bit further, we examine the performance of two parametric models for expected returns in jointly pricing returns on investment and equity. Specifically, we investigate the four-factor model of Hou, Xue, and Zhang (2015) and the five-factor specification of Fama and French (2015).

Hou, Xue, and Zhang (2015) propose a model that is explicitly motivated by the structural investment-based model. Specifically, the authors introduce an investment factor and a profitability factor motivated by the functional form of the return on investment. These factors are intimately related to the test portfolios in our paper. In order to generate their factors, Hou, Xue, and Zhang (2015) sort firms into terciles on the basis of return on equity and investment to assets, as in this paper. They also sort firms into above- and below-median market capitalization groups on the basis of June market capitalization. They form 18 value weighted portfolios on the intersections of these three characteristics.

The profitability factor ($R_{ROE,t}$) is the difference between the average return on the six high return on equity portfolios and the average return on the six low return on equity portfolios. Similarly, the investment factor ($R_{IA,t}$) is the difference in the average return on the six low investment portfolios in excess of the average return on the six high investment portfolios. Finally, the size factor ($R_{SMB,t}$) is the difference in the average return on the nine low market capitalization firms in excess of the average return on the nine high market capitalization firms. The authors suggest a four-factor model with stochastic discount factor

$$M_{t+1}^{HXZ} = d_0 + d_{MRP}R_{MRP,t+1} + d_{SMB}R_{SMB,t+1} + d_{IA}R_{IA,t+1} + d_{ROE}R_{ROE,t+1}, \quad (15)$$

where $R_{MRP,t+1}$ is the return on the value-weighted CRSP index in excess of the return on a one-month Treasury Bill.

The five-factor model in Fama and French (2015) uses similar factors but different motivation. The authors use the clean surplus accounting formulation and valuation formula to motivate two factors in addition to the size and book-to-market factor from Fama and French (1993). The two new factors are the difference in the return on a portfolio of robust (high) operating profitability and weak (low) operating profitability firms and the difference in the return on a portfolio of conservative (low) investment growth firms relative to a portfolio of agressive (high) investment growth firms. The resulting stochastic discount factor is

$$\begin{aligned} M_{t+1}^{FF} &= d_0 + d_{MRP}R_{MRP,t+1} + d_{SMB}R_{SMB,t+1} + d_{HML}R_{HML,t+1} \\ &\quad + d_{RMW}R_{RMW,t+1} + d_{CMA}R_{CMA,t+1}. \end{aligned} \quad (16)$$

$R_{MRP,t+1}$ above represents the excess return on the market portfolio and $R_{SMB,t+1}$ represents the excess return on the small cap portfolios relative to the large cap portfolios. The construction of the size factor differs in Fama and French (2015) and Hou, Xue, and Zhang (2015) because the remaining sorting variables differ. $R_{HML,t+1}$ is the return on a high book-to-market portfolio in excess of a low book-to-market portfolio, $R_{RMW,t+1}$ is the difference in the high and low profitability portfolio returns, and $R_{CMA,t+1}$ is the difference in the return on a low and high investment growth portfolio. We obtain data for these factors from Kenneth French's website.[7]

5.1 Empirical Methodology

One important aspect of the above factor-based pricing kernels is that the corresponding factors are traded assets, which should be priced by their models. Thus, in addition to the moment conditions for equity and investment returns, for this set of tests, we augment the GMM objective function to include the Euler equations for the factors. Further, since factors are excess returns, we cast the GMM moment conditions in terms of excess returns by subtracting the quarterly return of the 30-day Treasury Bill from investment and equity returns.

The GMM moment conditions for these tests are given by

$$\mathbf{h}_T = \frac{1}{T}\sum_t \begin{bmatrix} \left(1 - (\frac{1}{T}\sum_t M_{t+1}^{HXZ,FF}) + M_{t+1}^{HXZ,FF}\right)\mathbf{f}_{t+1} \\ \left(1 - (\frac{1}{T}\sum_t M_{t+1}^{HXZ,FF}) + M_{t+1}^{HXZ,FF}\right)(\mathbf{R}_{t+1}^{S} - R_{t+1}^{f}) \\ \left(1 - (\frac{1}{T}\sum_t M_{t+1}^{HXZ,FF}) + M_{t+1}^{HXZ,FF}\right)(\mathbf{R}_{t+1}^{I} - R_{t+1}^{f}) \end{bmatrix}.$$

The vector $\mathbf{f}_{t+1}$ denotes the factors in each model, $M_{t+1}^{HXZ,FF}$ denotes the Hou, Xue, and Zhang (2015) or Fama and French (2015) pricing kernels, and R_{t+1}^{f} is the risk-free rate. Consistent with Burnside's (2011) generic representation for linear factor models, we augment the stochastic discount factors by the term $1 - \frac{1}{T}\sum_t M_{t+1}^{HXZ,FF}$ to rule out a zero solution for risk prices since we are testing linear models on excess returns.[8]

To ensure that the asset pricing factors are perfectly explained by their models, we specify

[7] `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. We thank Ken French for making these data available.

[8] This specification implies that the sample mean of the effective stochastic discount factor, $1 - (\frac{1}{T}\sum_t M_{t+1}^{HXZ,FF}) + M_{t+1}^{HXZ,FF}$, is 1, and that risk premia are given by covariances with the stochastic discount factor, i.e., $E[\mathbf{R}_{t+1}^{S,I} - R_{t+1}^{f}] = -Cov(\mathbf{R}_{t+1}^{S,I} - R_{t+1}^{f}, M_{t+1}^{HXZ,FF})$. See also Kan and Robotti (2008).

a GMM weighting matrix, $\mathbf{W}_{(2N+M)\times(2N+M)}$, which estimates the parametric pricing kernels using the set of factors and tests their fit in the joint cross-section of equity and investment returns. Specifically, the GMM weighting matrix in these tests is a block diagonal matrix, where the elements that corresponds to the M moment conditions for factor returns are equal to the identity matrix and the remaining elements are zero:

$$\mathbf{W}_{(2N+M)\times(2N+M)} = \begin{bmatrix} \mathbf{I}_{M\times M} & \mathbf{0}_{M\times 2N} \\ \mathbf{0}_{2N\times M} & \mathbf{0}_{2N\times 2N} \end{bmatrix}.$$

The above weighting matrix constraints the parameters of the factor models to be consistent with the factor risk premia and the covariance matrix of factor returns.[9] As before, because we have a limited number of time series data points and a relatively large number of parameters, we utilize single-step GMM. Finally, we use investment returns as estimated in Section 4.1.

5.2 Estimation Results

In Panel A of Table 5, we present estimation results for the four-factor model in Hou, Xue, and Zhang (2015). Our earlier results suggest that it will be difficult for a stochastic discount factor in the linear span of equity returns to satisfy the Euler equation for both investment and equity returns. The results for the Hou, Xue, and Zhang (2015) model confirm this suspicion, as the model is rejected at the 5% probability level by the test of overidentifying restrictions. As expected, and enforced in estimation, estimates suggest positive and statistically significant prices of risk for all of the factors, excepting SMB.

In Table 5, we report two additional measures of fit. The first is the mean absolute Euler equation error, which is approximately 100 basis points per quarter. The second measure of fit, the adjusted R-square, is obtained from a two-stage cross-sectional regression. In the first stage, we regress the time series of excess returns on investment and equity on the estimated stochastic discount factor to obtain slope coefficients, as in Section 4.2. In the second stage, we cross-sectionaly regress risk premia on these slope coefficients imposing a zero intercept since

[9] The GMM weighting matrix above imposes the explicit solution for the pricing kernel parameters $\widehat{\mathbf{d}} = -\widehat{\boldsymbol{\Sigma}}_{\mathbf{f}}^{-1}\widehat{E}[\mathbf{f}_{t+1}]$, where $\widehat{\boldsymbol{\Sigma}}_{\mathbf{f}}$ is the covariance matrix of factor returns, and $\widehat{E}[\mathbf{f}_{t+1}]$ is the vector of factor risk premia.

our test payoffs are excess returns. *Adj.* $\bar{R}^2_S$ denotes the adjusted R-square in the set of equity returns, and *Adj.* $\bar{R}^2_{S,I}$ is the adjusted R-square in the joint set of equity and investment returns. In the set of equity returns, the exposures to the stochastic discount factor explain an impressive 79.8% of cross-sectional variation in average equity returns. However, the adjusted R-square in the joint cross-section is negative, *Adj.* $\bar{R}^2_{S,I} = -76.9\%$, indicating that the Hou, Xue, and Zhang (2015) discount factor cannot price investment returns.

We repeat the analysis with the Fama and French (2015) model in Panel B of Table 5. In terms of overall fit, the Fama and French (2015) model is rejected at the 5% probability level by the test of overidentifying restrictions. The price of market, profitability, and investment risk are precisely estimated, and the coefficients are all negative. In contrast, the size (SMB) and value (HML) parameters are not estimated accurately, which may result from the lack of power due to the number of parameters and relatively small sample size or from the fact that these factors are subsumed by investment and profitability.[10]

Comparing the results in Panels A and B, we conclude that the Hou, Xue, and Zhang (2015) model performs much better in pricing the cross-section of equity returns than the Fama and French (2015) specification, whose adjusted R-square in the set of equity returns is only 19.7%. In the joint cross-section of equity and investment returns the two models perform equally poorly with the Fama and French (2015) specification producing a large absolute pricing error of 110 basis points per quarter. Similarly, the cross-sectional regression of average returns on exposures to the estimated Fama and French (2015) stochastic discount factor generates a negative relationship with an adjusted R-square of -121.8%.

As we did for the stochastic discount factors in the linear spans of equity and investment returns, we look more closely at the exposure (β) of excess investment and equity returns to the estimated stochastic discount factors. These exposures are reported in Panel C of Table 5. Similar to the results from the nonparametric stochastic discount factors of Table 3, there is a surprising tendency for positive relations between investment returns and stochastic discount factors, suggesting that investment returns are high when stochastic discount factors are high,

[10] Fama and French (2015) also highlight the insignificance of HML in their five-factor model.

i.e., during bad states. Exposures are positive for six out of the nine investment returns using the Hou, Xue, and Zhang (2015) stochastic discount factor, and for five out of nine the investment returns using the Fama and French (2015) model. In contrast, the exposures for equity returns are all negative, implying that both stochastic discount factors capture the procyclicality of equity returns. Finally, the column t-statistic tests the equality of the two slopes for investment and equity excess returns. Contrary to the predictions of the investment model, we can reject equality of the slopes for six out of nine portfolios using the Hou, Xue, and Zhang (2015) discount factor, and for all portfolios using the Fama and French (2015) specification.

6 Robustness

The results in the preceding section suggest at least two conclusions. The first is that the central first order condition of the linear homogeneous investment-based CAPM fails to hold; a stochastic discount factor that prices equity returns does not satisfy the Euler equation for investment. The second conclusion is that parametric models inspired by this investment model may satisfy the Euler equation for equity returns, but are unlikely to satisfy the Euler equation for both equity and investment returns. Our results are critically dependent on the measurement and timing of investment returns. In this section, we examine whether our results are sensitive to alternative measurements of investment returns.

6.1 Cross-Sectional Variation in Production Parameters

For our baseline tests, we assume a common set of production coefficients (α, a) for all portfolios. However, it is quite possible that the investment share and adjustment cost parameters differ across firms, particularly, for example, across industries. If each portfolio has its own adjustment cost and capital share parameter, the estimation is underidentified, with 18 parameters and 9 moment conditions. Therefore, we choose instead to consider separately variation that depends only on profitability or investment tercile; that is, all three investment to assets (profitability) portfolios in a profitability (investment to asset) tercile will have the same parameters. This gives us nine moment conditions and six parameters to estimate. Results using groups based on return on equity tercile are presented in Table 6.[11] The message of the table is that mean returns are much better matched with flexible parameters at some degree of cost of precision of the point estimates.

We next examine tests of the equality of the minimum variance stochastic discount factors implied by investment and equity returns. We present these results in Table 7. The results indicate that cross-sectional parameter flexibility do not materially change the results of Euler equation tests for this investment-based model. The Euler equation conditions are statistically rejected, and slope coefficients obtained from regressing equity or investment returns on the minimum variance stochastic discount factors exhibit virtually no correlation.

Finally, in Table 8, we estimate the parametric stochastic discount factors of Hou, Xue, and Zhang (2015) and Fama and French (2015) in the set of investment returns with flexible production parameters. We continue to get results similar to those obtained when production parameters are cross-sectionally constant. The Euler equations are rejected, pricing errors are large, and while equity returns are strongly related to the stochastic discount factors, investment returns are not, and tend to have positive covariance with the parametric pricing kernels.

6.2 Timing of Investment and Equity Returns

Our main tests assume that returns to equity claims are measured contemporaneously with returns on investment; that is, for example, returns to equity from October 1, 2014 through December 31, 2014 are matched to financial statement information from October 1, 2014 through December 31, 2014. However, the financial statement information for December 31, 2014 will generally be available to investors only with a lag. As a consequence, Liu, Whited, and Zhang (2009) match variables in investment returns to equity returns with a six month lag. This means, for example, that financial statement information as of December 31, 2012 is matched to holding period returns from July 1, 2013 through June 30, 2014. In order to ensure that availability of financial statement information is not affecting our results, we repeat our analysis matching financial statement information to returns two quarters ahead.

[11] Mean returns are better matched using parameters constant across investment to asset terciles than return on equity terciles.

Results are reported in Table 9. The results suggest that average returns continue to increase in profitability and fall in investment. Point estimates of the capital share and adjustment cost parameters are of similar magnitude to results with concurrent timing assumed. However, there are two notable improvements. First, the adjustment cost parameter in this specification is more precisely estimated, with a point estimate more than two standard errors from zero. Further, while equality of means is statistically rejected, the error in estimation is substantially smaller than under the concurrent timing assumption. However, there is no improvement in Euler equation tests; the model is soundly rejected on the basis of all of the specification tests.

Alternatively, a possibility is that because investment plans take time to implement, returns on equity in period t reflect expectations of the results of an investment plan that will result in returns on investment at time $t + j$. A simple approach to this idea of time to build is to measure investment returns with a lead relative to equity returns. Results using investment returns leading equity returns by three years are shown in Table 10.[12] The table again suggests that we improve upon our ability to match mean returns, and that the moment conditions for the equality of investment return means and equity return means is at the edge of statistical significance. Production parameters are again more precisely estimated, but at a cost; the sign of the investment cost coefficient a is negative, violating the assumption of convex investment adjustment costs. Finally, the model continues to be rejected by tests of the equality of stochastic discount factors in the two sets of payoffs.

6.3 Annual Data

In their analysis of the equality of means on investment and stock returns, Liu, Whited, and Zhang (2009) use annual returns to permit the use of fiscal year end Compustat data. While this choice incurs a cost in terms of power of the tests, there are several reasons one may prefer to use the annual Compustat data. In particular, the quality of data and the number of firms reporting is greater for annual than quarterly Compustat. Therefore, we re-examine the model using annual, rather than quarterly data on firm characteristics and returns.

Results are shown in Table 11. One immediate point to note is that the use of annual data

[12]Results using a one- and two-year lead are very similar, and are available from the authors upon request.

affects the returns on profitability-sorted portfolios dramatically. Only firms that are also in the highest investment tercile exhibit average returns increasing in profitability.[13] However, many of the same patterns re-emerge. Capital share and adjustment cost parameters are similar to those using quarterly data, but more precisely estimated. Equality of mean investment and equity returns is rejected, as is the restriction that the stochastic discount factor that satisfies the Euler equation for equity returns also satisfies that for investment returns, and vice versa.

6.4 Leverage

Thus far, we have treated all firms as unlevered firms. However, when firms have leverage, the relevant condition is no longer that the return on investment should be equal to the return on equity. Rather, as discussed in Liu, Whited, and Zhang (2009), the levered return on a firm's assets should be equal to the equity return. We address this issue in two ways.

First, we limit our sample construction only to firms that have zero long term debt as indicated by Compustat item DLTTQ. Results for this approach are reported in Table 12. The equality of means of investment and equity return restriction is rejected, albeit at a weaker significance level than in our baseline tests. However, as in the case of leading investment returns relative to equity returns, the adjustment cost parameter is negative, though imprecisely estimated. Moreover, the restrictions implied by Euler equations continue to be sharply rejected.

A second possibility is to explicitly account for leverage, as in Liu, Whited, and Zhang (2009) , that takes into account debt returns and tax shields. Specifically, in the presences of debt financing and taxes, equation (12) for investment returns can be modified to account for leverage, debt returns, and taxes as follows:

$$R^S_{t+1} = R^{IL}_{t+1} = \frac{\frac{(1-\tau_{t+1})\left(\alpha\frac{Y_{t+1}}{K_{t+1}}+\frac{a}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right)+\tau_{t+1}\delta_{t+1}+(1-\delta_{t+1})\left(1+(1-\tau_{t+1})a\frac{I_{t+1}}{K_{t+1}}\right)}{1+(1-\tau_t)a\frac{I_t}{K_t}} - Lev_t \times R^{B,after}_{t+1}}{1-Lev_t}. \qquad (17)$$

Above, R^{IL}_{t+1} is the levered investment return, τ_t is the corporate tax rate, and Lev_t denotes

[13] The same general pattern holds in the same time period for 25 portfolios sorted on investment growth and profitability available from Kenneth French's website, `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

market leverage. $R_{t+1}^{B,after}$ is the after-tax gross return on debt, which is defined as $R_{t+1}^{B,after} = R_{t+1}^{B,pre} - \tau_{t+1}(R_{t+1}^{B,pre} - 1)$. We follow Liu, Whited, and Zhang (2009) in imputing debt returns from credit ratings.[14] Further, in accordance with our baseline tests, we estimate the production parameters of the investment model using GMM on the moment conditions implied by equation

$$\mathbf{e}_T = \frac{1}{T} \sum_t \left(\mathbf{R}_t^S - \mathbf{R}_t^{IL} \right). \tag{18}$$

This condition is equivalent to equation (13). However, in equation (18), investment returns from equation (12) are replaced by levered investment returns from equation (17).

Results of implementing this procedure are presented in Table 13. Once again, we obtain a negative, but statistically insignificant coefficient for the adjustment cost parameter. Equality of the mean returns is again rejected in the data with a fairly large mean absolute pricing error. Finally, with the exception of the likelihood ratio test using the equity stochastic discount factor, the null that the Euler equation for levered investment and equity returns holds with a common stochastic discount factor is rejected.

7 Conclusion

When firms invest optimally, their investment policy satisfies Euler equations for investment returns, given a stochastic discount factor that prices equity returns. According to Cochrane (1991), an implication of this result is that, under linear homogeneous production and investment technologies, returns on equity and investment will be equal in every state of nature. An immediate consequence of this equality is that when regressing equity and investment returns on some variable, one should obtain the same coefficient. Similarly, when regressing a variable on the returns on equity and investment, one should obtain the same coefficient.

This implication forms the basis for the tests conducted in this paper. Specifically, we focus on what we consider to be an especially economically interesting implication of this restriction; that one should obtain the same parameters when projecting the stochastic discount factor on

[14] See the Appendix for a detailed description of the debt return calculations.

equity and investment returns. One can interpret this implication as saying that when one retrieves a stochastic discount factor from equity returns using the minimum distance approach in Hansen and Jagannathan (1991), that this stochastic discount factor should satisfy the Euler equation for investment returns. We test this restriction and find that it is broadly rejected in the data. Thus, our results suggest inconsistencies in the implications of the commonly used structural investment-based model for equity returns.

These results are important in light of a number of recent innovations in the asset pricing literature. In advocating for the investment CAPM, Lin and Zhang (2013) suggest that the investment-based approach examined in this paper "changes the big picture of asset pricing." Motivated by this investment-based framework, Hou, Xue, and Zhang (2015) propose a four-factor model with investment- and profitability-sorted portfolios as factors. The authors claim that these factors are priced because of the implications of a structural model similar to that examined in this paper.

Our results suggest that this claim should be interpreted cautiously. Returns on investment, which are explicit functions of characteristics related to those used in forming return factors covary poorly with with stochastic discount factors known to price equity returns. Put differently, our results emphasize the fact that a stochastic discount factor that satisfies the Euler equation for equity returns will not necessarily imply optimal investment as embodied in the Euler equation for investment returns.

Our paper should not, however, be interpreted as invalidating the investment-based approach to asset pricing. This approach has significantly advanced our understanding of why firm characteristics might be related to equity returns. Moreover, we are focusing on a special case of the model with linear homogeneity and physical capital as the only factor of production. Although, we recognize that this model is limiting, it is the focus of our tests because of its prominence in the asset pricing literature. More recent literature, including Kuehn, Simutin, and Wang (2015) and Belo, Gala, Salamao, and Vitorino (2018) examine the role that other factors of production play in determining firm value and expected returns. Investigating the role of these alternative factors and modifying the production function represents a promising direction for future research.

Appendix

In the appendix, we discuss the data sources used to impute debt returns in the expression for levered investment returns of equation (17). We obtain information on market leverage and credit ratings from CRSP/Compustat. We define market leverage in equation (17) using the CRSP/Compustat items DLTTQ/(DLTTQ+PRC×SHROUT). Credit ratings are from Compustat item SPLTICRM. Following Blume, Lim, and MacKinlay (1998), we assign all firms with credit ratings to one of the following categories: AAA, AA, A, and BBB. For instance, firms whose rating is BBB+, BBB, BBB- or worse than BBB- are assigned a BBB rating. As in Blume, Lim, and MacKinlay (1998), we only consider investment grade ratings due to the limited time-series data for speculative grade bonds.

To fit credit ratings for firms with missing credit rating observations we run an ordered probit model as in Blume, Lim, and MacKinlay (1998). The independent variables are long-term book leverage (DLTTQ/ATQ), total book leverage ((DLTTQ+DLCQ)/ATQ), operational margin (IBQ/SALEQ), and log-market capitalization (PRCCQ × CSHOQ). We also include two measure of systematic risk: the CAPM beta and the CAPM residual volatility. To calculate these statistics, we regress daily firm-level returns (CRSP item RET) on the returns of the CRSP value-weighted aggregate index (CRSP item VWRETD). We require at least 50 daily returns per quarter, and normalize the estimated betas and residual volatilities by the corresponding cross-sectional, value-weighted (by market capitalization) averages in each quarter. Finally, we model heteroscedacity in the ordered probit model as a function of log-market capitalization.

After fitting credit ratings, we assign corporate bond yields. Corporate bond yields are obtained from Datastream. In merging credit ratings from Compustat (S&P ratings) to bond yields from Datastream (Moody's ratings), we match S&P ratings to the equivalent Moody's ratings, e.g., AA to Aa. To compute debt returns at the portfolio level, we value-weight bond yields at the firm-level by the market value of equity, similar to our methodology for aggregating equity returns at the portfolio level. Finally, we obtain historical corporate statutory tax rates from the Economic Policy Institute. Since corporate tax rates are reported annually, we assume that tax rates are the same for all quarters in a given year, and that they are portfolio invariant as in Liu, Whited, and Zhang (2009).

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Table 1: **Summary Statistics**

Table 1 presents summary statistics for nine portfolios sorted on the basis of investment to assets (IA) and return on equity (ROE). Panel A reports mean equity returns, $\overline{R^S}$ ($\times$100), Panel B reports mean ratios of output to capital, $\overline{Y/K}$ ($\times$100), Panel C reports mean ratios of investment to capital, $\overline{I/K}$ ($\times$100), and Panel D reports mean depreciation rates, $\overline{\delta}$ ($\times$100). Ratios are computed by summing the numerator across firms and dividing by the denominator summed across firms. Investment is the change in total assets, capital is beginning-of-period total assets, and output is net sales. The depreciation rate is calculated as the depreciation expense divided by beginning of period capital. Portfolio returns are value weighted, and data are sampled at the quarterly frequency from January, 1975 through December, 2014.

Panel A: $\overline{R^S}$

	IA1	IA2	IA3
ROE1	1.93	1.60	0.04
ROE2	4.12	3.08	2.70
ROE3	4.77	4.00	3.98

Panel B: $\overline{Y/K}$

	IA1	IA2	IA3
ROE1	22.57	22.13	20.28
ROE2	24.06	23.29	23.84
ROE3	27.97	27.76	27.90

Panel C: $\overline{I/K}$

	IA1	IA2	IA3
ROE1	-0.23	0.92	2.05
ROE2	1.14	1.97	2.91
ROE3	1.72	2.60	4.03

Panel D: $\overline{\delta}$

	IA1	IA2	IA3
ROE1	1.25	1.24	1.26
ROE2	1.24	1.22	1.12
ROE3	1.27	1.14	1.14

Table 2: **Investment Return Parameter Estimates**

Table 2 presents first-stage GMM results for the production parameters, α and a, from the sample moment conditions

$$\mathbf{e}_T = \frac{1}{T}\sum_{t=1}^{T}(\mathbf{R^I_t} - \mathbf{R^S_t}).$$

$\mathbf{R^I_t}$ are investment returns, which are function of the production parameters α and a, as shown in equation (12). The vector $\mathbf{R^S_t}$ is the vector of gross equity returns on the nine portfolios formed on the intersection of terciles of investment to assets (IA) and return on equity (ROE). Panel A presents point estimates of the production parameters α and a, the corresponding standard errors (SE), the mean absolute error of the GMM moment conditions, and the Hansen (1982) χ^2-test, degrees of freedom, and p-value for the null hypothesis that all the moment conditions are jointly zero. We calculate standard errors using the Newey and West (1987) (NW) correction with five lags. Panel B depicts mean investment returns $\overline{R^I}$ ($\times$100) for the nine portfolios, mean errors for the moment conditions ($\times$100), and associated t-statistics that each mean error is statistically equal to zero.

Panel A: Point Estimates

	α	a
Estimate	0.174	0.367
SE	(0.030)	(0.771)
χ^2-test (5 NW lags)	24.801	
d.o.f.	7	
p-value	(0.000)	
Mean Absolute Error	0.830	

Panel B: Mean Investment Returns and Pricing Errors

Portfolio	$\overline{R^I}$	Error	t-stat.
ROE1/IA1	2.73	-0.80	-2.61
ROE1/IA2	2.63	-1.03	-2.24
ROE1/IA3	2.16	-2.12	-3.92
ROE2/IA1	2.93	1.19	3.37
ROE2/IA2	2.74	0.34	1.76
ROE2/IA3	2.86	-0.16	-0.72
ROE3/IA1	3.59	1.17	2.85
ROE3/IA2	3.60	0.40	1.21
ROE3/IA3	3.70	0.27	0.74

Table 3: **Tests of Equivalence of Investment and Equity SDFs**

Table 3 presents tests of the hypotheses that stochastic discount factors in the linear span of equity returns (investment returns) can price investment returns (equity returns). Investment returns are calculated using the parameter estimates presented in Table 2. In Panel A, we present three sets of test statistics. In the columns "Equity SDF," we ask whether the stochastic discount factors in the linear span of equity returns can price investment returns using the Chen and Knez (1996) test statistic (CK), and the Wald and likelihood ratio tests (LRT) from Bekaert and Urias (1996). In the columns labeled "Investment SDF" we ask whether the stochastic discount factor in the linear span of investment returns can price equity returns. In Panel B, we present slope coefficients from regressions

$$\begin{aligned} R^S_{t+1} &= a^S_S + b^S_S M^S_{t+1} + e^S_{S,t+1} \\ R^I_{t+1} &= a^I_S + b^I_S M^S_{t+1} + e^I_{S,t+1} \\ R^S_{t+1} &= a^S_I + b^S_I M^I_{t+1} + e^S_{I,t+1} \\ R^I_{t+1} &= a^I_I + b^I_I M^I_{t+1} + e^I_{I,t+1}. \end{aligned}$$

M^S_{t+1} and M^I_{t+1} are the minimum variance stochastic discount factors that satisfy the Euler equations for equity and investment returns, respectively, at a mean equal to $\overline{1/R^f_t}$, where R^f_t is the quarterly gross return on a one-month Treasury Bill. t-stat. is the t-statistic that the two slopes are equal. Data are sampled at the quarterly frequency from January, 1975 through December, 2014.

Panel A: Specification Tests

	Equity SDF		
	CK	Wald	LRT
χ^2	73.535	193.336	99.814
p-value	(0.000)	(0.000)	(0.000)

	Investment SDF		
	CK	Wald	LRT
χ^2	35.264	44.483	35.351
p-value	(0.000)	(0.000)	(0.008)

Panel B: Slopes of Regressions of Returns on SDFs

Equity SDF			
Portfolio	Equity b^S_S	Investment b^I_S	t-stat.
ROE1/IA1	-0.94	0.01	-0.87
ROE1/IA2	-0.51	-0.06	-0.38
ROE1/IA3	1.54	0.06	1.28
ROE2/IA1	-3.80	0.04	-4.92
ROE2/IA2	-2.44	-0.09	-2.87
ROE2/IA3	-1.94	-0.14	-1.96
ROE3/IA1	-4.65	-0.01	-6.56
ROE3/IA2	-3.64	-0.16	-4.99
ROE3/IA3	-3.61	-0.10	-3.95

Investment SDF			
Portfolio	Equity b^S_I	Investment b^I_I	t-stat.
ROE1/IA1	0.12	-0.53	1.16
ROE1/IA2	0.38	-0.49	1.45
ROE1/IA3	0.30	-0.33	1.05
ROE2/IA1	-0.04	-0.60	1.30
ROE2/IA2	0.21	-0.53	1.73
ROE2/IA3	0.27	-0.57	1.77
ROE3/IA1	-0.22	-0.83	1.49
ROE3/IA2	0.08	-0.83	2.41
ROE3/IA3	0.43	-0.87	2.75

Table 4: **Equity SDF and Investment Variables**

Table 4 presents regressions of the minimum variance stochastic discount factor in the linear span of equity returns M^S_{t+1} on variables that enter the return on investment expression in equation (12). t-statistics are shown in parenthesis. *Adj.* $\bar{R}^2$ denotes the adjusted R-square. The minimum variance stochastic discount factor is chosen to have mean equal to that of the inverse of the quarterly gross return on a Treasury Bill with one month to maturity. The equity returns are from nine portfolios formed on the intersection of terciles of investment to assets (IA) and return on equity (ROE). Data are sampled at the quarterly frequency over the period January, 1975 through December, 2014.

	Dependent Variable: M^S_{t+1}			
Portfolio	Y_{t+1}/K_{t+1}	I_{t+1}/K_{t+1}	I_t/K_t	*Adj.* $\bar{R}^2$
ROE1/IA1	-2.39	5.86	0.99	0.01
	(-1.82)	(1.90)	(0.42)	
ROE1/IA2	-3.55	0.71	-1.00	0.04
	(-3.16)	(0.49)	(-0.88)	
ROE1/IA3	-1.73	2.13	-1.13	0.00
	(-1.51)	(1.14)	(-0.83)	
ROE2/IA1	-2.03	5.97	2.54	0.03
	(-1.93)	(2.32)	(0.83)	
ROE2/IA2	-1.36	1.93	0.17	-0.01
	(-1.22)	(0.61)	(0.06)	
ROE2/IA3	-1.58	-1.22	-1.12	0.00
	(-1.52)	(-0.60)	(-0.66)	
ROE3/IA1	-1.75	7.80	0.18	0.04
	(-1.96)	(2.81)	(0.06)	
ROE3/IA2	-2.12	-0.19	-1.48	0.01
	(-1.79)	(-0.07)	(-0.54)	
ROE3/IA3	-2.35	1.86	-0.32	0.01
	(-2.03)	(0.79)	(-0.14)	

Table 5: **Factor Model Tests**

Table 5 presents tests of the Euler equation moment conditions

$$\mathbf{h}_T = \frac{1}{T}\sum_t \left[\begin{array}{l} \left(1 - (\frac{1}{T}\sum_t M^{HXZ,FF}_{t+1}) + M^{HXZ,FF}_{t+1}\right)\mathbf{f}_{t+1} \\ \left(1 - (\frac{1}{T}\sum_t M^{HXZ,FF}_{t+1}) + M^{HXZ,FF}_{t+1}\right)(\mathbf{R}_{t+1} - R^f_{t+1}) \end{array} \right],$$

where $\mathbf{R}_t$ are a set of investment and equity returns, $\mathbf{R}_t = \{\mathbf{R}^I_t; \mathbf{R}^S_t\}$, R^f_t is the risk-free rate, and $\mathbf{f}_t$ is the set of candidate factors. The factors are suggested by two models. In Panel A, we present results for the Hou, Xue, and Zhang (2015) model (HXZ), where the factors are the excess return on the value-weighted CRSP portfolio, MRP, the return on a portfolio of small market capitalization stocks in excess of large capitalization stocks, SMB, the excess return on a portfolio of low investment to asset ratio stocks over a portfolio of high investment to asset ratio stocks, IA, and the difference in returns on a portfolio of stocks with high return on equity and low return on equity, ROE. Panel B presents results using the Fama and French (2015) five-factor model (FF), where in addition to market and size factors (MRP and SMB), the model uses the return on a portfolio of high book-to-market firms in excess of low book-to-market firms, HML, the return on a portfolio of high return on asset firms in excess of low return on asset firms, RMW, and a portfolio of high asset growth firms in excess of low return on asset firms, CMA. For these tests, we specify a block diagonal weighting matrix such that the GMM system estimates model parameters using the cross-section of factors in each model and tests model fit in the joint set of equity and investment excess returns. *Adj.* $\bar{R}^2_S$ denotes the cross-sectional adjusted R-square between risk premia and slope coefficients (β) in the set of equity excess returns imposing a zero intercept. *Adj.* $\bar{R}^2_{S,I}$ is the cross-sectional adjusted R-square between risk premia and slope coefficients in the joint set of equity and investment excess returns imposing a zero intercept. Panel C presents slope coefficients, β ($\times 100$), of regressions of excess returns on the stochastic discount factors. t-stat. is the t-statistic that the two slopes are equal. Panel C also presents the Euler equation errors ($\times 100$) in the set of investment and equity excess returns. Returns on investment are calculated using parameter estimates from Table 2. Data are sampled at the quarterly frequency from January, 1975 through December, 2014.

Panel A: HXZ Model

	d_{MRP}	d_{SMB}	d_{IA}	d_{ROE}
Estimate	-8.001	4.457	-19.538	-16.391
SE	(1.407)	(1.897)	(5.752)	(4.126)
χ^2-test (5 NW lags)		128.526		
d.o.f.		18		
p-value		(0.000)		
Mean Absolute Error		0.972		
Adj. $\bar{R}^2_S$		0.798		
Adj. $\bar{R}^2_{S,I}$		-0.769		

Panel B: FF Model

	d_{MRP}	d_{SMB}	d_{HML}	d_{RMW}	d_{CMA}
Estimate	-5.462	-0.747	1.088	-9.585	-10.518
SE	(1.561)	(0.785)	(3.335)	(3.292)	(3.950)
χ^2-test (5 NW lags)			245.838		
d.o.f.			18		
p-value			(0.000)		
Mean Absolute Error			1.090		
Adj. $\bar{R}^2_S$			0.197		
Adj. $\bar{R}^2_{S,I}$			-1.218		

Panel C: Exposures and Pricing Errors

HXZ	R^I		R^S		
	Error	β	Error	β	t-stat.
ROE1/IA1	1.62	0.12	-1.03	-1.96	2.11
ROE1/IA2	1.69	0.20	-0.63	-1.14	1.25
ROE1/IA3	1.39	0.51	-0.61	0.64	-0.12
ROE2/IA1	1.72	0.00	0.15	-3.10	4.23
ROE2/IA2	1.57	0.05	0.27	-1.79	2.41
ROE2/IA3	1.61	-0.03	0.58	-1.01	1.15
ROE3/IA1	2.30	-0.08	0.07	-3.90	5.72
ROE3/IA2	2.45	0.07	0.10	-3.01	4.77
ROE3/IA3	2.45	-0.04	1.26	-1.68	1.93

FF	R^I		R^S		
	Error	β	Error	β	t-stat.
ROE1/IA1	1.63	0.38	-1.89	-8.22	5.63
ROE1/IA2	1.49	0.25	-2.28	-8.40	5.20
ROE1/IA3	1.13	0.61	-2.50	-4.17	2.72
ROE2/IA1	1.70	-0.04	0.01	-9.11	8.43
ROE2/IA2	1.55	0.08	-0.26	-6.68	5.72
ROE2/IA3	1.52	-0.39	-0.25	-5.46	3.68
ROE3/IA1	2.31	-0.18	0.79	-8.69	8.30
ROE3/IA2	2.41	0.08	0.40	-7.52	7.70
ROE3/IA3	2.41	-0.24	0.97	-5.63	3.92

Table 6: **Flexible Investment Return Parameter Estimates**

Table 6 presents first-stage GMM results for the production parameters, α and a, from the sample conditions

$$\mathbf{e}_T = \frac{1}{T}\sum_{t=1}^{T}(\mathbf{R_t^I} - \mathbf{R_t^S}).$$

$\mathbf{R_t^I}$ above are investment returns, which are functions of the production parameters α and a, as shown in equation (12). In this set of tests, we allow α and a to vary by return on equity tercile. The vector $\mathbf{R_t^S}$ is the vector of gross equity returns on nine portfolios formed on the intersection of terciles of investment to assets (IA) and return on equity (ROE). Panel A reports point estimates of the production parameters α and a by return on equity terciles, the corresponding standard errors (SE), the mean absolute error of the GMM moment conditions, and the Hansen (1982) χ^2-test, degrees of freedom, and p-value for the null hypothesis that all the moment conditions are jointly zero. Panel B depicts mean investment returns $\overline{R^I}$ ($\times 100$) for the nine portfolios, mean errors for the moment conditions ($\times 100$), and associated t-statistics that each mean error is statistically equal to zero. The quarterly data span the period January, 1975 through December, 2014.

Panel A: Point Estimates

	α_{ROE1}	a_{ROE1}	α_{ROE2}	a_{ROE2}	α_{ROE3}	a_{ROE3}
Estimate	0.101	1.309	0.198	3.679	0.198	1.232
SE	(0.042)	(0.716)	(0.041)	(3.386)	(0.026)	(1.714)
χ^2-test (5 NW lags)				5.752		
$d.o.f.$				3		
p-value				(0.124)		
Mean Absolute Error				0.349		

Panel B: Mean Investment Returns and Pricing Errors

Portfolio	$\overline{R^I}$	Error	t-stat.
ROE1/IA1	1.32	0.62	1.66
ROE1/IA2	1.65	-0.04	-1.66
ROE1/IA3	0.69	-0.66	-1.66
ROE2/IA1	3.89	0.23	0.71
ROE2/IA2	2.91	0.17	0.72
ROE2/IA3	3.12	-0.42	-0.71
ROE3/IA1	4.27	0.49	1.97
ROE3/IA2	4.09	-0.09	-2.18
ROE3/IA3	4.39	-0.41	-1.93

Table 7: **Equivalence of Investment and Equity SDFs with Parameter Flexibility**

Table 7 presents tests of the hypotheses that stochastic discount factors in the linear span of equity returns (investment returns) can price investment returns (equity returns) when we allow the production parameters α and a to vary across assets. Investment returns are calculated using the production parameter estimates presented in Table 6. In Panel A, we present three sets of test statistics. In the columns "Equity SDF," we ask whether the stochastic discount factors in the linear span of equity returns can price investment returns using the Chen and Knez (1996) test statistic (CK), the Wald statistic from Bekaert and Urias (1996), and the likelihood ratio test (LRT) from Bekaert and Urias (1996). In the columns labeled "Investment SDF," we ask whether the stochastic discount factor in the linear span of investment returns can price equity returns. In Panel B, we present slope coefficients from regressions

$$
\begin{aligned}
R^S_{t+1} &= a^S_S + b^S_S M^S_{t+1} + e^S_{S,t+1} \\
R^I_{t+1} &= a^I_S + b^I_S M^S_{t+1} + e^I_{S,t+1} \\
R^S_{t+1} &= a^S_I + b^S_I M^I_{t+1} + e^S_{I,t+1} \\
R^I_{t+1} &= a^I_I + b^I_I M^I_{t+1} + e^I_{I,t+1}.
\end{aligned}
$$

M^S_{t+1} and M^{SI}_{t+1} are the minimum variance stochastic discount factors that satisfy the Euler equations for equity and investment returns, respectively, at a mean equal to $\overline{1/R^f_t}$, where R^f_t is the quarterly gross return on a one-month Treasury Bill. t-stat. is the t-statistic that the two slopes are equal. Data are sampled at the quarterly frequency from January, 1975 through December, 2014.

Panel A: Specification Tests

Equity SDF			
	CK	Wald	LRT
χ^2	79.453	165.834	41.322
p-value	(0.000)	(0.000)	(0.001)

Investment SDF			
	CK	Wald	LRT
χ^2	60.765	69.116	61.088
p-value	(0.000)	(0.000)	(0.000)

Panel B: Slopes of Regressions of Returns on SDFs

Equity SDF			
Portfolio	Equity b^S_S	Inv b^I_S	t-stat.
ROE1/IA1	-0.94	0.31	-1.10
ROE1/IA2	-0.51	0.76	-0.77
ROE1/IA3	1.54	0.55	0.76
ROE2/IA1	-3.80	1.37	-3.63
ROE2/IA2	-2.44	0.29	-2.19
ROE2/IA3	-1.94	-0.42	-0.96
ROE3/IA1	-4.65	0.48	-6.40
ROE3/IA2	-3.64	-0.14	-4.42
ROE3/IA3	-3.61	0.07	-3.75

Investment SDF			
Portfolio	Equity b^S_I	Inv b^I_I	t-stat.
ROE1/IA1	1.12	-0.09	1.27
ROE1/IA2	1.33	-0.40	1.25
ROE1/IA3	1.49	0.48	0.92
ROE2/IA1	0.80	-2.48	2.69
ROE2/IA2	1.17	-1.57	2.63
ROE2/IA3	1.37	-1.77	2.37
ROE3/IA1	0.66	-2.84	4.97
ROE3/IA2	0.92	-2.67	5.54
ROE3/IA3	1.49	-2.95	5.63

Table 8: **Factor Model Tests: Flexible Production Parameters**

Table 8 presents tests of the Euler equation moment conditions

$$\mathbf{h}_T = \frac{1}{T}\sum_t \left[\begin{array}{l} \left(1 - (\frac{1}{T}\sum_t M_{t+1}^{HXZ,FF}) + M_{t+1}^{HXZ,FF}\right)\mathbf{f}_{t+1} \\ \left(1 - (\frac{1}{T}\sum_t M_{t+1}^{HXZ,FF}) + M_{t+1}^{HXZ,FF}\right)(\mathbf{R}_{t+1} - R_{t+1}^f) \end{array} \right],$$

where $\mathbf{R}_t$ are a set of investment and equity returns, $\mathbf{R}_t = \{\mathbf{R}_t^I; \mathbf{R}_t^S\}$, R_t^f is the risk-free rate, and $\mathbf{f}_t$ is the set of candidate factors. The factors are suggested by two models. In Panel A, we present results for the Hou, Xue, and Zhang (2015) model (HXZ), where the factors are the excess return on the value-weighted CRSP portfolio, MRP, the return on a portfolio of small market capitalization stocks in excess of large capitalization stocks, SMB, the excess return on a portfolio of low investment to asset ratio stocks over a portfolio of high investment to asset ratio stocks, IA, and the difference in returns on a portfolio of stocks with high return on equity and low return on equity, ROE. Panel B presents results using the Fama and French (2015) five-factor model (FF), where in addition to market and size factors (MRP and SMB), the model uses the return on a portfolio of high book-to-market firms in excess of low book-to-market firms, HML, the return on a portfolio of high return on asset firms in excess of low return on asset firms, RMW, and a portfolio of high asset growth firms in excess of low return on asset firms, CMA. For these tests, we specify a block diagonal weighting matrix such that the GMM system estimates model parameters using the cross-section of factors in each stochastic discount factor and tests model fit in the joint set of equity and investment excess returns. *Adj.* $\bar{R}_S^2$ denotes the cross-sectional adjusted R-square between risk premia and slope coefficients (β) in the set of equity excess returns imposing a zero intercept. *Adj.* $\bar{R}_{S,I}^2$ is the cross-sectional adjusted R-square between risk premia and slope coefficients in the joint set of equity and investment excess returns imposing a zero intercept. Panel C presents slope coefficients, β ($\times 100$), of regressions of excess returns on the stochastic discount factors. t-stat. is the t-statistic that the two slopes are equal. Panel C also presents the Euler equation errors ($\times 100$) in the set of equity and investment excess returns. Returns on investment are calculated using the investment-based model with flexible production parameters from Table 6. Data are sampled at the quarterly frequency from January 1975 through December 2014.

Panel A: HXZ Model

	d_{MRP}	d_{SMB}	d_{IA}	d_{ROE}
Estimate	-8.001	4.457	-19.538	-16.391
SE	(1.407)	(1.897)	(5.752)	(4.126)
χ^2-test (5 NW lags)		92.501		
$d.o.f.$		18		
p-value		(0.000)		
Mean Absolute Error		1.023		
Adj. $\bar{R}_S^2$		0.792		
Adj. $\bar{R}_{S,I}^2$		-0.437		

Panel B: FF Model

	d_{MRP}	d_{SMB}	d_{HML}	d_{RMW}	d_{CMA}
Estimate	-5.462	-0.747	1.088	-9.585	-10.518
SE	(1.561)	(0.784)	(3.335)	(3.292)	(3.950)
χ^2-test (5 NW lags)			167.830		
$d.o.f.$			18		
p-value			(0.000)		
Mean Absolute Error			1.090		
Adj. $\bar{R}_S^2$			0.197		
Adj. $\bar{R}_{S,I}^2$			-0.596		

Panel C: Exposures and Pricing Errors

	HXZ				
	R^I		R^S		
	Error	β	Error	β	t-stat.
ROE1/IA1	0.43	0.37	-1.03	-1.96	2.25
ROE1/IA2	1.09	0.75	-0.63	-1.14	1.25
ROE1/IA3	0.71	1.38	-0.61	0.64	0.62
ROE2/IA1	2.99	0.36	0.15	-3.10	2.58
ROE2/IA2	1.99	0.33	0.27	-1.79	1.85
ROE2/IA3	1.50	-0.45	0.58	-1.01	0.38
ROE3/IA1	3.02	-0.04	0.07	-3.90	5.02
ROE3/IA2	3.01	0.16	0.10	-3.01	4.33
ROE3/IA3	3.06	-0.13	1.26	-1.68	1.66

	FF				
	R^I		R^S		
	Error	β	Error	β	t-stat.
ROE1/IA1	0.51	1.30	-1.89	-8.22	5.98
ROE1/IA2	0.70	0.87	-2.28	-8.40	3.82
ROE1/IA3	0.01	1.67	-2.50	-4.17	3.00
ROE2/IA1	2.57	-0.32	0.01	-9.11	4.03
ROE2/IA2	1.85	0.51	-0.26	-6.68	3.85
ROE2/IA3	1.13	-2.43	-0.25	-5.46	1.23
ROE3/IA1	3.03	-0.07	0.79	-8.69	7.13
ROE3/IA2	2.94	0.21	0.40	-7.52	6.76
ROE3/IA3	2.98	-0.62	0.97	-5.63	3.26

Table 9: **Tests with Nonconcurrent Equity and Financial Data: Lagged Financial Statement Information**

Table 9 presents results of tests on specification of the investment model using equations (10) and (11) in a sample where returns on equity are matched to investment returns that are based on financial statement data from two quarters earlier. Investment returns, which are a function of the production parameters α and a as shown in equation (12), are calculated using the moment conditions (13), where we estimate the production parameters via GMM. We present summary statistics for mean equity and investment returns in Panel A. Parameter estimates and specification tests of equation (13) are reported in Panel B. The results of the spanning specification tests are presented in Panel C. In the columns "Equity SDF," we ask whether the stochastic discount factors in the linear span of equity returns can price investment returns using the Chen and Knez (1996) test statistic (CK), and the Wald and likelihood ratio tests (LRT) from Bekaert and Urias (1996). In the columns labeled "Investment SDF," we ask whether the stochastic discount factor in the linear span of investment returns can price equity returns. Test assets are nine portfolios representing the intersection of return on equity (ROE) terciles and investment to asset (IA) terciles. The quarterly data span the period January, 1975 through December, 2014.

Panel A: Mean Returns

	Equity				Investment		
	IA1	IA2	IA3		IA1	IA2	IA3
ROE1	2.02	1.62	0.24	ROE1	3.28	2.51	0.92
ROE2	4.19	3.11	2.69	ROE2	3.50	2.91	2.34
ROE3	4.82	4.09	3.98	ROE3	4.59	3.99	3.16

Panel B: Investment Parameter Point Estimates

	α	a
Estimate	0.177	0.609
SE	(0.028)	(0.191)
χ^2-test (5 NW lags)	17.024	
$d.o.f.$	7	
p-value	(0.017)	
Mean Absolute Error	0.579	

Panel C: Specification Tests

	Equity SDF				Investment SDF		
	CK	Wald	LRT		CK	Wald	LRT
χ^2	102.740	216.479	111.578	χ^2	39.615	43.455	39.858
p-value	(0.000)	(0.000)	(0.000)	p-value	(0.000)	(0.001)	(0.002)

Table 10: **Tests with Nonconcurrent Equity and Financial Data: Time-to-Build**

Table 10 presents results of tests on specification of the investment model using equations (10) and (11) in a sample where returns on equity are matched to investment returns that are based on financial statement data from three years ahead, assuming that investment projects take on average three years to build. Investment returns, which are a function of the production parameters α and a as shown in equation (12), are calculated using the moment conditions (13), where we estimate the production parameters via GMM. We present summary statistics for mean equity and investment returns in Panel A. Parameter estimates and specification tests of equation (13) are reported in Panel B. The results of the spanning specification tests are presented in Panel C. In the columns "Equity SDF," we ask whether the stochastic discount factors in the linear span of equity returns can price investment returns using the Chen and Knez (1996) test statistic (CK), and the Wald and likelihood ratio tests (LRT) from Bekaert and Urias (1996). In the columns labeled "Investment SDF," we ask whether the stochastic discount factor in the linear span of investment returns can price equity returns. Test assets are nine portfolios representing the intersection of return on equity (ROE) terciles and investment to asset (IA) terciles. The quarterly data span the period January, 1975 through December, 2011.

Panel A: Mean Returns

	Equity				Investment		
	IA1	IA2	IA3		IA1	IA2	IA3
ROE1	2.05	1.50	-0.11	ROE1	2.16	2.51	1.68
ROE2	3.95	2.93	2.54	ROE2	2.81	2.37	2.72
ROE3	4.48	3.93	3.83	ROE3	3.88	3.83	3.45

Panel B: Investment Parameter Point Estimates

	α	a
Estimate	0.162	-1.235
SE	(0.034)	(0.654)
χ^2-test (5 NW lags)	14.045	
$d.o.f.$	7	
p-value	(0.050)	
Mean Absolute Error	0.654	

Panel C: Specification Tests

	Equity SDF				Investment SDF		
	CK	Wald	LRT		CK	Wald	LRT
χ^2	29.820	67.215	29.542	χ^2	43.133	53.696	55.516
p-value	(0.005)	(0.000)	(0.042)	p-value	(0.000)	(0.000)	(0.000)

Table 11: **Tests with Annual Portfolios**

Table 11 presents results of tests on specification of the investment model using equations (10) and (11) with annual data. Investment returns, which are a function of the production parameters a and α as shown in equation (12), are calculated using the moment conditions (13), where we estimate the production parameters via GMM. We present summary statistics for mean equity and investment returns ($\times$100) in Panel A. Parameter estimates and specification tests of equation (13) are reported in Panel B. The results of the spanning specification tests are presented in Panel C. In the columns "Equity SDF," we ask whether the stochastic discount factors in the linear span of equity returns can price investment returns using the Chen and Knez (1996) test statistic (CK), and the Wald and likelihood ratio tests (LRT) from Bekaert and Urias (1996). In the columns labeled "Investment SDF," we ask whether the stochastic discount factor in the linear span of investment returns can price equity returns. Test assets are nine portfolios representing the intersection of return on equity (ROE) terciles and investment to asset (IA) terciles. The annual data span the period December, 1975 through December, 2014.

Panel A: Mean Returns

	Equity		
	IA1	IA2	IA3
ROE1	17.32	14.41	6.41
ROE2	16.46	13.49	10.92
ROE3	14.88	12.92	13.68

	Investment		
	IA1	IA2	IA3
ROE1	14.04	11.11	8.07
ROE2	13.86	12.02	10.18
ROE3	17.90	15.54	16.52

Panel B: Investment Parameter Point Estimates

	α	a
Estimate	0.201	0.360
SE	(0.019)	(0.162)
χ^2-test (3 NW lag)	21.715	
$d.o.f.$	7	
p-value	(0.003)	
Mean Absolute Error	2.392	

Panel C: Specification Tests

	Equity SDF		
	CK	Wald	LRT
χ^2	578.146	184.542	185.827
p-value	(0.000)	(0.000)	(0.000)

	Investment SDF		
	CK	Wald	LRT
χ^2	53.940	116.580	55.569
p-value	(0.000)	(0.000)	(0.000)

Table 12: **Tests with Zero Debt Firms**

Table 12 presents results of tests on specification of the investment model using equations (10) and (11) on a set of firms with no debt, defined as those firms for which Compustat data item DLTTQ is zero. Investment returns, which are a function of the production parameters α and a as shown in equation (12), are calculated using the moment conditions (13), where we estimate the production parameters via GMM. We present summary statistics for mean equity and investment returns ($\times 100$) in Panel A. Parameter estimates and specification tests of equation (13) are reported in Panel B. The results of the spanning specification tests are presented in Panel C. In the columns "Equity SDF," we ask whether the stochastic discount factors in the linear span of equity returns can price investment returns using the Chen and Knez (1996) test statistic (CK), and the Wald and likelihood ratio tests (LRT) from Bekaert and Urias (1996). In the columns labeled "Investment SDF," we ask whether the stochastic discount factor in the linear span of investment returns can price equity returns. Test assets are nine portfolios representing the intersection of return on equity (ROE) terciles and investment to asset (IA) terciles. The data are sampled at the quarterly frequency and span the period March, 1976 through December, 2014.

Panel A: Mean Returns

Equity					Investment			
	IA1	IA2	IA3			IA1	IA2	IA3
ROE1	1.63	1.45	0.58		ROE1	1.03	1.27	0.72
ROE2	3.34	2.07	1.70		ROE2	3.06	3.23	3.18
ROE3	5.72	4.13	5.29		ROE3	4.54	4.65	4.33

Panel B: Investment Parameter Point Estimates

	α	a
Estimate	0.141	-0.561
SE	(0.027)	(0.349)
χ^2-test (5 NW lags)	15.529	
$d.o.f.$	7	
p-value	(0.030)	
Mean Absolute Error	0.722	

Panel C: Specification Tests

Equity SDF					Investment SDF			
	CK	Wald	LRT			CK	Wald	LRT
χ^2	187.490	168.458	118.550		χ^2	28.618	66.619	30.111
p-value	(0.000)	(0.000)	(0.000)		p-value	(0.001)	(0.000)	(0.036)

Table 13: **Tests with Levered Investment Returns**

Table 13 presents results of tests on specification of the investment model for levered investment returns using equations (10) and (11). Levered investment returns, which are a function of the production parameters α and a as shown in equation (17), are calculated using the moment conditions (18), where we estimate the production parameters via GMM. We present summary statistics for mean equity and levered investment returns ($\times 100$) in Panel A. Summary statistics for market leverage and after-tax portfolio debt returns are shown in Panel B. Parameter estimates and specification tests of equation (18) are reported in Panel C. The results of the spanning specification tests are reported in Panel D. In the columns "Equity SDF," we ask whether the stochastic discount factors in the linear span of equity returns can price investment returns using the Chen and Knez (1996) test statistic (CK), and the Wald and likelihood ratio tests (LRT) from Bekaert and Urias (1996). In the columns labeled "Investment SDF," we ask whether the stochastic discount factor in the linear span of investment returns can price equity returns. Test assets are nine portfolios representing the intersection of return on equity (ROE) terciles and investment to asset (IA) terciles. The data are sampled at the quarterly frequency and span the period January, 1975 through December, 2014.

Panel A: Mean Returns

Equity			
	IA1	IA2	IA3
ROE1	2.04	1.56	0.06
ROE2	4.00	3.10	2.63
ROE3	4.81	4.04	4.01

Levered Investment			
	IA1	IA2	IA3
ROE1	3.23	2.57	2.06
ROE2	3.36	2.67	2.71
ROE3	3.31	3.09	2.91

Panel B: Average Market Leverage and After-tax Debt Returns

Market Leverage			
	IA1	IA2	IA3
ROE1	41.79	40.29	36.36
ROE2	30.64	29.46	26.88
ROE3	24.79	17.00	16.17

After-tax Debt Returns			
	IA1	IA2	IA3
ROE1	1.21	1.19	1.20
ROE2	1.18	1.17	1.18
ROE3	1.16	1.13	1.15

Panel C: Investment Parameter Point Estimates

	α	a
Estimate	0.190	-0.708
SE	(0.042)	(1.012)
χ^2-test (5 NW lags)	36.628	
$d.o.f.$	7	
p-value	(0.000)	
Mean Absolute Error	0.989	

Panel D: Specification Tests

Equity SDF			
	CK	Wald	LRT
χ^2	38.141	93.884	21.864
p-value	(0.000)	(0.000)	(0.238)

Investment SDF			
	CK	Wald	LRT
χ^2	83.127	69.244	61.026
p-value	(0.000)	(0.000)	(0.000)

Figure 1: **Hansen-Jagannathan Bounds for Returns on Investment and Equity**

Figure 1 depicts the Hansen and Jagannathan (1991) bound on the minimum standard deviation stochastic discount factor in the linear span of equity returns. The bound is traced as

$$\sigma_M^2 \geq \left(\mathbf{1} - c\overline{\mathbf{R}}\right)' \widehat{\boldsymbol{\Sigma}}^{-1} \left(\mathbf{1} - c\overline{\mathbf{R}}\right),$$

where σ_M^2 is the variance of the stochastic discount factor, $\overline{\mathbf{R}}$ is a vector of average returns, and $\widehat{\boldsymbol{\Sigma}}$ is the sample covariance matrix of returns. The parameter c determines the constant mean of the stochastic discount factor, $E[M]$, which is varied to trace the bounds. We present bounds implied by the returns to equity portfolios, where portfolios are defined as the intersection of terciles of firms ranked independently on return on equity and investment to asset ratio. We also depict bounds on the returns on investment for these firms, where investment parameters are estimated in Table 2. Finally, we trace bounds for the combined set of equity and investment returns. Data are sampled at the quarterly frequency over the period January, 1975 through December, 2014.